JEE Main 4th April Session 1 - Memory Based Paper

4th April Session 1

Questions

Q.1. Five identical lenses are placed one after the other in close contact. The power of this arrangement is $25 \mathrm{~D}$. Then, the power of one such lens is

A) $5 \mathrm{~D}$

B) $10 \mathrm{~D}$

C) $20 \mathrm{~D}$

D) $125 \mathrm{~D}$

Answer: $5 \mathrm{~D}$

Solution: The net power of the combination of lenses is equal to the algebraic sum of powers of all lenses in the combination, that is

$$P_{net}=P_1+P_2+P_3+\ldots.$$
$$\Rightarrow 25=P+P+P+P+P$$
$$\Rightarrow P=\frac{25}{5}=5 \mathrm{~D}$$

Q.2. A metal wire of mass M and length l is bent to form a circle. A particle mass m is kept at the centre of the semi circle. Find the forced experienced by m.

A) $\frac{2\pi G M m}{L^2}$

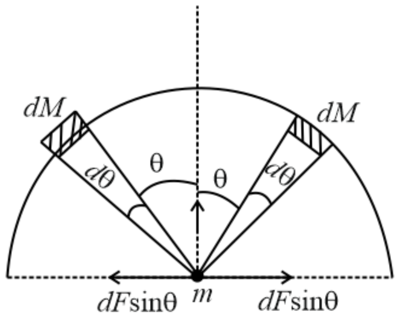
B) $\frac{\pi G M m}{L^2}$

C) $\frac{2\pi G M m}{3L^2}$

D) $\frac{3\pi G M m}{2L^2}$

Answer: $\frac{2\pi G M m}{L^2}$

Solution: Consider small element of mass $\mathrm{d}M$ on either sides of the vertical line, symmetrically, as shown in the image below.



Since, the length of the wire is L, we have,

$$\pi r = L$$
$$\Rightarrow r = \frac{L}{\pi}$$

The wire subtends an angle of $\pi\ \mathrm{rad}$ at the centre.

Hence, the mass per unit angle is $\frac{M}{\pi}$.

Hence, we have,

$$\mathrm{d}M = \frac{M}{\pi}\mathrm{d}\theta$$

The gravitational force due to this elemental mass is $\mathrm{d}F = G\frac{m.\mathrm{d}M}{r^2} = G\frac{m}{\pi r^2}M\mathrm{d}\theta$.

The gravitational force can be resolved into mutually perpendicular components.

The horizontal components cancel and the vertical ones add up.

The net vertical force due to the symmetric mass on either sides is

$$\mathrm{d}F_{\mathrm{net}} = 2\,\mathrm{d}\,F\cos(\theta) = 2G\frac{m}{\pi r^2}M\cos\theta\mathrm{d}\theta$$

To obtain the total force, we shall integrate the above equation between the limits 0 and $\frac{\pi}{2}$ rad.

$$F_{\mathrm{net}} = \int_0^{\frac{\pi}{2}} 2G\frac{m}{\pi r^2}M\cos\theta\mathrm{d}\theta = 2\frac{GMm}{\pi r^2}[\sin\theta]_0^{\frac{\pi}{2}} = 2\frac{GMm}{\pi r^2}$$

Substituting the value of radius in terms of length of the wire, we finally get,

$$F = 2\frac{GMm}{\pi r^2} = 2\frac{GMm}{\pi\left(\frac{L}{\pi}\right)^2} = 2\pi\frac{GMm}{L^2}.$$

Q.3. Position of a particle is related to time as given by equation $x = \left(t^4 + 6t^2 + 2t\right)$ m. Find its acceleration at $t = 5$ s in $\mathrm{m\ s^{-2}}$.

A) 212
B) 312
C) 112
D) 56

Answer: 312

Solution: Velocity of the particle is given by, $v = \frac{dx}{dt} = 4t^3 + 6\times(2t) + 2 = 4t^3 + 12t + 2$.

Now, acceleration of the particle can be written as,

$$a = \frac{dv}{dt} = 4\left(3t^2\right) + 12 = 12t^2 + 12$$
$$\text{At } t = 5\ \mathrm{s}$$
$$\Rightarrow \mathrm{a} = 12\times 5^2 + 12 = 312\ \mathrm{m\ s^{-2}}$$

Q.4. A mass m is dropped on a ground from height h and it rebounds to a height of $\frac{h}{2}$ after hitting the ground for the first time. Find the loss in energy during the first time it hits the ground and also speed with which it will hit the ground for the second time.

A) $\frac{mgh}{2}, \sqrt{gh}$

B) $mgh, \sqrt{2gh}$

C) $\frac{mgh}{4}, \sqrt{\frac{gh}{2}}$

D) $\frac{mgh}{8}, \sqrt{gh}$

Answer: $\frac{mgh}{2}, \sqrt{gh}$

Solution: Initial potential energy:

$U_i = mgh$

Potential energy when it reaches the height $\frac{h}{2}$:

$U_f = mg\frac{h}{2}$

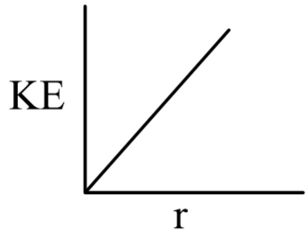
Therefore, loss in potential energy

$U_{loss} = U_i - U_f = \frac{mgh}{2}$.

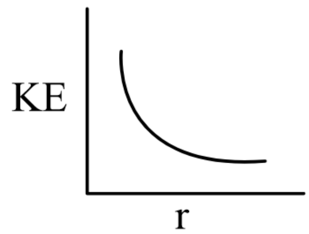
Speed after first rebound when it falls on the ground from height $\frac{h}{2}$, $v = \sqrt{2g\frac{h}{2}} = \sqrt{gh}$

Q.5. An electron is revolving around an infinite wire. Which graph correctly shows the relation between kinetic energy of electron and the distance from the wire.

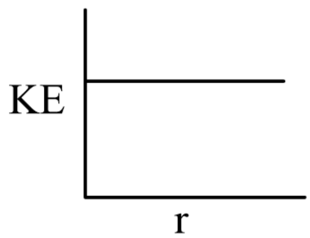
A)



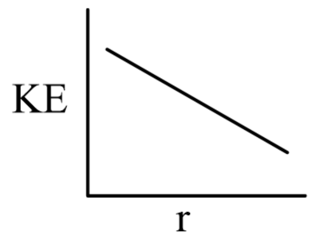
B)



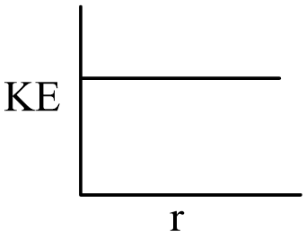
C)



D)



Answer:



Solution: Electric field due to infinite wire is given by, $E = \frac{2k\lambda}{r}$.

Now, electrostatic force acting on the electron will provide the required centripetal force.

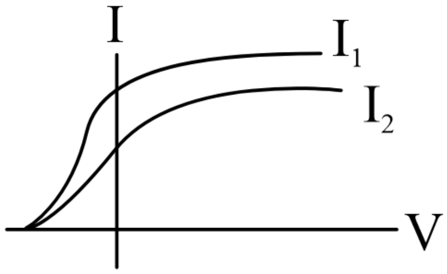
Therefore,

$$e \times \frac{2k\lambda}{r} = \frac{mv^2}{r}$$
$$\Rightarrow \frac{1}{2}mv^2 = ek\lambda$$

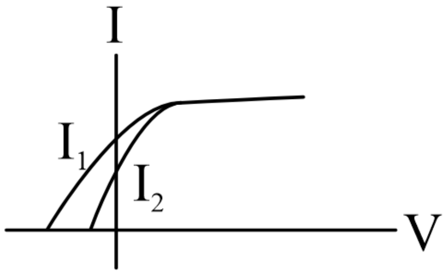
As we can see, the kinetic energy is a constant value and doesn't depend on r.

