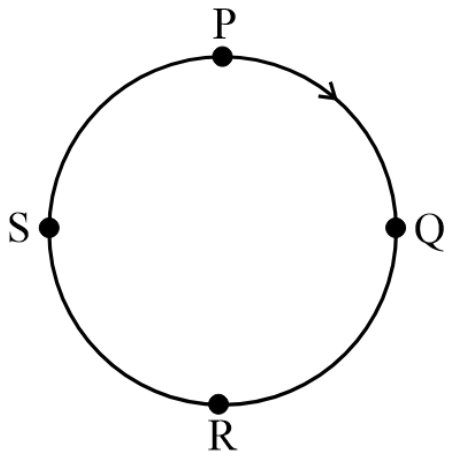
JEE Main 4th April Session 2 - Memory Based Paper

4th April Session 2

Questions

Q.1. P, Q, R, S are 4 symmetric points on a horizontal circle of radius of 4 km. What is the displacement when a car moves from P to R along the given circular path.



A) 4 km

B) 8 km

C) 4π km

D) $4\sqrt{2}$ km

Answer: 8 km

Solution: As displacement is the closest distance between final position and initial position, the displacement of the car is 8 km.

Q.2. One mole of an ideal monatomic gas is compressed adiabatically from volume $2V$ to V. If initially temperature of gas was T then the magnitude of work done in this process is

A) $\frac{3}{2}RT\left(2^{\frac{1}{2}}-1\right)$

B) $\frac{3}{2}RT\left(2^{\frac{2}{3}}-1\right)$

C) $\frac{2}{3}RT\left(2^{\frac{2}{3}}-1\right)$

D) $\frac{2}{3}RT\left(\sqrt{2}-1\right)$

Answer: $\frac{3}{2}RT\left(2^{\frac{2}{3}}-1\right)$

Solution: Work done in an adiabatic process is given by: $w=\frac{nR(T_2-T_1)}{\gamma-1}$.

For adiabatic process

$$\frac{T_2}{T_1}=\left[\frac{V_1}{V_2}\right]^{\gamma-1}$$

$$\Rightarrow T_2=T_1(2)^{\frac{5}{3}-1}$$

$$\Rightarrow T_2=2^{\frac{2}{3}}T$$

$$\Rightarrow w=\frac{3RT\left(2^{\frac{2}{3}}-1\right)}{2}$$

Q.3. A 2 kg brick is placed on an inclined plane of inclination 45°. The brick is at rest. The minimum coefficient of static friction is:

A) 0.5

B) $\sqrt{3}$

C) $\frac{1}{\sqrt{3}}$

D) 1

Answer: 1

Solution: The minimum coefficient of static friction required to keep a body from sliping on an incline is equal to

$\mu = \tan\theta$

$\Rightarrow \mu = \tan 45^\circ = 1$

Q.4. A satellite revolves around the earth. The distance of the satellite at which it should be placed from the earth depends on time period of the earth, mass of the earth and universal gravitational constant $G = \left[M^{-1}L^3T^{-2}\right]$. Find the Relation between distance & Time Period

A) $R \propto T^3$

B) $R \propto T^{\frac{2}{3}}$

C) $R \propto T^{-3}$

D) $R \propto T^{-\frac{2}{3}}$

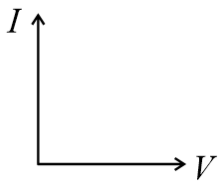
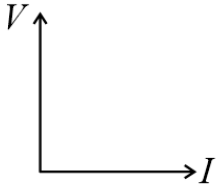
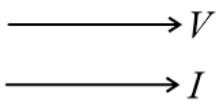
Answer: $R \propto T^{\frac{2}{3}}$

Solution: According to Kepler's law of orbits,

$T^2 \propto R^3$

$\Rightarrow R \propto T^{\frac{2}{3}}$

Q.5. Correct match for phasors of voltage and current for given elements is

(a) Inductive	(p) 
(b) Capacitive	(q) 
(c) Resistive	(r) 

A) $a \to p, b \to q,\ c \to r$

B) $a \to q,\ b \to p,\ c \to r$

C) $a \to p,\ b \to p,\ c \to r$

D) $a \to q,\ b \to q,\ c \to r$

Answer: $a \to q,\ b \to p,\ c \to r$

Solution: In an inductor, current lags behind the input voltage by a phase difference of $\frac{\pi}{2}$. Current and voltage are in same phase in resistor whereas current leads the voltage $\frac{\pi}{2}$ by in a capacitor.

Q.6. In YDSE, Width of one slit is 4 times the other. Find ratio of maximum intensity to minimum intensity of the interferenced pattern on the screen

A) $\frac{3}{1}$ B) $\frac{9}{1}$

C) $\frac{4}{1}$ D) $\frac{2}{1}$

Answer: $\frac{9}{1}$

Solution: Intensity of the light coming from the slits is proportional to the width of the slit.

Therefore, if $I_1 = I;\ I_2 = 4I$

Ratio of maximum intensity to the minimun intensity is given by

$$\frac{Imax}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$$

$$\frac{Imax}{I_{min}} = \frac{9}{1}$$

Q.7. Two wires A and B are made of same material and have same mass. Radius of A is $2\ \text{mm}$ and B is $4\ \text{mm}$. If resistance of B is $2\ \Omega$, resistance of A is

A) $4\ \Omega$ B) $16\ \Omega$

C) $8\ \Omega$ D) $32\ \Omega$

Answer: $32\ \Omega$

Solution: Resistance of a uniform wire is given by, $R = \rho\frac{l}{A}$.

As mass is same, we can write mass as $M \propto lA$.

Therefore,

$$\frac{R_A}{R_B} = \frac{\frac{l_A}{A_A}}{\frac{l_B}{A_B}} = \frac{\frac{M}{(A_A)^2}}{\frac{M}{(A_B)^2}} = \left(\frac{A_B}{A_A}\right)^2 = \left(\frac{\pi(r_B)^2}{\pi(r_A)^2}\right)^2 = 16$$

$$\Rightarrow R_A = 16R_B = 32\ \Omega$$

Q.8. In a system of $3\ \text{kg}$ and $2\ \text{kg}$, $3\ \text{kg}$ is moved by 2 units towards COM. How much should $2\ \text{kg}$ move so that COM does not change position.

A) $\frac{5}{3}$ unit B) $\frac{7}{3}$ unit

C) $\frac{7}{5}$ unit D) 3 unit

Answer: 3 unit

Solution: Let the $3\ \text{kg}$ mass is to the left and the $2\ \text{kg}$ mass is towards the right. Now, since due to shift of the $3\ \text{kg}$ mass, the position of COM remains unchanged, so we can write

$$m_1x_1 + m_2x_2 = 0$$
$$\Rightarrow 3 \times 2 + 2(x) = 0$$
$$\Rightarrow x = -3\ unit$$

So, the $2\ \text{kg}$ will shift by a distance $3\ unit$ towards left.

