JEE Main 6th April Session 1 - Memory Based Paper

6th April Session 1

Questions

Q.1. Which of the following does not explain the wave theory of particles?

A) Diffraction

B) Photoelectric Effect

C) Reflection

D) Interference

Answer: Photoelectric Effect

Solution: The photoelectric effect is a phenomenon in which electrons are emitted from a material surface when it is exposed to electromagnetic radiation, typically light. This effect occurs when photons, which are particles of light, transfer their energy to electrons in the material, causing them to be ejected from the surface. The energy of the ejected electrons depends on the frequency of the incident light.

This was the first experiment performed by Einstein to demonstrate the particle nature of light.

All the other phenomena can be explained by the wave nature of light.

Hence, this is the correct option.

Q.2. The speed of electromagnetic wave in a medium is $1.5 \times 10^{8}\ \mathrm{m\ s^{-1}}$. Relative permittivity of medium(ε_r) is 2. Find the value of relative permeability.

A) 2

B) 3

C) 1

D) 4

Answer: 2

Solution: The velocity of EM wave in vacuum is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \ldots(1)$$

And, that at the medium is

$$v = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \quad \ldots(2)$$

From the two equations above, it implies that

$$\frac{c}{v} = \frac{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}}{\frac{1}{\sqrt{\mu_m \varepsilon_m}}}$$

$$= \sqrt{\frac{\mu_m \varepsilon_m}{\mu_0 \varepsilon_0}}$$

$$\Rightarrow \frac{c^2}{v^2} = \frac{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}{\mu_0 \varepsilon_0}$$

$$\Rightarrow \mu_r = \frac{1}{\varepsilon_r} \frac{c^2}{v^2}$$

$$= \frac{1}{2} \times \frac{\left(3 \times 10^{8}\ \mathrm{m\ s^{-1}}\right)^2}{\left(1.5 \times 10^{8}\ \mathrm{m\ s^{-1}}\right)^2}$$

$$= 2$$

Q.3. In a photoelectric experiment of $2.48\ \mathrm{eV}$ irradiates a photo sensitive material and the stopping potential was measured to be $0.5\ \mathrm{V}$. What is the work function in eV of the photo sensitive material.

A) 0.5

B) 1.68

C) 2.48

D) 1.98

Answer: 1.98

Solution: The relation between the stopping potential and the work function is

$$E = \phi + eV_s \quad \ldots(1)$$

From equation (1), it follows that

$$2.48\ \mathrm{eV} = \phi + 0.5\ \mathrm{eV}$$
$$\Rightarrow \phi = 1.98\ \mathrm{eV}$$

Q.4. For a given single electron atom, the ratio of shortest wavelengths in Balmer and Lyman series is

A) $1:4$ B) $4:1$

C) $2:1$ D) $1:2$

Answer: $4:1$

Solution: The expression for the wavelength is given by

$$\frac{1}{\lambda}=Z^2R\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right)\quad \ldots(1)$$

The last line is the line of the shortest wavelength or highest energy.

For the shortest wavelength, from equation (1), it follows, by putting $n_2=\infty$,

$$\frac{1}{\lambda}=\frac{Z^2R}{{n_1}^2}\quad \ldots(2)$$

For the Balmer series,

$$\frac{1}{\lambda_B}=\frac{Z^2R}{2^2}=\frac{Z^2R}{4}$$

And, for the Lyman series,

$$\frac{1}{\lambda_L}=\frac{Z^2R}{1^2}=\frac{Z^2R}{1}$$

Hence,

$$\frac{\frac{1}{\lambda_L}}{\frac{1}{\lambda_B}}=\frac{4}{1}$$

$$\Rightarrow \lambda_B:\lambda_L=4:1$$

Q.5. Four particles $A,\ B,\ C$ and D have masses $\frac{m}{2}, m,\ 2m$ and $4m$ respectively. If they have equal momentum, the particle that has the highest kinetic energy is:

A) A B) B

C) C D) D

Answer: A

Solution: The kinetic energy of an object can be written as

$$K=\frac{1}{2}mv^2=\frac{p^2}{2m}\quad \ldots(1)$$

As can be seen from equation (1) that the kinetic energy is inversely proportional to the mass of the object, the particle with the least mass will have the maximum kinetic energy.

Hence, this is the correct option.

Q.6. A particle is performing SHM with $A=0.06\ \mathrm{m}$ and time period $T=3.14\ \mathrm{s}$. Find the maximum velocity in $\mathrm{m\ s^{-1}}$ unit.

A) 0.10 B) 0.15

C) 0.12 D) 0.18

Answer: 0.12

Solution: The angular velocity of the particle is given by

$$\omega = \frac{2\pi}{T} \quad \ldots(1)$$

The formula for the maximum velocity of the particle can be written as

$$v_{\max} = A\omega$$
$$= A\frac{2\pi}{T} \quad \ldots(2)$$

From equation (2), it follows that

$$v_{\max} = 0.06 \text{ m} \times \frac{2\pi}{3.14 \text{ s}}$$
$$= 0.12 \text{ m s}^{-1}$$

Q.7. Which of the following is not a semiconductor?

A) Graphite B) Copper oxide

C) Germanium D) Silicon

Answer: Graphite

Solution: Graphite is an electrical conductor not a semiconductor. The pi bonds in graphite gives the free electrons for electrical conduction for graphite.

Copper oxide is a semiconductor which is a natural p-type semiconductor.

Q.8. Bullet of mass 50 g fired with a speed 100 m s^{-1} on a plywood and emerges with 40 m s^{-1} the percentage of loss of kinetic energy is

A) 16% B) 32%

C) 44% D) 84%

Answer: 84%

Solution: Kinetic energy is given by, $\frac{1}{2}mv^2$.

So we get, $K_i = \frac{1}{2}\left(50 \times 10^{-3}\right) \times 100^2$

and $K_f = \frac{1}{2}\left(50 \times 10^{-3}\right) \times 40^2$

Therefore, loss in kinetic energy

$$\%loss = -\frac{\Delta KE}{KE_i} = \frac{KE_i - KE_f}{KE_i} = 84\%$$

Q.9. A sample contain mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen sample is

A) $\frac{1}{32}$ B) $\frac{1}{4}$

C) $\frac{1}{2\sqrt{2}}$ D) $2\sqrt{2}$

Answer: $2\sqrt{2}$

Solution: RMS velocity is given by, $V_{rms} = \sqrt{\frac{3RT}{M}}$.

Therefore,

$$\frac{V_{He}}{V_{O_2}} = \frac{\sqrt{M_{O_2}}}{\sqrt{M_{He}}} = \sqrt{\frac{32}{4}} = 2\sqrt{2} : 1$$

Q.10. A train starting from rest first accelerates uniformly up to speed 80 km h^{-1} for time t, then it moves with a constant speed for time $3t$. The average speed of the train for this duration of journey will be

A) 30 km h^{-1} B) 40 km h^{-1}

C) 70 km h^{-1} D) 80 km h^{-1}

Answer: 70 km h^{-1}

Solution:

Average speed $\frac{\text{total distance}}{\text{total time}} = \frac{\frac{1}{2}\left(\frac{80}{t}\right)t^2 + 80\times 3t}{t+3t} = \frac{280}{4} = 70 \text{ km h}^{-1}$

Q.11. For a spring block system, the error in time period calculation is 2% and the error in mass calculation is 1%. Find the percentage error in spring constant K.