Q.6. Which of the following graph correctly represents the effect of increase in intensity$(I_2 > I_1)$ of light falling on a metal in photoelectric effect.

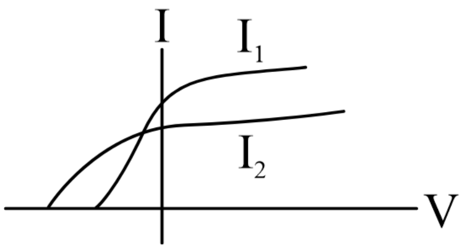
A)



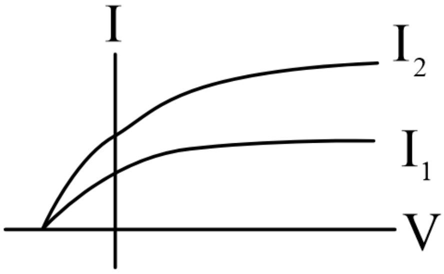
B)



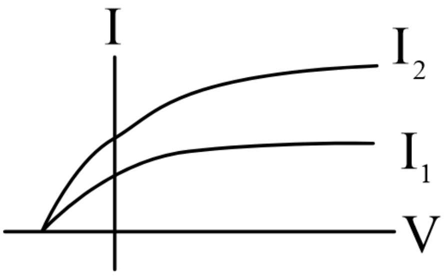
C)



D)

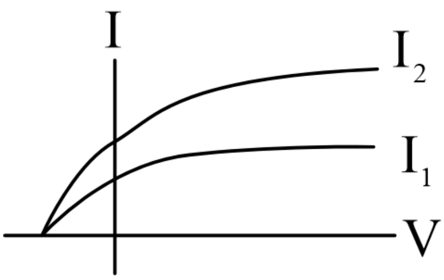


Answer:



Solution: Increasing the intensity while keeping frequency fixed, increases the number of photons hitting the metal and hence, it increases the rate at which electrons are emitted.

Therefore, graph would be as given below



Q.7. A hollow cylinder and solid cylinder of same mass and radius are rolling with same initial velocity v on a rough inclined plane. Find the ratios of their kinetic energies and maximum height reached by them.

A) $\frac{K_H}{K_S}=\frac{3}{4}\text{ and}\frac{h_H}{h_S}=\frac{4}{3}$

B) $\frac{K_H}{K_S}=\frac{1}{3}\text{ and}\frac{h_H}{h_S}=\frac{2}{3}$

C) $\frac{K_H}{K_S}=\frac{2}{3}\text{ and}\frac{h_H}{h_S}=\frac{1}{3}$

D) $\frac{K_H}{K_S}=\frac{4}{3}\text{ and}\frac{h_H}{h_S}=\frac{4}{3}$

Answer: $\frac{K_H}{K_S}=\frac{4}{3}\text{ and}\frac{h_H}{h_S}=\frac{4}{3}$

Solution: Kinetic energy of a rolling body is given by,

$k=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

For hollow cylinder $I=mR^2$

$\Rightarrow k_H=\frac{1}{2}mv^2+\frac{1}{2}mv^2=mv^2$

For solid cylinder, $I=\frac{1}{2}mR^2$

$\Rightarrow k_S=\frac{1}{2}mv^2+\frac{1}{4}mv^2=\frac{3}{4}mv^2$

$\Rightarrow \frac{k_H}{k_S}=\frac{4}{3}$

For maximum height applying conversation of mechanical energy we get,

$mgh_H=k_H$

$\Rightarrow mgh_H=mv^2$

For solid cylinder

$mgh_S=k_S$

$\Rightarrow mgh_s=\frac{3}{4}mv^2$

$\Rightarrow \frac{h_H}{h_S}=\frac{4}{3}$

Q.8. In a given equation $y=A\sin\omega t\cos\left(\frac{nx}{\lambda}\right)$. Find the dimensions of n

A) $[MLT]$

B) $\left[M^0L^0T^0\right]$

C) $\left[ML^0T\right]$

D) $\left[MLT^{-2}\right]$

Answer: $\left[M^0L^0T^0\right]$

Solution: A term in a cosine function is dimension less.

Therefore,

$\left[M^0L^0T^0\right]=\frac{[n]\times[x]}{[\lambda]}$

$\Rightarrow \left[M^0L^0T^0\right]=\frac{[L]\times[x]}{[L]}$

$\left[M^0L^0T^0\right]=[x]$

Q.9. In YDSE if the m^{th} maximum of $\lambda_1=\ 450\ \mathrm{nm}$ coincides with n^{th} maximum of $\lambda_2=650\ \mathrm{nm}$, then find the minimum possible value of $m(m\neq 0)$.

A) 11

B) 7

C) 13

D) 9

Answer: 13

Solution: For both maxima to coincide, position of both maxima from the axis will be the same. Therefore,

$y_1=y_2$

$\Rightarrow m\beta_1=n\beta_2$

$\Rightarrow m\frac{\lambda_1D}{d}=n\frac{\lambda_2D}{d}$

$\Rightarrow m\times 450=n\times 650$

$\Rightarrow \frac{m}{n}=\frac{13}{9}$

So for minimum integral value of m, we get $m=13$

Q.10. If the current is given by $i = 6 + \sqrt{56}\sin\omega t\ \mathrm{A}$. Then what will be the RMS current?

A) $64\ \mathrm{A}$

B) $8\ \mathrm{A}$

C) $32\ \mathrm{A}$

D) $\sqrt{28}\ \mathrm{A}$

Answer: $8\ \mathrm{A}$

Solution: If the current is given as $i = A + B\sin\omega t$

The RMS current is given by $i_{RMS} = \sqrt{A^2 + \frac{B^2}{2}}$

Therefore, $i_{RMS} = \sqrt{6^2 + \frac{56}{2}} = 8\ \mathrm{A}$

Q.11. For a particle in motion in plane x & y coordinates can be expressed as $x = 2 + 4t,\ y = 3 + 8t^2$. Here x and y are in meters and t is in seconds. Then which of the following is false?

A) Uniform accelerated motion

B) Constant velocity along x

C) Parabolic trajectory

D) Particle will pass through origin

Answer: Particle will pass through origin

Solution: Let the particle passed through the origin, then

$y = 3 + 8t^2$
$\Rightarrow 0 = 3 + 8t^2$
$\Rightarrow t = \sqrt{\frac{-3}{8}}$

which is not possible. So for no value of time, y co-ordinate of the particle can be zero. Hence, this statement is false.

Q.12. If the electric field vector at a point in an electromagnetic wave is given by, $\vec{E} = 40\cos\omega\left(t - \frac{Z}{C}\right)\hat{\imath}$, then corresponding magnetic field vector will be

A) $\vec{B} = \frac{40}{3} \times 10^{-8}\cos\omega\left(t - \frac{Z}{C}\right)\hat{\jmath}$

B) $\vec{B} = \frac{40}{3} \times 10^{-8}\cos\omega\left(t - \frac{Z}{C}\right)\hat{k}$

C) $\vec{B} = \frac{40}{3} \times 10^{-8}\cos\omega\left(t - \frac{Z}{C}\right)\left(\hat{\jmath} + \hat{k}\right)$

D) $\vec{B} = \frac{40}{3} \times 10^{-8}\cos\omega\left(t - \frac{Z}{C}\right)\left(\hat{\jmath} - \hat{k}\right)$

Answer: $\vec{B} = \frac{40}{3} \times 10^{-8}\cos\omega\left(t - \frac{Z}{C}\right)\hat{\jmath}$

Solution: As we know,

$\frac{E}{B} = C$
$\Rightarrow B = \frac{E}{C} = \frac{40}{3 \times 10^8}$

Also, direction vector will follow the relation

$\widehat{E} \times \widehat{B} = \widehat{C}$.

