# JEE Main 5th April Session 2 - Memory Based Paper

## 5th April Session 2

## Questions

Q.1. Which of the following is not correct about stopping potential?

A) It depends on the frequency of the incident light.

B) It depends on the nature of the material.

C) It is equal to $\frac{K.E.}{e}$.

D) It increases with intensity of the incident light.

Answer: It increases with intensity of the incident light.

Solution: The expression for the stopping potential of a metal can be written as

$eV_s = h\nu - \phi \quad \ldots(1)$

From the above equation, it can be concluded that the stopping potential depends on the nature of the material and the frequency of the incident light.

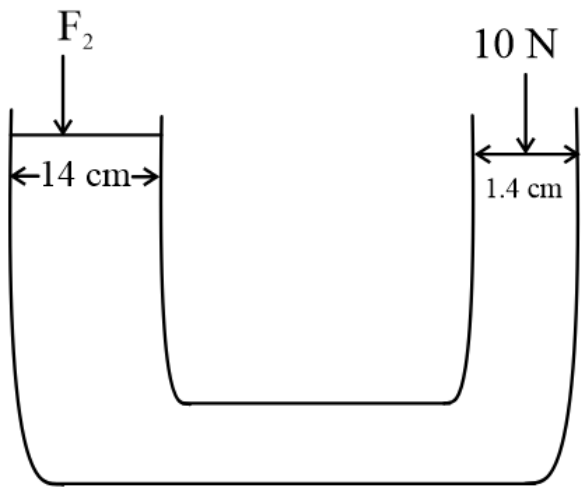
Also, the kinetic energy of the electron is given by

$K.E. = eV_s$

$\Rightarrow V_s = \frac{K.E.}{e}$

The stopping potential does not depend on the intensity of the light. Hence, this is the correct option.

Q.2. Find the value of $F_2$ in newton unit for the following diagram.




A) 100

B) 10

C) 1000

D) 10000

Answer: 1000

**Solution:** For the left arm of the tube, from Pascal's law, it follows that

$$P=\frac{F_2}{\frac{\pi\left(14\times10^{-2}\right)^2}{4}}$$

$$=\frac{4F_2}{196\times10^{-4}\pi}\quad\ldots(1)$$

For the right arm of the tube, it follows that

$$P=\frac{10\ \mathrm{N}}{\frac{\pi\left(1.4\times10^{-2}\right)^2}{4}}$$

$$=\frac{40\ \mathrm{N}}{1.96\times10^{-4}\pi}\quad\ldots(2)$$

From equations (1) and (2),

$$\frac{4F_2}{196\pi}=\frac{40\ \mathrm{N}}{1.96\pi}\Rightarrow F_2=1000\ \mathrm{N}$$

Q.3. Given that the current varies with time through an inductor as $I=(3t+8)\ \mathrm{A}$. If the inductor is connected across a source of emf $\varepsilon=12\ \mathrm{V}$, find the self inductance (in $\mathrm{H}$) of the inductor.

A) $2$ B) $12$

C) $4$ D) $14$

**Answer:** $4$

**Solution:** Given,

$$I=3t+8\quad\ldots(1)$$

The formula to calculate the self-inductance of the inductor is given by

$$\varepsilon=L\frac{dI}{dt}\quad\ldots(2)$$

From equations (1) and (2), it follows that

$$12=L\times\frac{d}{dt}(3t+8)$$

$$\Rightarrow L=\frac{12}{3}$$

$$=4\ \mathrm{H}$$

Q.4. The maximum height reached by a projectile is $64\ \mathrm{m}$. If the initial velocity is halved, then calculate the new maximum height of the projectile in meters.

A) $12$ B) $10$

C) $16$ D) $20$

**Answer:** $16$

**Solution:** The formula to calculate the maximum height attained by the projectile is

$$H=\frac{v^2\sin^2\theta}{2g} \quad \ldots(1)$$

where, $v$ is the initial velocity and $\theta$ is the angle of projection.

Substitute the new value of velocity into equation (1) to obtain the new value $(H')$ of maximum height.

$$\begin{aligned} H' &= \frac{\left(\frac{v}{2}\right)^2\sin^2\theta}{2g} \\ &= \frac{1}{4}\frac{v^2\sin^2\theta}{2g} \\ &= \frac{H}{4} \\ &= \frac{64\text{ m}}{4} \\ &= 16\text{ m} \end{aligned}$$

Q.5. The Van der Waal's gas equation is expressed as $\left(P-\frac{a}{V^2}\right)(V-b)=nRT$,where symbol have their usual meaning. Then dimension of $\frac{a}{b^2}$ is

A) $\left[\mathrm{ML^{-1}T^{-2}}\right]$ B) $\left[M^{-1}L\mathrm{T}^{-2}\right]$

C) $\left[\mathrm{ML^{-2}T^{-2}}\right]$ D) $\left[\mathrm{ML^{-1}T^{-1}}\right]$

**Answer:** $\left[\mathrm{ML^{-1}T^{-2}}\right]$

**Solution:** Only similar nature quantities can be added or subtracted , So

It means $\mathrm{V}$ and $\mathrm{b}$ must have same dimensions, so

$$[b]=[V]=\left[L^3\right]$$

Similarly $\mathrm{p}$ and $\frac{\mathrm{a}}{\mathrm{V}^2}$ have same dimensions,so

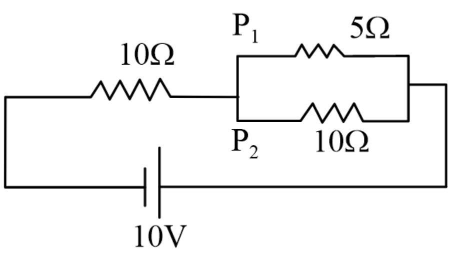
$$\left[p\right]=\frac{[\mathrm{a}]}{[\mathrm{V}]^2}=\left[\mathrm{ML^{-1}T^{-2}}\right]$$

$$[a]=\left[ML^5T^{-2}\right]$$

Hence,

$$\begin{aligned} \left[\frac{a}{b^2}\right] &= \frac{\left[\mathrm{ML^5T^{-2}}\right]}{\left[\mathrm{L^6}\right]} \\ &= \left[\mathrm{ML^{-1}T^{-2}}\right] \end{aligned}$$

Q.6. The ratio of heat dissipated $\left(\frac{P_1}{P_2}\right)$ per second through $5\ \Omega$ and $10\ \Omega$ is given as




A) $1:2$ B) $1:1$

C) $2:1$ D) $4:1$

**Answer:** $2:1$

**Solution:** Since both branches are in parallel, therefore potential drop will be equal.

So for power, we can write

$$\frac{P_1}{P_2}=\frac{\left(\frac{V^2}{R_1}\right)}{\left(\frac{V^2}{R_2}\right)}=\frac{R_2}{R_1}=\frac{10}{5}=2:1$$

Q.7. During an adiabatic process, if pressure of gas is found to be proportional to cube of its absolute temperature, then $\frac{Cp}{Cv}$ for gas is

A) $\frac{3}{2}$

B) $\frac{5}{3}$

C) $\frac{7}{5}$

D) $\frac{9}{7}$

**Answer:** $\frac{3}{2}$

**Solution:** Given:

$$P \propto T^3$$
$$\Rightarrow P \propto (PV)^3$$
$$\Rightarrow P^2V^3 = \text{constant}$$
$$\Rightarrow PV^{\frac{3}{2}} = \text{constant}$$

Now, comparing the above equation with the standard equation($PV^{\gamma}=\text{constant}$) for adiabatic process, we get $\gamma=\frac{3}{2}=\frac{Cp}{Cv}$.