A) 2%

B) 4%

C) 10%

D) 5%

Answer: 5%

Solution: The time period of a spring block system is given by

$$T = 2\pi\sqrt{\frac{m}{K}} \quad \ldots(1)$$

The percentage error of the spring constant can, then, be calculated as follows:

$$T^2 = 4\pi^2\frac{m}{K}$$
$$\Rightarrow K = 4\pi^2\frac{m}{T^2}$$
$$\Rightarrow \frac{\Delta K}{K} = \frac{\Delta m}{m} + 2\frac{\Delta T}{T}$$
$$\Rightarrow \frac{\Delta K}{K} \times 100\% = \left(\frac{\Delta m}{m} + 2\frac{\Delta T}{T}\right) \times 100\%$$
$$= 1\% + 2 \times 2\%$$
$$= 5\%$$

Q.12. While measuring the diameter of a wire using a screw gauge, the following readings were noted: Main scale reading is 1 mm and circular scale reading is equal to 42 division. Pitch of the screw gauge is 1 mm and it has 100 divisions on circular scale. The diameter of wire is $\frac{x}{50} \text{ mm}$. The value of x is

Answer: 71

Solution: The least count of the screw gauge is given by

$$\text{L. C.} = \frac{\text{Pitch}}{\text{No. of divisions}}$$
$$= \frac{1 \text{ mm}}{100}$$
$$= 0.01 \text{ mm}$$

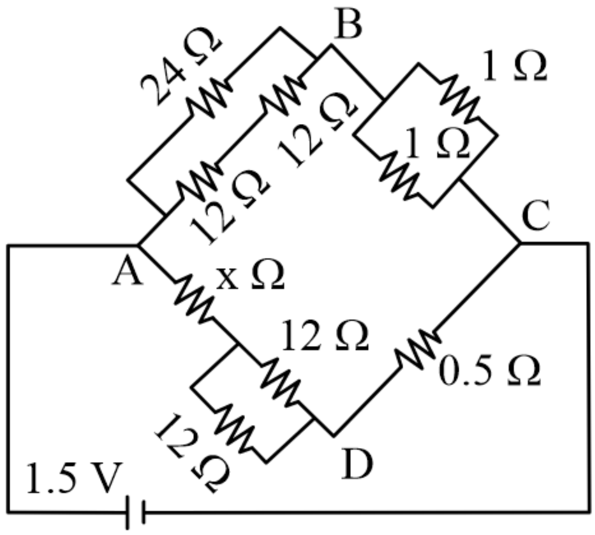
The total reading from the screw gauge can be written as

$$D = \text{M. S. R.} + n \times \text{L. C.}$$
$$= 1 \text{ mm} + 42 \times 0.01 \text{ mm}$$
$$= 1.42 \text{ mm}$$

By comparing the above value with the given expression, it follows that

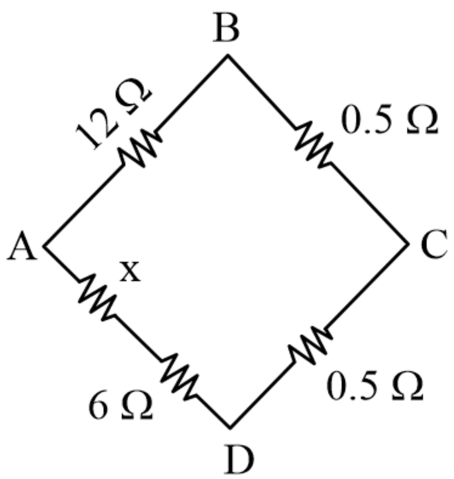
$$\frac{x}{50} = 1.42$$
$$= \frac{142}{100}$$
$$\Rightarrow x = 71$$

Q.13. The value of unknown resistance x for which potential difference between point B and D is zero is (in Ω)



Answer: 6

Solution: The equivalent circuit diagram is given as follows:



With reference to the above equivalent circuit diagram, from the balanced condition of a Wheatstone Bridge, it follows that

$\frac{12}{x+6} = \frac{0.5}{0.5}$

$\Rightarrow x + 6 = 12$

$\Rightarrow x = 6\ \Omega$

Q.14. The ratio of the angle of a prism and the minimum deviation is one for a prism whose refractive index is $\sqrt{3}$. Then the angle of the prism is x°. What is the value of x?

Answer: 60

Solution: According to the given question, $A=\delta_m$.

The formula for the refractive index of the prism is

$$n=\frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad \ldots(1)$$

From equation (1), it follows that

$$\sqrt{3}=\frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$=\frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$=2\cos\left(\frac{A}{2}\right)$$

$$\Rightarrow \cos\left(\frac{A}{2}\right)=\frac{\sqrt{3}}{2}$$

$$=\cos 30^{\circ}$$

$$\Rightarrow A=60^{\circ}$$

Q.15. A wire of resistance R and radius r is stretched till its radius becomes $\frac{r}{2}$. If now resistance of the stretched wire is xR, then the value of x is

Answer: 16

Solution: Resistance of the wire is given by, $R=\rho\frac{l}{A}$. Now, volume will remain same and $V=lA$.

Now,

$$R=\rho\frac{l}{A}=\rho\frac{\frac{V}{A}}{A}=\rho\frac{V}{A^2}=\rho\frac{V}{\left(\pi r2\right)^2}$$

Therefore, $R\propto\frac{1}{r^4}$.

Required ratio

$$\frac{R_1}{R_2}=\left(\frac{r_2}{r_1}\right)^4=\left(\frac{\frac{r}{2}}{r}\right)^4=\frac{1}{16}$$

$$\Rightarrow R_2=16R_1=16R$$

Therefore, $x=16$.

Q.16. If the radius of the earth is reduced to three fourth of its present value without change in its mass then value of duration of the day of will be _____ hours 30 minutes.

Answer: 13

Solution: Applying conservation of angular momentum, we get

$$I_1\omega_1 = I_2\omega_2$$
$$\Rightarrow \frac{2}{5}m(r_1)^2 \times \frac{2\pi}{T_1} = \frac{2}{5}m(r_2)^2 \times \frac{2\pi}{T_2}$$
$$\Rightarrow \frac{(r_1)^2}{T_1} = \frac{(r_2)^2}{T_2}$$
$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^2$$
$$\Rightarrow \frac{24}{T_2} = \left(\frac{r}{\frac{3r}{4}}\right)^2 = \frac{16}{9}$$
$$\Rightarrow T_2 = \frac{24\times9}{16} = 13.5$$

Q.17. Among the given molecules, identify the one which undergoes nucleophilic addition reaction at fastest rate:

A) $HCHO$

B) CH_3CHO

C) CH_3CH_2CHO

D) $CH_3CH_2CH_2CHO$

Answer: $HCHO$

Solution: A nucleophilic addition reaction is a chemical addition reaction in which a nucleophile forms a sigma bond with an electron-deficient species. These reactions are considered very important in organic chemistry since they enable the conversion of carbonyl groups into a variety of functional groups.

For the nucleophilic addition reaction, bulky substituents around the carbonyl carbon can hinder the approach of nucleophiles, reducing the reactivity. More is the electrophilicity of the carbon atom the more will be the rate of the nucleophilic addition reaction.

Q.18. Which compound will absorb light at more frequency?

A) $[Cr(H_2O)_6]^{3+}$

B) $[CrCl_6]^{3-}$

C) $[Cr(CN)_6]^{3-}$

D) $[CrCl_3(H_2O)_3]$

Answer: $[Cr(CN)_6]^{3-}$

Solution: Ligands can be classified as strong field or weak field based on their ability to cause d-orbital splitting in the metal ion. Strong field ligands typically lead to larger energy differences between the d-orbitals compared to weak field ligands. If the coordination complex contains strong field ligands, the d-orbital splitting will be larger, resulting in higher energy transitions between these orbitals. This corresponds to absorption of light at higher frequencies (shorter wavelengths) in the visible spectrum. Among the given complexes, CN^- is the strong field ligand and hence, $[Cr(CN)_6]^{3-}$ absorbs at higher frequency in the visible spectrum.