Therefore,

$\hat{\imath} \times \widehat{B} = \hat{k}$
$\Rightarrow \widehat{B} = \hat{\jmath}$

Hence, required expression is $\vec{B} = \frac{40}{3} \times 10^{-8}\cos\omega\left(t - \frac{Z}{C}\right)\hat{\jmath}$

Q.13. Find out the electric field at the centre of a hollow hemisphere with the surface charge density σ on the sphere

A) $\frac{\sigma}{\varepsilon_0}$

B) $\frac{\sigma}{2\varepsilon_0}$

C) $\frac{\sigma}{3\varepsilon_0}$

D) $\frac{\sigma}{4\varepsilon_0}$

Answer: $\frac{\sigma}{4\varepsilon_0}$

Solution: Electric field at the centre of a hollow hemisphere is given by $\frac{\sigma}{4\varepsilon_0}$

Q.14. For the 4^{th} orbit of the Hydrogen atom the De-Broglie wavelength of the electron is $\alpha \pi r_0$, where r_0 is Bohr radius. The value of α is

Answer: 8

Solution: According to Bohr's quantisation law, $mvr = \frac{nh}{2\pi}$.

Using de Broglie law, momentum is given by $mv = \frac{h}{\lambda}$

Therefore,

$$\frac{hr}{\lambda} = \frac{nh}{2\pi}$$
$$\Rightarrow r = \frac{n\lambda}{2\pi}$$
$$\Rightarrow \lambda = \frac{2\pi r}{n}$$

Now, radius of orbit in Hydrogen atom is given by, $r = n^2 r_0$, where r_0 is the radius of the ground orbit.

Therefore, $\lambda = \frac{2\pi r}{n} = \frac{2\pi n^2 r_0}{n} = 2\pi n r_0$.

For $n = 4$, we get $\lambda = 8\pi r_0$, so $\alpha = 8$.

Q.15. Two forces F_1 & F_2 having magnitude as $|F_1| = 3|F_2|$, the resultant of the force is equal to F_1. If the angle between the forces be $\cos^{-1}\left(\frac{-1}{n}\right)$, write the value of n.

Answer: 6

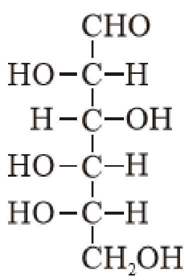
Solution: As we know, the resultant of the force using parallelogram law is given by,

$$F_{net} = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2\cos\theta}$$
$$\Rightarrow 3F_2 = \sqrt{(3F_2)^2 + (F_2)^2 + 2 \times 3F_2F_2\cos\theta}$$
$$\Rightarrow \cos\theta = \frac{-1}{6}$$

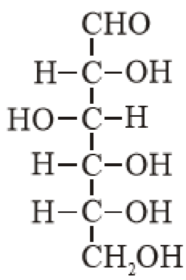
Therefore, $\theta = \cos^{-1}\left(\frac{-1}{6}\right)$. Hence, $n = 6$.

Q.16. Which of the following is the correct structure of L-Glucose?

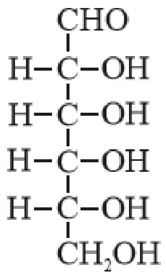
A)



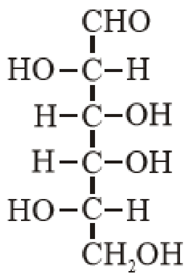
B)



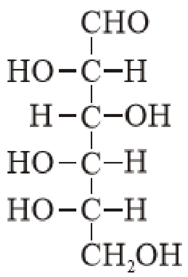
C)



D)

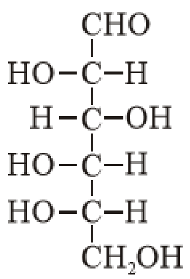


Answer:



Solution: It is a simple sugar having a molecular formula $C_6H_{12}O_6$. In simple terms, we can say that it is made up of six carbon atoms, twelve hydrogen atoms and six oxygen atoms. Glucose is a widely available monosaccharide and is also known as dextrose and blood sugar.

L-glucose is a short form of Levorotatory-glucose. It is an enantiomer of D-Glucose.



Q.17. Which of the following has the highest dipole moment?

A) NH_3

B) NF_3

C) PF_5

D) CH_4

Answer: NH_3

Solution: The molecules with perfect geometry and all the surrounding atoms have the same dipole moment zero. Hence, PCl_5 (trigonal bipyramidal) and CH_4 (tetrahedral) have dipole moment zero.

In NH_3, the dipole moment vector of the bond and the lone pairs are in the same direction. But in NF_3, the dipole moment vector of lone pairs and bond pairs are opposite in direction. So, the net dipole moment will be the subtractive effect between the two. Hence, the dipole moment of NH_3 is larger than NF_3.

Q.18. Which one out of the following shows one oxidation state other than its elemental state?

A) Ti

B) Sc

C) Co D) Ni

Answer: Sc

Solution: The oxidation state, or oxidation number, is the hypothetical charge of an atom if all of its bonds to other atoms were fully ionic.

Scandium usually exhibits an oxidation state of +3 in its compounds. In its elemental state, it exists as Scandium metal (Sc), and its most common oxidation state is +3 in compounds.

Ti can exhibit +2,+4 apart from +3. Co and Ni exhibit +2 apart from +3 state.

Q.19. Among the following, decreasing order of basic strength will be:

$$\underset{(\mathrm{I})}{\mathrm{OH}^-},\ \underset{(\mathrm{II})}{\mathrm{H}^-},\ \underset{(\mathrm{III})}{\mathrm{HCOO}^-},\ \underset{(\mathrm{IV})}{\mathrm{CH_3COO}^-},\ \underset{(\mathrm{V})}{\mathrm{RO}^-}$$

A) $\mathrm{II}>\mathrm{V}>\mathrm{III}>\mathrm{I}>\mathrm{IV}$ B) $\mathrm{III}>\mathrm{IV}>\mathrm{I}>\mathrm{V}>\mathrm{II}$

C) $\mathrm{II}>\mathrm{V}>\mathrm{I}>\mathrm{IV}>\mathrm{III}$ D) $\mathrm{V}>\mathrm{I}>\mathrm{IV}>\mathrm{II}>\mathrm{III}$

Answer: $\mathrm{II}>\mathrm{V}>\mathrm{I}>\mathrm{IV}>\mathrm{III}$

Solution: A strong base is a base, which ionises completely in an aqueous solution. The resonance stabilised anions are less basic than anions with no resonance. According to bronsted concept, if the acid is strong, its conjugate base is weak.

The acidic nature order for acids of given conjugate bases is

$\mathrm{H_2}<\mathrm{ROH}<\mathrm{HOH}<\mathrm{CH_3COOH}<\mathrm{HCOOH}$

Hence, the correct basic nature order is

$\mathrm{H}^->\mathrm{RO}^->\mathrm{OH}^->\mathrm{CH_3COO}^->\mathrm{HCOO}^-$

Q.20. Which of the following is the correct order of first ionization enthalpy?

A) $\mathrm{Be}<\mathrm{B}<\mathrm{O}<\mathrm{F}<\mathrm{N}$ B) $\mathrm{B}<\mathrm{Be}<\mathrm{O}<\mathrm{N}<\mathrm{F}$

C) $\mathrm{B}<\mathrm{Be}<\mathrm{N}<\mathrm{F}<\mathrm{O}$ D) $\mathrm{Be}<\mathrm{B}<\mathrm{N}<\mathrm{F}<\mathrm{O}$

Answer: $\mathrm{B}<\mathrm{Be}<\mathrm{O}<\mathrm{N}<\mathrm{F}$

Solution: Ionization enthalpy is defined as the minimum amount of energy that is required to remove the most loosely bounded electrons that is electron present in the outermost shell from an isolated gaseous atom.

The ionization enthalpy increases from left to right in a period and decreases down the group.

Ionisation energy of boron being unexpectedly less than that for beryllium due to the 2s orbital being totally filled in beryllium, whereas boron has one electron in a 2p orbital as well, and the 2s orbital is shielded much more than the 2p orbital.