Q.9. Arrange the following in ascending order of wavelength.
1) Gamma Rays
2) X-Rays
3) Infra-Red Rays
4) Microwaves

A) 1, 2, 3, 4 B) 4, 3, 2, 1

C) 1, 4, 3, 2 D) 3, 1, 2, 4

Answer: 1, 2, 3, 4

Solution: According to electromagnetic spectrum, ascending order of wavelengths is :

(Gamma Rays)<(X-Rays)<(Infra-Red Rays)<(Microwaves)

Q.10. A bar magnet has a magnetic moment of $0.5\ \text{A m}^2$ and its is placed in an external magnetic field of $8 \times 10^2\ \text{T}$. Find the work done by external agent when it is rotated from most stable position to unstable position

A) $-2MB$ B) $-MB$

C) MB D) $2MB$

Answer: $2MB$

Solution: In case of stable position: $U_i = -MB$

In case of unstable position: $U_f = +MB$

Change in potential energy is $\Delta U = U_f - U_i = 2MB$

Hence, work done on the magnet is $\Delta U = 2MB$

Q.11. A heater of rating $50\ \text{W},\ 200\ \text{V}$ is connected with source voltage of $100\ \text{V}$. Power consumed by the heater is

A) $12.5\ \text{W}$ B) $25\ \text{W}$

C) $50\ \text{W}$ D) $100\ \text{W}$

Answer: $12.5\ \text{W}$

Solution: Power is given by,

$$P = \frac{V^2}{R}$$

$$\Rightarrow R = \frac{V^2}{P} = \frac{200^2}{50} = 800\ \Omega$$

Now, power consumed by heater for the given supply

$$P' = \frac{\left(V'\right)^2}{R} = \frac{100^2}{800} = 12.5\ \text{W}$$

Q.12. A metallic rod of length $4\ \text{m}$ is rotating about perpendicular bisector of the rod with angular velocity of $2\ \text{rad s}^{-1}$ in presence of transverse magnetic field of $0.5\ \text{T}$. Potential difference developed across ends of rod is

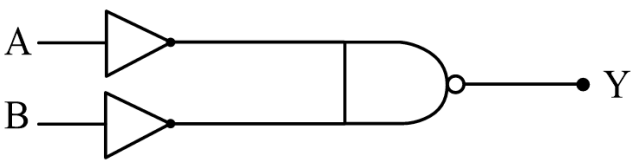
A) $16\ \text{V}$ B) $8\ \text{V}$

C) $0\ \text{V}$ D) $32\ \text{V}$

Answer: $0\ \text{V}$

Solution: Since the rod is rotating about its centre, the end points of the rods will be at the same potential. Therefore, the potential difference between the ends of the rod will be $0\ \text{V}$.

Q.13. The circuit diagram shown is equivalent to

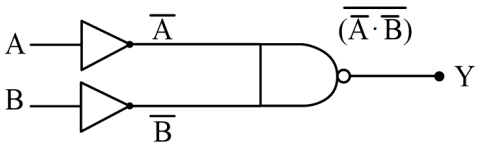


A) OR B) NOR

C) AND D) $NAND$

Answer: OR

Solution:



First two elements are NOT gates and then subsequent output is fed to AND gate.

So, output will be $Y = \overline{\left(\overline{A} \cdot \overline{B}\right)} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$, which is equivalent to OR gate.

Q.14. Assertion (A): The contact angle depends on material of solid and liquid.
Reason (R): Height of the liquid in a capillary tube is independent of the radius of the tube.

A) A is true, R is true and R explains A

B) A is true, R is true but R does not explain A

C) A is true and R is false

D) A is false and R is true

Answer: A is true and R is false

Solution: Contact angle depends on the cohesive and adhesive forces. Hence, it depends on the material of solid and liquid. Therefore, the Assertion is true.

The height in a capilary tube is given by

$$h = \frac{2T\cos\theta}{\rho r g}$$

Hence, it depends on the radius of the tube. Therefore, the Reason is false.

Q.15. A body of mass $4\ \text{kg}$ is at a height of R (radius of earth) from the surface of the earth. The weight of the body is _____ N.

(Take $g = 10\ \text{m s}^{-2}$)

Answer: 10

Solution: Acceleration due to gravity at a height h from the surface of the earth is given by, $g' = g\left(\frac{R}{R+h}\right)^2$.

Therefore, we can write

$$g' = g\left(\frac{R}{R+R}\right)^2 = \frac{g}{4}.$$

Weight of the body at the given height, $W = mg' = 4 \times \frac{g}{4} = 10\ \text{N}$

Q.16. Which of the following has a pyramidal shape?

A) $S_2O_3^{2-}$

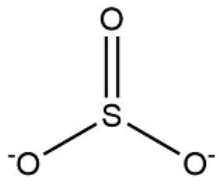
B) SO_3^{2-}

C) SO_4^{2-}

D) $S_2O_7^{2-}$

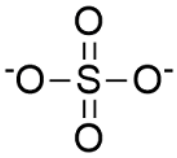
Answer: SO_3^{2-}

Solution: The molecule in which central atom has three bond pairs and a lone pair are known to have pyramidal shape.

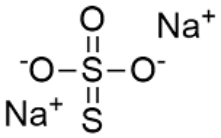


Sulfite ion

The above structure has pyramidal shape. Rest of structures have tetrahedral shape around sulphur.

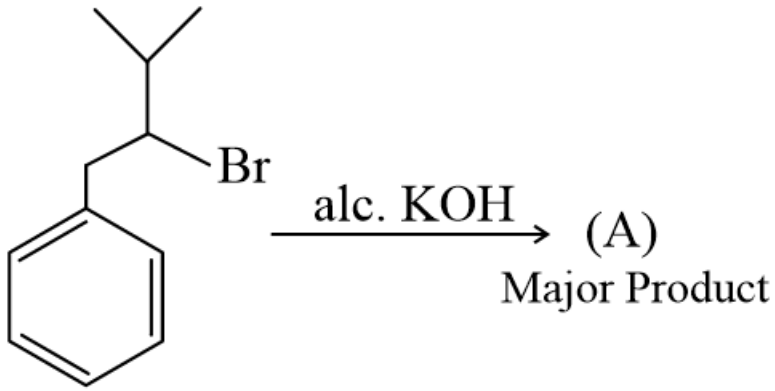


sulfate ion

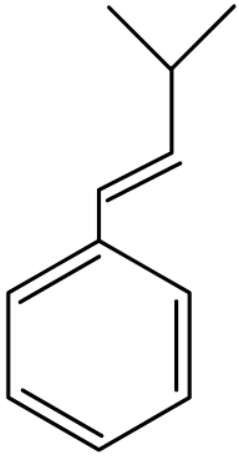


Sodium thiosulfate

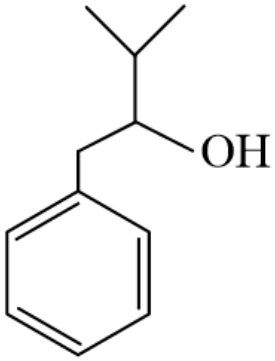
Q.17. Major product A is:



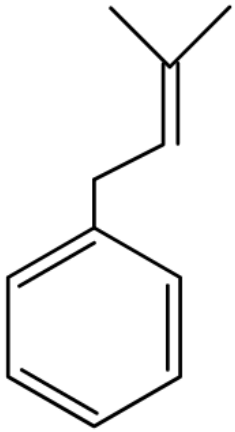
A)



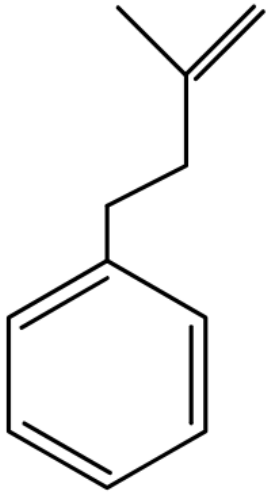
B)



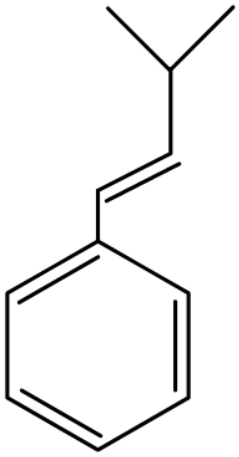
C)



D)



Answer:



Solution: Alcoholic KOH is a strong base.