Q.19. Match column I with column II

Column I	Column II
1. SF_4	P. Tetrahedral
2. BrF_3	Q. Pyramidal
3. BrO_3^-	R. Sea-saw
4. NH_4^+	S. T-shape

A) $1-R,\ 2-S,\ 3-Q,\ 4-P$

B) $1-R,\ 2-S,\ 3-P,\ 4-Q$

C) $1-S,\ 2-R,\ 3-Q,\ 4-P$

D) None of the above

Answer: $1-R,\ 2-S,\ 3-Q,\ 4-P$

Solution: 1. The central atom in Sulphur tetrafluoride molecule has 10 electrons. There are 4 bond pairs and one single lone pair. Hence, the steric number is 5. Using VSEPR theory, we can predict the shape and geometry of the molecule. Clearly, SF_4 has a trigonal bipyramidal geometry (or structure). The lone pair is in the equatorial plane, giving an overall see-saw shape.

2. The hybridisation of BrF_3 is sp^3d and its structure is T-shaped or trigonal bipyramidal. Bromine trifluoride is a nonpolar molecule.

3. BrO_3^- is sp^3 hybridized. The bromine atom forms single bonds with two oxygen atoms and a double bond with one oxygen atom. This results in a trigonal pyramidal molecular geometry around the bromine atom, with one lone pair of electrons. Therefore, the molecular geometry around the bromine atom in the bromate ion, BrO_3^-, is trigonal pyramidal.

4. Hybridisation of central N atom is sp^3. The shape of NH_4^+ ion is tetrahedral.

Q.20. Find the ratio of shortest wave lengths in Lyman and Balmer series for H-atom.

A) 4

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) 2

Answer: $\frac{1}{4}$

Solution: Shortest wavelength means highest energetic line. i.e, $n_2 = \infty$ and $n_1 =$Series starting energy level

For the shortest wavelength line in Lyman series $n_1 = 1,\ n_2 = \infty$

$$\therefore \frac{1}{\lambda_L} = R_H\left[\frac{1}{1^2} - \frac{1}{\infty}\right] \text{ ...... (i)}$$

For the shortest wavelength line in Balmer series, $n_1 = 2,\ n_2 = \infty$

$$\therefore \frac{1}{\lambda_B} = R_H\left[\frac{1}{2^2} - \frac{1}{\infty}\right] \text{ ...... (ii)}$$

$$\therefore \frac{\lambda_L}{\lambda_B} = \frac{1}{4}$$

Q.21. Match column I with column II

Column I	Column II
1. Iodoform	P. Fire extinguisher
2. CCl_4	Q. Insecticides
3. CFC	R. Antiseptic
4. DDT	S. Refrigerant

A) $1 - R,\ 2 - P,\ 3 - S,\ 4 - Q$

B) $1 - R,\ 2 - P,\ 3 - Q,\ 4 - S$

C) $1 - P,\ 2 - R,\ 3 - S,\ 4 - Q$

D) None of the above

Answer: $1 - R,\ 2 - P,\ 3 - S,\ 4 - Q$

Solution: 1. Iodoform, a yellow, crystalline solid belonging to the family of organic halogen compounds, used as an antiseptic component of medications for minor skin diseases.

2. CCl_4 is used as fire extinguishers under the name of pyrene. The dense vapours of carbon tetrachloride forms a protective layer on the burning objects and avoids the oxygen or air to come in contact with the fire from the burning objects and provides incombustible vapours.

3. Chlorofluorocarbons (CFCs)Their other important property is their volatility, having boiling points close to zero degrees Centigrade. These physical properties make them ideal for use as refrigerant gases in air conditioners, freezers and refrigerators.

4. Dichlorodiphenyltrichloroethane (DDT) is an insecticide used in agriculture.

Q.22. Find the sum of magnetic moments of basic and amphoteric oxides of Cr.
$CrO,\ Cr_2O_3,\ CrO_3$

A) 4.89 BM

B) 8.76 BM

C) 11.73 BM

D) 3.89

Answer: $8.76\ \mathrm{BM}$

Solution: The magnetic moments of basic and amphoteric oxides of chromium (Cr) are as follows:

1. **Chromium(II) Oxide (CrO)**:
   - Oxidation State: $+2\ (Cr^{2+}\left(3d^4\right))$
   - Magnetic Moment: $\sqrt{n\,(n+2)}BM = \sqrt{4(4+2)} = 4.89\ \ BM$
2. **Chromium(III) Oxide (Cr_2O_3)**:
   - Oxidation State: $+3\left(Cr^{3+}\left(3d^3\right)\right)$
   - Magnetic Moment: $\sqrt{n\,(n+2)}BM = \sqrt{3(3+2)} = 3.87\ \ BM$

The sum of the magnetic moments of CrO and Cr_2O_3 is: Total Magnetic Moment$= 4.89 + 3.87 = 8.76\,BM$

Q.23. Match colum I with column II

Column I	Column II
1. sp^3	P. Octahedral
2. dsp^2	Q. Trigonal bipyramidal
3. sp^3d	R. Tetrahedral
4. sp^3d^2	S. Square planar

A) $1-R,\ 2-S,\ 3-P,\ 4-Q$

B) $1-R,\ 2-S,\ 3-Q,\ 4-P$

C) $1-S,\ 2-R,\ 3-Q,\ 4-P$

D) None of the above

Answer: $1-R,\ 2-S,\ 3-Q,\ 4-P$

Solution: 1. For sp^3 hybridized central atoms the only possible molecular geometry is tetrahedral. If all the bonds are in place the shape is also tetrahedral.

2. dsp^2 type of hybridization is seen specially in case of transition metal ions. The orbitals involved in this type of hybridization are $d_{x^2-y^2}$, s and two p orbitals. The four dsp^2 hybrid orbitals adopt square planar geometry.

3. For sp^3d hybridized central atoms the only possible molecular geometry is trigonal bipyramidal. If all the bonds are in place the shape is also trigonal bipyramidal.

4. In octahedral geometry, six electrons are involved in making sp^3d^2 hybridisation. In this case, the orbital makes a $90°$ degree angle.

Q.24. Which of the following is the sulphonic functional group?

A) SO_3H

B) SO_2

C) $O=S-OH$

D) SO_3

Answer: SO_3H

Solution: Sulphonic acid is a type of organosulphur compound that contains a sulphonyl functional group $(-SO_2OH)$. It is an inorganic substance that has a very high acidity level.

A functional group characterised by a sulphur atom doubly bonded to two oxygen atoms, a hydroxyl group, and a carbon atom of any hybridization.

Q.25. Match the following:

Column I	Column II
1. Pb^{2+}	P. $NH_4OH + NH_4Cl$
2. Al^{3+}	Q. $H_2S\ +\ dil\ \ HCl$
3. Sr^{2+}	R. $H_2S\ +\ NH_4OH$
4. Mn^{2+}	S. $(NH_4)_2CO_3\ +\ NH_4OH$

A) $1-Q,\ 2-P, 3-S,\ 4-R$

B) $1-P,\ 2-Q, 3-S,\ 4-R$

C) $1-Q,\ 2-P, 3-R,\ 4-S$ D) None of the above

Answer: $1-Q,\ 2-P, 3-S,\ 4-R$

Solution: A reagent used to identify a group of radicals is called a Group radical. Every group has its separate group reagent for testing the radicals of the respective groups.

Group reagent for the precipitation of group II basic radicals for the qualitative analysis is $\text{dil. } HCl + H_2S$. In II group, sulphides are precipitated in acidic medium.

The group reagent of $3rd$ group is ammonium sulphide solution or hydrogen sulphide gas in the presence of ammonia and ammonium chloride. When we add group reagent to the filtrate we will get precipitate of $3rd$ group cations

Group V cations are precipitated in form of carbonates by $(NH_4)_2CO_3$.

In group IV, cations are $Ni^{2+},\ Co^{2+},\ Mn^{2+},\ Zn^{2+}$ and group reagent is H_2S in the presence of NH_4OH .

Q.26. Assertion (A): Ga is used in thermometer
Reason (R): Melting point of Ga is low where as boiling point is high

A) Both A and R are correct and R is the correct explanation of A.