Q.8. A block of mass $50\ \text{kg}$ is moving with speed of $10\ \text{m s}^{-1}$ on a rough horizontal surface(friction coefficient $=0.3$). Find the kinetic friction acting on the object.

A) $16\ \text{N}$

B) $150\ \text{N}$

C) $167\ \text{N}$

D) $500\ \text{N}$

**Answer:** $150\ \text{N}$

**Solution:** Normal force acting on the object will be $N=mg$.

Now, kinetic friction is given by the expression, $f=\mu N=\mu mg=0.3\times 50\times 10=150\ \text{N}$.

Q.9. A truck is moving from rest with constant power $P$. If the displacement of the truck is proportional to $t^n$, where is $t$ is time, find $n$.

A) $\frac{1}{2}$

B) $2$

C) $\frac{3}{2}$

D) $\frac{5}{2}$

**Answer:** $\frac{3}{2}$

**Solution:** Given:

$$P = C$$
$$\Rightarrow Fv = C$$
$$\Rightarrow m\frac{dv}{dt}v = C$$
$$\Rightarrow \int_0^v vdv = \frac{C}{m}\int_0^t dt$$
$$\Rightarrow \frac{v^2}{2} = \frac{C}{m}t$$
$$\Rightarrow v = C_1\sqrt{t}$$
$$\Rightarrow \frac{ds}{dt} = C_1\sqrt{t}$$
$$\Rightarrow \int_0^s ds = \int_0^t C_1\sqrt{t}$$
$$\Rightarrow s = C_1\frac{t^{\frac{3}{2}}}{\frac{3}{2}}$$
$$\Rightarrow s = C_2 t^{\frac{3}{2}}$$
$$\Rightarrow s \propto t^{\frac{3}{2}}$$

Therefore, $n = \frac{3}{2}$

Q.10. A solid sphere is rolling without slipping. Find the ratio of rotational kinetic energy to total kinetic energy of sphere.

A) $\frac{3}{7}$

B) $\frac{2}{7}$

C) $\frac{4}{7}$

D) $\frac{5}{7}$

**Answer:** $\frac{2}{7}$

**Solution:** Translational kinetic energy of the rolling sphere can be written as,

$$\mathrm{K}_{\text{trans}} = \frac{1}{2}\mathrm{M}v^2$$

Rotational kinetic energy of the rolling sphere can be expressed as,

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5}MR^2 \times \frac{v^2}{R^2} = \frac{1}{5}Mv^2$$

Hence, the total $K.E.$ of the rolling sphere,

$$(K.E.)_{\text{Total}} = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$
$$\left(K.E.\right)_{\text{Total}} = \frac{1}{2}Mv^2 + \frac{1}{2}\cdot\frac{2}{5}MR^2\cdot\frac{v^2}{R^2}$$
$$= \frac{1}{2}\mathrm{M}v^2 + \frac{1}{5}\mathrm{M}v^2$$
$$= \frac{7}{10}Mv^2$$
$$\therefore \frac{(\mathrm{K.E.})_{\text{rot}}}{(\mathrm{K.E.})_{\text{Total}}} = \frac{\frac{1}{5}\mathrm{Mv}^2}{\frac{7}{10}\mathrm{Mv}^2} = \frac{2}{7}$$

Q.11. On vehicles containing flammable fluid, metallic chains are provided touching the earth, the correct option is

A) It is custom

B) alert for another vehicle

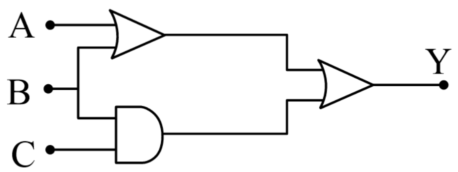
C) for discharging the static charges developed due to friction

D) it is fashion

**Answer:** for discharging the static charges developed due to friction

**Solution:** Moving vehicle gets charged due to friction. The inflammable material may catch fire due to the spark produced by charged vehicle. When metallic rope is used, the charge developed on the vehicle is transferred to the ground and so the fire is prevented.

Q.12. For what Boolean values of A, B & C, the given logic gate gives an output of zero.




A) $A=1,\ B=0,\ C=1$

B) $A=0,\ B=1,\ C=1$

C) $A=1,\ B=1,\ C=1$

D) $A=0,\ B=0,\ C=1$

Answer: $A=0,\ B=0,\ C=1$

Solution: The output $(X)$ of the first OR gate is given by

$X=A+B$

The output $(X')$ of the AND gate is given by

$X'=B\cdot C$

Hence, the net output $(Y)$ is given by

$$\begin{aligned}Y&=X+X'\\&=A+B+B\cdot C\\&=A+B(1+C)\\&=A+B\end{aligned}$$

From the above calculation, it is clear that the value of the final output does not depend $C$. So, for the output to be zero, the values of $A$ and $B$ have to be zero.

Hence, this is the correct option.

Q.13. In sonometer, fundamental frequency changes from $400\ \text{Hz}$ to $500\ \text{Hz}$ keeping the same tension. Find the percentage change in length.

A) $10\%$

B) $5\%$

C) $20\%$

D) $40\%$

Answer: $20\%$

Solution: For the old fundamental frequency,

$$F=\frac{v}{2l_1}$$
$$\Rightarrow l_1=\frac{v}{2F}$$
$$=\frac{v}{800}\quad\ldots(1)$$

For the new fundamental frequency,

$$F'=\frac{v}{2l_2}$$
$$\Rightarrow l_2=\frac{v}{1000}\quad\ldots(2)$$

Hence, the required percentage error $(p)$ in length can be calculated as follows:

$$\begin{aligned}|p|&=\left|\frac{l_2-l_1}{l_1}\times100\%\right|\\&=\left|\frac{\frac{v}{1000}-\frac{v}{800}}{\frac{v}{800}}\times100\%\right|\\&=\left[1-\frac{8}{10}\right]\times100\%\\&=20\%\end{aligned}$$

Q.14. The Electrostatic force $\left(\vec{F}_1\right)$ and magnetic force $\left(\vec{F}_2\right)$ acting on charge $q$ moving with a velocity $\vec{v}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$ is given by

A) $\vec{F}_1 = \vec{v} \times \vec{E}$, $\vec{F}_2 = q\left(\vec{B} \times \vec{v}\right)$

B) $\vec{F}_1 = q\vec{B}$, $\vec{F}_2 = q\left(\vec{E} \times \vec{v}\right)$

C) $\vec{F}_1 = q\vec{E}$, $\vec{F}_2 = q\left(\vec{v} \times \vec{B}\right)$

D) $\vec{F}_1 = q\vec{E}$, $\vec{F}_2 = q\left(\vec{B} \times \vec{v}\right)$

Answer: $\vec{F}_1 = q\vec{E}$, $\vec{F}_2 = q\left(\vec{v} \times \vec{B}\right)$

Solution: The force due to the electric field on a charge is built into its definition, $\vec{F}_1 = q\vec{E}$.