Alcoholic solution of KOH is used for dehydrohalogenation of an haloalkane and results in the formation of Alkenes.

Answer is option A because of presence of alpha hydrogen and also due to conjugation with benzene ring.

Q.18. The correct order of ionisation enthalpy of $\mathrm{Li},\ \mathrm{Na},\ \mathrm{Cl},\ \mathrm{F}$ is:

A) $\mathrm{Na} < \mathrm{Li} < \mathrm{Cl} < \mathrm{F}$

B) $\mathrm{Li} < \mathrm{Na} < \mathrm{Cl} < \mathrm{F}$

C) $\mathrm{Na} < \mathrm{Li} < \mathrm{F} < \mathrm{Cl}$

D) $\mathrm{F} < \mathrm{Cl} < \mathrm{Li} < \mathrm{Na}$

Answer: $\mathrm{Na} < \mathrm{Li} < \mathrm{Cl} < \mathrm{F}$

Solution: Ionization enthalpy is defined as the minimum amount of energy that is required to remove the most loosely bounded electrons that is electron present in the outermost shell from an isolated gaseous atom.

Ionisation enthalpy decreases down the group due to an increase in the number of the shells and it increases in a period from left to right due to increase in the nuclear charge.

Hence, A is the answer.

Q.19. The IUPAC name of catechol is

A) Benzene-1,2-diol

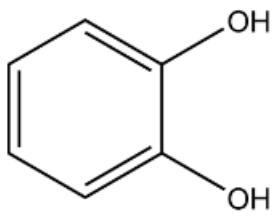
B) Benzene-1,3-diol

C) Benzene-1,4-diol

D) 3-hydroxyphenol

Answer: Benzene-1,2-diol

Solution: The structure of the catechol is



catechol

The IUPAC name is benzene-1,2-diol

1. "benzene" indicates the parent hydrocarbon structure, which is a six-carbon aromatic ring with alternating single and double bonds, commonly known as benzene.
2. "1,2-diol" tells us the positions and nature of the substituents on the benzene ring. "1,2" indicates that the hydroxy (OH) groups are attached to the first and second carbon atoms of the benzene ring. "Diol" indicates that there are two hydroxy (OH) groups present in the compound.

Q.20. Consider the following statements:
Statement I: The number of emitted photoelectrons increases with increase in frequency of incident light.
Statement II: Kinetic energy of emitted photoelectrons increases with an increase in frequency of incident light

A) Both Statement-1 and Statement-2 are true

B) Statement-1 true and Statement-2 are false

C) Statement-1 false and Statement-2 are true

D) Both Statement-1 and Statement-2 are false

Answer: Statement-1 false and Statement-2 are true

Solution: When the intensity of light is increased then it means that the number of photons incident on the metal surface increases which leads to greater production of photoelectrons which means that the photo current is increasing due to increase in intensity of incident radiation and not due increase in frequency.

We know: $\mathrm{h}\nu = \frac{1}{2}\mathrm{mv}^2 + \Phi$

Or, $\frac{1}{2}\mathrm{mv}^2 = \mathrm{h}\nu - \Phi$

Thus, from the above relation we can see that $\mathrm{K.E} = \left(\frac{1}{2}\mathrm{mv}^2 = \mathrm{h}\nu - \Phi\right)$ is directly proportional to frequency (ν).

Note: Φ is the work function.

Q.21. Which of the following statements is incorrect?

A) Atoms are indivisible particles, which cannot be created or destroyed chemical reaction.

B) Atoms combine in any ratio to form compounds.

C) Atoms reorganise in a chemical reaction.

D) Atoms of different elements have different masses and chemical properties

Answer: Atoms combine in any ratio to form compounds.

Solution: According to Dalton's atomic theory, all matter, whether an element, a compound or a mixture is composed of small particles called atoms. The postulates of this theory may be stated as follows:
(i) All matter is made of very tiny particles called atoms.
(ii) Atoms are indivisible particles, which cannot be created or destroyed in a chemical reaction.
(iii) Atoms of a given element are identical in mass and chemical properties.
(iv) Atoms of different elements have different masses and chemical properties.
(v) Atoms combine in the ratio of small whole numbers to form compounds.
(vi) The relative number and kinds of atoms are constant in a given compound.

Q.22. Which of the following is used as an adsorbent in adsorption chromatography?

A) Silica gel

B) Alumina

C) Cellulose

D) All of these

Answer: All of these

Solution: In adsorption chromatography, two phases are used. These are called the stationary phase and the mobile phase. The stationary phase is also called adsorbent.
Out of several forms of adsorption chromatography, two forms are column chromatography and paper chromatography. Both forms use the same principle.
You can use either silica gel or alumina as the stationary phase. The stationary phase is also called adsorbent. You can use cellulose as a stationary phase (adsorbent) in paper chromatography. Filter paper contains cellulose fibers. The pores of cellulose fibers contain moisture / water. This serves the purpose of the stationary phase. A suitable solvent is used as a mobile phase.
Silica, alumina and cellulose can be used as adsorbent in adsorption chromatography.
In paper chromatography adsorbent is cellulose.

Q.23. Arrange the following in increasing order of their first ionisation enthalpy:

A) $\mathrm{Tl} < \mathrm{In} < \mathrm{Ga} < \mathrm{Al} < \mathrm{B}$

B) $\mathrm{In} < \mathrm{Al} < \mathrm{Ga} < \mathrm{Tl} < \mathrm{B}$

C) $\mathrm{In} < \mathrm{Ga} < \mathrm{Al} < \mathrm{B} < \mathrm{Tl}$

D) $\mathrm{B} < \mathrm{Al} < \mathrm{Ga} < \mathrm{In} < \mathrm{Tl}$

Answer: $\mathrm{In} < \mathrm{Al} < \mathrm{Ga} < \mathrm{Tl} < \mathrm{B}$

Solution: Ionization energy, in chemistry and physics, the amount of energy required to remove an electron from an isolated atom or molecule.

On moving down a group, ionization enthalpy generally decreases due to an increase in the atomic size and shielding. Thus, on moving down group 13, ionization enthalpy decreases from B to Al. But, Ga has higher ionization enthalpy than Al. Al follows immediately after s–block elements, whereas Ga follows after d–block elements. The shielding provided by d-electrons is not very effective. These electrons do not shield the valence electrons very effectively. As a result, the valence electrons of Ga experience a greater effective nuclear charge than those of Al. Further, moving from Ga to In, the ionization enthalpy decreases due to an increase in the atomic size and shielding. But, on moving from In to Tl, the ionization enthalpy again increases. In the periodic table, Tl follows after 4f and 5d electrons. The shielding provided by the electrons in both these orbitals is not very effective. Therefore, the valence electron is held quite strongly by the nucleus. Hence, the ionization energy of Tl is on the higher side.