The p orbital in nitrogen is more stable than in oxygen, which has one electron more than the half-filled configuration. So, the ionisation energy of nitrogen is more than that of oxygen

Q.21. Which of the following compounds does not show Lassaigne's test for nitrogen?

A) Hydrazine B) Phenylhydrazine

C) Glycine D) Urea

Answer: Hydrazine

Solution: The organic compounds (which have carbon atoms) along with nitrogen give Lassaigne’s test. It is because the carbon present will react with nitrogen and sodium metal added during this test to give NaCN(sodium cyanide). Sodium cyanide is converted to sodium ferrocyanide on treating with ferrous sulphate. On further treating it with ferric chloride, a prussian blue complex, ferricferrocyanide is formed.

Hydrazine ($\mathrm{NH_2NH_2}$) does not contain carbon. On fusion with Na metal, it cannot form NaCN. So, hydrazine does not show Lassaigne's test.

Q.22. In the precipitation of the iron group III in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to:

A) Prevent interference by phosphate ions B) Increase concentration of Cl^- ions

C) Decrease concentration of OH^- ions D) Increase concentration of $\mathrm{NH_4^+}$ ions

Answer: Decrease concentration of OH^- ions

Solution: $NH_4OH \leftrightharpoons NH_4^+ + OH^-$

$NH_4Cl \leftrightharpoons NH_4^+ + Cl^-$

The common ion here is NH_4^+

The common ion effect is an effect that suppresses the ionization of an electrolyte when another electrolyte (which contains an ion which is also present in the first electrolyte, i.e. a common ion) is added. It is considered to be a consequence of Le Chatlier's principle (or the Equilibrium Law).

Hence, here the concentration of OH^- decreases.

Q.23. Statement-1: Aldol condensation reaction is proceed due to acidic nature of alpha hydrogen.

Statement-2: Benzaldehyde and ethanal will not give cross aldol product.

A) Both Statement-1 and Statement-2 are true

B) Statement-1 is true and Statement-2 is false

C) Statement-1 is false and Statement-2 is true

D) Both Statement-1 and Statement-2 are false

Answer: Statement-1 is true and Statement-2 is false

Solution: The hydrogen atom on the $\alpha-$carbon of the carbonyl compound is relatively acidic due to the presence of the adjacent carbonyl group. This hydrogen can be abstracted by a base to generate the enolate ion, facilitating the aldol condensation reaction.

In the case of benzaldehyde (C_6H_5CHO) and ethanal (CH_3CHO), they can undergo a cross aldol condensation reaction to form a product. Benzaldehyde contains an aromatic ring, while ethanal is an aliphatic aldehyde. When these two reactants undergo aldol condensation, the resulting product can be a mixture of different isomeric forms.

Q.24. What is the pressure in bar of hydrogen gas, if $E_{H^+/H_2,Pt} = 0$ at $25^\circ C$?

Assume $[H^+] = 1M$

A) 0

B) 0.06

C) 0.03

D) 1

Answer: 0

Solution: The reaction at electrode is,

$2H^+(aq) + 2e^- \rightarrow H_2(g)$

Now, the Nernst equation is

$$E_{cell} = E^\circ_{cell} - \frac{0.06}{2}\log\frac{P_{H_2}}{[H^+]^2}$$

$$\Rightarrow E_{Cell} = 0 - 0.03\log\frac{P_{H_2}}{1}$$

So, the pressure of hydrogen gas is zero bar.

Q.25. Compare ligand strength of $F^-,\ OH^-,\ SCN^-,\ CO$

A) $CO > OH^- > F^- > SCN^-$

B) $CO > F^- > OH^- > SCN^-$

C) $SCN^- > OH^- > F^- > CO$

D) $F^- > CO > OH^- > SCN^-$

Answer: $CO > OH^- > F^- > SCN^-$

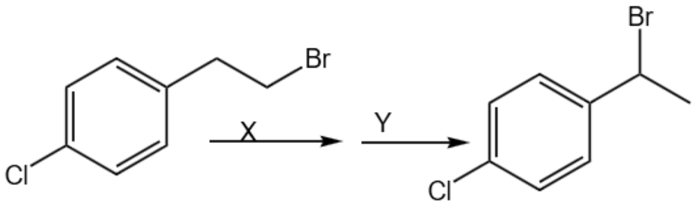
Solution: A ligand is an ion or molecule which donates a pair of electrons to the central metal atom or ion to form a coordination complex.

A strong ligand is able to pair the electrons due to large splitting in the d – orbital, while weak ligands do not pair up the electrons as they have less splitting.

The correct order of given ligands according spectrochemical series is

$CO > OH^- > F^- > SCN^-$

Q.26.



Identify X and Y in the above conversion from the following.

A) $\text{dil. NaOH}/20^\circ C$; HBr/CH_3COOH

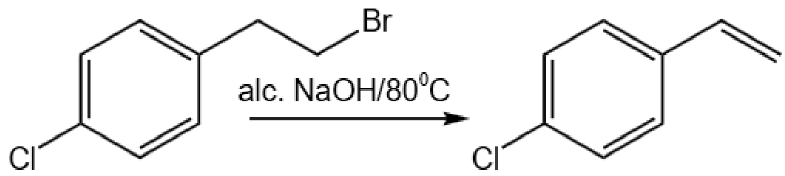
B) $\text{dil. NaOH}/20^\circ C$; Br_2/CH_3COOH

C) $\text{alc. NaOH}/80^\circ C$; HBr/CH_3COOH

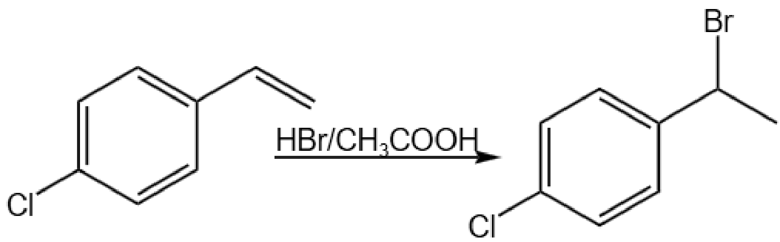
D) $\text{alc. NaOH}/80^\circ C$; $HBr/\text{Peroxide}$

Answer: $\text{alc. NaOH}/80^\circ C$; HBr/CH_3COOH

Solution: In the first step styrene derivative is prepared using alcoholic sodium hydroxide at high temperature by beta elimination mechanism.



Now, the styrene derivative is treated with HBr in the presence of acetic acid to get addition product according to Markonikov's rule as shown below.



Q.27.

$$K_2MnO_4 \xrightarrow{\text{Neutral or acidic}} KMnO_4 + MnO_2$$

Find the sum of spin only magnetic moment of the central metal ion in both products.

A) 0

B) 2.8

C) 3.8

D) 4.9

Answer: 3.8

Solution: $\overset{+7}{KMnO_4}$

$Mn^{7+}:\ 3d^0$

There is no unpaired electron.

$\mu = \sqrt{n(n+2)}$ where $n =$ Number of unpaired electrons

$\overset{+4}{MnO_2}$

$Mn^{+4}: 3d^3$

$\mu = \sqrt{3(3+2)}$
$= \sqrt{15} = 3.8$

Sum of spin only magnetic moment$= 0 + 3.8 = 3.8$

Hence, the answer is option C.

Q.28. Calculate the molarity of $NaCl$ solution, if $5.85\ g$ of $NaCl$ is dissolved in $500\ mL$ of the solution.

A) $0.2\ M$ B) $0.1\ M$

C) $0.5\ M$ D) $1\ M$

Answer: $0.2\ M$

Solution: Mass of sodium chloride, $w = 5.85\ g$

Molar mass of $NaCl$, $M = 58.5g$

Volume of the solution $V = 500\ mL$

Therefore, molarity of the solution $= \frac{w \times 1000}{M \times V\ in\ mL} = \frac{5.85 \times 1000}{58.5 \times 500} = 0.2\ M$.