Magnetic fields can exert a force on an electric charge only if it moves and the force is given by, $\vec{F}_2 = q\left(\vec{v} \times \vec{B}\right)$.

Q.15. $20\ \Omega$ resistance is divided into $10$ equal parts and they are connected in parallel. The overall resistance (in $\Omega$) is

A) $0.2$

B) $0.8$

C) $0.4$

D) $0.6$

Answer: $0.2$

Solution: When the $20\ \Omega$ resistor is divided into ten parts, each resistance becomes $2\ \Omega$.

Hence, the equivalent resistance $(R)$ can be calculated as follows:

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{2} + \ldots\ldots .10 \text{ terms}$$
$$= \frac{10}{2}$$
$$\Rightarrow R = \frac{2}{10}$$
$$= 0.2\ \Omega$$

Q.16. For a series LCR circuit having AC source of $220\ \text{V}$, $50\ \text{Hz}$, it is observed that the voltage across the inductor which has self inductance of $10\ \text{mH}$ is $31.4\ \text{V}$. Find the current in circuit.

A) $10\ \text{mA}$

B) $63\ \text{mA}$

C) $10\ \text{A}$

D) $63\ \text{A}$

Answer: $10\ \text{A}$

Solution: Voltage across the inductor,

$$V_L = IX_L$$
$$\Rightarrow 31.4 = I(\omega L)$$
$$\Rightarrow 31.4 = I\left(2\pi \times 50 \times 10 \times 10^{-3}\right)$$
$$\Rightarrow I = 10\ \text{A}$$

Q.17. In the hydrogen spectrum, the wavelength of the shortest line from the Lyman series is $916\ \text{Å}$, then find the longest wavelength of the Balmer series.

A) $65\ \text{Å}$

B) $95\ \text{Å}$

C) $6595\ \text{Å}$

D) $9565\ \text{Å}$

Answer: $6595\ \text{Å}$

**Solution:** For the shortest wavelength in Lyman series, it can be written that

$$\frac{1}{\lambda_L}=R\left(\frac{1}{1^2}-\frac{1}{\infty}\right)$$
$$=R \quad \ldots(1)$$

For the longest wavelength in Balmar series, it follows that

$$\frac{1}{\lambda_B}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$
$$=\frac{5R}{36} \quad \ldots(2)$$

Equations (1) and (2) imply that

$$\frac{1}{\lambda_B}=\frac{5}{36\lambda_L}$$
$$\Rightarrow \lambda_B=\frac{36}{5}\times 916\ \mathrm{\mathring{A}}$$
$$\approx 6595\ \mathrm{\mathring{A}}$$

Q.18. Find out $\mathrm{E\ \ cell}$ of the given cell $\mathrm{M}\left|\mathrm{M}^{2+}\right\| \mathrm{X}^{2-}\left|\mathrm{X}\right.$

$$\mathrm{E^{\circ}}_{\mathrm{M^{2+}|M}}=0.34\ \mathrm{V}$$
$$\mathrm{E^{\circ}}_{\mathrm{X|X^{2-}}}=0.46\ \mathrm{V}$$

A) $0.80\ \mathrm{V}$ B) $0.12\ \mathrm{V}$

C) $-0.12\ \mathrm{V}$ D) $-0.80\ \mathrm{V}$

**Answer:** $0.12\ \mathrm{V}$

**Solution:** $\mathrm{E_{cell}}$ is the difference between standard reduction potential of catode and standard reduction potential of anode.

$$\mathrm{E^{\circ}}\left(\mathrm{X|X^{2-}}\right)-\mathrm{E^{\circ}}\left(\mathrm{M^{2+}|M}\right)$$
$$=0.46-0.34$$
$$=0.12\ \mathrm{V}$$

Q.19. Angular momentum of an electron in an orbit of radius R of a hydrogen atom is directly proportional to

A) $\frac{1}{\mathrm{R}}$ B) $\mathrm{R}$

C) $\frac{1}{\sqrt{\mathrm{R}}}$

D) $\sqrt{\mathrm{R}}$

**Answer:** $\sqrt{\mathrm{R}}$

**Solution:** The angular momentum of Bohr's orbit is

$$\mathrm{L=mvr=\frac{nh}{2\pi}}$$

The $\mathrm{n^{th}}$ orbit radius can be calculated as follows:

$$\mathrm{R}=0.529\times\frac{\mathrm{n^2}}{\mathrm{Z}}\mathrm{\mathring{A}}$$
$$\mathrm{n} \propto \sqrt{\mathrm{ZR}}$$
$$\Rightarrow \mathrm{L} \propto \sqrt{\mathrm{R}}$$

Q.20. Which of the following is true regarding coagulation of the egg?

A) Primary structure does not change B) Secondary structure does not change

C) Tertiary structure does not change D) Denaturation of protein does not occur

**Answer:** Primary structure does not change

Solution: Denaturation of proteins protein present in egg white has an unique three dimensional structure. When it is subjected to physical change like change in temperature. i.e., on boiling, coagulation of egg white occurs due to denaturation of protein. During denaturation hydrogen bonds are disturbed due to this globules unfold and helix gets uncoiled and protein looses its biological activity.

During denaturation, secondary and tertiary structures are destroyed but primary structure remains intact. The coagulation of egg white on bolling is a common example of denaturation.

Q.21. Find out value of $\frac{C_p}{C_v}$ for an ideal gas undergoing reversible adiabatic process for which $P \propto T^3$.

A) $3/2$ B) $5/4$

C) $4/3$ D) $5/3$

Answer: $3/2$

Solution: For an adiabatic reversible process,

$PV^\gamma = k(\text{constant})$

$\frac{C_p}{C_v} = \gamma$ for an ideal gas.

From the ideal gas equation, $PV = nRT$

Given that $\frac{P}{T^3} = \text{constant}$

$\Rightarrow \frac{P}{(PV)^3} = \text{constant}$

$\Rightarrow \frac{1}{P^2V^3} = \text{constant}$

$\Rightarrow PV^{3/2} = \text{constant}$

Hence, $\frac{C_p}{C_v} = 3/2$

Q.22. For the reaction,

$CH_4 + O_2 \rightarrow CO_2 + H_2O$

How many moles of methane is required for the formation of $11\ g$ of $CO_2$.

A) $0.25$ B) $0.45$

C) $0.32$ D) $0.36$

Answer: $0.25$

Solution: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
$16\ g \quad 32\ g \quad 44\ g \quad 36\ g$

From the above reaction, we can say that one mole of methane gives one mole of carbon dioxide.

Number of mole of methane required for the formation of $11\ g$ of $CO_2 = \frac{11}{44} = \frac{1}{4} = 0.25\ \text{moles}$

Q.23. Assertion: $NF_3$ have less dipole moment than $NH_3$.

Reason: $F$ is more electronegative and bond moment of $N - H$ bond is in the same direction of lone pair.