B) Both A and R are correct, but R is not the correct explanation of A.

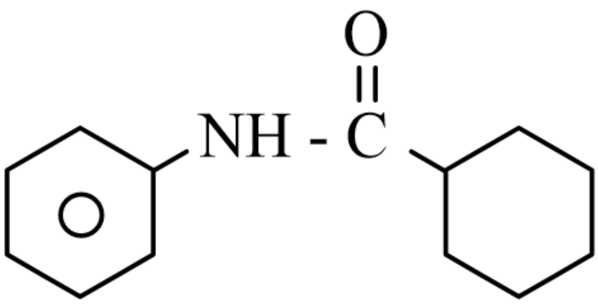
C) A correct but, R is incorrect.

D) R is correct but, A is incorrect.

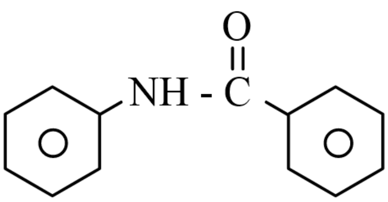
Answer: Both A and R are correct and R is the correct explanation of A.

Solution: Gallium (Ga) is indeed used in certain types of thermometers, particularly those designed for measuring high temperatures. The melting point of Ga is low whereas the boiling point is high. This reason correctly states that gallium has a relatively low melting point (29.76°C or 85.57°F), which is useful for applications requiring a liquid metal at or near room temperature. However, gallium's boiling point is relatively high compared to its melting point. Gallium boils at approximately 2204°C (3999°F), which is considerably high.

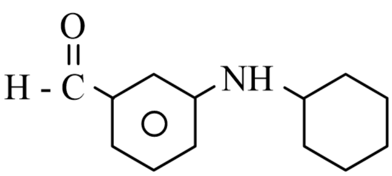
Q.27. The correct metamer of the following compound is:



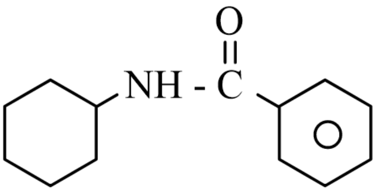
A)



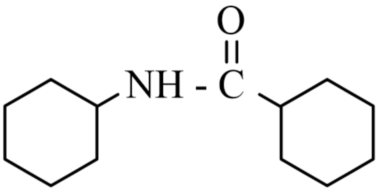
B)



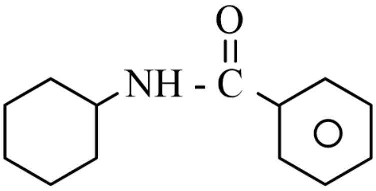
C)



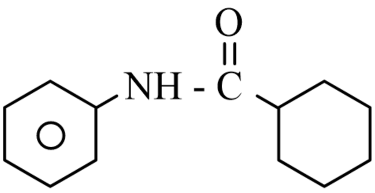
D)



Answer:



Solution: Metamers are the isomers having the same molecular formula but different alkyl groups on two sides of functional groups. This phenomenon of isomerism is called metamerism.



In the above compound benzene ring is attached to -NH group and cyclohexane ring is attached to -CO group. So, in the metamer benzene ring is attached to -CO group and cyclohexane ring is attached to -NH group.

Hence, the correct answer here is option C.

Q.28. Which of the following is not a semiconductor?

A) Graphite
B) Si
C) CuO
D) Ge

Answer: Graphite

Solution: Semiconductors are materials which have a conductivity between conductors (generally metals) and nonconductors or insulators (such as most ceramics). Semiconductors can be pure elements, such as silicon or germanium, or compounds such as gallium arsenide or cadmium selenide.

Graphite is an electrical conductor not a semiconductor.

Cupric oxide (CuO) is a semiconductor that exhibits p-type conductivity, crystallizes in the monoclinic structure and has a band gap of $1.5\ \mathrm{eV}$.

Hence, the answer is option A.

Q.29. The density of $3\ \mathrm{M}$ solution of NaCl is $1.25\ \mathrm{g/mL}$. Calculate molality of the solution.

Give the answer to the nearest integer value.

A) 3
B) 5
C) 6
D) 8

Answer: 3

Solution: 3 Molar solution means there are 3 moles of NaCl salt in 1 Liter.

Molecular weight of $\mathrm{NaCl} = 58.44$. Hence, there are 3×58.44 gms in 1 Litre of water.

$$\mathrm{Density} = \frac{\mathrm{Mass}}{\mathrm{Volume}}$$

Mass of 1 litre of solution
$= 1.25\ \mathrm{gm}/\mathrm{mL} \times 1000\ \mathrm{mL}$
$= 1250\ \mathrm{gms}$

$\mathrm{V} =$ Volume of water added to make the solution or volume of solvent

Mass of solute $+$ Mass of solvent $=$ Mass of solution

$175.32\ \mathrm{gms} + \mathrm{mass\ \ of\ solvent} = 1250\ \mathrm{gm}$

mass of solvent $= 1250 - 175.32 = 1074.8$

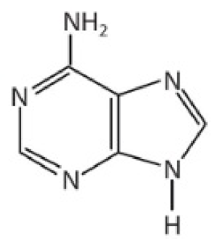
So $1074.68\ \mathrm{gms}$ of water is mixed with 3 moles of NaCl to make the $3\ \mathrm{M}$ solution.

$$\mathrm{Molality} = \frac{\mathrm{mass\ \ of\ \ solute\ \ in\ \ number\ \ of\ \ moles}}{\mathrm{mass\ \ of\ \ solvent\ \ in\ \ kg}}$$

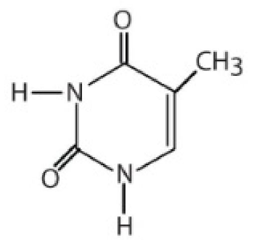
$$= \frac{3}{1.0746} = 2.79\ \mathrm{Molal} \approx 3\ \ \mathrm{Molal}$$

Q.30. Which nitrogen base is not present in DNA?

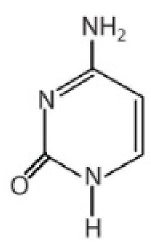
A)



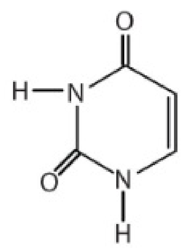
B)



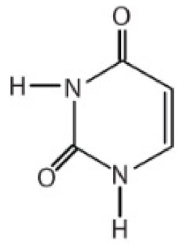
C)



D)

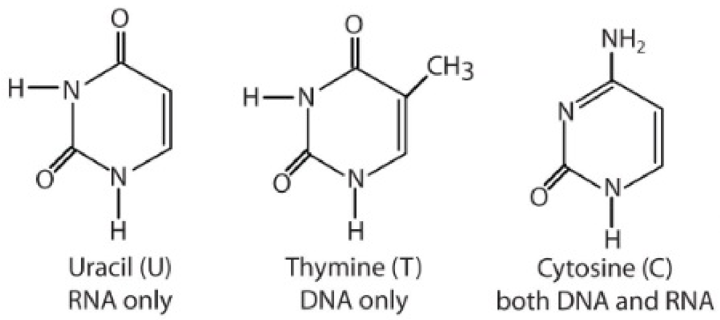


Answer:

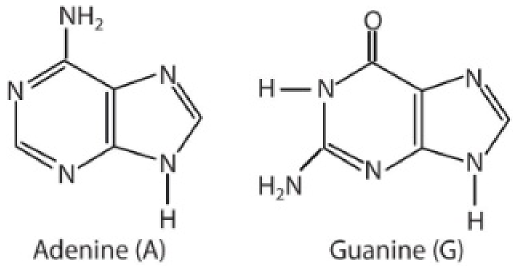


Solution: Nucleic acids have two kinds of bases pyramidine bases and purine bases.