Q.29. In how many of the following molecules, the central atom is involved in sp^3 hybridization?

$NO_3^-, BCl_3, ClO_2^-, ClO_3^-$

Answer: 2

Solution: The number of hybrid orbital in sp^3 hybridisation is equal to 4.

The number of hybrid orbitals can be calculated as follows,

$H = \frac{1}{2}\left(V + M - C + A\right)$

H= The number of hybrid orbitals

V = The number of valence electrons at central atom

M = number of monovalent surrounding atoms

C = Positive charge on the molecule

A = Negative charge on the molecule

For NO_3^-, $H = \frac{1}{2}(5+1) = 3$,

For BCl_3, $H = \frac{1}{2}(3+3) = 3$,

For ClO_2^- and ClO_3^-, $H = \frac{1}{2}\left(7+1\right) = 4$

Q.30. The number of different chain isomers in C_7H_{16}.

Answer: 9

Solution: Heptane (C_7H_{16}) has a total of 9 chain isomers named n-heptane, 2-methylhexane, 3-methylhexane, 2,3-dimethylpentane, 2,4-dimethylpentane, 2,2-dimethylpentane, 3,3-dimethylpentane, 3-ethylpentane and 2,2,3-trimethylbutane.

1. $CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - CH_2 - CH_3$

n-heptane

2. $CH_3 - \overset{\overset{\displaystyle CH_3}{|}}{CH} - CH_2 - CH_2 - CH_2 - CH_3$

2-methyl hexane

3. $CH_3 - CH_2 - \underset{\underset{\displaystyle CH_3}{|}}{CH} - CH_2 - CH_2 - CH_3$

3-methyl hexane

4. $CH_3 - \overset{\overset{\displaystyle CH_3}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}} - CH_2 - CH_2 - CH_3$

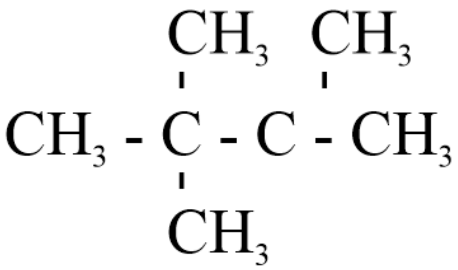
2, 2-dimethyl pentane

5. $CH_3 - CH_2 - \overset{\overset{\displaystyle CH_3}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}} - CH_2 - CH_3$

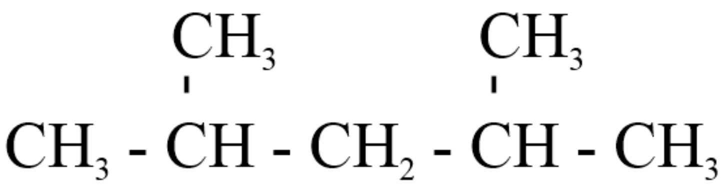
3, 3-dimethyl pentane

6. $CH_3 - \overset{\overset{\displaystyle CH_3}{|}}{CH} - \overset{\overset{\displaystyle CH_3}{|}}{CH} - CH_2 - CH_3$

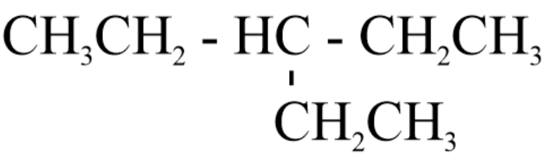
2, 3-dimethyl pentane

7. 

2, 2, 3-trimethyl butane

8. 

2, 4-dimethyl pentane

9. 

3-ethyl pentane

Q.31. The number of complexes from the following have an even number of unpaired d electrons

$[V(H_2O)_6]^{3+}$, $[Cr(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{3+}$, $[Ni(H_2O)_6]^{2+}$, $[Cu(H_2O)_6]^{2+}$

Answer: 3

Solution: In $[V(H_2O)_6]^{3+}$ vanadium ion has $3d^2$ electronic configuration and the two electrons are unpaired electrons.

In $[Cr(H_2O)_6]^{2+}$ chromium ion has $3d^4$ electronic configuration and the four electrons are unpaired electrons.

In $[Fe(H_2O)_6]^{3+}$ iron ion has $3d^5$ electronic configuration and the five electrons are unpaired electrons.

In $[Ni(H_2O)_6]^{2+}$ nickel ion has $3d^8$ electronic configuration and it has two unpaired electrons.

In $[Cu(H_2O)_6]^{2+}$ chromium ion has $3d^9$ electronic configuration and it has one unpaired electron.

Q.32. The de Broglie wavelength of an electron in 4^{th} orbit.............. πa_0

Where a_0 Bohr's orbit radius.

Answer: 8

Solution: De Broglie wavelength $(\lambda) = 2\pi a_0 \times \frac{n}{z}$

$= 2\pi a_0 \times \frac{4}{z}$

$= \frac{8\pi ao}{z}$

Assuming $z = 1$

De Broglie wavelength $(\lambda) = 8\pi R_o$

Q.33. How many species have unpaired electrons?

O_2, O_2^-, O_2^{2-}, CN^-

Answer: 2

Solution: Oxygen molecule (O_2): Each oxygen atom has 8 electrons. Hence, O_2 molecule has a total of 16 electrons. The electronic configuration of O_2 molecule therefore is:

$$O_2:(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2pz)^2$$
$$(\pi 2px)^2=(\pi 2py)^2(\pi^* 2px)^1=(\pi^* 2py)^1$$

As we can see from the above electronic configuration, there are two unpaired electrons in the antibonding molecular π - orbital which leads to oxygen exhibiting paramagnetic properties.

O_2^- has one unpaired electron.

O_2^{2-} and CN^- have all electrons paired.

Q.34. If $f(x)=\begin{cases} x-2, & 0\le x\le 2 \\ -2, & -2\le x\le 0\end{cases}$ and $h(x)=f(|x|)+|f(x)|$, then $\int_0^k h(x)dx$ is equal to ______. $(k>0)$

A) k

B) 0

C) $\frac{k}{2}$

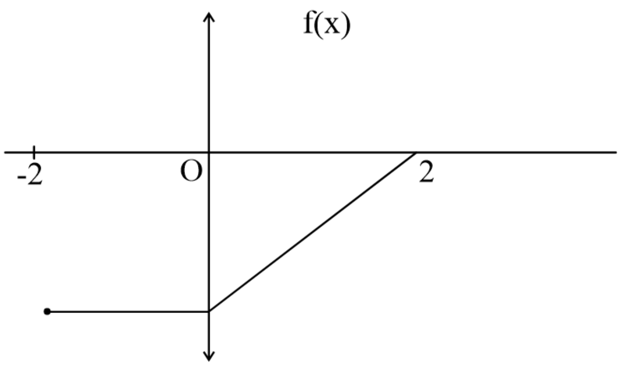
D) $2k$

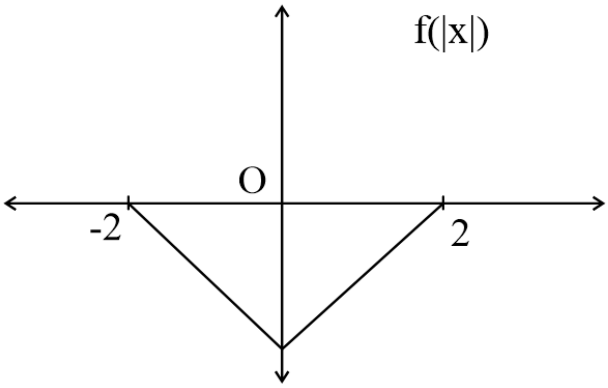
Answer: 0

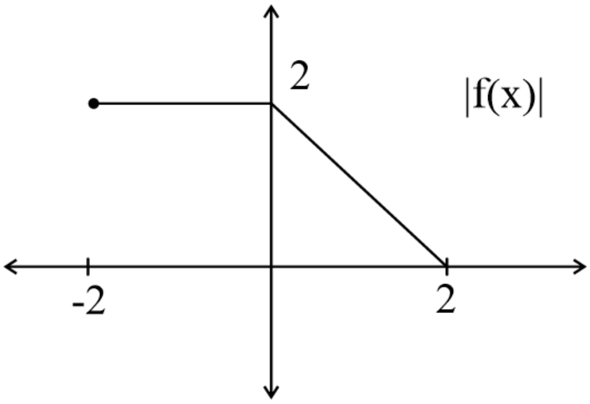
Solution: Given:

$$f(x)=\begin{cases} x-2, & 0\le x\le 2 \\ -2, & -2\le x\le 0\end{cases}$$ and $h(x)=f(|x|)+|f(x)|$

Now, plotting the diagram of $f(x),\ f(|x|)\ \&\ |f(x)|$ we get,.