A) Both assertion and reason are correct and reason is the correct explanation for the assertion

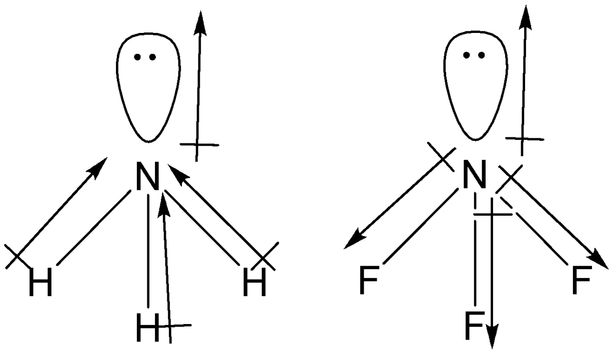
B) Both assertion and reason are correct and reason is not the correct explanation for the assertion

C) Assertion is correct and reasoon is incorrect

D) Assertion is incorrect and reason is correct

Answer: Both assertion and reason are correct and reason is the correct explanation for the assertion

**Solution:** In $NH_3$, the dipole moment vector of the bond and the lone pairs are in the same direction. But in $NF_3$, the dipole moment vector of lone pairs and bond pairs are opposite in direction. So, the net dipole moment will be the substractive effect of the two. Hence, the dipole moment of $NH_3$ is larger than $NF_3$.



Fluorine is more electronegative than nitrogen so all the electron will move towards fluorine. Due to this all of the vectors will cancel each other and the summation will come minimum.

Q.24. Given elementary reaction

$2A + B \leftrightharpoons C$

The initial pressures of A and B respectively are $1.5$ atm and $0.7$ atm. After time t, the pressure of C is $0.5$ atm. Find the ratio of initial rate and rate after time t.

A) $\frac{63}{2}$

B) $\frac{33}{2}$

C) $\frac{23}{2}$

D) $\frac{31}{2}$

**Answer:** $\frac{63}{2}$

**Solution:**

| | $2A$ | $+$ | $B$ | $\leftrightharpoons$ | $C$ |
|---|---|---|---|---|---|
| initial pressure | $1.5$ | | $0.7$ | | |
| time t | $1.5 - x$ | | $0.7 - x/2$ | | $x/2$ |

Given that
$x/2 = 0.5$ atm
$\Rightarrow x = 1$ atm

Now partial pressures of A & B after time t is $0.5$ atm and $0.2$ atm respectively.

Now, rate law can be written as

$R = k[A]^2[B]$

$R_1$ (initial rate) $= k[1.5]^2[0.7]$

$R_2$ (rate at time t $= k[0.5]^2[0.2]$

$$\frac{R_1}{R_2} = \frac{[1.5]^2[0.7]}{[0.5]^2[0.2]} = 31.5$$

Q.25. Correct order of increasing atomic size of $13^{th}$ group element will be:

A) $Tl > In > Al > Ga > B$

B) $Tl < In > Al > Ga > B$

C) $Tl > In < Al > Ga > B$

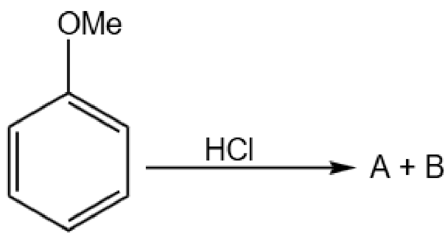
D) $In > Tl > Ga > Al > B$

**Answer:** $Tl > In > Al > Ga > B$

**Solution:** Atomic and ionic radii of group 13 elements are lower than those of alkaline earth metals of group 2 primarily due to greater nuclear charge of group 13 elements as compared to group 2 elements. On moving down the group the atomic radius of $Ga$ is slightly lower than that of $Al$. This is due to the presence of d - electrons in $Ga$ which do not shield the nucleus effectively. As a result, the electrons in $Ga$ experience greater force of attraction by the nucleus than in $Al$ and hence the atomic radius of $Ga$ 135 $pm$ is slightly less than that of $Al$ 143 $pm$. Thus, the increasing order of atomic radii of group 13 elements is $B < Ga < Al < In < Tl$.

Hence, the correct option is A

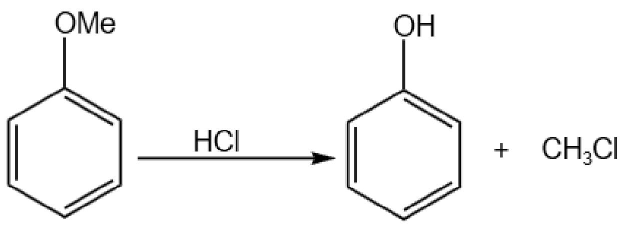
Q.26. Identify the products A and B in the following reaction.




A) Phenol and methyl chloride
B) Phenol and methanol
C) Chlorobenzene and methanol
D) Chlorobenzene and methyl chloride

Answer: Phenol and methyl chloride

Solution: Anisole on reaction with hydrogen chloride gives phenol and methyl chloride as the major products. Phenol is major product because of double character between carbon of benzene ring and oxygen.




Q.27. Match Column I with column II

| Column I | Column II |
|---|---|
| 1.$ICl$ | P. T-shape |
| 2.$ICl_3$ | Q. Pentagonal bipyramidal |
| 3. $ClF_5$ | R. Linear |
| 4. $IF_7$ | S. Square pyramidal |

A) $1-R,\ 2-P,\ 3-S,\ 4-Q$
B) $1-R,\ 2-P,\ 3-Q,\ 4-S$
C) $1-P,\ 2-R,\ 3-S,\ 4-Q$
D) $1-R,\ 2-S,\ 3-P,\ 4-Q$

Answer: $1-R,\ 2-P,\ 3-S,\ 4-Q$

Solution: $ICl$ has linear shape due to lone of electrons.

The given compound is $ICl_3$ . Due to the presence of two lone pairs on iodine, the molecular geometry of this compound is T-shaped. The electron pair geometry is trigonal bipyramidal.

$ClF_5$If the ligand atoms were connected, the resulting shape would be that of a pyramid with a square base.

$IF_7$, or Iodine heptafluoride is an interhalogen compound. It has a coordination number of $7$. It was predicted by the VSEPR theory that $IF_7$ has an unusual pentagonal bipyramidal structure as the central iodine atom undergoes $sp^3d^3$ hybridisation.

Q.28. From the given information, calculate the enthalpy of formation of 2 moles of $C_6H_6\,(l)$ at $25^oC$.

Given $\Delta_C H_{C_6H_6(l)} = -3264.6\ kJ/mol,\ \Delta_C H_{C(s)} = -393.5\ kJ/mol,\ \Delta_C H_{H_2O(l)} = -285.83\ kJ/mol$

A) $246.11\ kJ/mol$
B) $46.11\ kJ/mol$
C) $-124.5\ kJ/mol$
D) $124.5\,kJ/mol$

**Answer:** $46.11\ \mathrm{kJ/mol}$

**Solution:** The formation reaction of benzene is

$6C_{(s)} + 3H_{2(g)} \rightarrow C_6H_6(l)$

The enthalpy of formation of benzene can be calculated as follows,

$\Delta H_f(C_6H_6(l)) = 6 \times \Delta H_c\left(C_{(s)}\right) + 3 \times \Delta H_c\left(H_{2(g)}\right) - \Delta H_c(C_6H_6(l))$

$\Rightarrow \Delta H_f(C_6H_6(l)) = 6 \times (-393.5) + 3 \times (-285.83) - (-3264.6)$
$\Rightarrow \Delta H_f(C_6H_6(l)) = 46.11\ \mathrm{kJ/mol}$

Q.29. Which of the following is an acidic oxide?