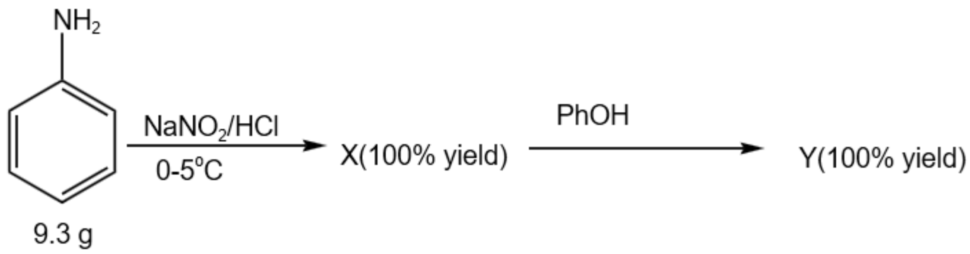
The pyramidine bases present in DNA are thymine and cyctocine.



The purine bases present in DNA are adenine and guanine.



Q.31.



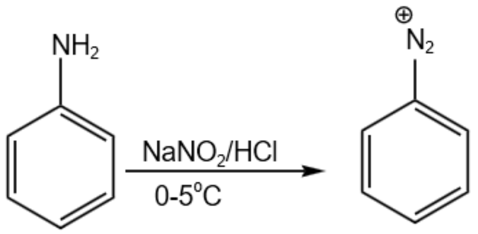
Find the mass of the compound Y in the above reaction.

A) 9.3 g B) 19.8 g

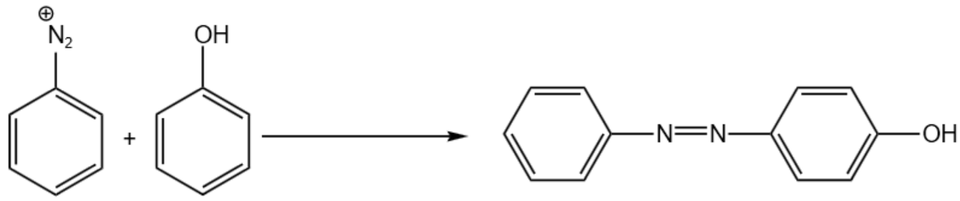
C) 18.6 g D) 15.6 g

Answer: 19.8 g

Solution: The process of conversion of a primary aromatic amino compound into a diazonium salt is known as diazotization. This process is carried out by adding an aqueous solution of sodium nitrite to a solution of primary aromatic amine (e.g., aniline) in excess of HCl at a temperature below 5°C.



In a mildly basic solution, benzene diazonium chloride reacts with phenol to produce p-hydroxy azobenzene.



One mole of aniline gives one mole of p-hydroxy azobenzene(Y). Hence, $9.3\ g$ of aniline gives $19.8\ g$ of p-hydroxy azobenzene.

Q.32. The dark purple colour of $KMnO_4$ disappears in the titration with oxalic acid in acidic medium. The overall change in the oxidation number of manganese in the reaction is

Answer: 5

Solution: The overall reaction takes place in the process is

$$2\,KMnO_4 + 3H_2SO_4 + 5(COOH)_2 \rightarrow K_2SO_4 + 2\,MnSO_4 + 8H_2O + 10\,CO_2 \uparrow$$

Potassium permanganate is a strong oxidising agent and in the presence of sulfuric acid it acts as a powerful oxidising agent. In acidic medium the oxidising ability of $KMnO_4$ is represented by the following equation.

$$MnO_4{}^{-} + 8H^{+} + 5e^{-} \rightarrow Mn^{2+} + 4H_2O$$

Q.33. The number of molecules which can show hydrogen bonding from the following.

$CH_3OH, H_2O, C_2H_6, C_6H_6, o-\text{nitrophenol},\ NH_3, HF,$

Answer: 5

Solution: To determine which molecules can exhibit hydrogen bonding, we need to consider their molecular structure and the presence of hydrogen atoms bonded directly to highly electronegative atoms such as fluorine (F), oxygen (O), or nitrogen (N). Hydrogen bonding occurs between a hydrogen atom attached to one of these electronegative atoms and a lone pair of electrons on another electronegative atom.

In the given list $CH_3OH, H_2O, o-\text{nitrophenol},\ NH_3, HF$ are satisfied with given condition.

Q.34. Find the ratio between $t_{99.9\%}$ and $t_{90\%}$ for the first order reactions.

Answer: 3

Solution: For a first order reaction, $t = \frac{2.303}{k}\log\left(\frac{a}{a-x}\right)$, where a is the initial concentration and $(a-x)$ is the concentration at time t.

Given, $x = 0.999\ a$ at $t_{99\%}$ and $x = 0.9\ a$ at $t_{90\%}$.

Therefore, $$\frac{t_{99.9\%}}{t_{90\%}} = \frac{\frac{2.303}{k}\log\left(\frac{a}{a-0.999a}\right)}{\frac{2.303}{k}\log\left(\frac{a}{a-0.9a}\right)} = \frac{\log\left(\frac{1}{0.001}\right)}{\log\left(\frac{1}{0.1}\right)} = 3$$

Hence, $t_{99.9\%} = 3\ t_{90\%}$

Q.35. Which of the following elements belongs to the lanthanod series?

$\mathrm{Eu, Cm, Cr, Yb, Lu, Cd}$

Answer: 3

Solution: The lanthanide series consists of the 14 elements from Cerium (Ce) to lutetium (Lu) on the periodic table.

- Eu (Europium) is a lanthanide
- Cm (Curium) is an actinide.
- Cr (Chromium) is a transition metal.
- Yb (Ytterbium) is part of the lanthanide series.
- Lu (Lutetium) is the last element of the lanthanide series.
- Cd (Cadmium) is a transition metal.

Q.36. Find the number of processes in which the electron gain enthalpy is negative.

$\left(\mathrm{A}\right)\ \mathrm{Al}\left(\mathrm{g}\right)\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{Al}^{-}\ \left(\mathrm{g}\right)$

$(\mathrm{B})\ \mathrm{Be}(\mathrm{g})\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{Be}^{-}\ (\mathrm{g})$

$\left(\mathrm{C}\right)\ \mathrm{O}\left(\mathrm{g}\right)\ +\ 2\mathrm{e}^{-}\ \rightarrow \mathrm{O}^{2-}\ \left(\mathrm{g}\right)$

$(\mathrm{D})\ \mathrm{Na}(\mathrm{g})\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{Na}^{-}\ (\mathrm{g})$

$(\mathrm{E})\ \mathrm{N}(\mathrm{g})\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{N}^{-}\ (\mathrm{g})$

Answer: 2

Solution: Among the given elements beryllium and nitrogen have stable electronic configuration, so they have positive electron gain enthalpies. The sum of the two electron gain enthalpies for oxygen is also positive.

$\left(\mathrm{A}\right)\ \mathrm{Al}\left(\mathrm{g}\right)\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{Al}^{-}\ \left(\mathrm{g}\right)\ \ \Delta\ \mathrm{H_{eg}} = -\mathrm{ve}$

$(\mathrm{B})\ \mathrm{Be}(\mathrm{g})\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{Be}^{-}\ (\mathrm{g})\ \ \Delta\ \mathrm{H_{eg}} = +\mathrm{ve}$

$\left(\mathrm{C}\right)\ \mathrm{O}\left(\mathrm{g}\right)\ +\ 2\mathrm{e}^{-}\ \rightarrow \mathrm{O}^{2-}\ \left(\mathrm{g}\right)\ \ \Delta\ \mathrm{H_{eg}} = +\mathrm{ve}$

$(\mathrm{D})\ \mathrm{Na}(\mathrm{g})\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{Na}^{-}\ (\mathrm{g})\ \ \Delta\ \mathrm{H_{eg}} = -\mathrm{ve}$

$(\mathrm{E})\ \mathrm{N}(\mathrm{g})\ +\ \mathrm{e}^{-}\ \rightarrow \mathrm{N}^{-}\ (\mathrm{g})\ \ \Delta\ \mathrm{H_{eg}} = +\mathrm{ve}$

Q.37. If $n \in [100,\ 700]$ then how many numbers are there that are neither multiples of 3 nor 4

A) 300

B) 200

C) 252

D) 315

Answer: 300

Solution: Let a numbers be multiples of 3.

