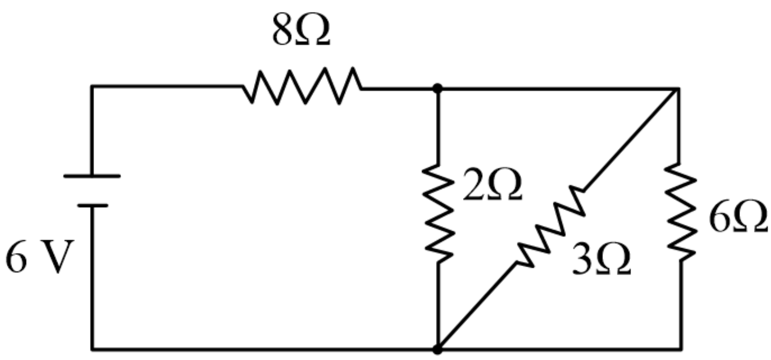
JEE Main 5th April Session 1 - Memory Based Paper

5th April Session 1

Questions

Q.1. Find the current in the following circuit in ampere unit.



A) 2 B) 0.6

C) 6 D) 3

Answer: 0.6

Solution: From the given diagram, it can be concluded that the resistances $2\ \Omega,\ 3\ \Omega,\ 6\ \Omega$ are in parallel combination with each other.

So, their equivalent resistance (R) can be calculated as follows:

$$\frac{1}{R}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$$
$$=\frac{6}{6}$$
$$=1$$
$$\Rightarrow R=1\ \Omega$$

Hence, the net equivalent resistance (R_e) of the entire circuit can be written as

$$R_e=8\ \Omega+1\ \Omega$$
$$=9\ \Omega$$

Thus, the current through the circuit is given by

$$I=\frac{6\ \mathrm{V}}{9\ \Omega}$$
$$\approx 0.6\ \mathrm{A}$$

Q.2. Two wires, A and B have the same length and same material. Cross-sectional radii of wire A and B are $2\ \mathrm{mm}$ and $4\ \mathrm{mm}$ respectively. If the resistance of wire B is $2\ \Omega$, then what is the resistance of wire A?

A) $8\ \Omega$ B) $1\ \Omega$

C) $4\ \Omega$ D) $16\ \Omega$

Answer: $8\ \Omega$

Solution: The resistance of a wire can be written as

$$R = \rho\frac{l}{A}$$
$$= \rho\frac{l}{\pi r^2} \quad \ldots(1)$$

Hence, for both the given wires, it follows that

$$\frac{R_A}{R_B} = \frac{\rho\frac{l}{\pi r_A{}^2}}{\rho\frac{l}{\pi r_B{}^2}}$$
$$= \frac{r_B{}^2}{r_A{}^2} \quad \ldots(2)$$

Substituting the values into equation (2), we have

$$\frac{R_A}{R_B} = \frac{4^2}{2^2}$$
$$= 4$$
$$R_A = 4 \times 2\ \Omega$$
$$= 8\ \Omega$$

Q.3. Match the following columns:

A. Escape velocity	P. $\sqrt{gh}$
B. Orbital Velocity	Q. $\sqrt{2gh}$
C. Gravitational Potential energy	R. $-\frac{GM_1M_2}{r}$

A) $A \rightarrow P,\ B \rightarrow Q,\ C \rightarrow R$

B) $A \rightarrow Q,\ B \rightarrow R,\ C \rightarrow P$

C) $A \rightarrow Q,\ B \rightarrow P,\ C \rightarrow R$

D) $A \rightarrow R,\ B \rightarrow P,\ C \rightarrow Q$

Answer: $A \rightarrow Q,\ B \rightarrow P,\ C \rightarrow R$

Solution: Escape velocity or escape speed is the minimum speed needed for an object to escape from the gravity of Earth. It depends on the acceleration due to gravity and the height from the surface of the Earth. Mathematically,

$$v_e = \sqrt{2gh}$$

Orbital speed depends on the gravitational pull and the distance of the object. It can be expressed as

$$v_o = \sqrt{gh}$$

Also, the gravitational potential energy between two objects can be written as

$$U = -\frac{GM_1M_2}{r}$$

Hence, this is the correct option.

Q.4. The ratio of radius of gyration of the uniform hollow sphere and the uniform solid sphere about its diameter is (both have the same radius)

A) $\sqrt{\frac{5}{3}}$

B) $\sqrt{\frac{3}{2}}$

C) $\sqrt{\frac{4}{3}}$

D) $\sqrt{\frac{3}{5}}$

Answer: $\sqrt{\frac{5}{3}}$

Solution: The moment of inertia of a solid sphere is

$I_S = \frac{2}{5} M R^2 \quad \ldots(1)$

And, the same for a hollow sphere is

$I_h = \frac{2}{3} M R^2 \quad \ldots(2)$

Hence, the ratio of radius of gyration of the two spheres can be calculated as follows:

$$\frac{K_h}{K_s} = \frac{\sqrt{\frac{I_h}{M_h}}}{\sqrt{\frac{I_S}{M_S}}}$$

$$= \frac{\sqrt{\frac{2}{3} R^2}}{\sqrt{\frac{2}{5} R^2}}$$

$$= \sqrt{\frac{5}{3}}$$

Q.5. If the time period of a pendulum at height R (where R is radius of earth) from surface of earth is T_1, and at height $2R$ it is T_2, then

A) $2T_1 = 3T_2$

B) $T_1 = 3T_2$

C) $3T_1 = 2T_2$

D) $3T_1 = 4T_2$

Answer: $3T_1 = 2T_2$

Solution: The value of acceleration due to gravity changes with the height from the surface of the Earth as follows:

$g' = \frac{GM}{(R+h)^2} \quad \ldots(1)$

Hence, for $h = R$,

$g_R = \frac{GM}{4R^2} \quad \ldots(2)$

And, for $h = 2R$,

$g_{2R} = \frac{GM}{9R^2} \quad \ldots(3)$

Hence, the relation between the time periods can be calculated as follows:

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{l}{g_R}}}{2\pi\sqrt{\frac{l}{g_{2R}}}}$$

$$= \sqrt{\frac{g_{2R}}{g_R}}$$

$$= \sqrt{\frac{\frac{GM}{9R^2}}{\frac{GM}{4R^2}}}$$

$$= \frac{2}{3}$$

$$\Rightarrow 3T_1 = 2T_2$$

Q.6. A point source of light is placed at focus of convex lens, then what is the shape of wavefront after passing through the lens

A) planar

B) elliptical

C) spherical

D) cynlindrical

Answer: planar

Solution: Wavefronts are imaginary surfaces that represent the spatial distribution of a wave's amplitude at a given instant in time. They are used to visualize and understand the propagation of waves in various mediums, such as sound waves, light waves, and water waves.

When a light source is kept at the focus, the light beams are parallel to the principal axis, after passing through the lens.

Hence, the nature of the wavefronts will be planar.

Q.7. Find the dimension of $\sqrt{G\mu}$, where G is the gravitational constant and μ is the energy gradient.