A) $N_2O_3$

B) $NO$

C) $CO$

D) $CaO$

**Answer:** $N_2O_3$

**Solution:** An oxide that exhibits the properties of acids is called acidic oxide. Since they are acidic, they react with a base to form a salt. All the nitrogen oxides are acidic except nitric oxide and nitrous oxide, which are neutral oxides. Carbon monoxide is a neutral oxide and calcium oxide is a basic oxide.

Q.30. If square planar complex $M(ABXL)$ has all four unidentate ligands, then find out its total number of geometrical isomers.

A) $2$

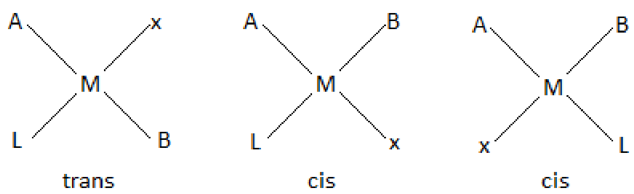
B) $3$

C) $4$

D) $0$

**Answer:** $3$

**Solution:** The square planar complex of the type MABXL (where A, B, X, L are unidentates) exhibit three geometrical isomers. Out of three isomers, two isomers are cis and one isomer is trans.




Q.31. How many of the following have zero dipole moment?

$H_2S,\ CH_4,\ NH_3,\ BF_3,\ SO_2,\ NF_3$

**Answer:** 2

**Solution:** The bond moments act from Hydrogen towards sulphur and the moment from sulphur acts towards the lone pair of electrons. Both the bond moments act in the same direction and thus the resultant moment adds up vectorially.

The dipole moment of methane is zero because the molecule is a symmetrical tetrahedron. Therefore, the dipole moments between carbon and hydrogen cancel out each other, leading to a net dipole moment of zero.

The $NH_3$ molecule has a dipole moment because the three N–H bond dipoles add to give a net dipole pointing upward to the $NH_3$ structure.

The fluorine is the most electronegative atom, so the movement of dipole is towards the fluorine atom. As the magnitude is the same for the two B-F bonds which are opposite to the third B-F bond, it will cancel the dipole of the third B-F bond and as a result the dipole will be zero.

The geometry of $SO_2$ is bent because the lone pair on the central sulphur atom pushes the two bonds down for a bond angle of due to the electrons repulsion. This is what makes $SO_2$ a polar molecule because the dipole moments of both bonds of not cancel out,

In case of $NF_3$ the dipole moment is not zero because of presence of a lone pair of electrons on Nitrogen atom.

Q.32. In an atom, how many electrons can have:

(i) $n=4$ (ii) $m_l=1$ (iii) $m_s=\frac{1}{2}$

Answer: 3

Solution:
$4s,\ m_l=0$
$4p,\ m_l=-1,0,+1$
$4d,\ m_l=-2,-1,0,+1,+2$
$4f,\ m_l=-3,-2,-1,0,+1,+2,+3$

Hence, the answer is $3$

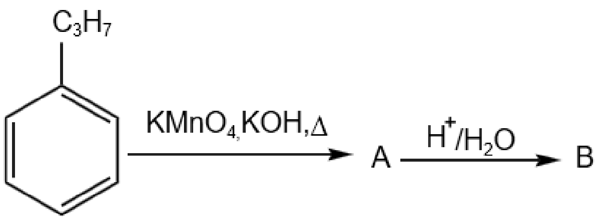
Q.33. One coulomb charge is passed through $AgNO_3$ solution during electrolysis. Find mass of silver (in mg) deposited at the electrode. Give answer in nearest integer.

Answer: 1

Solution:
$m=ZIt$
$q=It$
$\frac{\text{equivalent mass}}{\text{Faraday}}=\frac{\text{mass}}{n_{\text{factor}}\times 96500}$
$=\frac{108}{96500}\times 10^{-3}\ g$
$=\frac{108}{96.5}g=1.11g$

Hence, the answer is $1$

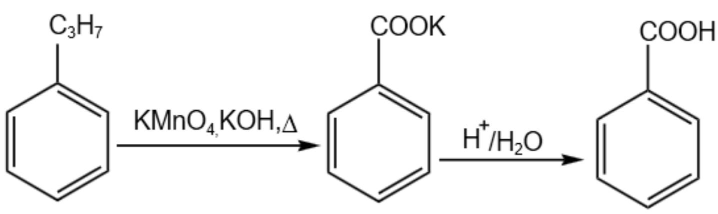
Q.34. Consider the following reaction




The number of pi bonds in product B is

Answer: 4

Solution: The propyl benzene on reaction with $KMnO_4/KOH$ give potassium benzoate. Potassium benzoate in acidic medium results benzoic acid.




The number of pi bonds in benzoic acid is 4.

Q.35. How many can give $H_2$ gas from dilute acid of $Ti^{2+},\ Cr^{2+},\ V^{2+},\ Mn^{2+},\ Fe^{2+},\ Co^{2+}$

Answer: 5

Solution: $Mn^{2+}: 3d^5$

It is in a stable state. Hence, it will not willing to liberate hydrogen gas.

$Ti^{2+},\ Cr^{2+},\ V^{2+},\ Fe^{2+},\ Co^{2+}$ will be able to liberate hydrogen gas.

Hence, the answer is $5$

Q.36. Find the coefficient of $x^0$ in the expansion of $\left(\frac{3^{\frac{1}{5}}}{x}+\frac{x}{5^{\frac{1}{3}}}\right)^{12}$

A) $^{12}C_7\left(\frac{3^{6/5}}{5^2}\right)$

B) $^{12}C_6\left(\frac{3^{6/5}}{5^2}\right)$

C) $^{12}C_6\left(\frac{3}{5^2}\right)$

D) $^{12}C_5\left(\frac{3^2}{5^2}\right)$

Answer: $^{12}C_6\left(\frac{3^{6/5}}{5^2}\right)$

Solution: Given,

Binomial expression $\left(\frac{3^{\frac{1}{5}}}{x}+\frac{x}{5^{\frac{1}{3}}}\right)^{12}$

Now, $T_{r+1}={}^{12}C_r\left(\frac{3^{\frac{1}{5}}}{x}\right)^{12-r}\left(\frac{x}{5^{\frac{1}{3}}}\right)^{r}$

$\Rightarrow T_{r+1}={}^{12}C_r\left(3^{\frac{1}{5}}\right)^{12-r}\left(5^{\frac{-1}{3}}\right)^{r}x^{2r-12}$

Now, for constant term we get, $r=6$

So, $T_7={}^{12}C_6\left(3^{1/5}\right)^{12-6}\left(5^{-1/3}\right)^{6}$

$\Rightarrow T_7={}^{12}C_6\left(\frac{3^{6/5}}{5^2}\right)$

Q.37. Find the minimum value of $k$ so that $4^{x+1}+4^{1-x},\ \frac{k}{2},\ 16^x+16^{-x}$ are in AP.

A) $5$

B) $16$

C) $10$

D) $12$

Answer: $10$

Solution: Given: $4^{x+1}+4^{1-x},\ \frac{k}{2},\ 16^x+16^{-x}$ are in AP.