Hence, the answer is option B.

Q.24. Which of the following are the correct statements about fuel cell?
S-1: It is a galvanic cell
S-2: Fuel cell have efficiency of 40%
S-3: It is eco friendly
S-4: Aluminium as a catalyst used in this cell

A) S-1 and S-2 only

B) S-1 and S-3 only

C) S-2 and S-3 only

D) All statements are correct

Answer: S-1 and S-3 only

Solution: Galvanic cells that are designed to convert the energy of combustion of fuels like hydrogen, methane, methanol, etc. directly into electrical energy are called fuel cells. Fuel cells produce electricity with an efficiency of about 70 % compared to thermal plants whose efficiency is about 40%. Fuel cells are pollution free and in view of their future importance, a variety of fuel cells have been fabricated and tried. Catalysts like finely divided platinum or palladium metal are incorporated into the electrodes for increasing the rate of electrode reactions

Q.25. Find out the number of unpaired electrons in d-subshell for $\left[\mathrm{Co}\left(\mathrm{H_2O}\right)_6\right]^{3+}$

A) 0

B) 2

C) 3

D) 4

Answer: 0

Solution: In the complex $\left[\mathrm{Co}\left(\mathrm{H_2O}\right)_6\right]^{3+}$, Co^{3+} has $3\mathrm{d}^6$ electronic configuration. In the case of this complex, the splitting energy is more than pairing energy. Hence, all the electrons get paired against Hund's rule. Hence, it has zero unpaired electrons.

Q.26. Match Column I and Column II

Column I	Column II
1. $\alpha-$Glucose and $\alpha-$Galactose	a. Homologues
2. $\alpha-$Glucose and $\alpha-$Fructose	b.Epimer
3.$\alpha-$Glucose and $\beta-$Glucose	c.Anomer
4. $\alpha-$Ribose and $\alpha-$Glucose	d. Functional isomers

A) $1-\mathrm{b},\ 2-\mathrm{d},\ 3-\mathrm{a},\ 4-\mathrm{c}$

B) $1-\mathrm{b},\ 2-\mathrm{d},\ 3-\mathrm{c},\ 4-\mathrm{a}$

C) $1-d,\ 2-b,\ 3-c,\ 4-a$ D) $1-a,\ 2-c,\ 3-d,\ 4-b$

Answer: $1-b,\ 2-d,\ 3-c,\ 4-a$

Solution: Epimers are carbohydrates which vary in one position for the placement of the -OH group.

Anomers are diastereoisomers of cyclic forms of sugars or similar molecules differing in the configuration at the anomeric carbon (C-1 atom of an aldose or the C-2 atom of a 2-ketose).

Functional isomers are structural isomers that have the same molecular formula (that is, the same number of atoms of the same elements), but the atoms are connected in different ways so that the groupings are dissimilar. These groups of atoms are called functional groups, functionalities.

Hence, the answer is option B.

Q.27. We have a complex of Fe^{3+} ion having electronic configuration according to crystal field theory is $t_{2g}^5 e_g^0$. If the complex is $[Fe(NH_3)_X(CN)_Y]$, then value of $(X+Y)$ is

A) 6 B) 4

C) 3 D) 5

Answer: 6

Solution: Fe^{3+} ion has d^5 configuration and pairing of electrons occurred. To have +3 oxidation state on iron, three cyanide ligands are required. With strong field ligands iron generally forms octahedral complex. Hence, the values of $X \text{ and } Y$ respectively are $3 \text{ and } 3$.

Q.28. Arrange in increasing wave length order:

Gamma rays, X-rays, UV rays, IR rays

A) Gamma rays<X-rays<UV rays<IR rays B) Gamma rays<X-rays<IR rays<UR rays

C) IR rays<X-rays<UV rays<Gamma rays D) Gamma rays<UV rays<X rays<IR rays

Answer: Gamma rays<X-rays<UV rays<IR rays

Solution: Electromagnetic radiation is a form of energy that propagates as both electrical and magnetic waves traveling in packets of energy called photons. There is a spectrum of electromagnetic radiation with variable wavelengths and frequency, which in turn imparts different characteristics.

Electromagnetic radiation is classified into types according to the frequency of the wave. These types include, in order of increasing frequency, radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.

Q.29. An element of d-Block (Z) of 4th period has spin only magnetic moment of its Z^{3+} form is $3.9\ BM$, Then find atomic number of
element (Z).

A) 22 B) 23

C) 24 D) 25

Answer: 24

Solution: The spin only magnetic momentum $3.9\ BM$ means the ions has three unpaired electrons. That means Z^{3+} can have $3d^3 \text{ or } 3d^7$ electronic configuration. Hence, the atom Z can be Chromium(24) or Cobalt(27).

Q.30. Find the atomic number of the element having 3 unpaired electrons, and it belongs to a transition series with $+2$ Oxidation state.

A) 23 B) 22

C) 24 D) 25

Answer: 23

Solution: Transition elements (also known as transition metals) are elements that have partially filled d orbitals.

In transition elements

$Sc^{2+} = 4s^2 3d^1$
$Ti^{2+} = 4s^2 3d^2$
$V^{2+} = 4s^2 3d^3$

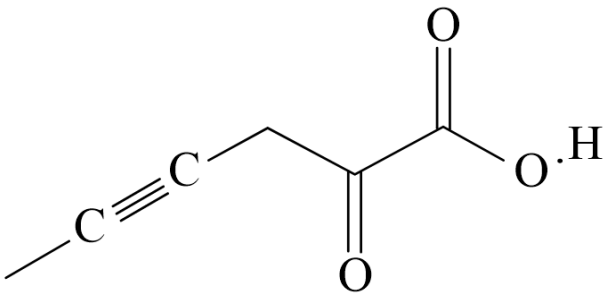
Hence, Vanadium has 3 unpaired electrons in $+2$ oxidation state.

Hence, the answer is option A.

Q.31. What is the sum of the number of σ and π bonds present in 2-oxo-hex-4-ynoicacid?

Answer: 18

Solution: Sigma bonds (σ) are the first type of covalent bond, formed by overlap of atomic orbitals head-to-head. They are found in single, double, and triple bonds. Pi bonds (π) are the second and third types of covalent bonds, formed by overlap of p orbitals side-to-side.