A) $\left[\mathrm{LT}^{-3}\right]$

B) $\left[\mathrm{LT}^{-1}\right]$

C) $\left[\mathrm{LT}^{-2}\right]$

D) $\left[L^{2}T^{-2}\right]$

Answer: $\left[L^{2}T^{-2}\right]$

Solution: The SI unit of the quantity $G\mu$ can be expressed as follows:

$$\text{Unit of } G\mu = \frac{\mathrm{Nm}^2}{\mathrm{kg}^2} \times \frac{\mathrm{J}}{\mathrm{m}}$$
$$= \frac{\mathrm{kg\ m\,s}^{-2} \times \mathrm{m}^2}{\mathrm{kg}^2} \times \frac{\mathrm{kg\ m}^2\,\mathrm{s}^{-2}}{\mathrm{m}}$$
$$= \frac{\mathrm{m}^4}{\mathrm{s}^4}$$

Hence, using dimensional analysis, it follows that

$$[G\mu] = \left[\frac{\mathrm{L}^4}{\mathrm{T}^4}\right]$$
$$\Rightarrow \left[\sqrt{G\mu}\right] = \left[\mathrm{L}^2\mathrm{T}^{-2}\right]$$

Q.8. The correct relation between kinetic energy $(\mathrm{K.E.})$ and the total energy $(\mathrm{T.E.})$ of a satellite orbiting around a planet is

A) $\mathrm{K.E.} = |\mathrm{T.E.}|$

B) $\mathrm{K.E.} = 2|\mathrm{T.E.}|$

C) $3\,\mathrm{K.E.} = |\mathrm{T.E.}|$

D) $\mathrm{K.E.} = \frac{|\mathrm{T.E.}|}{2}$

Answer: $\mathrm{K.E.} = |\mathrm{T.E.}|$

Solution: The gravitational potential energy between the Earth and the satellite can be given by the formula:

$$\mathrm{P.E.} = -\frac{GMm}{r} \quad \ldots(1)$$

And, the formula for the kinetic energy is given by

$$\mathrm{K.E.} = \frac{1}{2}\frac{GMm}{r} \quad \ldots(2)$$

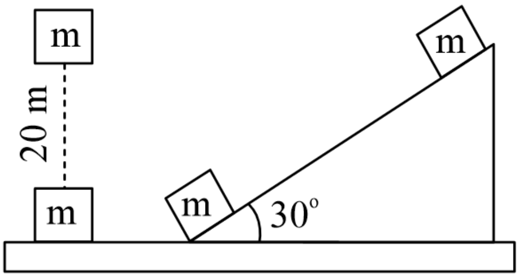
Thus, the total energy can be calculated as

$$\mathrm{T.E.} = \mathrm{K.E.} + \mathrm{P.E.}$$
$$= \frac{1}{2}\frac{GMm}{r} + \left(-\frac{GMm}{r}\right)$$
$$= -\frac{GMm}{2r} \quad \ldots(3)$$

From equations (2) and (3), it can be concluded that

$\mathrm{K.E.} = |\mathrm{T.E.}|$

Q.9. A block of mass $m = 50\ \mathrm{kg}$ is lifted from ground to a height of $20\ \mathrm{m}$ in two different ways as shown in the figure. Find the ratio of work done by gravity in both cases.



A) $1:2$

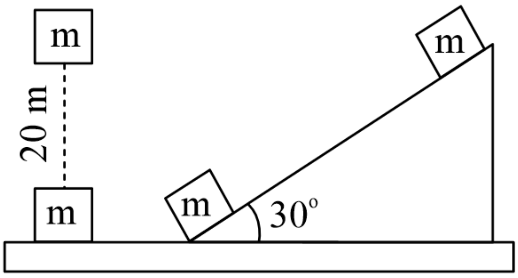
B) $2:1$

C) $1:1$

D) $1:5$

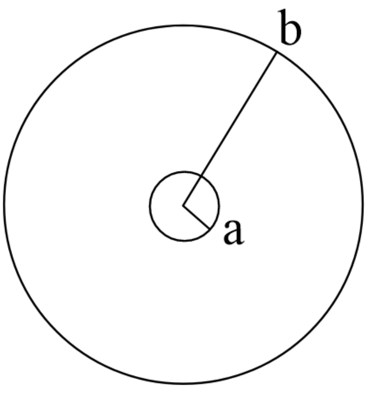
Answer: $1:1$

Solution:



Work done by gravity depends only on the height covered. Since in both cases, height covered is the same therefore work done in both cases will be the same and hence ratio will be $1:1$.

Q.10. Two concentric conducting rings of radius a and b are placed as shown in diagram $(a \ll b)$. Find coefficient of mutual inductance of rings



A) $\frac{\mu_0 a^2}{2b}$

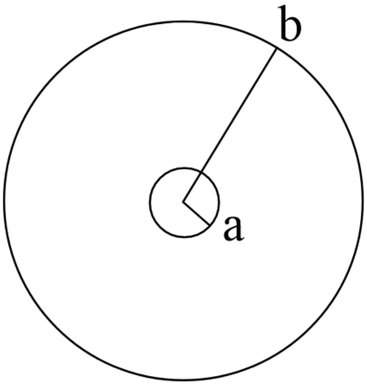
B) $\frac{\mu_0 a^3}{2\pi b^2}$

C) $\frac{\mu_0 \pi b^2}{2a}$

D) $\frac{\mu_0 \pi a^2}{2b}$

Answer: $\frac{\mu_0 \pi a^2}{2b}$

Solution:



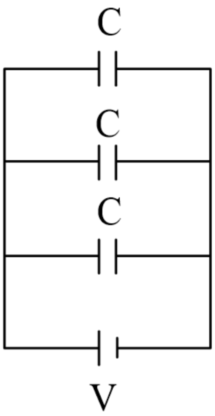
Let the current flows through the large ring in clock-wise direction, then magnetic field due to large ring at the common centre will be inside the plane of the paper. Also $B_{large} = \frac{\mu_0 i}{2b}$.

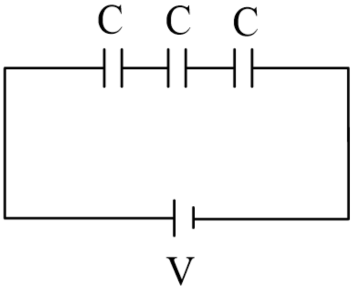
Now, magnetic flux passing through the small ring due to magnetic field of the large ring will be,
$\phi = B_{large} \times \text{area of small ring}$
$\Rightarrow \phi = \frac{\mu_0 i}{2b} \times \pi a^2 = \left(\frac{\mu_0}{2b} \times \pi a^2\right) i$

Now, magnetic flux in terms of mutual inductance can be written as, $\phi = Mi$.

By comparing we get, $M = \left(\frac{\mu_0 \pi a^2}{2b}\right)$

Q.11. Find the ratio of energy stored in the two given capacitor systems.





A) $1:1$ B) $2:1$

C) $4:1$ D) $9:1$

Answer: $9:1$

Solution: In the left circuit, all capacitors are connected in parallel, therefore equivalent capacitance will be, $C_{eq,1} = 3C$.

Hence, energy stored $E_1 = \frac{1}{2}(3C)V^2$

Now, in the right circuit, all capacitors are connected in series, therefore equivalent capacitance will become $C_{eq,2} = \frac{C}{3}$

Hence, energy stored will be $E_2 = \frac{1}{2}\left(\frac{C}{3}\right)V^2$

Therefore, required ratio will be

$$\frac{E_1}{E_2} = 9:1$$

Q.12. Three helium atoms form carbon at high temperature due to fusion. Masses of helium and carbon nuclei in a.m.u are 4.0002 and 12 respectively. Find energy released in the process.