$\Rightarrow 4^{x+1}+4^{1-x}+16^x+16^{-x}=\frac{k}{2}\times 2$

$\Rightarrow k=4\left(4^x+\frac{1}{4^x}\right)+\left(4^{2x}+\frac{1}{4^{2x}}\right)$

We know that, $AM\geq GM$

$\Rightarrow \frac{4^x+\frac{1}{4^x}}{2}\geq 4^x\times\frac{1}{4^x},\ \frac{4^{2x}+\frac{1}{4^{2x}}}{2}\geq 4^{2x}\times\frac{1}{4^{2x}}$

$\Rightarrow 4^x+\frac{1}{4^x}\geq 2,\ 4^{2x}+\frac{1}{4^{2x}}\geq 2$

$\Rightarrow k=4\,(2)+(2)$

$\Rightarrow k\geq 10$

Hence, the minimum value of $k$ is $10$.

Q.38. If $x\,|x|+2\,|x-5|+2x+3=0$. Find the value of $x$.

A) $-\sqrt{13}$

B) $\sqrt{13}$

C) $5$ D) $12$

Answer: $-\sqrt{13}$

Solution: Given: $x|x|+2|x-5|+2x+3=0$

Case-I: When $x \leq 0$

$\Rightarrow -x^2+10-2x+2x+3=0$

$\Rightarrow x^2=13$

$\Rightarrow x=-\sqrt{13}$

Case-II: When $0<x<5$

$\Rightarrow x^2+10-2x+2x+3=0$

$\Rightarrow x^2=-13$

So, no solution in this case.

Case-III: When $x \geq 5$

$\Rightarrow x^2+2x-10+2x+3=0$

$\Rightarrow x^2+4x-7=0$

$\Rightarrow x=\frac{-4 \pm \sqrt{16+28}}{2}$

$\Rightarrow x=\frac{-4 \pm 2\sqrt{11}}{2}$

So, no solution in this case.

Hence, $x=-\sqrt{13}$

Q.39. The $50^{th}$ word in the dictionary using the letters $B,\ B,\ H,\ J,\ O$

A) $JHBBO$ B) $OBBJH$

C) $BBJOH$ D) $OBBHJ$

Answer: $OBBJH$

Solution: Given,

Letters $B,\ B,\ H,\ J,\ O$

Now, word starting with $B$ we have,

$B\ \_\ \_\ \_\ \_ \rightarrow 4!=24$

Similarly, with $H$ we get,

$H\ \_\ \_\ \_\ \_ \rightarrow \frac{4!}{2}=12\ \{\text{as } 2\ B \text{ are repeating}\}$

And with $J$ we get,

$J\ \_\ \_\ \_\ \_ \rightarrow \frac{4!}{2}=12\ \{\text{as } 2\ B \text{ are repeating}\}$

So, $48$ words have already come, now $49^{th}$ and $50^{th}$ word will be $OBBHJ$ & $OBBJH$ respectively,

Hence, $OBBJH$ is required answer.

Q.40. Find the area bounded by $y=-2|x|$ and $y=x|x|$

A) $\frac{1}{2}$ B) $\frac{2}{3}$

C) $\frac{4}{3}$ D) $\frac{1}{3}$

Answer: $\frac{4}{3}$

**Solution:** Given: $y=-2|x|$ and $y=x|x|$

$\Rightarrow -2|x|=x|x|$

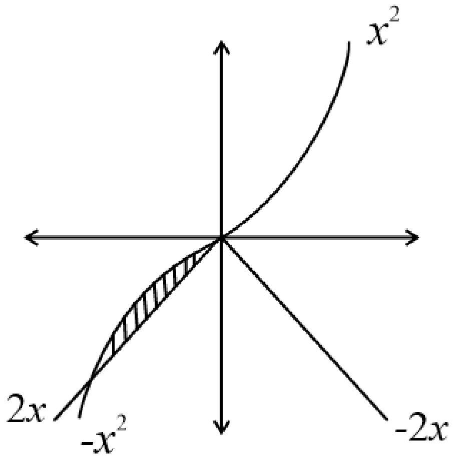
$\Rightarrow |x|(x+2)=0$

$\Rightarrow x=0,-2$

$\Rightarrow y=0,-4$

So, the points of intersection are $(0,0)$ and $(-2,-4)$.

$\Rightarrow y=\begin{cases}-2x & x\geq 0\\ 2x & x<0\end{cases}$ and $y=\begin{cases}x^2 & x\geq 0\\ -x^2 & x<0\end{cases}$




So, the required area is given by,

$A=\left|\int_{-2}^{0}\left(-x^2-(2x)\right)dx\right|$

$\Rightarrow A=\left|\left[\frac{-x^3}{3}-x^2\right]_{-2}^{0}\right|$

$\Rightarrow A=\left|\frac{8}{3}-4\right|$

$\Rightarrow A=\frac{4}{3}$ square units.

Q.41. $ABCD$ and $AEFG$ are two squares of side length $4$ and $2$ respectively. If $E$ lies on line segment $AB$ and $F$ lies on diagonal $AC$, find radius of circle passing through $F$ and touching $BC$ and $CD$

A) $4+2\sqrt{2}$

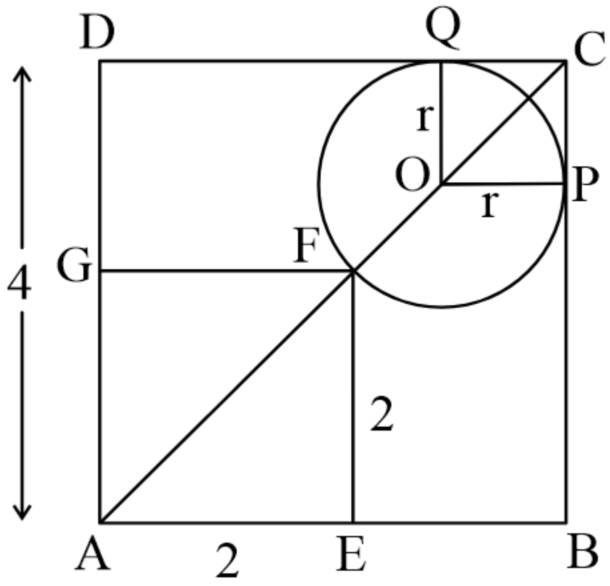
B) $4-2\sqrt{2}$

C) $2\sqrt{2}$

D) $\sqrt{2}$

**Answer:** $4-2\sqrt{2}$

Solution:




Using pythagoras theorem in $\triangle AEF$ and $\triangle ABC$

$AF = \sqrt{2^2+2^2} = 2\sqrt{2},\ AC = \sqrt{4^2+4^2} = 4\sqrt{2}$

Also, $OPCQ$ is a square with side $r$.

