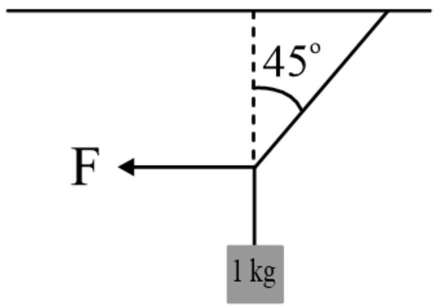
JEE Main 2024

9th April Session 2

Physics

Q.1. Calculate the value of the force F for the following arrangement. Given the mass of the block is $1\ \text{kg}$. Take $g = 10\ \text{m s}^{-2}$.



A) $0.1\ \text{N}$

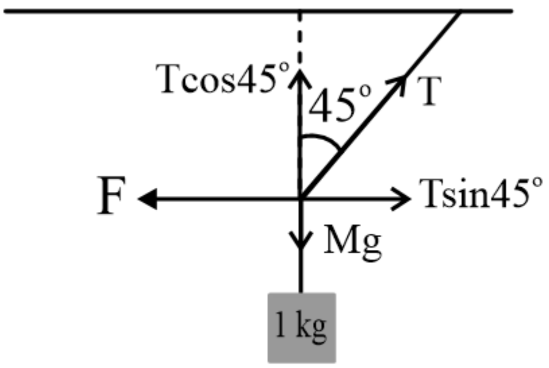
B) $1\ \text{N}$

C) $10\ \text{N}$

D) $0.01\ \text{N}$

Answer: $10\ \text{N}$

Solution: Let's consider the following figure:



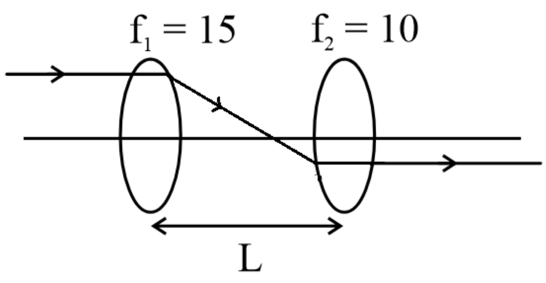
With respect to the above figure, it can be written that

$T\cos 45^\circ = Mg \quad \ldots(1)$
$T\sin 45^\circ = F \quad \ldots(2)$

Dividing equation (2) by equation (1), we have

$\tan 45^\circ = \frac{F}{Mg}$
$\Rightarrow F = Mg\tan 45^\circ$
$= 1\ \text{kg} \times 10\ \text{m s}^{-2} \times 1$
$= 10\ \text{N}$

Q.2. Find the distance L for the given figure:



A) $10\ \text{cm}$

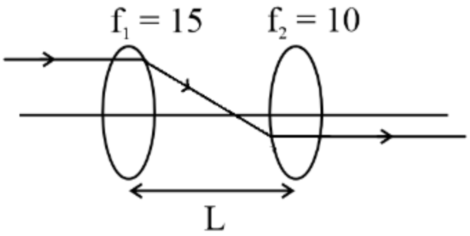
B) $15\ \text{cm}$

C) $20\ \text{cm}$

D) $25\ \text{cm}$

Answer: 25 cm

Solution: As can be seen from the given diagram, the light is incident on the first lens parallel to the principal axis and also, from the second lens, it comes out parallel to its principal axis.



This indicates that the point in between the lenses is nothing but the focal point for both the lenses.

Thus, the distance between the lenses can simply be written as

$$L = 15 \text{ cm} + 10 \text{ cm}$$
$$= 25 \text{ cm}$$

Q.3. If a nucleus, initially at rest, disintegrates in the mass ratio of $1:2$, what is the ratio of the velocities of the products?

A) $1:1$

B) $1:2$

C) $2:1$

D) None of the above

Answer: $2:1$

Solution: Let's assume that initially the mass of the bigger nucleus is $3m$.

Also, assume that after the disintegration, the lower mass, which, according to the problem, is m, will have a velocity of v_s and the larger mass $2m$ will have a velocity of v_l.