As can be seen from the diagram above:

Number of sigma bonds are 14

Number of pi bonds are 4

Sum$= 4 + 14 = 18$

Q.32. $SO_{2(g)} + \frac{1}{2}O_{2(g)} \leftrightharpoons SO_3(g);\ Kc = 5 \times 10^{-2}$
$2SO_{3(g)} \leftrightharpoons 2SO_{2(g)} + O_{2(g)};\ Kc = ?$

Answer: 400

Solution: $SO_{2(g)} + \frac{1}{2}O_{2(g)} \leftrightharpoons SO_3(g);\ Kc = 5 \times 10^{-2}$
$2SO_{3(g)} \leftrightharpoons 2SO_{2(g)} + O_{2(g)};$

$K = \left(\frac{1}{Kc}\right)^2 = \left(\frac{100}{5}\right)^2$
$= \left(\frac{100}{5}\right)^2 = 400$

Hence, the answer is 400

Q.33. Find out magnitude of heat (q) for an isothermal irreversible expansion against external pressure of 8 bar if volume increases by10 L (in Joule)

Answer: 8000

Solution: According to First law of thermodynamics,

$\Delta U = q + W$

For isothermal process,

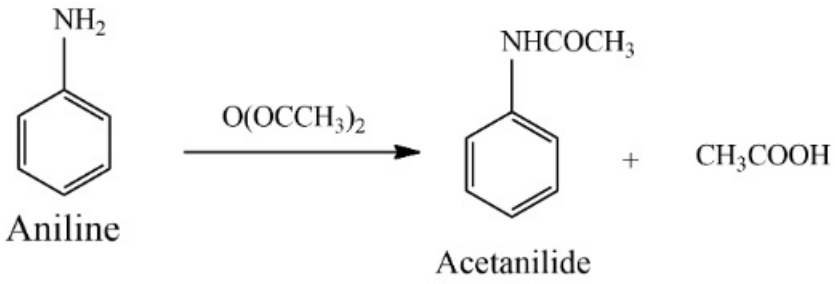
$\Delta U = 0$, hence $q = -W = p\,\Delta\,V$

$\Rightarrow q = 8 \times 10\ L\ bar$
$\Rightarrow q = 8000J\ (100J = 1L\ bar)$

Q.34. What is the maximum amount of acetanilide formed when acetic anhydride in excess is treated with 18 gm of aniline. (nearest Integer).

Answer: 26

Solution:



From the above reaction it is clear that one mole of aniline gives, one mole of acetanilide. i.e., 93 grams of aniline gives 135 grams of acetanilide. Hence, 18 grams of aniline gives

$18 \times \frac{135}{93}$ grams of acetanilide.

The maximum amount of acetanilide formed = 26 grams.

Q.35. Find the number of compounds with non zero dipole moment:

$BeCl_2,\ BCl_3,\ NF_3,\ H_2O,\ H_2S,\ CCl_4,\ XeF_4,\ CO_2,\ HBr$

Answer: 4

Solution: The bond between Be and Cl is considered polar because there is a large electronegative difference between the two said atoms. However, the molecule has no dipole moment because the molecular geometry is linear which cancels out the dipole moment that comes from the Be-Cl bond.

The BCl_3 molecule is non-polar. This is because BCl_3 is trigonal planar in shape. It is a symmetrical molecule. Hence, the respective dipole-moments of the B–Cl bond cancel each other, thereby causing a zero-dipole moment.

In case of NF_3 the dipole moment is not zero because of presence of a lone pair of electrons on Nitrogen atom.

H_2O does have a dipole moment because oxygen is more electronegative than hydrogen.

Hydrogen and sulfur have a large electronegativity difference, so their bond is polar.

Carbon tetrachloride molecule has zero dipole moment even though C and Cl have different electronegativities and each of the C - Cl bond is polar and has some dipole moment. This is because the individual dipole moments cancel out because of the symmetrical tetrahedral shape of the molecule.

Xenon Tetrafluoride is a basic molecule. The dipole moment inside the bonds is zero, making it a non-polar molecule.

The arrangement of atoms in the CO_2 molecule is liner and symmetrical, and the bond polarities within the molecule are canceled out.

Hydrogen bromide molecules are polar molecules with partial charges on either end. Therefore, the oppositely charged poles of hydrogen bromide molecules attract each other in what is called a dipole-dipole interaction. Thus, the predominant intermolecular force present in hydrogen bromide is dipole-dipole.

Q.36. If $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx = \frac{2}{\pi}$, then find α.

A) $\frac{\pi}{2}$

B) 2

C) π

D) $\frac{\pi}{4}$

Answer: $\frac{\pi}{2}$

Solution: Let, $I = \int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx \quad \ldots (i)$

We know that, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\Rightarrow I = \int_{-1}^{1} \frac{\cos \alpha x}{1+3^{-x}} dx$

$\Rightarrow I = \int_{-1}^{1} \frac{(\cos \alpha x)(3^x)}{1+3^x} dx \quad \ldots (ii)$

Adding (i) and (ii),

$\Rightarrow 2I = \int_{-1}^{1} \frac{\cos \alpha x + 3^x \times \cos \alpha x}{1+3^x} dx$

$\Rightarrow 2I = \int_{-1}^{1} \cos \alpha x dx$

$\Rightarrow 2I = \left[\frac{\sin \alpha x}{\alpha}\right]_{-1}^{1}$

$\Rightarrow 2I = \frac{\sin \alpha}{\alpha} + \frac{\sin \alpha}{\alpha}$

$\Rightarrow I = \frac{\sin \alpha}{\alpha} = \frac{2}{\pi}$

$\Rightarrow \alpha = \frac{\pi}{2}$

Q.37. Find the area bounded by the curves $y^2 \leq 2x$ and $y \geq 4x - 1$.

A) $\frac{32}{9}$

B) $\frac{9}{32}$

C) 32

D) $\frac{1}{9}$

Answer: $\frac{9}{32}$

Solution: Given: $y^2 \leq 2x$ and $y \geq 4x - 1$

$\Rightarrow 2x = (4x-1)^2$

$\Rightarrow 2x = 16x^2 + 1 - 8x$

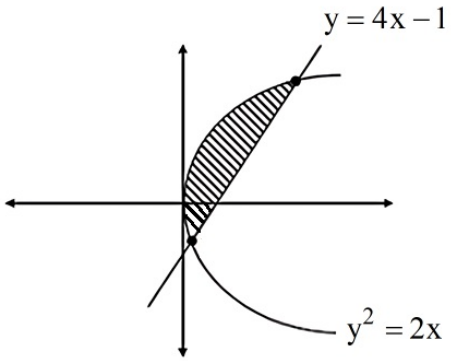
$\Rightarrow 16x^2 - 10x + 1 = 0$

$\Rightarrow (8x-1)(2x-1) = 0$

$\Rightarrow x = \frac{1}{2}, \frac{1}{8}$

$\Rightarrow y = 1, \frac{-1}{2}$

So, the points of intersection are $\left(\frac{1}{2}, 1\right)$ and $\left(\frac{1}{8}, \frac{-1}{2}\right)$.