A) $0.56\ \text{MeV}$

B) $0.10\ \text{MeV}$

C) $0.18\ \text{MeV}$

D) $21.3\ \text{keV}$

Answer: $0.56\ \text{MeV}$

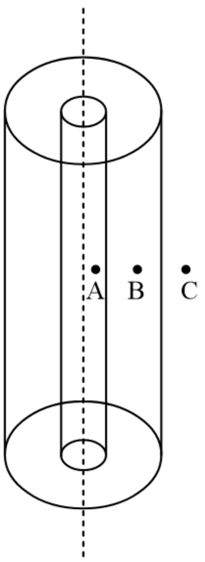
Solution: The formula to calculate the Q-value of the given nuclear process can be written as

$$Q = \left(3 \times M_{He} - M_C\right) \times 931\ \text{MeV} \quad \ldots(1)$$

Substituting the values of the known parameters into equation (1), we have

$$\begin{aligned} Q &= (3 \times 4.0002 - 12) \times 931\ \text{MeV} \\ &\approx 0.56\ \text{MeV} \end{aligned}$$

Q.13. The figure shows two long, co-axial cylindrical cables, carrying the same current along their walls, in opposite directions. The magnetic field will be zero at



A) A and B

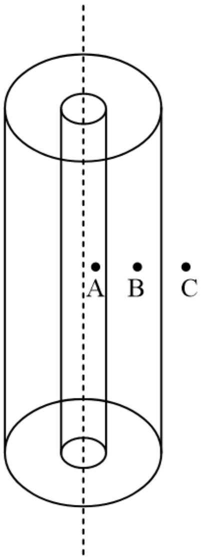
B) A and C

C) B and C

D) None of these

Answer: A and C

Solution:



If we consider an Amperian loop inside the smaller cylindrical cable, then current flowing through that loop would be zero and hence the magnetic field will be zero.

If we consider an Amperian loop in the region C, then as current in both cylindrical shells are flowing in opposite directions, net current will be zero and hence magnetic field will be zero.

For region B, magnetic will be present due to current flowing through the smaller cylindrical shell in the Amperian loop.

Q.14. An electron and a proton are placed at a certain distance apart. The ratio of the coulombic force to gravitational force between them is of the order

A) 10^{32}

B) 10^{42}

C) 10^{39}

D) 10^{36}

Answer: 10^{39}

Solution: The formula to calculate the Coulomb's force between the electron and the proton is given by

$$F_c = k\frac{e^2}{r^2} \quad \ldots(1)$$

And, the gravitational force between them is given by

$$F_G = G\frac{m_e m_p}{r^2} \quad \ldots(2)$$

Hence, the required ratio can be calculated as follows:

$$\frac{F_C}{F_G} = \frac{ke^2}{Gm_e m_p}$$

$$= \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{6.67\times10^{-11}\times9.1\times10^{-31}\times1.67\times10^{-27}}$$

$$\approx 10^{39}$$

Q.15. A particle is moving in a straight line with constant acceleration with zero initial velocity. If the ratio of distance travelled by the particle in $\left(n-1\right)^{th}$ sec to that in n^{th} sec, where $n=10$ is $\frac{A}{B}$, then $(A+B)$ is

Answer: 36

Solution: Given: $u = 0$.

Distance travelled by particle in n^{th} second is given by, $S_n = u + \frac{a}{2}[2n - 1] = \frac{a}{2}[2n - 1]$

Now, distance travelled in $(n-1)^{th}$ second can be written as, $S_{n-1} = u + \frac{a}{2}[2(n-1) - 1] = u + \frac{a}{2}[2n - 3] = \frac{a}{2}[2n - 3]$

Therefore, we can write

$\frac{S_{n-1}}{S_n} = \frac{(2\times10-1)}{(2\times10-3)} = \frac{19}{17}$

Hence, $A + B = 19 + 17 = 36$

Q.16. Number of σ and π bonds in ethylene:

A) $5\ \sigma\ and\ 1\pi$ B) $6\ \sigma\ and\ 1\ \pi$

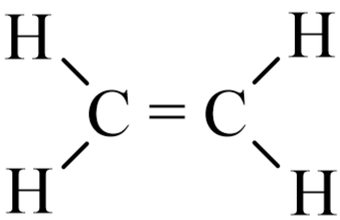
C) $4\ \sigma\ and\ 2\ \pi$ D) $4\ \sigma\ and\ 1\ \pi$

Answer: $5\ \sigma\ and\ 1\pi$

Solution: A sigma(σ) bond is formed by the overlapping of half-filled atomic orbitals of two atoms along their internuclear axis.

A pi (π) bond is formed by the lateral or sidewise overlapping of half-filled atomic orbitals present in the valence shells of two atoms.

The shape of ethylene(C_2H_4) molecule is:



The compound consists of 6 bonds. The bonds are $4\ C-H$ sigma bonds, $1\ C-C$ sigma bond and a pi bond between carbon and carbon.

Q.17. Which transition metal exhibits the highest range of oxidation states, including the maximum observed oxidation state?

A) Mn B) Fe

C) Co D) Cr

Answer: Mn

Solution: The oxidation state, or oxidation number, is the hypothetical charge of an atom if all of its bonds to other atoms were fully ionic.

Manganese shows $+2,\ +3,\ +4,\ +5\ ,\ +6,\ +7$

Hence, $+7$ is the highest oxidation state exhibited by Manganese.

Q.18. Consider the reaction:
$Fe_2O_3(s) + 3CO(g) \leftrightharpoons 2Fe(s) + 3CO_2(g)$
Which of the following will not affect the equilibrium state:
(I) Addition of Fe_2O_3 (II) Addition of CO_2
(III) Decreasing mass of Fe_2O_3 (IV) Removal of CO

A) (II) and (IV) B) (I) and (IV)

C) (I) and (III) D) All will affect the equilibrium

Answer: (I) and (III)

Solution: $Fe_2O_3(s) + 3CO(g) \leftrightharpoons 2Fe(s) + 3CO_2(g)$

The active mass of solid substances is considered to be 1. Solid species in equilibrium will not affect equilibrium. In the above equilibrium Fe_2O_3 and Fe are solid substances, so they will not affect equilibrium.

Q.19. Assertion: Trans But-2-ene is less polar than cis But-2-ene.

Reason: Trans But-2-ene has zero dipole moment.

A) Both assertion and reason are correct and reason is the correct explanation for the assertion.

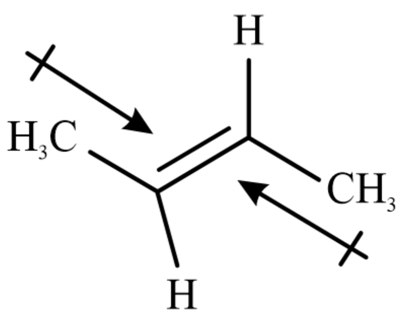
B) Both assertion and reason are correct and reason is not the correct explanation for the assertion.

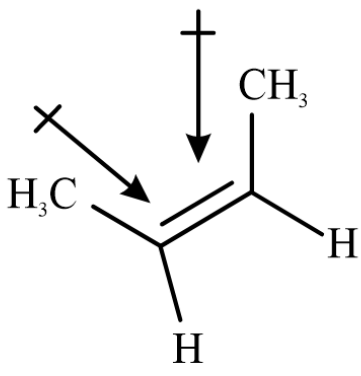
C) Assertion is true and reason is false

D) Assertion is false and reason is true

Answer: Both assertion and reason are correct and reason is the correct explanation for the assertion.

Solution: Trans isomer has no or less dipole moment than cis isomer due to the cancellation of opposite bond moments in case of but-2-ene.
Cis isomer has more dipole moment than trans isomer, because it has two similar groups on same side of double bond. So the dipole gets added, thus cis isomer is more polar than trans.
In the trans isomer, the terminal groups are on the opposite sides of the double bond. So, here the dipole moment of the trans isomer is zero.





Hence, the answer is A.

Q.20. Arrange the following according to the strength of the ligands:

Br^{-}, F^{-}, H_2O, NH_3

A) $F^{-} < Br^{-} < H_2O < NH_3$ B) $Br^{-} < F^{-} < H_2O < NH_3$

C) $Br^{-} < F^{-} < NH_3 < H_2O$ D) None of the above

Answer: $Br^{-} < F^{-} < H_2O < NH_3$

Solution: A spectrochemical series represents an ordered list of ligands based on their strength, as experimentally determined by the interaction between ligands and metal ions.