$\Rightarrow 102+(a-1)3=699$

$\Rightarrow a-1=\frac{597}{3}=199$

$\Rightarrow a=200$

Let b numbers be multiples of 4.

$\Rightarrow 100+(b-1)4=700$

$\Rightarrow b-1=150$

$\Rightarrow b=151$

Let c numbers be multiples of 12 $(\text{i.e., } 3 \text{ and } 4 \text{ both})$.

$\Rightarrow 108+(c-1)12=696$

$\Rightarrow c-1=\frac{588}{12}$

$\Rightarrow c=50$

So, the numbers that are neither multiple of 3 nor 4 are,

$601-(200+151-50)=300$

Q.38. If $y=y(x)$ be the solution of the differential equation $(2x\log_e x)\frac{dy}{dx}+2y=\frac{3}{x}\log_e x,\ x>0$ and $y\left(e^{-1}\right)=0$ then the value of $y(e)$ will be

A) $\frac{-3}{2e}$

B) $\frac{2}{e}$

C) $\frac{-3}{e}$

D) $\frac{-1}{e}$

Answer: $\frac{-3}{e}$

Solution: Given,

Differential equation,

$$((2x\log_e x))\frac{dy}{dx}+2y=\frac{3}{x}\log_e x$$

$$\Rightarrow \frac{dy}{dx}+\frac{2}{(2x\log_e x)}y=\frac{3}{x(2x\log_e x)}\log_e x$$

$$\Rightarrow \frac{dy}{dx}+\frac{1}{x\log_e x}y=\frac{3}{2x^2}$$

Which is a linear differential equation,

Now, finding $IF=e^{\int \frac{1}{x\log_e x}dx}=\ln x$

Now, the solution is given by

$$y\cdot\ln x=\int\frac{3}{2x^2}\ln x dx$$

$$\Rightarrow y\cdot\ln x=\frac{3}{2}\int\frac{\ln x}{x^2}dx$$

Now, let $x=e^t\Rightarrow dx=e^t dt$

$$\Rightarrow y\cdot\ln x=\frac{3}{2}\int te^{-t}dt$$

$$\Rightarrow y\cdot\ln x=\frac{3}{2}\left[-te^{-t}-e^{-t}\right]+c$$

$$\Rightarrow y\cdot\ln x=\frac{-3}{2x}[\ln x+1]+c\quad\left\{\text{as } x=e^{-t}\right\}$$

Now, using $y\left(e^{-1}\right)=0$ we get,

$$\Rightarrow 0=\frac{-3}{2e^{-1}}[-1+1]+c\Rightarrow c=0$$

$$\Rightarrow y\cdot\ln x=\frac{-3}{2x}[\ln x+1]$$

Now, finding the value of $y(e)$,

$$\Rightarrow y(e)\cdot\ln e=\frac{-3}{2e}[\ln e+1]$$

$$\Rightarrow y(e)=\frac{-3}{e}$$

Q.39. If C be the circle of minimum area touching the parabola $2y=6-x^2$ and the lines $y=\sqrt{3}|x|$, then which one of the following points lie on the circle C

A) $(2,4)$

B) $(1,2)$

C) $(2,2)$

D) $(1,1)$

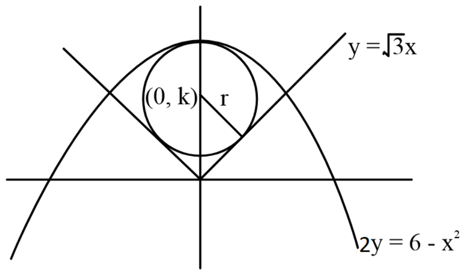
Answer: $(1,2)$

Solution: Given,

C is the circle of minimum area touching the parabola $2y=6-x^2$ and the lines $y=\sqrt{3}|x|$,

Now, let $x^2+(y-k)^2=r^2$ be the circle,

Now, plotting the diagram we get,



Now, using the perpendicular distance formula from centra of circle we get,

$$\frac{k}{\sqrt{1+\left(\sqrt{3}\right)^2}}=r$$

$\Rightarrow k=2r$

So, the equation of a circle will be,

$$x^2+(y-k)^2=\frac{k2}{4}$$

Now, the circle is also touching the parabola $2y=6-x^2$, so we get,

$$6-2y+(y-k)^2=\frac{k2}{4}$$

$$\Rightarrow y^2-y(2k+2)+\frac{3k2}{4}+6=0$$

Now, taking discriminant $D=0$ we get,

$$(2k+2)^2-\left(3k^2+24\right)=0$$

$\Rightarrow k=2\ \{\text{as } k\neq-10\}$

So, radius will be $r=1$

Hence, the equation of the circle will be,

$$x^2+(y-2)^2=1$$

Hence, the point $(1,2)$ will satisfy.

Q.40. If $f(x)=\begin{cases} x^3\sin\left(\frac{1}{x}\right), & x\neq0 \\ 0, & x=0 \end{cases}$, then

A) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$

B) $f''(0)=0$

C) $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$

D) $f''(0)=1$

Answer: $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$

Solution:

Given: $f(x)=\begin{cases} x^3\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$

$\Rightarrow f'(x)=x^3\cos\left(\frac{1}{x}\right)\times\frac{-1}{x^2}+\sin\left(\frac{1}{x}\right)\times 3x^2$

$\Rightarrow f'(x)=-x\cos\left(\frac{1}{x}\right)+3x^2\sin\left(\frac{1}{x}\right)$

$\Rightarrow f''(x)=-\left[x\times\sin\left(\frac{1}{x}\right)\times\frac{1}{x^2}+\cos\left(\frac{1}{x}\right)\right]+3\left[x^2\cos\left(\frac{1}{x}\right)\times\frac{-1}{x^2}+\sin\left(\frac{1}{x}\right)\times 2x\right]$

$\Rightarrow f''(x)=-\sin\left(\frac{1}{x}\right)\times\frac{1}{x}-\cos\left(\frac{1}{x}\right)-3\cos\left(\frac{1}{x}\right)+6x\sin\left(\frac{1}{x}\right)$

$\Rightarrow f''(x)=-\sin\left(\frac{1}{x}\right)\times\frac{1}{x}-4\cos\left(\frac{1}{x}\right)+6x\sin\left(\frac{1}{x}\right)$

$\Rightarrow f''\left(\frac{2}{\pi}\right)=-\sin\left(\frac{\pi}{2}\right)\times\frac{\pi}{2}-4\cos\left(\frac{\pi}{2}\right)+\frac{12}{\pi}\sin\left(\frac{\pi}{2}\right)$

$\Rightarrow f''\left(\frac{2}{\pi}\right)=-\frac{\pi}{2}+\frac{12}{\pi}$

$\Rightarrow f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$

Q.41.

If $A_r=\begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n}{2}(3n-1) \end{vmatrix}$ then the value of $2A_{10}-A_8$ is equal to:

A) $2\alpha+4\beta$

B) $4\alpha+2\beta$

C) $2n$

D) 0

Answer: $4\alpha+2\beta$

Solution:

Given: $A_r=\begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n}{2}(3n-1) \end{vmatrix}$

Applying, $C_1\to C_1-rC_2$

$\Rightarrow A_r=\begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & \frac{n}{2}(3n-1) \end{vmatrix}$

Expanding along C_1,

$\Rightarrow A_r=-2\left[\left(n^2-\beta\right)-2\left(\frac{n^2}{2}+\alpha\right)\right]$

$\Rightarrow A_r=2(2\alpha+\beta)$

$\Rightarrow A_r=4\alpha+2\beta$

Since, A_r is independent of r, therefore, $A_r=A_{10}=A_8$.