So, the required area is given by,

$A = \int_{-\frac{1}{2}}^{1} \left(\frac{y+1}{4} - \frac{y^2}{2}\right) dy$

$\Rightarrow A = \left[\frac{(y+1)^2}{8} - \frac{y^3}{6}\right]_{-\frac{1}{2}}^{1}$

$\Rightarrow A = \left(\frac{1}{2} - \frac{1}{6}\right) - \left(\frac{1}{32} + \frac{1}{48}\right)$

$\Rightarrow A = \frac{1}{3} - \frac{5}{96} = \frac{32-5}{96}$

$\Rightarrow A = \frac{9}{32}$

Q.38. If $f(x) = \int_0^x t + \sin\left(1 - e^t\right) \mathrm{d}\, t,\ f(0) = 0$, then find the value of $\lim_{x \to 0} \frac{f(x)}{x^3}$

A) $\frac{1}{12}$

B) $\frac{1}{6}$

C) $\frac{-1}{6}$

D) $\frac{-1}{12}$

Answer: $\frac{-1}{6}$

Solution: Given,

$f(x)=\int_0^x t+\sin\left(1-e^t\right)\mathrm{d}\,t,\ f(0)=0$

Now, using Newton Leibnitz Theorem we get,

$\Rightarrow f'(x)=x+\sin\left(1-e^x\right)$

Now, solving limit $\lim\limits_{x\to0}\frac{f(x)}{x^3}$ we get, $\frac{0}{0}$ form,

So, applying L-hospital rule we get,

$\lim\limits_{x\to0}\frac{f(x)}{x^3}=\lim\limits_{x\to0}\frac{f'(x)}{3x^2}$

Now, again limit is form of $\frac{0}{0}$ as $f'(0)=0+\sin\left(1-e^0\right)=0$

Now, again using L-hospital rule we get,

$\lim\limits_{x\to0}\frac{f(x)}{x^3}=\lim\limits_{x\to0}\frac{f'(x)}{3x^2}=\lim\limits_{x\to0}\frac{f''(x)}{6x}$

Now, finding $f''(x)=1+\cos\left(1-e^x\right)\left(-e^x\right)$

$\Rightarrow f''(0)=1+\cos\left(1-e^0\right)\left(-e^0\right)=0$

So, $\lim\limits_{x\to0}\frac{f''(x)}{6x}$ is again $\frac{0}{0}$ form

So, again using L-hospital rule we get,

$\lim\limits_{x\to0}\frac{f(x)}{x^3}=\lim\limits_{x\to0}\frac{f'(x)}{3x^2}=\lim\limits_{x\to0}\frac{f''(x)}{6x}=\lim\limits_{x\to0}\frac{f'''(x)}{6}$

Now, finding $f'''(x)=0-\sin\left(1-e^x\right)\left(-e^x\right)\left(-e^x\right)+\cos\left(1-e^x\right)\left(-e^x\right)$

$\Rightarrow f'''(0)=0-\sin\left(1-e^0\right)\left(-e^0\right)\left(-e^0\right)+\cos\left(1-e^0\right)\left(-e^0\right)=-1$

Hence, $\lim\limits_{x\to0}\frac{f(x)}{x^3}=\lim\limits_{x\to0}\frac{f'(x)}{3x^2}=\lim\limits_{x\to0}\frac{f''(x)}{6x}=\lim\limits_{x\to0}\frac{f'''(x)}{6}=\frac{-1}{6}$

Q.39. If for $f(x)=3\sqrt{x-2}+\sqrt{4-x}$ the maximum value is α and the minimum value is β, then $\alpha^2+\beta^2$ is

A) 22

B) 18

C) 10

D) 2

Answer: 22

Solution: Given: $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$

$\Rightarrow x - 2 \geq 0,\ 4 - x \geq 0$

$\Rightarrow x \in [2, 4] \quad \ldots(i)$

Now, $f'(x) = \frac{3}{2\sqrt{x-2}} + \frac{-1}{2\sqrt{4-x}} = 0$

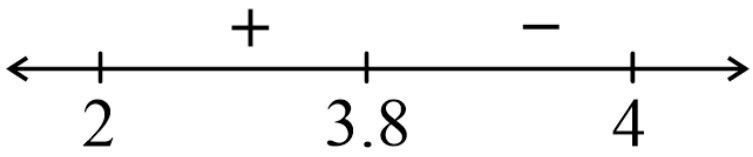
$\Rightarrow \frac{3}{2\sqrt{x-2}} = \frac{1}{2\sqrt{4-x}}$

$\Rightarrow 9(4 - x) = x - 2$

$\Rightarrow 36 - 9x = x - 2$

$\Rightarrow 10x = 38$

$\Rightarrow x = 3.8$



So, the extreme values will be at $x = (2 \text{ or } 4) \text{ and } 3.8$

$\Rightarrow f(2) = \sqrt{2},\ f(4) = 3\sqrt{2}$

$\Rightarrow f(3.8) = 3\sqrt{1.8} + \sqrt{0.2}$

$\Rightarrow f(3.8) = \frac{9\sqrt{2}+\sqrt{2}}{\sqrt{10}}$

$\Rightarrow f(3.8) = \sqrt{20}$

$\Rightarrow \alpha^2 + \beta^2 = 2 + 20 = 22$

Q.40. If $\sin^{-1}x + \cos^{-1}y = \alpha,\ \alpha \in \left(\frac{-\pi}{2}, \pi\right)$ then find the value of $x^2 + y^2 - 2xy\sin\alpha$

A) $\sin^2\alpha$

B) $\cos^2\alpha$

C) $\sin 2\alpha$

D) $\cos 2\alpha$

Answer: $\cos^2\alpha$

Solution: Let, $\sin^{-1}x = A\ \&\ \cos^{-1}y = B$

Given, $A + B = \alpha$

$\Rightarrow \sin(A + B) = \sin\alpha$

$\Rightarrow \sin A\cos B + \sin B\cos A = \sin\alpha$

$\Rightarrow xy + \sqrt{1-x^2}\sqrt{1-y^2} = \sin\alpha$

$\Rightarrow \sqrt{1-x^2}\sqrt{1-y^2} = \sin\alpha - xy$

$\Rightarrow \left(1-x^2\right)\left(1-y^2\right) = (\sin\alpha - xy)^2$

$\Rightarrow 1 - x^2 - y^2 + x^2y^2 = \sin^2\alpha + x^2y^2 - 2xy\sin\alpha$

$\Rightarrow 1 - x^2 - y^2 = \sin^2\alpha - 2xy\sin\alpha$

$\Rightarrow x^2 + y^2 - 2xy\sin\alpha = 1 - \sin^2\alpha = \cos^2\alpha$

Q.41. Let $f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos 2x}}, & x \neq 0 \\ a\log 2 \times \log 3, & x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$, then the value of a is

A) $6\sqrt{2}$

B) $8\sqrt{2}$

C) $4\sqrt{2}$

D) $2\sqrt{2}$

Answer: $6\sqrt{2}$

Solution:

Given: $f(x)=\begin{cases}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos 2x}}, & x\neq 0\\ a\log 2\times\log 3, & x=0\end{cases}$ is continuous at $x=0$.

$\Rightarrow f(0)=\lim\limits_{x\to 0} f(x)$

$\Rightarrow a\log 2\times\log 3=\lim\limits_{x\to 0}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos 2x}}$

$\Rightarrow a\log 2\times\log 3=\lim\limits_{x\to 0}\frac{\left(9^x-1\right)\left(8^x-1\right)\left(\sqrt{2}+\sqrt{1+\cos 2x}\right)}{1-\cos 2x}$

$\Rightarrow a\log 2\times\log 3=\lim\limits_{x\to 0}\frac{\left(9^x-1\right)\left(8^x-1\right)\left(\sqrt{2}+\sqrt{1+\cos 2x}\right)}{\frac{2\sin^2 x}{x^2}\times x^2}$

$\Rightarrow a\log 2\times\log 3=\lim\limits_{x\to 0}\frac{\left(\frac{9^x-1}{x}\right)\left(\frac{8^x-1}{x}\right)\left(\sqrt{2}+\sqrt{1+\cos 2x}\right)}{\frac{2\sin^2 x}{x^2}}$

$\Rightarrow a\log 2\times\log 3=\frac{1}{2}\log 9\times\log 8\times 2\sqrt{2}$

$\Rightarrow a\log 2\times\log 3=\frac{1}{2}(2\log 3)\times(3\log 2)\times 2\sqrt{2}$

$\Rightarrow a=6\sqrt{2}$

Q.42. A parabola $y^2=12x$ has a chord PQ with midpoint $(4,1)$ then equation of PQ passes through:

A) $\left(-10,\frac{-1}{2}\right)$

B) $\left(\frac{1}{2},-10\right)$

C) $\left(10,\frac{1}{2}\right)$

D) $\left(\frac{1}{2},-20\right)$

Answer: $\left(\frac{1}{2},-20\right)$

Solution:

Given,

$y^2=12x$ has chord PQ with midpoint $(4,1)$

Now, using the formula $T=S_1$ we get,

$y-6(x+4)=1-48$

$\Rightarrow y-6x+23=0$

Now, from given options only $\left(\frac{1}{2},-20\right)$ satisfy the given equation.