Applying the conservation of linear momentum, it follows that

$$(3m) \times 0 = mv_s - 2mv_l$$
$$\Rightarrow v_s = 2v_l$$
$$\Rightarrow \frac{vs}{v_l} = \frac{2}{1}$$

Q.4. What is the dimension of Planck's constant?

A) $[M^1L^1T^2]$

B) $[M^1L^{-1}T^{-2}]$

C) $[M^1L^2T^{-1}]$

D) $[M^0L^1T^{-2}]$

Answer: $[M^1L^2T^{-1}]$

Solution: The dimension of Planck's constant is the product of energy multiplied by time, a quantity called action. Planck's constant is often defined, therefore, as the elementary quantum of action.

It is known that

$E = h\nu \quad \ldots(1)$

Hence, the required dimension can be calculated as follows:

$$[h] = \frac{[E]}{[v]}$$
$$= \frac{[ML^2T^{-2}]}{[T^{-1}]}.$$
$$= [M^1L^2T^{-1}]$$

Q.5. Two cars A and B are moving towards each other with speed 20 m s^{-1} each. When 300 m apart, they both apply breaks which causes deceleration of 2 m s^{-2}. The distance between them when they stop will be:

A) 50 m

B) 100 m

C) 150 m

D) 200 m

Answer: $100\ \text{m}$

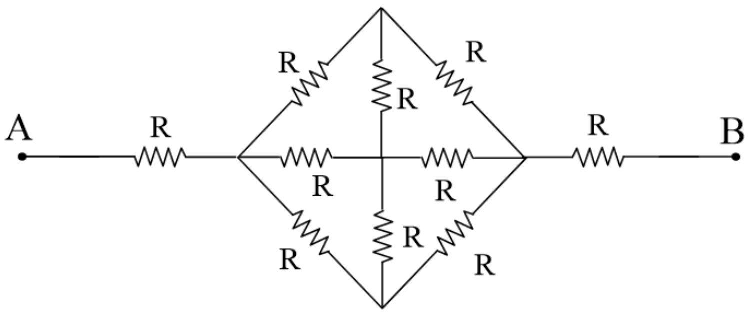
Solution: Distance covered by A before it stops,

$0^2 = 20^2 - 2 \times 2s$
$\Rightarrow s = 100\ \text{m}$

Similarly, B will cover $100\ \text{m}$ before it stops.

Therefore, distance between A and B, when both stop will be $300 - 100 - 100 = 100\ \text{m}$

Q.6. Find the equivalent resistance between the terminals A and B.



A) $\frac{8R}{3}$

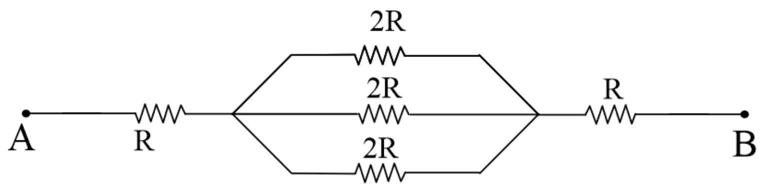
B) $\frac{4R}{3}$

C) $3R$

D) $\frac{5R}{3}$

Answer: $\frac{8R}{3}$

Solution: The equivalent circuit for the given diagram can be drawn as follows:



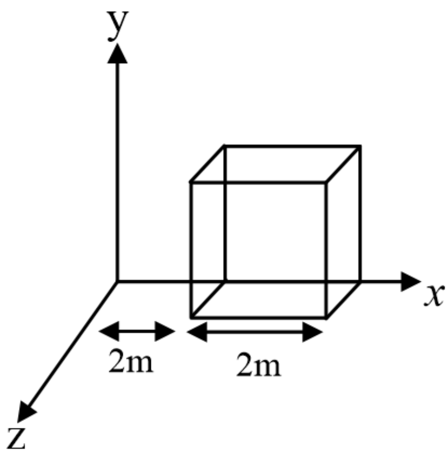
With respect to the above diagram, the equivalent resistance for the middle parallel combination (R') is given by

$\frac{1}{R'} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R}$
$= \frac{3}{2R}$
$\Rightarrow R' = \frac{2R}{3}$