The strength of a ligand depends upon the manner in which electrons fill the orbitals of an atom. Each atom possesses a certain number of electrons, or negatively charged particles, distributed in an ordered manner amongst the subshells surrounding each atom.

In the spectrochemical series the field strength of ligands order is

Carbon based ligands > Nitrogen ligands>Oxygen ligands > Halogen ligands

The correct answer is option B.

Q.21. Assertion: For the 13^{th} group, the $+1$ oxidation state increases down the group.

Reason: The atomic size of Ga is greater than that of Al.

A) Both assertion and reason are correct and reasopn is correct explanation for assertion

B) Both assertion and reason are correct and reasopn is not the correct explanation for assertion

C) Assertion is true but reason is false

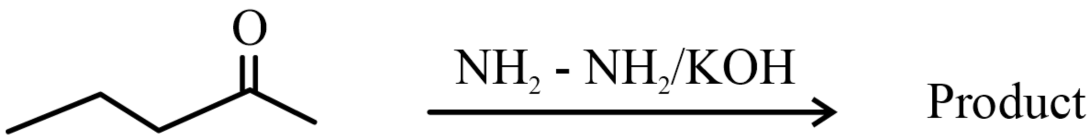
D) Assertion is false and reason is true

Answer: Assertion is true but reason is false

Solution: The stability of $+1$ oxidation state progressively increases for the heavier element of group 13 due to inert pair effect in which on moving down the group due to poor shielding effect of intervening d and f orbitals the increased effective nuclear charge holds ns electrons tightly.

Gallium has smaller atomic radius than aluminium because the d orbital which is highly diffused offers poor shielding effect which results in increased nuclear charge. Due to increased charge by nucleus, outer electrons get attracted, decreasing the radius.

Q.22.



The product will be:

A) Alcohol

B) Alkane

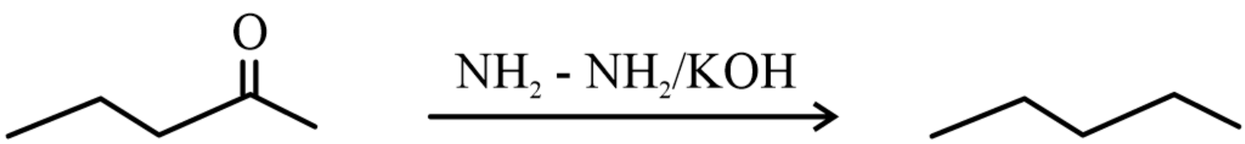
C) Alkene

D) Ester

Answer: Alkane

Solution: The Wolff-Kishner Reduction is a method to reduce aldehydes and ketones into alkanes. It is used explicitly for aldehydes and ketones attached to a benzene ring. The reactants of the Wolff-Kishner reaction include the aldehyde or ketone to be reduced, hydrazine, and a base (with heat).

Wolff-Kishner reduction mechanism begins with the formation of a hydrazone anion which then releases the nitrogen molecule to form a carbanion. This carbanion then reacts with the water in the system to give a hydrocarbon.



Hence, the answer is option B.

Q.23. Which postulates of the Dalton's theroy is wrong?

A) Matter consists of indivisible atoms

B) All atoms of a given element have identical properties but different masses.

C) Compounds are formed when atoms of different elements combine in a fixed ratio.

D) Chemical reaction involves reorganisation of atoms.

Answer: All atoms of a given element have identical properties but different masses.

Solution: Postulates of Dalton's Atomic Theory

- All matter is made up of tiny, indivisible particles called atoms.
- All atoms of a specific element are identical in mass, size, and other properties. However, atoms of different element exhibit different properties and vary in mass and size.
- Atoms can neither be created nor destroyed. Furthermore, atoms cannot be divided into smaller particles.
- Atoms of different elements can combine with each other in fixed whole-number ratios in order to form compounds.
- Atoms can be rearranged, combined, or separated in chemical reactions.
- Hence, the answer is option B.

Q.24. If the number of neutrons in the most abundant isotope of boron X and its highest oxidation state in unsaturated compound is Y then find the value of $X+Y$.

A) 6

B) 9

C) 3

D) 4

Answer: 9

Solution: The most abundant isotope of boron is ${}^{11}_{5}B$. The number of neutrons is equal to difference in mass number and atomic number. So the number of neutrons in the most abundant isotope of boron is $11-5=6\,(X)$. Boron exhibits the highest oxidation state $+3\,(Y)$ in any compound. Hence, $X+Y=9$

Q.25. Arrange the following carbocations in the increasing order of stability

$I)\ (CH_3)_2CH^+ \quad II)\ CH_3^+ \quad III)\ (CH_3)_3C^+ \quad IV)\ CH_3-CH_2^+$

A) $I < II < III < IV$ B) $III < I < IV < II$

C) $II < IV < I < III$ D) $II < III < I < IV$

Answer: $II < IV < I < III$

Solution: The stability of given carbocations can be explained by hyperconjugation effect. More the number of hyperconjugation structures, more is the stability of carbocation. So, as the number of alpha hydrogen increase, the stability of carbocation increases. So, the order of the stability of carbocation is:

$$CH_3{}^+ < CH_3-CH_2{}^+ < (CH_3)_2CH^+ < (CH_3)_3C^+$$

$\alpha-H$ 0 3 6 9

Q.26. Ninhydrin test is used for:

A) Starch B) Cellulose

C) PVC D) Egg albumin

Answer: Egg albumin

Solution: The ninhydrin test is a chemical test useful to identify ammonia, primary/secondary amines, or amino acids. In this test, ninhydrin reagent is added to the test material, resulting in the production of deep blue colour, also known as Ruhemann's purple, in the presence of an amino group.

Egg albumin, also known as egg white, is primarily composed of proteins. It serves as a significant source of essential amino acids.

Hence, the answer is D.

Q.27. In which of the following compounds Mn exhibit the highest oxidation state?

A) MnO_4^- B) MnO_4^{2-}

C) Mn_2O_3 D) MnO_2

Answer: MnO_4^-

Solution: The sum of the oxidation states of all the atoms in a species is equal to the charge present on that species.

Oxygen exhibits -2 oxidation state in oxyanions.

The oxidation state of manganese in MnO_4^- is $+7$.

The oxidation state of manganese in MnO_4^{2-} is $+6$

The oxidation state of manganese in Mn_2O_3 is $+3$.

The oxidation state of manganese in MnO_2 is $+4$.

Q.28. The molar conductivities of a divalent cation (M^{2+}) and monovalent anion (A^-) are $57\ S\ cm^{-1} mol^{-1}$ and $73\ Scm^{-1}\ mol^{-1}$ respectively. Then find the total molar conductivity shown by their compound in $S\ cm^{-1} mol^{-1}$.

A) 130 B) 203

C) 187 D) 41

Answer: 203

Solution: Kohlrausch's law states that the equivalent conductivity of an electrolyte at infinite dilution is equivalent to the sum of the conductances of cations and anions.

$M^{2+} + 2A^- \rightarrow MA_2$

Now, the molar conductivity of the compound is equal to $\mu_{M^{2+}} + 2\mu_{A^-}$

$\Rightarrow \mu_{MA_2} = 57 + (73 \times 2) = 203\ S\ cm^{-1} mol^{-1}$

Q.29. Which is the least paramagnetic?