Q.43. $\frac{1\times 2^2+2\times 3^2+3\times 4^2+.....+100\times 101^2}{1^2\times 2+2^2\times 3+3^2\times 4+.....+100^2\times 101}=$

A) $\frac{305}{301}$

B) $\frac{301}{305}$

C) $\frac{300}{301}$

D) $\frac{301}{300}$

Answer: $\frac{305}{301}$

Solution: General term of $\frac{1\times2^2+2\times3^2+3\times4^2+.....+100\times101^2}{1^2\times2+2^2\times3+3^2\times4+.....+100^2\times101}$ is given by,

$$S=\frac{\sum_{r=1}^{100} r(r+1)^2}{\sum_{r=1}^{100} r^2(r+1)}$$

$$\Rightarrow S=\frac{\sum_{r=1}^{100} r\left(r^2+1+2r\right)}{\sum_{r=1}^{100} r^3+\sum_{r=1}^{100} r^2}$$

$$\Rightarrow S=\frac{\sum_{r=1}^{100} r^3+2\sum_{r=1}^{100} r^2+\sum_{r=1}^{100} r}{\sum_{r=1}^{100} r^3+\sum_{r=1}^{100} r^2}$$

$$\Rightarrow S=\frac{\left[\frac{100(100+1)}{2}\right]^2+\frac{200(100+1)(200+1)}{6}+\frac{100(100+1)}{2}}{\left[\frac{100(100+1)}{2}\right]^2+\frac{100(100+1)(200+1)}{6}}$$

$$\Rightarrow S=\frac{\frac{100^2\times101^2}{4}+\frac{100\times101\times201}{3}+\frac{100\times101}{2}}{\frac{100^2\times101^2}{4}+\frac{100\times101\times201}{6}}$$

$$\Rightarrow S=\frac{\frac{100\times101}{4}+\frac{201}{3}+\frac{1}{2}}{\frac{100\times101}{4}+\frac{201}{6}}$$

$$\Rightarrow S=\frac{2525+67+\frac{1}{2}}{2525+\frac{67}{2}}$$

$$\Rightarrow S=\frac{305}{301}$$

Q.44. If centre of circle is $(0,0)$ and radius is $\sqrt{10}$ and $x+y=2$ is a chord and another chord of slope $m=-1$ has length 2 unit then least possible distance between this chord and the line will be

A) $\frac{\sqrt{2}-1}{\sqrt{2}}$

B) $\frac{3\sqrt{2}-2}{\sqrt{2}}$

C) $\frac{3-\sqrt{2}}{\sqrt{2}}$

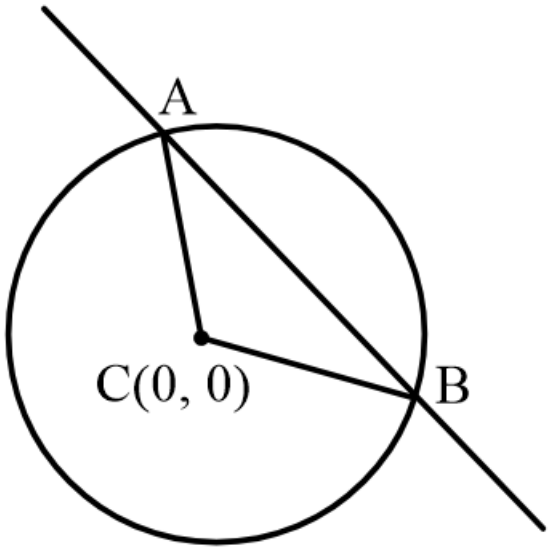
D) $\frac{4\sqrt{2}-1}{\sqrt{2}}$

Answer: $\frac{3\sqrt{2}-2}{\sqrt{2}}$

Solution: Given,

Circle with centre $(0,0)$ and radius $\sqrt{10}$

Now, plotting the diagram we get,



Now, line with slope $m=-1$ will be AB and its equation will be $x+y=c$

And its distance from origin will be 3 as radius is given as $\sqrt{10}$ and length of chord is 2

So, using distance formula we get,

$\left|\frac{c}{\sqrt{2}}\right|=3$

$\Rightarrow c=\pm 3\sqrt{2}$

So, equation of line will be $x+y=\pm 3\sqrt{2}$

And other equation of line is $x+y=2$,

Then minimum distance is given by, $\left|\frac{3\sqrt{2}-2}{\sqrt{2}}\right|$

Q.45. If $\int(\operatorname{cosec}\theta+\cot\theta)^5 d\theta=\alpha(f(x))^4+\beta(f(x))^2+\gamma\log\left|1+(f(x))^2\right|+C$, where C is constant of integration, then find $|2\alpha+\beta+\gamma|$.

A) 1

B) 2

C) 3

D) 4

Answer: 1

Solution: Let, $I = \int (\operatorname{cosec}\theta + \cot\theta)^5 d\theta$

Putting $\operatorname{cosec}\theta + \cot\theta = t \quad \ldots(i)$

$\Rightarrow \left(-\operatorname{cosec}\theta\cot\theta - \operatorname{cosec}^2\theta\right)d\theta = dt$

$\Rightarrow -\operatorname{cosec}\theta \times t = \frac{dt}{d\theta}$

Now, by using (i), $\operatorname{cosec}\theta - \cot\theta = \frac{1}{t} \quad \ldots(ii)$

Adding (i) and (ii),

$\Rightarrow \cos ec\theta = \frac{1}{2}\left(t + \frac{1}{t}\right)$

$\Rightarrow -\left(\frac{t^2+1}{2t}\right) \times t = \frac{dt}{d\theta}$

$\Rightarrow d\theta = \left(\frac{-2}{1+t^2}\right)dt$

$\Rightarrow I = \int t^5\left(\frac{-2}{1+t^2}\right)dt$

$\Rightarrow I = \int -2\left(\frac{t^5+t^3-t^3+t-t}{1+t^2}\right)dt$

$\Rightarrow I = \int -2\left(t^3 - t + \frac{t}{1+t^2}\right)dt$

$\Rightarrow I = \int -2\left(\frac{t^4}{4} - \frac{t^2}{2} + \frac{\log|1+t^2|}{2}\right)dt$