$\Rightarrow f(|x|) = \begin{cases} -2-x, & x<0 \\ x-2, & x>0 \end{cases}$ and $|f(x)| = \begin{cases} 2, & x<0 \\ 2-x, & x>0 \end{cases}$

$\Rightarrow h(x) = f(|x|) + |f(x)|$

$\Rightarrow h(x) = \begin{cases} -x, & x<0 \\ 0, & x>0 \end{cases}$

$\Rightarrow \int_0^k h(x)dx = \int_0^k (0)dx$

$\Rightarrow \int_0^k h(x)dx = 0$

Q.35. There are three bags $A,\ B$ and C. Bag A contains 7 black balls and 5 red balls, bag B contains 5 red and 7 black balls and bag C contains 7 red and 7 black balls. A ball is drawn and found to be black, find the probability that it is drawn from bag A.

A) $\frac{3}{14}$

B) $\frac{6}{7}$

C) $\frac{7}{12}$

D) $\frac{7}{18}$

Answer: $\frac{7}{18}$

Solution: Given balls are: $\text{Bag } A \rightarrow 7B,\ 5R,\ \text{Bag } B \rightarrow 5B,\ 7R,\ \text{Bag } C \rightarrow 7B,\ 7R$

The probabilty of drawing a black ball is given by,

$P(\text{black ball}) = \frac{1}{3} \times \frac{7}{12} + \frac{1}{3} \times \frac{5}{12} + \frac{1}{3} \times \frac{7}{14}$

So, by using Baye's theorem, the probability that ball is drawn from bag A is given by,

$$P(A|\text{black ball}) = P(E) = \frac{\frac{1}{3}\times\frac{7}{12}}{\frac{1}{3}\times\frac{7}{12}+\frac{1}{3}\times\frac{5}{12}+\frac{1}{3}\times\frac{7}{14}}$$

$$\Rightarrow P(E) = \frac{\frac{7}{12}}{\frac{7}{12}+\frac{5}{12}+\frac{7}{14}}$$

$$\Rightarrow P(E) = \frac{\frac{7}{12}}{1+\frac{1}{2}}$$

$$\Rightarrow P(E) = \frac{7}{18}$$

Q.36. If $x^2 - ax + b = 0$ has roots $2,\ 6$ and $\alpha = \frac{1}{2a+1};\ \beta = \frac{1}{2b-a}$ then find equation having roots $\alpha,\ \beta$

A) $x^2 - 33x + 272 = 0$

B) $272x^2 - 33x + 1 = 0$

C) $x^2 - 272x + 33 = 0$

D) $33x^2 - 272x + 33 = 0$

Answer: $272x^2 - 33x + 1 = 0$

Solution: Given,

$x^2 - ax + b = 0$ has roots 2 & 6

So, by using the sum of roots and product of roots, we get $a = 8$ & $b = 12$

Now, finding $\alpha = \frac{1}{2a+1} = \frac{1}{17}$ & $\beta = \frac{1}{2b-a} = \frac{1}{16}$

Now, finding an equation having roots α & β we get,

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - \left(\frac{1}{17} + \frac{1}{16}\right)x + \frac{1}{16 \times 17} = 0$

$\Rightarrow x^2 - \left(\frac{17+16}{272}\right)x + \frac{1}{272} = 0$

$\Rightarrow 272x^2 - 33x + 1 = 0$

Q.37. If $f(x) = x^5 + 2e^{\frac{x}{4}}$ $\forall\, x \in R$, consider a function $g(x)$ such that $gof(x) = x$ $\forall\, x \in R$, then the value of $8g'(2)$ is

A) 2

B) 16

C) 8

D) 4

Answer: 16

Solution: Given,

$gof(x) = x$

Now, differentiating both side we get,

$g'(f(x))f'(x) = 1$

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}$

Now, taking $f(x) = 2$

$\Rightarrow x^5 + 2e^{\frac{x}{4}} = 2$

$\Rightarrow x = 0$

Now, finding $g'(2) = \frac{1}{f'(0)}$

$\Rightarrow g'(2) = \frac{1}{\left(5x^4 + \frac{2}{4}e^{\frac{x}{4}}\right)_{x=0}} = \frac{4}{2} = 2$

$\Rightarrow 8g'(2) = 8 \times 2 = 16$

Q.38. Find the value of $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\sin x \cos x} dx$.

A) $\frac{\pi}{6\sqrt{3}} - \frac{1}{2}\log\left(\frac{3}{2}\right)$

B) $\frac{\pi}{6\sqrt{3}} + \frac{1}{2}\log\left(\frac{3}{2}\right)$

C) $\frac{\pi}{6\sqrt{3}} - \frac{1}{2}\log\left(\frac{2}{3}\right)$

D) $\frac{\pi}{6\sqrt{3}} + \frac{1}{2}\log\left(\frac{2}{3}\right)$

Answer: $\frac{\pi}{6\sqrt{3}} - \frac{1}{2}\log\left(\frac{3}{2}\right)$

Solution:

Let, $I=\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\sin x \cos x} dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{2(1+\sin x \cos x)} dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{2+2\sin x \cos x} dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{2+\sin 2x} dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \frac{1}{2+\sin 2x} dx-\int_0^{\frac{\pi}{4}} \frac{\cos 2x}{2+\sin 2x} dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \frac{1}{2+\frac{2\tan x}{1+\tan^2 x}} dx-\frac{1}{2}\int_0^1 \frac{dt}{2+t} dx$

(By putting $\sin 2x=t \Rightarrow 2\cos 2x dx=dt$)

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{2\left(1+\tan^2 x+\tan x\right)} dx-\left[\frac{1}{2}\log|2+t|\right]_0^1$

In the first integral putting $\tan x=z \Rightarrow \sec^2 x dx=dz$

$\Rightarrow I=\int_0^1 \frac{dz}{2\left(z^2+z+1\right)}-\frac{1}{2}\log\left|\frac{3}{2}\right|$

$\Rightarrow I=\frac{1}{2}\int_0^1 \frac{dz}{\left(z+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}-\frac{1}{2}\log\left|\frac{3}{2}\right|$

$\Rightarrow I=\left[\frac{1}{2}\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)\right]_0^1-\frac{1}{2}\log\left|\frac{3}{2}\right|$

$\Rightarrow I=\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right)\right]_0^1-\frac{1}{2}\log\left|\frac{3}{2}\right|$

$\Rightarrow I=\frac{1}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\right)-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]-\frac{1}{2}\log\left|\frac{3}{2}\right|$

$\Rightarrow I=\frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)-\frac{1}{2}\log\left(\frac{3}{2}\right)$

$\Rightarrow I=\frac{\pi}{6\sqrt{3}}-\frac{1}{2}\log\left(\frac{3}{2}\right)$

Q.39. If $\alpha,\ \beta \in R$ and the mean and variance of 6 observations $-3,\ 4,\ 7,-6,\ \ \alpha,\ \beta$ be 2 & 23 respectively, then the mean deviation about the mean of these 6 observations will be