Hence, the equivalent resistance between A and B is fiven by

$R_{eq} = R + \frac{2R}{3} + R$
$= \frac{8R}{3}$

Q.7. There is an imaginary cube of side $2 \mathrm{~m}$ shown where edges are along axes. The electrostatic field varies as $\vec{E}\left(x\right)=2 x \hat{i}$, then the flux through the cube, in $\mathrm{N} \mathrm{~m}^2 \mathrm{C}^{-1}$, is



A) 12 B) 14

C) 16 D) 18

Answer: 16

Solution: As the electric field is along the $x-$ axis, therefore only the surfaces parallel to the $yz-$ plane will contribute to the net flux through the cube.

The surface area of each face of the cube is given by

$$A=2^2 \mathrm{~m}^2$$
$$=4 \mathrm{~m}^2$$

Hence, using Gauss' theorem, the net flux through the cube can be calculated as follows:

$$\phi=E_4 A-E_2 A$$
$$=[2 \times 4] \times 4-[2 \times 2] \times 4$$
$$=16 \mathrm{~N} \mathrm{~m}^2 \mathrm{C}^{-1}$$

Here, E_4, E_2 represent the values of the electric fields at $x=4 \mathrm{~m}$ and $x=2 \mathrm{~m}$ respectively.

Q.8. For a wire, the original resistance was 50Ω at the initial temperature of $27^{\circ} C$. When the temperature is increased, its resistance becomes 62Ω. If the thermal coefficient of resistivity of the wire is $2.4 \times 10^{-2} \mathrm{~K}^{-1}$, find the final temperature.

A) $32{ }^{\circ} C$ B) $37{ }^{\circ} C$

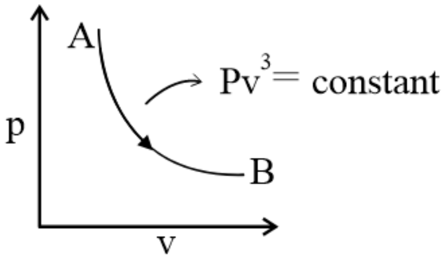
C) $45{ }^{\circ} C$ D) $48{ }^{\circ} C$

Answer: $37{ }^{\circ} C$

Solution: Change in resistance due to temperature change is given by,

$$R=R_0(1+\alpha \Delta T)$$
$$\Rightarrow 62=50\left(1+2.4 \times 10^{-2} \Delta T\right)$$
$$\Rightarrow \Delta T=\frac{0.24}{2.4 \times 10^{-2}}=10$$
$$\Rightarrow T_2-T_1=10$$
$$\Rightarrow T_2=10+27=37{ }^{\circ} \mathrm{C}$$

Q.9. Find the work done by a monoatomic gas from A to B. Here the temperature of gas($1\ \text{mol}$) decreases by $30\ \text{K}$.



A) $125\ \text{J}$

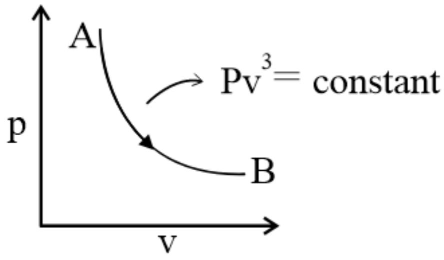
B) $250\ \text{J}$

C) $500\ \text{J}$

D) $1000\ \text{J}$

Answer: $125\ \text{J}$

Solution:



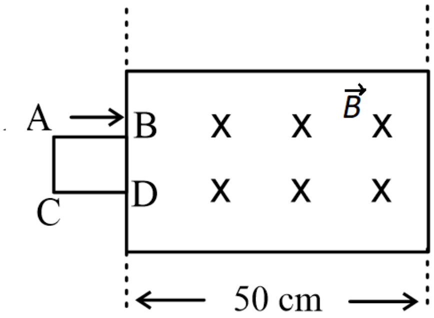
In a polytropic process, work done is given by

$W = \frac{nR\Delta T}{1-n}$.

Comparing the given expression with the standard form of polytropic process, $PV^n = \text{constant}$, we get $n = 3$.