$\Rightarrow \alpha = \frac{-1}{2},\ \beta = 1,\ \gamma = -1$

$\Rightarrow |2\alpha + \beta + \gamma| = |-1 + 1 - 1| = 1$

Q.46. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = I + (adjA) + \left(adjA^2\right) + \ldots\ldots 10$ terms then B is

A) $\begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

B) $11\begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

C) $10\begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

D) $10\begin{bmatrix} 1 & -11 \\ 0 & 1 \end{bmatrix}$

Answer: $11\begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

Solution: Given,

$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Now, finding $adjA = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

And $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ and $adjA^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

Now, solving

$B = I + (adjA) + (adjA)^2 + \ldots\ldots 10 \text{ terms}$

$\Rightarrow B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} + \ldots\ldots + \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} 11 & -2-4-\ldots 20 \\ 0 & 11 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} 11 & -10 \times 11 \\ 0 & 11 \end{bmatrix}$

$\Rightarrow B = 11\begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

Q.47. If a relation $(x_1, y_1)R(x_2, y_2)$ is defined as $\{(x, y) \in N,\ x_1 \leq x_2,\ y_1 \leq y_2\}$, then the relation is

A) reflexive and symmetric

B) symmetric and transitive

C) transitive and reflexive

D) None of these

Answer: symmetric and transitive

Solution: We know that $x_1 \leq x_1$ & $y_1 \leq y_1$

So, the relation is reflexive.

Also, $x_1 \leq x_2 \not\Rightarrow x_2 \leq x_1$

i.e. the relation is not symmetric.

Now, if $x_1 \leq x_2$ & $x_2 \leq x_3 \Rightarrow x_1 \leq x_3$

i.e. the relation is transitive.

Q.48. If $a,\ b$ & c are in $A.P.$, $a+1,\ b$ & $c+3$ are in $G.P.$ where $a > 10$ and $A.M.$ of a, b & c is 8 then find the $(G.M.)^3$ of $a,\ b$ & c

Answer: 120

Solution: Given,

$a,\ b\ \&\ c$ are in $A.P.$

So, $2b = a + c\ \ .....(i)$

$a+1,\ b\ \&\ c+3$ are in $G.P.$

So, $b^2 = (a+1)(c+3)\ \ ....(ii)$

And $A.M.$ of $a, b\ \&\ c$ is 8

So, $\frac{a+c}{2} = b = 8\ ...(iii)$

Now, solving above equations we get,

$b^2 = (a+1)(c+3)$

$\Rightarrow \left(\frac{a+c}{2}\right)^2 = ac + 3a + c + 3$

$\Rightarrow 64 = a(16-a) + 2a + 16 + 3$

$\Rightarrow a^2 - 18a + 45 = 0$

$\Rightarrow (a-3)(a-15) = 0$

$\Rightarrow a = 15\ \{\text{as } a > 10\}$

Now, from equation (iii) we get, $c = 1$

Hence, $(G.M.) = (a \times b \times c)^{\frac{1}{3}} = 120^{\frac{1}{3}}$

$\Rightarrow (G.M.)^3 = 120$

Q.49. If in expansion of $(1+x)^n$, coefficient of $x^4,\ x^5\ \&\ x^6$ are in A.P then maximum possible value of n is

Answer: 14

Solution: Given,

Coefficient of $x^4,\ x^5\ \&\ x^6$ are in A.P in the expansion of $(1+x)^n$

So, ${}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$

$\Rightarrow \frac{n!}{5!(n-5)!} - \frac{n!}{4!(n-4)!} = \frac{n!}{6!(n-6)!} - \frac{n!}{5!(n-5)!}$

$\Rightarrow \frac{1}{5!(n-5)!} - \frac{1}{4!(n-4)!} = \frac{1}{6!(n-6)!} - \frac{1}{5!(n-5)!}$

$\Rightarrow \frac{1}{4!(n-5)!}\left(\frac{1}{5} - \frac{1}{n-4}\right) = \frac{1}{5!(n-6)!}\left(\frac{1}{6} - \frac{1}{n-5}\right)$

$\Rightarrow \frac{1}{(n-5)}\left(\frac{1}{5} - \frac{1}{n-4}\right) = \frac{1}{5}\left(\frac{1}{6} - \frac{1}{n-5}\right)$

$\Rightarrow \frac{1}{(n-5)}\left(\frac{n-4-5}{5(n-4)}\right) = \frac{1}{5}\left(\frac{n-5-6}{n-5}\right)$

$\Rightarrow 6(n-9) = (n-11)((n-4))$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow (n-7)(n-14) = 0$

Hence, the maximum value of $n = 14$

Q.50. If $\frac{dy}{dx} = \frac{1}{(x+y+2)^2}$ and $f(0) = 0$, then $f(x) = \tan^{-1}\left(\frac{x+y}{2x+2y+\lambda}\right)$. Then find λ.

Answer: 5

Solution: Given: $\frac{dy}{dx}=\frac{1}{(x+y+2)^2}$

$\Rightarrow \frac{dx}{dy}=(x+y+2)^2$

Let, $x+y+2=t$

$\Rightarrow 1+\frac{dx}{dy}=\frac{dt}{dy}$

$\Rightarrow \frac{dt}{dy}-1=t^2$

$\Rightarrow \frac{dt}{1+t^2}=dy$

$\Rightarrow \int\frac{dt}{1+t^2}=\int dy$

$\Rightarrow \tan^{-1}t=y+C$

$\Rightarrow \tan^{-1}(x+y+2)=y+C$

It is given that, $f(0)=0$.

$\Rightarrow \tan^{-1}(2)=C$

$\Rightarrow \tan^{-1}(x+y+2)=y+\tan^{-1}(2)$

$\Rightarrow \tan^{-1}\left(\frac{x+y}{1+2(x+y+2)}\right)=y$

$\Rightarrow \tan^{-1}\left(\frac{x+y}{2x+2y+5}\right)=y=f(x)$

$\Rightarrow \lambda=5$

Q.51. Team A has 4 men and 5 women. Team B has 4 women and 5 men. In how many ways can we pick 4 from each team such that there are 4 men and 4 women?

Answer: 5626

Solution: Given: $\text{Team } A\rightarrow 4M,\ 5W\ ;\ \text{Team } B\rightarrow 5M,\ 4W$

If x men are chosen from team A then $(4-x)$ men are to be chosen from team B. Similarly, if x women are chosen from team A then $(4-x)$ women are to be chosen from team B.

So, the required number of ways is given by,

$N=\left({}^4C_x\times{}^5C_{4-x}\right)\times\left({}^4C_x\times{}^5C_{4-x}\right)$

$\Rightarrow N=\left({}^4C_x\times{}^5C_{4-x}\right)^2$

Putting, $x=0,\ 1,\ 2,\ 3 \text{ and } 4$

$\Rightarrow N=\left({}^4C_0\times{}^5C_4\right)^2+\left({}^4C_1\times{}^5C_3\right)^2+\left({}^4C_2\times{}^5C_2\right)^2+\left({}^4C_3\times{}^5C_1\right)^2+\left({}^4C_4\times{}^5C_0\right)^2$

$\Rightarrow N=25+(4\times10)^2+(6\times10)^2+(4\times5)^2+(1)^2$

$\Rightarrow N=25+1600+3600+400+1$

$\Rightarrow N=5626$