A) $\frac{16}{3}$

B) $\frac{13}{3}$

C) $\frac{11}{3}$

D) $\frac{14}{3}$

Answer: $\frac{13}{3}$

Solution: Given,

The mean of observations is 2

So, $\frac{-3+4+7-6+\alpha+\beta}{6}=2$

$\Rightarrow \alpha+\beta=10 \ldots .(1)$

And variance is given as 23

So, $\frac{(-3)^2+4^2+7^2+6^2+\alpha^2+\beta^2}{6}-2^2=23$

$\Rightarrow \alpha^2+\beta^2=52 \quad \ldots .(2)$

Now, on solving both equations we get, $\alpha=6$ & $\beta=4$

Now, finding mean deviation about mean we get,

$\frac{\Sigma\left|x_i-\bar{x}\right|}{6}=\frac{5+2+5+8+4+2}{6}=\frac{26}{6}=\frac{13}{3}$

Q.40. If α & β be the sum and product of all the non-zero solution of the equation $(\bar{z})^2+|z|=0,\ z \in C$ then $4\left(\alpha^2+\beta^2\right)$ is equal to

A) 2 B) 4

C) 6 D) 8

Answer: 4

Solution: Given,

$(\bar{z})^2+|z|=0 \quad \ldots \ldots (i)$

$\Rightarrow |z|=-(\bar{z})^2$

Now, taking modulus both side we get,

$|z|=|z|^2$

$\Rightarrow |z|=1\ \{\text{as } \ |z| \neq 0\}$

Now, using equation (i) we get,

$(\bar{z})^2+1=0$

$\Rightarrow z=i, \ -i$

Now, finding $\alpha=i-i=0$ & $\beta=-i^2=1$

Hence, $4\left(\alpha^2+\beta^2\right)=4$

Q.41. If the system of equations $x+(2\sin 2\theta)y+2\cos 2\theta=0,\ x+(\sin \theta)y+\cos \theta$ and $x+(\cos \theta)y-\sin \theta=0$ has non-trivial solutions then find θ. $\left(\cos^{-1}\frac{1}{2\sqrt{2}}=\alpha\right)$

A) $n\pi \pm \alpha+\frac{\pi}{4}$ B) $2n\pi \pm \alpha+\frac{\pi}{2}$

C) $2n\pi \pm \alpha+\frac{\pi}{4}$ D) $2n\pi \pm \alpha-\frac{\pi}{4}$

Answer: $2n\pi \pm \alpha+\frac{\pi}{4}$

Solution: Let, $\triangle=\begin{vmatrix} 1 & 2\sin 2\theta & 2\cos 2\theta \\ 1 & \sin\theta & \cos\theta \\ 1 & \cos\theta & -\sin\theta \end{vmatrix}=0$

$\Rightarrow -\sin^2\theta-\cos^2\theta-2\sin 2\theta\,(-\sin\theta-\cos\theta)+2\cos 2\theta\,(\cos\theta-\sin\theta)=0$

$\Rightarrow -1+2\sin 2\theta\,(\sin\theta+\cos\theta)+2\cos 2\theta\,(\cos\theta-\sin\theta)=0$

$\Rightarrow -1+2\sin 2\theta\sin\theta+2\sin 2\theta\cos\theta+2\cos 2\theta\cos\theta-2\cos 2\theta\sin\theta=0$

$\Rightarrow -1+2\,(\sin 2\theta\sin\theta+\cos 2\theta\cos\theta)+2\,(\sin 2\theta\cos\theta-\cos 2\theta\sin\theta)=0$

$\Rightarrow -1+2\cos\theta+2\sin\theta=0$

$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta-\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta-\frac{\pi}{4}\right)=\cos\alpha$

$\Rightarrow \theta-\frac{\pi}{4}=2n\pi\pm\alpha$

$\Rightarrow \theta=2n\pi\pm\alpha+\frac{\pi}{4}$, where $\alpha=\cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)$

Q.42. Let $f(x)=\frac{2x^2-3x+9}{2x^2+3x+4}$. If maximum value of $f(x)$ is m and minimum value is n then find $m+n$.

A) 2

B) 10

C) $\frac{122}{23}$

D) 0

Answer: $\frac{122}{23}$

Solution: Let, $y=\frac{2x^2-3x+9}{2x^2+3x+4}$

$\Rightarrow 2x^2y+3xy+4y-2x^2+3x-9=0$

$\Rightarrow x^2(2y-2)+x(3y+3)+4y-9=0$

Now, $D\geq 0$ for real solutions.

$\Rightarrow (3y+3)^2-4\,(2y-2)\,(4y-9)\geq 0$

$\Rightarrow 9y^2+9+18y-4\left(8y^2-18y-8y+18\right)\geq 0$

$\Rightarrow 9y^2+9+18y-32y^2+104y-72\geq 0$

$\Rightarrow -23y^2+122y-63\geq 0$

$\Rightarrow 23y^2-122y+63\leq 0$

We know that, sum of maximum and minimum values is given by sum of roots.

$\Rightarrow m+n=\frac{122}{23}$

Q.43. If a differential equation satisfies $\frac{dy}{dx}-y=\cos x$ at $x=0,\ y=\frac{-1}{2}$.then find $y\left(\frac{\pi}{4}\right)$.

A) 0

B) 1

C) 2

D) 4

Answer: 0

Solution: Given: $\frac{dy}{dx} - y = \cos x$

$\Rightarrow IF = e^{-\int dx} = e^{-x}$

So, solution of the equation is given by,

$y \times e^{-x} = \int e^{-x} \cos x dx$

$\Rightarrow ye^{-x} = -e^{-x} \cos x - \int e^{-x} \sin x dx$

$\Rightarrow ye^{-x} = -e^{-x} \cos x - \left[-e^{-x} \sin x + \int e^{-x} \cos x dx\right]$

$\Rightarrow ye^{-x} = -e^{-x} \cos x + e^{-x} \sin x - ye^{-x} + c$

$\Rightarrow 2ye^{-x} = -e^{-x} \cos x + e^{-x} \sin x + c$

It is given that, $y(0) = \frac{-1}{2}$

$\Rightarrow 2 \times \frac{-1}{2} = -\cos 0 + 0 + c$

$\Rightarrow c = 0$

$\Rightarrow 2ye^{-x} = -e^{-x} \cos x + e^{-x} \sin x$

Putting $x = \frac{\pi}{4}$.

$\Rightarrow 2ye^{-\frac{\pi}{4}} = -\frac{e^{-\frac{\pi}{4}}}{\sqrt{2}} + \frac{e^{-\frac{\pi}{4}}}{\sqrt{2}}$

$\Rightarrow 2ye^{-\frac{\pi}{4}} = 0$

$\Rightarrow y = 0$

Q.44. Find the number of rational numbers in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$

A) 2 B) 3

C) 4 D) 5

Answer: 2

Solution: Given,

Binomial expression $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$

Now, finding general term we get,

$T_{r+1} = {}^{15}C_r \cdot \left(2^{\frac{1}{5}}\right)^{15-r} \left(5^{\frac{1}{3}}\right)^r$

$\Rightarrow T_{r+1} = {}^{15}C_r \cdot 2^{3-\frac{r}{5}} \cdot 5^{\frac{r}{3}}$

Now, for rational number r should be multiple of 3 & 5 where $0 \leq r \leq 15$

So, $r = 0$ & 15 will satisfy

Hence, only 2 rational number are possible.

Q.45. If $2,$ p and q are distinct terms of a GP and in an AP, 2 is third term , p is 7^{th} term and q is 8^{th} term, then find $2p + 8q$.

Answer: 2

Solution: Given: $2,\ p,\ q \to GP$

Let, r be the common ratio of this GP then $p = 2r,\ q = 2r^2$.