A) $\left[Co\left(H_2O\right)_6\right]^{2+}$

B) $\left[Cr\left(H_2O\right)_6\right]^{2+}$

C) $\left[Mn\left(H_2O\right)_6\right]^{2+}$

D) $\left[Fe(H_2O)_6\right]^{2+}$

Answer: $\left[Co\left(H_2O\right)_6\right]^{2+}$

Solution: Paramagnetic substances are materials that exhibit a weak attraction to an external magnetic field. This attraction arises due to the presence of unpaired electrons within the atoms or molecules of the substance.

$Co: \ 27: \ 3d^7 4s^2$
$Co^{2+}: 3d^7$

The number of unpaired electrons are 3

$Cr: \ 24: \ 3d^5 4s^1$
$Cr^{2+}: 3d^4$

The number of unpaired electrons are 4

$Mn: 25: 3d^5 4s^2$
$Mn^{+2}: \ 3d^5$

The number of unpaired electrons will be 5

$Fe: 26: \ 3d^6 4s^2$
$Fe^{2+}: 3d^6$

The number of unpaired electrons will be 6

Hence, the least paramagnetic will be $\left[Co\left(H_2O\right)_6\right]^{2+}$

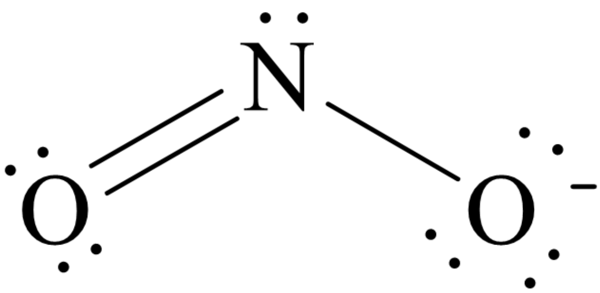
Q.30. In the lewis dot structure of NO_2^-, total number of valence electrons is:

Answer: 18

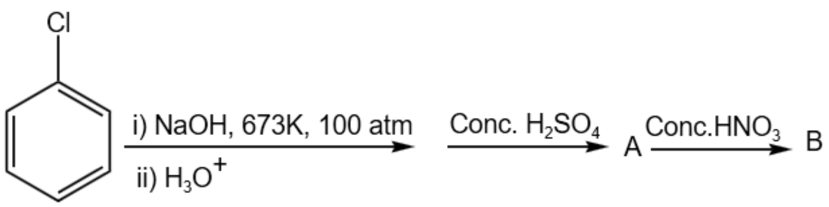
Solution: Lewis structures, also known as Lewis-dot diagrams, show the bonding relationship between atoms of a molecule and the lone pairs of electrons in the molecule.

In NO_2^-

Total number of valence electrons$= 5+12+1=18 \text{ electrons}$



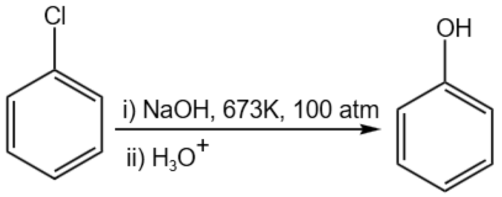
Q.31.



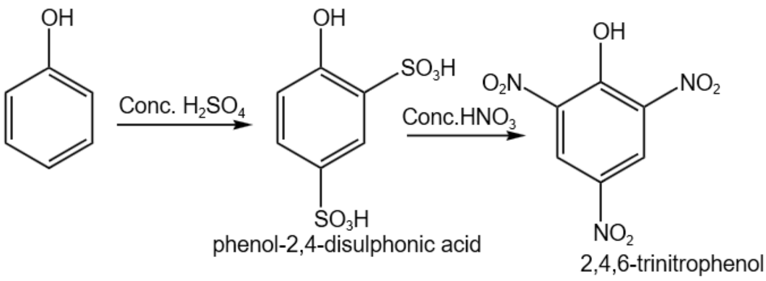
The sum of the oxygen atoms in A and B products.

Answer: 14

Solution: Chlorobenzene is fused with NaOH at 623K and 320 atmospheric pressure. Phenol is obtained by acidification of sodiumphenoxide so produced.



Picric acid is prepared by treating phenol first with concentrated sulphuric acid which converts it to phenol-2,4-disulphonic acid, and then with concentrated nitric acid to get 2,4,6-trinitrophenol.



Q.32. How many of the given cations will give green colour in Borax bead test?

$\mathrm{Fe},\ \mathrm{Co},\ \mathrm{Mn},\ \mathrm{Ni}$

Answer: 1

Solution: Borax bead test is a preliminary test for detection of cation radicals in inorganic qualitative determination. It is one of the most traditional tests used for the detection.

Iron gives green colour in reducing flame in borax bead test

Q.33. One litre solution of $0.2\ \mathrm{M}$ glucose is separated with its pure solvent with semi-permeable membrane, $0.1\ \mathrm{mol}$ of NaCl is added to the solution. The change in osmotic pressure of solution will be at $300\ \mathrm{K}$ (take $\mathrm{R} = 0.083\ \mathrm{L-bar\ K^{-1} mol^{-1}}$)

Answer: 5

Solution: The mathematical formula for calculating osmotic pressure

$\pi = \mathrm{iCRT}$

$\pi = \text{osmotic pressure}$
$\mathrm{i} = \text{Van't Hoff factor}$
$\mathrm{C} = \text{Molarity}$
$\mathrm{T} = \text{Absolute temperature}$

The osmotic pressure of $0.2\ \mathrm{M}$ glucose is

$\pi_1 = 0.2\mathrm{RT}$

After addition of $0.1\ \mathrm{mol}$ sodium chloride, the osmotic pressure is $\pi_2 = (\mathrm{i_1C_1 + i_2C_2})\mathrm{RT} = 0.4\mathrm{RT}$

Now the change in osmotic pressure is $0.2\ \mathrm{RT}$

i.e., the change in osmotic pressure is $4.98\ \mathrm{bar}$

Q.34. If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in R$ is continuous at $x = 0$, then $f(0)$ is

A) -4

B) -2

C) 2

D) 4

Answer: -4

Solution: Given: $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$

Now, using the expansion of trigonometric functions we get,

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{3x - \frac{27x^3}{6} + \ldots + \alpha x - \frac{\alpha x^3}{6} + \ldots - \beta\left(1 - \frac{9x^2}{2} + \ldots\right)}{x^3}$$

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{(3+\alpha)x - \frac{27x^3}{6} + \ldots + -\frac{\alpha x^3}{6} + \ldots - \beta\left(1 - \frac{9x^2}{2} + \ldots\right)}{x^3}$$

Now, for limit to exist $3 + \alpha = 0$ and $\beta = 0$

$\Rightarrow \alpha = -3,\ \beta = 0$

So, the value of limit will be,

$\Rightarrow f(0) = -\frac{27}{6} - \frac{\alpha}{6}$

$\Rightarrow f(0) = -\frac{27}{6} + \frac{3}{6} = \frac{-9}{2} + \frac{1}{2} = \frac{-8}{2}$

$\Rightarrow f(0) = -4$

Q.35. Let $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$ then $\cos\theta$ is equal to

A) $\frac{6-\sqrt{6}}{3\sqrt{6}-2}$

B) $\frac{4}{3\sqrt{6}+2}$

C) $\frac{6-\sqrt{6}}{3\sqrt{6}+2}$

D) $\frac{4}{3\sqrt{6}-2}$

Answer: $\frac{4}{3\sqrt{6}-2}$

Solution: Given,

$4\cos\theta - 3\sin\theta = 1$

$\Rightarrow 4\cos\theta - 1 = 3\sin\theta$

$\Rightarrow 16\cos^2\theta + 1 - 8\cos\theta = 9\sin^2\theta$

$\Rightarrow 16\cos^2\theta + 1 - 8\cos\theta = 9 - 9\cos^2\theta$

$\Rightarrow 25\cos^2\theta - 8\cos\theta - 8 = 0$

$\Rightarrow \cos\theta = \frac{8 \pm \sqrt{64+800}}{50}$

$\Rightarrow \cos\theta = \frac{8 \pm 12\sqrt{6}}{50}$

$\Rightarrow \cos\theta = \frac{4 \pm 6\sqrt{6}}{25}$

$\Rightarrow \cos\theta = \frac{4+6\sqrt{6}}{25}\ \left\{\text{ignoring negative sign as } \theta \in \left[0, \frac{\pi}{4}\right]\right\}$