So, $OC = r\sqrt{2}$

$\Rightarrow AF + FO + OC = AC$

$\Rightarrow 2\sqrt{2} + r + r\sqrt{2} = 4\sqrt{2}$

$\Rightarrow r\left(1+\sqrt{2}\right) = 2\sqrt{2}$

$\Rightarrow r = \frac{2\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$\Rightarrow r = 4 - 2\sqrt{2}$

Q.42. If $f(x) = |x-1|,\ g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x < 0 \end{cases}$ then $f(g(x))$ is

A) One-one and onto

B) One-one but not onto

C) Onto but not one-one

D) Neither one-one nor onto

Answer: Onto but not one-one

Solution: Given,

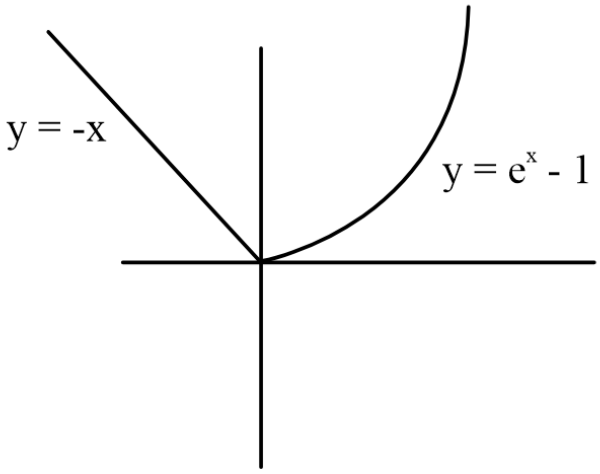
$f(x)=|x-1|,\ g(x)=\begin{cases} e^{x},\ x\geq 0 \\ x+1,\ x<0 \end{cases}$

Now, finding $f(g(x))=\begin{cases} g(x)-1,\ g(x)\geq 1 \\ 1-g(x),\ g(x)<1 \end{cases}$

$\Rightarrow f(g(x))=\begin{cases} e^{x}-1,\ x\geq 0 \\ 1-(1+x),\ x<0 \end{cases}$

$\Rightarrow f(g(x))=\begin{cases} e^{x}-1,\ x\geq 0 \\ -x,\ x<0 \end{cases}$

Plotting the diagram we get,




Now, range of the above function will be $[0,\infty)$

And also from above diagram function is not one-one but it is onto.

Q.43. If $A=\begin{bmatrix} \alpha & \alpha & \alpha \\ \beta & \alpha & -\beta \\ -\alpha & \alpha & \alpha \end{bmatrix}$ and $B=(adjA)^{T}$ then find the determinant of $AB$

A) $\alpha^{6}(\alpha+\beta)^{3}$

B) $8\alpha^{6}(\alpha+\beta)^{3}$

C) $8\alpha^{3}(\alpha+\beta)^{3}$

D) $27\alpha^{6}(\alpha+\beta)^{3}$

Answer: $8\alpha^{6}(\alpha+\beta)^{3}$

**Solution:** Given,

$$A=\begin{bmatrix}\alpha & \alpha & \alpha\\ \beta & \alpha & -\beta\\ -\alpha & \alpha & \alpha\end{bmatrix}$$

And $B=(adjA)^T$

Now, to find $|AB|=|A|\,|B|$

$\Rightarrow |AB|=|A|\left|(adjA)^T\right|$

$\Rightarrow |AB|=|A|\,|adjA|$

$\Rightarrow |AB|=|A||A|^2=|A|^3$

$\Rightarrow |AB|=\left(\alpha\left(\alpha^2+\alpha\beta\right)-\alpha(0)+\alpha\left(\alpha^2+\alpha\beta\right)\right)^3$

$\Rightarrow |AB|=\left(\alpha^3+\alpha^2\beta+\alpha^3+\alpha^2\beta\right)^3$

$\Rightarrow |AB|=\left(2\alpha^3+2\alpha^2\beta\right)^3$

$\Rightarrow |AB|=8\alpha^6(\alpha+\beta)^3$

Q.44. If $y=\frac{2\cos 2\theta+\cos\theta}{\cos 3\theta+\cos^2\theta+\cos\theta}$ then the value of $y''+y'+y$ is

A) $\sec^3\theta$

B) $\tan\theta+2\sec^2\theta$

C) $\sec\theta\left(\tan\theta+\sec^2\theta\right)$

D) $\sec\theta\left(\tan\theta+2\sec^2\theta\right)$

**Answer:** $\sec\theta\left(\tan\theta+2\sec^2\theta\right)$

**Solution:** Given: $y=\frac{2\cos 2\theta+\cos\theta}{\cos 3\theta+\cos^2\theta+\cos\theta}$

We know that, $\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\Rightarrow y=\frac{2\cos 2\theta+\cos\theta}{2\cos 2\theta\cos\theta+\cos^2\theta}$

$\Rightarrow y=\frac{2\cos 2\theta+\cos\theta}{\cos\theta(2\cos 2\theta+\cos\theta)}$

$\Rightarrow y=\sec\theta$

$\Rightarrow y'=\sec\theta\tan\theta$

$\Rightarrow y''=\sec\theta\times\sec^2\theta+\tan^2\theta\sec\theta$

$\Rightarrow y''+y'+y=\sec\theta\tan\theta+\sec\theta\times\sec^2\theta+\tan^2\theta\sec\theta+\sec\theta$

$\Rightarrow y''+y'+y=\sec\theta\tan\theta+\sec\theta\times\sec^2\theta+\sec\theta\left(\tan^2\theta+1\right)$

$\Rightarrow y''+y'+y=\sec\theta\tan\theta+\sec^3\theta+\sec^3\theta$

$\Rightarrow y''+y'+y=\sec\theta\tan\theta+2\sec^3\theta$

$\Rightarrow y''+y'+y=\sec\theta\left(\tan\theta+2\sec^2\theta\right)$

Q.45. Consider a equation $P(x)=ax^2+bx+c=0$, if $a,\ b,\ c\in A$ where $A=\{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ then find the probability that $P(x)$ has real and distinct roots ?

A) $\frac{1}{4}$

B) $\frac{25}{108}$

C) $\frac{19}{108}$

D) $\frac{1}{16}$

**Answer:** $\frac{19}{108}$

Solution: Given,

Equation $P(x) = ax^2 + bx + c = 0$,

Now, for real and distinct roots $b^2 > 4ac$ where $a, b, c \in A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$

Now, for $b = 1\ \&\ 2$ there will be no cases,

For $b = 3$ we get, $ac = \{(1,1),\ (1,2),\ (2,1)\} \rightarrow 3 \text{ ways}$

For $b = 4$ we get, $ac = \{(1,1),\ (1,2),\ (2,1),\ (1,3),\ (3,1)\} \rightarrow 5 \text{ ways}$

For $b = 5$ we get,

$ac = \{(1,1),\ (1,2),\ (2,1),\ (1,3),\ (3,1), (2,2),\ (1,4),\ (1,4),\ (2,3),\ (3,2), (5,1),\ (1,5),\ (1,6),\ (6,1)\} \rightarrow 14 \text{ ways}$

And for $b = 6$ we get,

$ac = \{(1,1),\ (1,2),\ (2,1),\ (1,3),\ (3,1), (2,2),\ (1,4),\ (1,4),\ (2,3),\ (3,2), (5,1),\ (1,5),\ (1,6),\ (6,1), (2,4),\ (4,2)\} \rightarrow 16 \text{ ways}$

So, total possible ways are $3 + 5 + 14 + 16 = 38$

Hence, probability is given by $\frac{38}{6^3} = \frac{19}{108}$

Q.46.

If $2x^2 - x + 2 = 0$ and its one root is $a$, then $\lim_{x \to \frac{1}{a}} \frac{16\left(1-\cos\left(2x^2-x+2\right)\right)}{(ax-1)^2}$ is equal to:

A) $\frac{20\left(1-a^2\right)^2}{a^3}$

B) $\frac{8\left(1-a^2\right)^2}{a^3}$

C) $\frac{32\left(1-a^2\right)^2}{a^4}$

D) $\frac{16\left(1-a^2\right)^2}{a^4}$

Answer: $\frac{32\left(1-a^2\right)^2}{a^4}$

Solution: Given: $2x^2 - x + 2 = 0$ and one of its roots is $a$.