Now, $a + 2d = 2,\ a + 6d = p$ and $a + 7d = q$

$\Rightarrow a + 2d = 2,\ a + 6d = 2r$ and $a + 7d = 2r^2$

$\Rightarrow a + 2d - a - 6d = 2 - 2r,\ a + 2d - a - 7d = 2 - 2r^2$

$\Rightarrow -4d = 2 - 2r,\quad -5d = 2 - 2r^2$

$\Rightarrow d = \frac{2-2r}{-4} = \frac{2-2r^2}{-5}$

$\Rightarrow -10 + 10r = -8 + 8r^2$

$\Rightarrow 8r^2 - 10r + 2 = 0$

$\Rightarrow 4r^2 - 5r + 1 = 0$

$\Rightarrow (r-1)(4r-1) = 0$

$\Rightarrow r = 1,\ \frac{1}{4}$

But $r \neq 1$.

$\Rightarrow r = \frac{1}{4}$

$\Rightarrow p = \frac{1}{2}$ and $q = \frac{1}{8}$

$\Rightarrow 2p + 8q = 2$

Q.46. If the length of focal chord of $y^2 = 12x$ is 15 and if the distance of focal chord from origin is p then the value of $10p^2$ is

Answer: 72

Solution: Given,

The equation of parabola is $y^2 = 12x$ and on comparing with $y^2 = 4ax$ we get, $a = 3$

So, the focus will be $(3, 0)$ and parametric points will be $\left(3t^2, 6t\right)$

Now, let AB be focal chord of the given parabola,

And we know that, if t_1 & t_2 are parametric points of focal chord then $t_1 t_2 = -1$

So, the points of $A \equiv \left(3t^2, 6t\right)$ & $B\left(\frac{3}{t^2}, \frac{-6}{t}\right)$

Now, using the given distance $AB = 15$ and by distance formula we get,

$$\left(3t^2 - \frac{3}{t^2}\right)^2 + \left(6t + \frac{6}{t}\right)^2 = 225$$

$$\Rightarrow 9\left(t^2 - \frac{1}{t^2}\right)^2 + 36\left(t + \frac{1}{t}\right)^2 = 225$$

$$\Rightarrow 9\left(t + \frac{1}{t}\right)^2\left(\left(t - \frac{1}{t}\right)^2 + 4\right) = 225$$

$$\Rightarrow 9\left(t + \frac{1}{t}\right)^2\left(\left(t + \frac{1}{t}\right)^2\right) = 225$$

$$\Rightarrow \left(t + \frac{1}{t}\right)^4 = \frac{225}{9}$$

$$\Rightarrow \left(t + \frac{1}{t}\right)^4 = 25$$

$$\Rightarrow \left(t + \frac{1}{t}\right) = \sqrt{5}$$

$$\Rightarrow \left(t - \frac{1}{t}\right)^2 = \left(t + \frac{1}{t}\right)^2 - 4$$

$$\Rightarrow t - \frac{1}{t} = 1$$

Now, finding the equation of AB we get,

$$y - 0 = \frac{2}{t - \frac{1}{t}}(x - 3)$$

$$\Rightarrow y = 2x - 6$$

Now, finding the distance from origin we get,

$$p = \frac{6}{\sqrt{5}}$$

$$\Rightarrow 10p^2 = 72$$

Q.47. If $f(x) = \begin{cases} \frac{1 - \cos \alpha x}{x^2}, & x < 0 \\ 2, & x = 0 \\ \frac{\beta\sqrt{1 - \cos x}}{x}, & x > 0 \end{cases}$ is continuous at $x = 0$ then find the value of $\alpha^2 + \beta^2$

Answer: 12

Solution: Given,

$$f(x)=\begin{cases}\frac{1-\cos\alpha x}{x^2}, & x<0\\ 2, & x=0\\ \frac{\beta\sqrt{1-\cos x}}{x}, & x>0\end{cases}$$ is continuous at $x=0$

So, $f\left(0^{-}\right)=f(0)=f\left(0^{+}\right)$

$$\lim_{x\to 0^{-}}\frac{1-\cos\alpha x}{x^2}=2$$

$$\Rightarrow \lim_{h\to 0}\frac{1-\cos\alpha(0-h)}{(0-h)^2}=2$$

$$\Rightarrow \lim_{h\to 0}\frac{1-\cos(\alpha h)}{(\alpha h)^2}\cdot\alpha^2=2$$

$$\Rightarrow \frac{1}{2}\alpha^2=2$$

$$\Rightarrow \alpha^2=4$$

Now, solving $\lim_{x\to 0^{+}}\frac{\beta\sqrt{1-\cos x}}{x}=2$

$$\Rightarrow \lim_{h\to 0}\frac{\beta\sqrt{1-\cos(0+h)}}{0+h}=2$$

$$\Rightarrow \lim_{h\to 0}\frac{\beta\sqrt{\frac{1-\cos h}{h^2}\cdot h^2}}{h}=2$$

$$\Rightarrow \lim_{h\to 0}\frac{\beta\sqrt{\frac{h^2}{2}\cdot}}{h}=2$$

$$\Rightarrow \frac{\beta}{\sqrt{2}}=2$$

$$\Rightarrow \beta=2\sqrt{2}$$

$$\Rightarrow \beta^2=8$$

Hence, $\alpha^2+\beta^2=12$

Q.48. If $\lim_{x\to 1}\frac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}}=\frac{m(5)^{\frac{1}{2}}}{n(2n)^{\frac{2}{3}}}$ then find the value of $8m+12n$

Answer: 100

Solution: Given,

$$\lim_{x\to1}\frac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}}=\frac{m(5)^{\frac{1}{2}}}{n(2n)^{\frac{2}{3}}}$$

Now, using L-hospital rule in LHS we get,

$$\lim_{x\to1}\frac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}}$$

$$=\lim_{x\to1}\frac{\frac{1}{3}(5x+1)^{\frac{-2}{3}}\cdot5-\frac{1}{3}(x+5)^{\frac{-2}{3}}}{\frac{1}{2}\cdot2\cdot(2x+3)^{\frac{-1}{2}}-\frac{1}{2}(x+4)^{\frac{-1}{2}}}$$

$$=\frac{\frac{1}{3}(5+1)^{\frac{-2}{3}}\cdot5-\frac{1}{3}(1+5)^{\frac{-2}{3}}}{\frac{1}{2}\cdot2\cdot(2+3)^{\frac{-1}{2}}-\frac{1}{2}(1+4)^{\frac{-1}{2}}}$$

$$=\frac{\frac{5}{3}(6)^{\frac{-2}{3}}-\frac{1}{3}(6)^{\frac{-2}{3}}}{(5)^{\frac{-1}{2}}-\frac{1}{2}(5)^{\frac{-1}{2}}}$$

$$=\frac{8(6)^{\frac{-2}{3}}}{3(5)^{\frac{-1}{2}}}$$

$$=\frac{8(5)^{\frac{1}{2}}}{3(2\cdot3)^{\frac{2}{3}}}$$

Now, comparing $\frac{8(5)^{\frac{1}{2}}}{3(2\cdot3)^{\frac{2}{3}}}$ with $\frac{m(5)^{\frac{1}{2}}}{n(2n)^{\frac{2}{3}}}$ we get, $m=8\ \&\ n=3$

Hence, $8m+12n=64+36=100$

Q.49. In a ΔABC, line AB has 5 points, BC has 6 points and CA has 7 points. Find the number of triangles formed by these points.

Answer: 751

Solution: We know that to form a triangle we require three non-collinear points.

Now, $AB,\ BC$ and CA have $5,\ 6$ and 7 collinear points respectively.

So, the total number of required triangles is given by,

$$N={}^{18}C_3-\left({}^{5}C_3+{}^{6}C_3+{}^{7}C_3\right)$$

$$\Rightarrow N=\frac{18\times17\times16}{6}-\left(\frac{5\times4}{2}+\frac{6\times5\times4}{6}+\frac{7\times6\times5}{6}\right)$$

$$\Rightarrow N=3\times17\times16-(10+20+35)$$

$$\Rightarrow N=751$$