$\Rightarrow \cos\theta = \frac{2\left(2+3\sqrt{6}\right)}{25} \cdot \frac{3\sqrt{6}-2}{3\sqrt{6}-2}$

$\Rightarrow \cos\theta = \frac{4}{3\sqrt{6}-2}$

Q.36. The value of $\int_{-\pi}^{\pi} \frac{2y(1+\sin y)}{1+\cos^2 y} dy$ is

A) π

B) $-\pi$

C) π^2

D) π^3

Answer: π^2

Solution: Let $I=\int_{-\pi}^{\pi} \frac{2y(1+\sin y)}{1+\cos^2 y} dy$

$\Rightarrow I=\int_{-\pi}^{\pi} \frac{2y}{1+\cos^2 y} dy+\int_{-\pi}^{\pi} \frac{2y \sin y}{1+\cos^2 y} dy$

$2y \rightarrow \text{Odd function}, \ \left(1+\cos^2 y\right) \rightarrow \text{Even function}$

We know that $\int_{-a}^{a} \text{Odd function}=0$

$\Rightarrow I=\int_{-\pi}^{\pi} \frac{2y \sin y}{1+\cos^2 y} dy$

$\Rightarrow I=\int_0^{\pi} \frac{4y \sin y}{1+\cos^2 y} dy \quad \ldots(i)$

Now, $\int_a^b f(x) dx=\int_a^b f(a+b-x) dx$

$\Rightarrow I=4 \int_0^{\pi} \frac{(\pi-y) \sin y}{1+\cos^2 y} dy \quad \ldots(ii)$

Adding (i) and (ii),

$\Rightarrow 2I=4\pi \int_0^{\pi} \frac{\sin y}{1+\cos^2 y} dy$

Putting $\cos y=t \Rightarrow-\sin y dy=dt$

$\Rightarrow I=2\pi \int_{-1}^{1} \frac{dy}{1+t^2}$

$\Rightarrow I=2\pi\left[\tan^{-1} t\right]_{-1}^{1}$

$\Rightarrow I=2\pi\left[\frac{\pi}{4}+\frac{\pi}{4}\right]$

$\Rightarrow I=\pi^2$

Q.37. If $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\ldots\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}=m$ and $\frac{1}{1\times2}+\frac{1}{2\times3}+\ldots\ldots+\frac{1}{99\times100}=n$ then point (m, n) lies on the line

A) $y=11x$

B) $100x=11y$

C) $11x=100y$

D) $x=11y$

Answer: $11x=100y$

Solution: Given: $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\ldots\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}=m$

$\Rightarrow m=\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\ldots\ldots+\sqrt{100}-\sqrt{99}$

$\Rightarrow m=\sqrt{100}-1$

$\Rightarrow m=9$

Also, $\frac{1}{1\times2}+\frac{1}{2\times3}+\ldots\ldots+\frac{1}{99\times100}=n$

$\Rightarrow n=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots\ldots+\frac{1}{99}-\frac{1}{100}$

$\Rightarrow n=1-\frac{1}{100}$

$\Rightarrow n=\frac{99}{100}$

$\Rightarrow (m,n)=\left(9,\frac{99}{100}\right)$

So, (m,n) satisfies the line $11x=100y$.

Q.38. The value of $\int_0^{\frac{\pi}{4}}\frac{dx}{1+\tan x}$ equals to:

A) $\frac{\pi}{4}+\log 2$

B) $\frac{\pi}{8}+\frac{1}{4}\log 2$

C) $\frac{\pi}{8}+\frac{1}{2}\log 2$

D) $\frac{\pi}{8}+\log 2$

Answer: $\frac{\pi}{8}+\frac{1}{4}\log 2$

Solution: Let, $I=\int_0^{\frac{\pi}{4}}\frac{dx}{1+\tan x}$

We know that, $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\frac{dx}{1+\tan\left(\frac{\pi}{4}-x\right)}$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\frac{dx}{1+\frac{1-\tan x}{1+\tan x}}$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\frac{1+\tan x}{2}dx$

$\Rightarrow I=\frac{[x+\log|\sec x|]_0^{\frac{\pi}{4}}}{2}$

$\Rightarrow I=\frac{\frac{\pi}{4}+\log|\sqrt{2}|}{2}$

$\Rightarrow I=\frac{\pi}{8}+\frac{1}{4}\log 2$

Q.39. If $y=y(x)$ and the differential equation is given as $\frac{dy}{dx}+2y=\sin 2x,\ y(0)=\frac{3}{4}$, then find the value of $y\left(\frac{\pi}{8}\right)$

A) $e^{\frac{\pi}{4}}$

B) $e^{\frac{-\pi}{4}}$

C) $e^{\frac{\pi}{2}}$

D) $e^{\frac{-\pi}{2}}$

Answer: $e^{\frac{-\pi}{4}}$

Solution: Given,

$$\frac{dy}{dx}+2y=\sin 2x$$

Which is a linear differential equation

Now, finding the integrating factor we get,

$$IF=e^{\int 2dx}=e^{2x}$$

Now, the solution of the differential equation is given by,

$$ye^{2x}=\int e^{2x}\sin 2x dx$$

Now, let $2x=t \Rightarrow 2dx=dt$ we get,

$$ye^{2x}=\frac{1}{2}\int e^{t}\sin t dt$$

$$\Rightarrow ye^{2x}=\frac{1}{2}\left[e^{t}\sin t-\int \cos t e^{t}\ dt\right]$$

$$\Rightarrow ye^{2x}=\frac{1}{2}\left[e^{t}\sin t-\left(\cos t e^{t}+\int e^{t}\sin t\ dt\right)\right]+c$$

$$\Rightarrow ye^{2x}=\frac{1}{2}\left[e^{t}\sin t-\left(\cos t e^{t}+2ye^{2x}\right)\right]+c$$

$$\Rightarrow ye^{2x}=\frac{1}{4}\left[e^{t}\sin t-\cos t e^{t}\right]+c$$

Now, using $y(0)=\frac{3}{4}$ we get,

$$\Rightarrow \frac{3}{4}=\frac{1}{4}[0-1]+c$$

$$\Rightarrow c=1$$

So,

$$\Rightarrow ye^{2x}=\frac{1}{4}\left[e^{t}\sin t-\cos t e^{t}\right]+1$$

$$\Rightarrow ye^{\frac{\pi}{4}}=\frac{1}{4}\left[e^{\frac{\pi}{4}}\sin\frac{\pi}{4}-\cos\frac{\pi}{4}e^{\frac{\pi}{4}}\right]+1$$

$$\Rightarrow ye^{\frac{\pi}{4}}=1$$

$$\Rightarrow y=e^{\frac{-\pi}{4}}$$

Q.40. If $f(x)=\sin x+3x-\frac{2}{\pi}\left(x^2+x\right)$ then in the interval $\left[0,\frac{\pi}{2}\right]$, $f(x)$ is