$\Rightarrow 2A_{10}-A_8=4\alpha+2\beta$

Q.42.

The value of integral $\int_0^{\frac{\pi}{4}}\frac{\cos^2x\sin^2x}{\left(\cos^3x+\sin^3x\right)^2}dx$ will be

A) $\frac{1}{2}$

B) $\frac{1}{6}$

C) $\frac{1}{3}$

D) 1

Answer: $\frac{1}{6}$

Solution: Given,

$$\int_0^{\frac{\pi}{4}} \frac{\cos^2 x \sin^2 x}{\left(\cos^3 x+\sin^3 x\right)^2} \mathrm{d} x$$

$$=\int_0^{\frac{\pi}{4}} \frac{\cos^2 x \sin^2 x}{\cos^6 x\left(1+\tan^3 x\right)^2} \mathrm{d} x$$

$$=\int_0^{\frac{\pi}{4}} \frac{\tan^2 x \sec^2 x}{\left(1+\tan^3 x\right)^2} \mathrm{d} x$$

Now, let $1+\tan^3 x=t \Rightarrow \tan^2 x \sec^2 x d x=\frac{d t}{3}$

$$=\frac{1}{3} \int_1^2 \frac{1 d t}{t^2}$$

$$=\frac{1}{3}\left[\frac{-1}{t}\right]_1^2=\frac{-1}{3}\left[\frac{1}{2}-1\right]=\frac{1}{6}$$

Q.43. Let α, β be distinct roots of the quadratic equation $x^2-\left(t^2-5t+6\right)x+1=0$ and $a_n=\alpha^n+\beta^n$, then the minimum value of $\frac{a_{2023}+a_{2025}}{a_{2024}}$ is

A) $\frac{-1}{2}$

B) $\frac{1}{2}$

C) $\frac{-1}{4}$

D) $\frac{1}{4}$

Answer: $\frac{-1}{4}$

Solution: Given: $x^2 - \left(t^2 - 5t + 6\right)x + 1 = 0$

And α & β are roots,

$\Rightarrow \alpha^2 - \left(t^2 - 5t + 6\right)\alpha + 1 = 0$ and $\beta^2 - \left(t^2 - 5t + 6\right)\beta + 1 = 0$

$\Rightarrow \alpha^2 + 1 = \left(t^2 - 5t + 6\right)\alpha$

And $\beta^2 + 1 = \left(t^2 - 5t + 6\right)\beta$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = \frac{\alpha^{2025}+\beta^{2025}+\alpha^{2023}+\beta^{2023}}{\alpha^{2024}+\beta^{2024}}$$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = \frac{\alpha^{2023}\left(\alpha^2+1\right)+\beta^{2023}\left(\beta^2+1\right)}{\alpha^{2024}+\beta^{2024}}$$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = \frac{\alpha^{2023}\left(t^2-5t+6\right)\alpha+\beta^{2023}\left(t^2-5t+6\right)\beta}{\alpha^{2024}+\beta^{2024}}$$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = \left(t^2 - 5t + 6\right)\frac{\alpha^{2024}\alpha+\beta^{2024}}{\alpha^{2024}+\beta^{2024}}$$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = \left(t^2 - 5t + 6\right)$$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = t^2 - 5t + \frac{25}{4} - \frac{25}{4} + 6$$

$$\Rightarrow \frac{a_{2025}+a_{2023}}{a_{2024}} = \left(t - \frac{5}{2}\right)^2 + \left(\frac{-1}{4}\right)$$

Now, $\left(t - \frac{5}{2}\right)^2 \geq 0$

So, the minimum value of $\frac{a_{2025}+a_{2023}}{a_{2024}}$ is $\frac{-1}{4}$.

Q.44. Find the shortest distance between two lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$

A) $8\sqrt{3}$

B) $4\sqrt{3}$

C) $6\sqrt{3}$

D) $2\sqrt{3}$

Answer: $4\sqrt{3}$

Solution: Given,

Equations of line,

$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ & $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$

Now, we know that,

The shortest distance is given by,

$$\text{S.D.}=\left|\frac{\begin{vmatrix}-1-3 & 1+15 & 0\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}}{\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}}\right|$$

$$\Rightarrow \text{S.D.}=\left|\frac{\begin{vmatrix}-4 & 16 & 0\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}}{\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}}\right|$$

$$\Rightarrow \text{S.D.}=\left|\frac{-4(21-5)-16(-6-10)}{\left|\hat{\imath}(16)-\hat{\jmath}(-16)+\hat{k}(2+14)\right|}\right|$$

$$\Rightarrow \text{S.D.}=\left|\frac{-64+256}{\sqrt{256+256+256}}\right|$$

$$\Rightarrow \text{S.D.}=\left|\frac{192}{16\sqrt{3}}\right|=\left|\frac{12}{\sqrt{3}}\right|=4\sqrt{3}$$

Q.45. Solve the differential equation $\frac{dy}{dx}+\frac{y}{1+x^2}=e^{-\tan^{-1}x}$

A) $ye^{\tan^{-1}x}=2x+c$

B) $ye^{\tan^{-1}x}=x+c$

C) $ye^{\tan^{-1}x}=-x+c$

D) $ye^{\tan^{-1}x}=3x+c$

Answer: $ye^{\tan^{-1}x}=x+c$

Solution: Given,

Differential equation $\frac{dy}{dx}+\frac{y}{1+x^2}=e^{-\tan^{-1}x}$

Which is a linear form,

So, finding $IF=e^{\int\frac{1}{1+x^2}}=e^{\tan^{-1}x}$

Now, solution of the differential equation is given by,

$ye^{\tan^{-1}x}=\int e^{\tan^{-1}x}e^{-\tan^{-1}x}dx$

$\Rightarrow ye^{\tan^{-1}x}=\int 1dx$

$\Rightarrow ye^{\tan^{-1}x}=x+c$

Q.46. R is defined on set $X=\{1,2,\ldots,20\}$ and $R_1=\{(x,y):2x-3y=2\}$, $R_2=\{(x,y):5x-4y=0\}$. If M,N represents the number of elements to be added to make R_1 and R_2 symmetric respectively, then find the value of $M+N$

A) 8

B) 11

C) 12

D) 10

Answer: 10

Solution: Given: $R_1 = \{(x, y) : 2x - 3y = 2\}$

So, $2x \text{ and } 3y$ has to be either odd or even simultaneously.

Since, $2x$ can't be odd therefore, $3y$ has to be even.

$\Rightarrow R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$

So, 6 elements are required to make it symmetric.

$\Rightarrow M = 6$

Now, $R_2 = \{(x, y) : 5x - 4y = 0\}$

$\Rightarrow 5x = 4y$

$\Rightarrow R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$

So, 4 elements are required to make it symmetric.

$\Rightarrow N = 4$

$\Rightarrow M + N = 10$

Q.47. Find the interval in which x^x is strictly increasing.