Let the other root be $\alpha$

$\Rightarrow a \times \alpha = \frac{2}{2}$

$\Rightarrow \alpha = \frac{1}{a}$

Let, $y = \lim_{x \to \frac{1}{a}} \frac{16\left(1-\cos\left(2x^2-x+2\right)\right)}{(ax-1)^2}$

$\Rightarrow y = \lim_{x \to \frac{1}{a}} \frac{16\left(1-\cos\left(2(x-a)\left(x-\frac{1}{a}\right)\right)\right)}{(ax-1)^2}$

$\Rightarrow y = \lim_{x \to \frac{1}{a}} \frac{16\left(1-\cos\left(2(x-a)\left(x-\frac{1}{a}\right)\right)\right)}{a^2\left(x-\frac{1}{a}\right)^2} \times \frac{4(x-a)^2}{4(x-a)^2}$

We know that, $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

$\Rightarrow y = \lim_{x \to \frac{1}{a}} \frac{1-\cos\left(2(x-a)\left(x-\frac{1}{a}\right)\right)}{\left[2(x-a)\left(x-\frac{1}{a}\right)\right]^2} \times \frac{64(x-a)^2}{a^2}$

$\Rightarrow y = \lim_{x \to \frac{1}{a}} \frac{32(x-a)^2}{a^2}$

$\Rightarrow y = \frac{32}{a^2}\left(\frac{1}{a} - a\right)^2$

$\Rightarrow y = \frac{32\left(1-a^2\right)^2}{a^4}$

Q.47. $\beta(m, n) = \int_0^1 x^m (1 - x^m)^{n-1} dx$ and $a \times \beta(-b, c) = \int_0^1 \left(1 - x^{10}\right)^{20} dx$, then $(a + b + c)$ is equal to

A) $210$ B) $250$

C) $270$ D) $230$

Answer: $210$

Solution: Let, $I = \int_0^1 \left(1 - x^{10}\right)^{20} dx$

$\Rightarrow I = \left[x\left(1 - x^{10}\right)^{20}\right]_0^1 - \int_0^1 20\left(1 - x^{10}\right)^{19}\left(-10x^9\right) x dx$

$\Rightarrow I = 200 \int_0^1 x^{10}\left(1 - x^{10}\right)^{19} dx$

$\Rightarrow a \times \beta(-b, c) = 200 \int_0^1 x^{10}\left(1 - x^{10}\right)^{19} dx$

Comparing it with $\beta(m, n) = \int_0^1 x^m (1 - x^m)^{n-1} dx$

$\Rightarrow a = 200,\ n - 1 = 19,\ \ -b = 10$

$\Rightarrow a = 200,\ c = n = 20,\ b = -10$

$\Rightarrow a + b + c = 200 + 20 - 10$

$\Rightarrow a + b + c = 210$

Q.48. If the image of the point $(8, 5, 7)$ with respect to line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$ is $(\alpha,\ \beta,\ \gamma)$ then find the value of $\alpha + \beta + \gamma$

**Answer:** 14

**Solution:** Given,

Equation of line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{5}$

Now, any point on the above line is given by,

$Q\left(2r+1,\ 3r-1,\ 5r+2\right)$

Now, the direction ratio of line from point $P(8,5,7)$ to point $Q\left(2r+1,\ 3r-1,\ 5r+2\right)$ is given by $\left(2r-7,\ 3r-6,\ 5r-5\right)$

Now, using the perpendicular condition as $PQ$ is perpendicular to the given line we get,

$2\left(2r-7\right)+3\left(3r-6\right)+5\left(5r-5\right)=0$

$\Rightarrow 38r=57$

$\Rightarrow r=\frac{3}{2}$

So, point $Q\left(2r+1,\ 3r-1,\ 5r+2\right)=Q\left(4,\ \frac{7}{2},\ \frac{19}{2}\right)$

Now, using the midpoint formula to find the image $P'\left(\alpha,\ \beta,\ \gamma\right)$ we get,

$\frac{\alpha+8}{2}=4,\ \frac{\beta+5}{2}=\frac{7}{2},\ \frac{\gamma+7}{2}=\frac{19}{2}$

$\Rightarrow \alpha=0,\ \beta=2,\ \gamma=12$

$\Rightarrow \alpha+\beta+\gamma=14$

Q.49. If line $L$ is perpendicular to $y=2x+10$ such that it touches the parabola $y^2=4\left(x-9\right)$ then find the square of distance between the point of contract and origin

**Answer:** 185

**Solution:** Given,

A line $L$ is perpendicular to $y=2x+10$ such that it touches the parabola $y^2=4\left(x-9\right)$,

So, let the line $2y+x=c$ be line perpendicular to the given line,

Now, given $2y+x=c$ touches the parabola $y^2=4\left(x-9\right)$

So, $\left(\frac{c-x}{2}\right)^2=4\left(x-9\right)$

$\Rightarrow x^2-2\left(c+8\right)x+c^2+144=0$

Making discriminant $D=0$ we get,

$\Rightarrow 4(c+8)^2-4\left(c^2+144\right)=0$

$\Rightarrow c=5$

So, the equation of line will be $2y+x=5$ and point of contact will be $(13,-4)$

Hence, distance of point of point $(13,-4)$ from origin is $\sqrt{169+16}=\sqrt{185}$.

Q.50. If $\left|\vec{a}\right|=2,\ \left|\vec{b}\right|=3$ & $\vec{a}=\vec{b}\times\vec{c}$, then find the minimum value of $9\left|\vec{c}-\vec{a}\right|^2$

**Answer:** 40

Solution: Given,

$\left|\vec{a}\right| = 2,\ \left|\vec{b}\right| = 3,$

And $\vec{a} = \vec{b} \times \vec{c}$

$\Rightarrow \vec{a} \cdot \vec{b} = 0\ \&\ \vec{a} \cdot \vec{c} = 0$

Now, solving $\left|\vec{c} - \vec{a}\right|^2 = \left|\vec{c}\right|^2 + \left|\vec{a}\right|^2 + 2\vec{c} \cdot \vec{a}$

$\Rightarrow \left|\vec{c} - \vec{a}\right|^2 = \left|\vec{c}\right|^2 + 4 \ldots\ldots\ldots (i)$

Now, solving $\left|\vec{a}\right| = \left|\vec{b} \times \vec{c}\right|$

$\Rightarrow \left|\vec{a}\right| = \left|\vec{b}\right|\left|\vec{c}\right| \sin\theta$

$\Rightarrow \left|\vec{c}\right| = \frac{2}{3\sin\theta}$

$\Rightarrow \left|\vec{c}\right|^2 = \frac{4}{9\sin^2\theta} \ldots\ldots (ii)$

From equation $(i)\ \&\ (ii)$ we get,

$\left|\vec{c} - \vec{a}\right|^2 = \frac{4}{9\sin^2\theta} + 4$

$\Rightarrow \left|\vec{c} - \vec{a}\right|^2 \geq \frac{4}{9} + 4$

$\Rightarrow \left|\vec{c} - \vec{a}\right|^2 \geq \frac{40}{9}$

$\Rightarrow 9\left|\vec{c} - \vec{a}\right|^2 \geq 40$