A) Neither increasing nor decreasing

B) Increasing

C) Decreasing

D) First increasing then decreasing

Answer: Increasing

Solution: Given: $f(x)=\sin x+3x-\frac{2}{\pi}\left(x^2+x\right)$

$\Rightarrow f'(x)=\cos x+3-\frac{2}{\pi}(2x+1)$

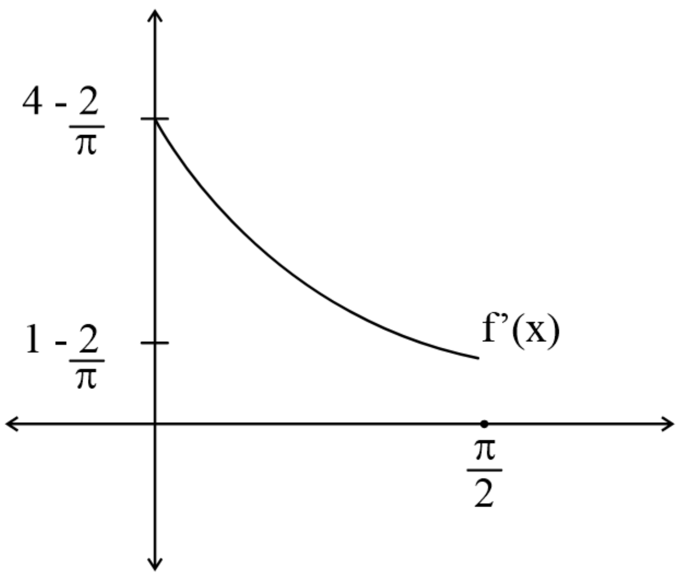
$\Rightarrow f'(x)=\cos x-\frac{4x}{\pi}-\frac{2}{\pi}+3$

$\Rightarrow f'(0)=4-\frac{2}{\pi},\ f'\left(\frac{\pi}{2}\right)=1-\frac{2}{\pi}$

Now, $f''(x)=-\sin x-\frac{4}{\pi}$

$\Rightarrow f''(x)<0$

So, $f'(x)$ is always decreasing,



So, from the above graph we can say that $f'(x)>0$

Hence, $f(x)$ is an increasing function.

Q.41. Find the area bounded by $y=x^2-5x$ and $y=7x-x^2$

A) 144

B) 36

C) 216

D) 72

Answer: 72

Solution: Given: $y=x^2-5x$ and $y=7x-x^2$

$\Rightarrow x^2-5x=7x-x^2$

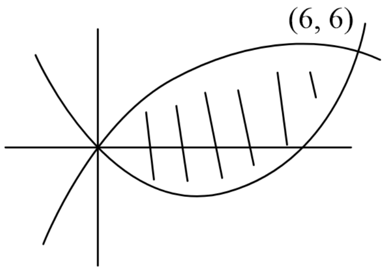
$\Rightarrow 2x^2-12x=0$

$\Rightarrow x(x-6)=0$

$\Rightarrow x=0,\ 6$

$\Rightarrow y=0,\ 6$

So, the intersection points are $(0,0)$ and $(6,6)$.



Now, the required area is given by,

$A=\int_0^6\left(7x-x^2-x^2+5x\right)dx$

$\Rightarrow A=\int_0^6\left(12x-2x^2\right)dx$

$\Rightarrow A=\left[6x^2-\frac{2x^3}{3}\right]_0^6$

$\Rightarrow A=216-144$

$\Rightarrow A=72$ square units

Q.42. If the length of focal chord of $y^2=12x$ is 'l' and the distance of the focal chord from origin is d then ld^2 is

A) 108

B) 12

C) 27

D) 49

Answer: 108

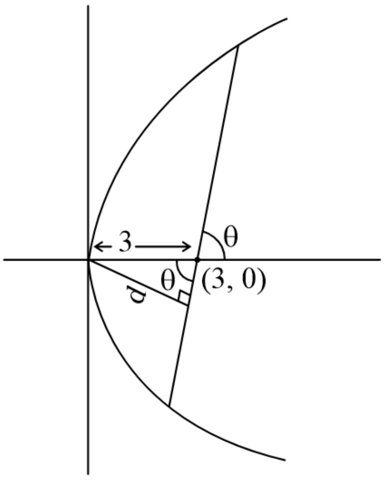
Solution: Given: $y^2 = 12x$

$\Rightarrow a = 3$

So, the focus is $(3, 0)$.

Now, length of focal chord is given by, $l = 4a \operatorname{cosec}^2 \theta$

$\Rightarrow l = \dfrac{12}{\sin^2 \theta}$



$\Rightarrow l = \dfrac{12}{\frac{d^2}{9}}$

$\Rightarrow l = \dfrac{108}{d^2}$

$\Rightarrow ld^2 = 108$

Q.43. Consider the equation $ax^2 + bx + c = 0$. Find probability that the equation has equal roots if $a, b, c \in A$ where $A = \{1, 2, 3, \ldots, 8\}$.

A) $\frac{1}{8}$

B) $\frac{1}{512}$

C) $\frac{1}{4}$

D) $\frac{1}{64}$

Answer: $\frac{1}{64}$

Solution: Given: $ax^2 + bx + c = 0$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac$

So, ac must be a perfect square.

$\Rightarrow ac = \{(1,1), (1,4), (2,2), (2,8), (3,3), (4,1), (4,4), (5,5), (6,6), (7,7), (8,8)\}$

$\Rightarrow (a, b, c) = \{(1,2,1), (1,4,4), (2,4,2), (2,8,8), (3,6,3), (4,4,1), (4,8,4), (8,8,2)\}$

So, the required probability is given by,

$P(E) = \dfrac{8}{8^3}$

$\Rightarrow P(E) = \dfrac{1}{64}$

Q.44. If 4 dices are rolled then find the probability of getting a sum of 16

A) $\frac{25}{6^4}$

B) $\frac{125}{6^4}$

C) $\frac{5^4}{6^4}$

D) $\frac{125}{6^3}$

Answer: $\frac{125}{6^4}$

Solution: Given,

4 dices are rolled,

Now, to sum 16 following cases will be,

$6\ \ 6\ \ 2\ \ 2 \rightarrow \frac{4!}{2!2!} = 6$ ways

$6\ \ 6\ \ 3\ \ 1 \rightarrow \frac{4!}{2!} = 12$ ways

$6\ \ 5\ \ 1\ \ 4 \rightarrow 4! = 24$ ways

$6\ \ 5\ \ 3\ \ 2 \rightarrow 4! = 24$ ways

$6\ \ 4\ \ 4\ \ 2 \rightarrow \frac{4!}{2!} = 12$ ways

$6\ \ 3\ \ 3\ \ 4 \rightarrow \frac{4!}{2!} = 12$ ways

$5\ \ 5\ \ 2\ \ 4 \rightarrow \frac{4!}{2!} = 12$ ways

$5\ \ 5\ \ 3\ \ 3 \rightarrow \frac{4!}{2!2!} = 6$ ways

$5\ \ 5\ \ 5\ \ 1 \rightarrow \frac{4!}{3!} = 4$ ways

$5\ \ 4\ \ 3\ \ 4 \rightarrow \frac{4!}{2!} = 12$ ways

$4\ \ 4\ \ 4\ \ 4 \rightarrow 1! = 1$ way

Hence, total ways of getting a sum of 16 will be$= 125$ ways,

Hence, probability is will be $= \frac{125}{6^4}$

Q.45. A rectangle $ABCD$ with $AB = 2\ cm$ and $BC = 4\ cm$ is inscribed in a rectangle $PQRS$ such that vertices of $ABCD$ lies on the sides of $PQRS$, then the maximum possible area (in sq. units) of rectangle $PQRS$ is:

A) 18

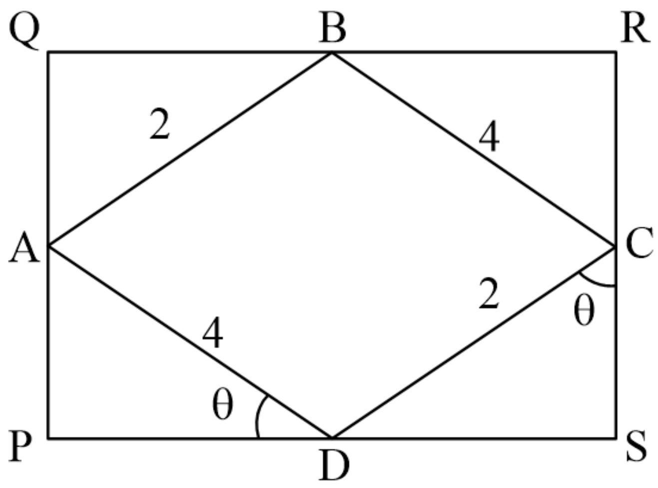
B) 20

C) 12

D) 9

Answer: 18

Solution:



In $\triangle APD$ and $\triangle CDS$, $PD = 4\cos\theta$, $DS = 2\sin\theta$

$\Rightarrow PS = 4\cos\theta + 2\sin\theta$

Similarly, in $\triangle APD$ and $\triangle AQB$, $AP = 4\sin\theta$, $QA = 2\cos\theta$

$\Rightarrow PQ = 4\sin\theta + 2\cos\theta$

So, area of rectangle $PQRS$ is given by,

$A = (4\cos\theta + 2\sin\theta)(4\sin\theta + 2\cos\theta)$

$\Rightarrow A = 16\sin\theta\cos\theta + 8\cos^2\theta + 8\sin^2\theta + 4\sin\theta\cos\theta$

$\Rightarrow A = 20\sin\theta\cos\theta + 8$

$\Rightarrow A = 10\sin 2\theta + 8$

$\Rightarrow A_{max}$ if $\sin 2\theta = 1$

$\Rightarrow \theta = 45^\circ$

So, $A_{max} = 18$ square units.

Q.46. If $f(x) = x^5 + 2x^3 + 3x + 1$ and $g(f(x)) = x$, then $\frac{g(1)}{g'(1)}$ is equal to

Answer: 0

Solution: Given: $f(x) = x^5 + 2x^3 + 3x + 1$ and $g(f(x)) = x$

$\Rightarrow f'(x) = 5x^4 + 6x^2 + 3$

So, $f(x)$ is an increasing function.

$\Rightarrow g'(f(x)) \times f'(x) = 1$

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}$

Putting $f(x) = 1$

$\Rightarrow x^5 + 2x^3 + 3x + 1 = 1$

$\Rightarrow x^5 + 2x^3 + 3x = 0$

Since, $f(x)$ is an increasing function so $x = 1$ is the only solution.

$\Rightarrow g(1) = 0$ as $(f(x) = 1 \text{ at } x = 0)$

$\Rightarrow \frac{g(1)}{g'(1)} = 0$

Q.47. If $f(1) = 1$ and $\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ then find the value of $2f(2) + 3f(3)$

Answer: 24

Solution: Given,

$f(1)=1$

And $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t-x} = 1 \left\{\frac{0}{0} \text{ form}\right\}$

Now, using L-hospital rule we get,

$\lim_{t \to x} \frac{2t f(x) - x^2 f'(t)}{1} = 1$

$\Rightarrow 2x f(x) - x^2 f'(x) = 1$

$\Rightarrow x^2 f'(x) - 2x f(x) + 1 = 0$

$\Rightarrow x^2 \frac{dy}{dx} - 2xy = -1 \left\{\text{where } \frac{dy}{dx} = f'(x)\right\}$

$\Rightarrow \frac{dy}{dx} - 2\frac{y}{x} = -\frac{1}{x^2}$

Which is linear differential equation, now finding integrating factor we get,

$IF = e^{\int \frac{-2}{x}} = \frac{1}{x^2}$

So, solution of the differential equation is given by,

$y \cdot \frac{1}{x^2} = \int -\frac{1}{x^2} \cdot \frac{1}{x^2} dx$

$\Rightarrow y \cdot \frac{1}{x^2} = \int -\frac{1}{x^4} dx$

$\Rightarrow y \cdot \frac{1}{x^2} = \frac{1}{3x^3} + c$

Now. using $f(1)=1$ we get, $c = \frac{2}{3}$

So, $y = \frac{1}{3x} + \frac{2}{3}x^2$

Hence, $2f(2) + 3f(3) = 2\left[\frac{1}{6} + \frac{8}{3}\right] + 3\left[\frac{1}{9} + 6\right] = 2\left[\frac{17}{6}\right] + 3\left[\frac{55}{9}\right]$

$\Rightarrow 2f(2) + 3f(3) = \frac{17}{3} + \frac{55}{3} = \frac{72}{3} = 24$

Q.48. If the system of linear equations,

$11x + y + \lambda z = 5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$, has infinite many solution then find the value of $|\lambda^{\mu} - \mu|$

Answer: 2059

Solution: Given,

System of linear equations,

$11x + y + \lambda z = 5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$, has infinite many solution

So, $\triangle = \triangle_1 = \triangle_2 = \triangle_3 = 0$

$\triangle = \begin{vmatrix} 11 & 1 & \lambda \\ 2 & 3 & 5 \\ 8 & -19 & -39 \end{vmatrix} = 0$

$\Rightarrow 11(-117 + 95) - 1(-78 - 40) + \lambda(-38 - 24) = 0$

$\Rightarrow -242 + 118 - 62\lambda = 0$

$\Rightarrow \lambda = -2$

Now, solving $\triangle_3 = \begin{vmatrix} 11 & 1 & 5 \\ 2 & 3 & 3 \\ 8 & -19 & \mu \end{vmatrix} = 0$

$\Rightarrow 11(3\mu + 57) - 1(2\mu - 24) + 5(-38 - 24) = 0$

$\Rightarrow 33\mu + 627 - 2\mu + 24 - 310 = 0$

$\Rightarrow 31\mu = 341$

$\Rightarrow \mu = 11$

Hence, the value of $|\lambda^{\mu} - \mu| = \left|(-2)^{11} - 11\right| = |-2048 - 11| = 2059$

Q.49. If the constant term in the expansion of $\left(1 + 2x - 3x^3\right)\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ is k then find the value of $108k$

Answer: 54

Solution: Given,

$$\left(1+2x-3x^3\right)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$$

Now, finding the general term in the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ we get,

$$T_{r+1}={}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(\frac{-1}{3x}\right)^r$$

$$\Rightarrow T_{r+1}={}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(\frac{-1}{3}\right)^r x^{18-3r}$$

Now, solving

$$\left(1+2x-3x^3\right)\left[{}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(\frac{-1}{3}\right)^r x^{18-3r}\right]$$

For the constant term there will be two cases when $r=6$ and when $r=7$ we get,

$${}^9C_6\left(\frac{3}{2}\right)^3\left(\frac{-1}{3}\right)^6-3\left[{}^9C_7\left(\frac{3}{2}\right)^2\left(\frac{-1}{3}\right)^7\right]$$

$$=\frac{9\times8\times7}{6}\left(\frac{27}{8}\right)\left(\frac{-1}{3}\right)^6-3\left[\frac{9\times8}{2}\left(\frac{9}{4}\right)\left(\frac{-1}{3}\right)^7\right]$$

$$=\left(\frac{7}{2}\right)\left(\frac{1}{9}\right)-3\left[\left(\frac{-1}{27}\right)\right]$$

$$=\frac{7}{18}+\frac{1}{9}=\frac{9}{18}=\frac{1}{2}$$

Hence, $k=\frac{1}{2}\Rightarrow 108k=54$