A) $\left(0, \frac{1}{e}\right]$

B) $\left[\frac{1}{e^2}, \infty\right)$

C) $(0, \infty)$

D) $\left(\frac{1}{e}, \infty\right)$

Answer: $\left(\frac{1}{e}, \infty\right)$

Solution: Let, $y = x^x$

$\Rightarrow \log y = x \log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$

$\Rightarrow \frac{dy}{dx} = x^x (1 + \log x)$

For x^x to be strictly increasing, $\frac{dy}{dx} > 0$

$\Rightarrow x^x (1 + \log x) > 0$

$\Rightarrow 1 + \log x > 0$

$\Rightarrow x > \frac{1}{e}$

$\Rightarrow x \in \left(\frac{1}{e}, \infty\right)$

Q.48. A company produces automobiles. It has two factories. Factory A produces 60% of the automobiles and rest is produced by the factory B. 80% of the automobiles producted by A is upto the standards and 90% of the automobiles produced by B is upto the standards. If an automobile is selected we found it as standard, the probability that it came from factory B is P, then find $126P$

A) 52

B) 54

C) 27

D) 48

Answer: 54

Solution: The probability of choosing an automobile that is upto the standard and it came from factory B is given by,

$P=\frac{P(B)\times 90\%}{P(A)\times 80\%+P(B)\times 90\%}$ {using bayes theorem}

$\Rightarrow P=\frac{\frac{4}{10}\times\frac{90}{100}}{\frac{6}{10}\times\frac{80}{100}+\frac{4}{10}\times\frac{90}{100}}$

$\Rightarrow P=\frac{360}{480+360}$

$\Rightarrow P=\frac{360}{840}=\frac{3}{7}$

$\Rightarrow 126P=54$

Q.49. If $\sigma^2=4$ and $\overline{x}=10$ of 20 observations. One term was taken wrong i.e., instead of 12 they have taken 8. Find the correct standard deviation.

A) $\sqrt{3.84}$

B) $\sqrt{3.96}$

C) 1.93

D) 1.8

Answer: $\sqrt{3.96}$

Solution:

Incorrect mean, $\overline{x}=\frac{\sum x_i}{10}=20$

Incorrect sum, $x_1+x_2+\ldots\ldots+x_{20}=200$

Correct sum, $S_{20}=200-8+12=204$

Correct mean, $M=\frac{204}{20}=10.2$

We know that, $\sigma^2=\frac{\sum (x_i)^2}{n}-(\overline{x})^2=4$

$\Rightarrow \frac{(x_1)^2+(x_2)^2+\ldots\ldots+(x_{20})^2}{20}-10^2=4$

$\Rightarrow (x_1)^2+(x_2)^2+\ldots\ldots+(x_{20})^2=2080$, which is incorrect sum.

So, correct sum is $(x_1)^2+(x_2)^2+\ldots\ldots+(x_{20})^2-8^2+12^2=2080-64+144$

$\Rightarrow (x_1)^2+(x_2)^2+\ldots\ldots+(x_{20})^2-8^2+12^2=2160$

So, correct standard deviation is given by,

$\sigma^2=\frac{2160}{20}-(10.2)^2$

$\Rightarrow \sigma^2=108-104.04$

$\Rightarrow \sigma=\sqrt{3.96}$

Q.50. Let a point $P(10,-2,-1)$ and $R(1,7,6)$. If Q is a foot of the perpendicular from R to the line joining $(2,-5,11)$ and $(6,-7,5)$. Then find the value of PQ^2.

A) $\frac{3600}{7}$

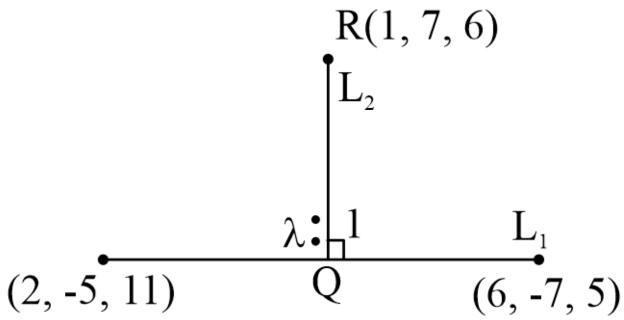
B) $\frac{2941}{14}$

C) $\frac{3409}{7}$

D) $\frac{3509}{7}$

Answer: $\frac{2941}{14}$

Solution:



Using section formula to find the coordinates of Q.

$\Rightarrow Q \equiv \left(\frac{6\lambda+2}{\lambda+1}, \frac{-7\lambda-5}{\lambda+1}, \frac{5\lambda+11}{\lambda+1}\right)$

Now, direction ratio of line $L_1 = 4\hat{i} - 2\hat{j} - 6\hat{k}$.

Similarly, for $L_2 = \left(\frac{6\lambda+2}{\lambda+1} - 1\right)\hat{i} + \left(\frac{-7\lambda-5}{\lambda+1} - 7\right)\hat{j} + \left(\frac{5\lambda+11}{\lambda+1} - 6\right)\hat{k}$

$\Rightarrow L_2 = \left(\frac{5\lambda+1}{\lambda+1}\right)\hat{i} + \left(\frac{-14\lambda-12}{\lambda+1}\right)\hat{j} + \left(\frac{-\lambda+5}{\lambda+1}\right)\hat{k}$

Since, $L_1 \perp L_2$, therefore, $L_1 . L_2 = 0$

$\Rightarrow 4\left(\frac{5\lambda+1}{\lambda+1}\right) - 2\left(\frac{-14\lambda-12}{\lambda+1}\right) - 6\left(\frac{-\lambda+5}{\lambda+1}\right) = 0$

$\Rightarrow 20\lambda + 4 + 28\lambda + 24 + 6\lambda - 30 = 0$

$\Rightarrow 54\lambda - 2 = 0$

$\Rightarrow \lambda = \frac{1}{27}$

$\Rightarrow Q \equiv \left(\frac{\frac{6}{27}+2}{\frac{1}{27}+1}, \frac{-\frac{7}{27}-5}{\frac{1}{27}+1}, \frac{\frac{5}{27}+11}{\frac{1}{27}+1}\right)$

$\Rightarrow Q \equiv \left(\frac{15}{7}, \frac{-71}{14}, \frac{151}{14}\right)$

$\Rightarrow PQ^2 = \left(10 - \frac{15}{7}\right)^2 + \left(-2 + \frac{71}{14}\right)^2 + \left(-1 - \frac{151}{14}\right)^2$

$\Rightarrow PQ^2 = \left(\frac{55}{7}\right)^2 + \left(\frac{43}{14}\right)^2 + \left(\frac{165}{14}\right)^2$

$\Rightarrow PQ^2 = \frac{3025}{49} + \frac{1849}{196} + \frac{27225}{96}$

$\Rightarrow PQ^2 = \frac{12100+29074}{196}$

$\Rightarrow PQ^2 = \frac{41174}{196}$

$\Rightarrow PQ^2 = \frac{2941}{14}$

Q.51. If $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$ then find the value of n

Answer: 47

Solution: Given,

$\cot^{-1}3+\cot^{-1}4+\cot^{-1}5+\cot^{-1}n=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{12}}\right)+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{7}{11}\right)+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{7}{11}+\frac{1}{5}}{1-\frac{7}{55}}\right)+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{46}{48}\right)+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{23}{24}+\frac{1}{n}}{1-\frac{23}{24n}}\right)=\frac{\pi}{4}$

$\Rightarrow \left(\frac{23n+24}{24n-23}\right)=\tan\frac{\pi}{4}$

$\Rightarrow \left(\frac{23n+24}{24n-23}\right)=1$

$\Rightarrow 23n+24=24n-23$

$\Rightarrow n=47$

Q.52. Find the number of triangles formed whose vertices are also vertices of a regular octagon but the side of triangle is not common with sides of octagon.

Answer: 16

Solution: Total number of triangles that can be formed by using any three vertices of the octagon are,

$^{8}C_{3}=\frac{8\times7\times6}{3\times2}=56$

Case-I: Number of triangles when all three sides are common with octagon.

Since, no such triangle is possible so number of triangles is 0.

Case-II: Number of triangles when two sides are common with octagon.

In this case one vertex has to be common with octagon i.e., $^{8}C_{1}=8$.

Case-III: Number of triangles when one side is common with octagon.

In this case the adjacent vertex of the chosen side can't be used, so the third vertex of triangle needs to be chosen from remaining 4 sides i.e.,

$^{8}C_{1}\,(\text{for chosing any 1 side})\times{}^{4}C_{1}\,(\text{for the third vertex})$

$=32$

So, the total number of required triangles are, $56-(0+8+32)=16$.