Therefore, work done will be $W = \frac{nR\Delta T}{1-n} = \frac{1\times 8.31\times(-30)}{1-3} \approx 125\ \text{J}$

Q.10. Find the induced emf in the square loop of side $15\ \text{cm}$, moving with $2\ \text{cm s}^{-1}$ after 10 sec.



A) $0\ \text{V}$

B) $3\ \text{V}$

C) $6\ \text{V}$

D) $9\ \text{V}$

Answer: $0\ \text{V}$

Solution: After $t = 10\ \text{s}$, distance covered by the loop will be $10 \times 2 = 20\ \text{cm}$. Therefore, the loop will be completely inside the magnetic field region.

As the total flux will remain same, therefore no EMF will be induced.

Q.11. A spring exerts a force on a bock $\vec{F} = -50\vec{x}$, where x is the change in length of the spring. Find the time period of oscillations. (Given: $m = 0.5$ kg)

A) 0.31 s

B) 0.63 s

C) 1.57 s

D) 3.14 s

Answer: 0.63 s

Solution: Comparing the given expression with standard form, $\vec{F} = -k\vec{x}$, we get $k = 50$.

Now, time period of oscillation of spring block system is given by $T = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 \times \sqrt{\frac{0.5}{50}} \approx 0.63$ s

Q.12. Two bubbles of radii r_A and r_B are having excess pressure P_A and P_B respectively. If $P_A = 3P_B$, find $\frac{r_A}{r_B}$

A) $1:3$

B) $3:1$

C) $1:9$

D) $9:1$

Answer: $1:3$

Solution: Excess pressure in a bubble is given by, $\Delta P = \frac{4T}{r}$,

Given:

$P_A = 3P_B$

$\Rightarrow \frac{4T}{r_A} = 3\frac{4T}{r_B}$

$\Rightarrow \frac{r_A}{r_B} = 1:3$

Q.13. The work done by force $F = 3x^2 + 2x - 5$ in moving a particle from $x = 2$ to $x = 4$ is found to be α J. Find the value of α.

Answer: 58

Solution: The work done by the force can be calculated as follows:

$$W = \int_2^4 F dx$$
$$= \int_2^4 \left(3x^2 + 2x - 5\right) dx$$
$$= \left[x^3 + x^2 - 5x\right]_2^4$$
$$= 58 \text{ J}$$

Hence, $\alpha = 58$.

Chemistry

Q.14. Correct order of bond angle of the following compounds is:

BF_3, PF_3, ClF_3

A) $BF_3 > PF_3 > ClF_3$

B) $PF_3 > ClF_3 > BF_3$

C) $ClF_3 > PF_3 > BF_3$

D) $BF_3 > ClF_3 > PF_3$

Answer: $BF_3 > PF_3 > ClF_3$

Solution: BF_3 is trigonal planar which makes the angle as $120°$. PF_3 is trigonal bipyramidal making the angle slightly less than $109°$. ClF_3 has 2 lone pairs of electrons and has a bond angle close to $90°$, hence, it is T-shaped making the least angle.

Q.15. Ca^{2+} makes which type of complex with EDTA?

A) Trigonal bipyramidal

B) Square planar

C) Tetrahedral

D) Octahedral

Answer: Octahedral

Solution: EDTA is a hexadentate ligand and therefore only one EDTA molecule is required to form octahedral complex.

An octahedral complex has 6 bonds around the central atom . One EDTA moledule complexes with one molecule of calcium cation to form octahedral complex.

Q.16. Identify the correct electronic configuration of Einsteinium is

A) $[Rn]\,5f^{14}6d^{1}7s^{2}$ B) $[Rn]\,5f^{11}7s^{2}$

C) $[Rn]\,5f^{10}6d^{1}7s^{2}$ D) $[Rn]\,5f^{11}6d^{1}7s^{2}$

Answer: $[Rn]\,5f^{11}7s^{2}$

Solution: The correct electronic configuration of Einsteinium (Es) is $[Rn]\,5f^{11}7s^{2}$.

Here's a breakdown of this configuration:

- [Rn] represents the electron configuration of radon (Rn) which precedes einsteinium in the periodic table.
- $5f^{11}$ indicates that there are 11 electrons occupying the 5f orbitals.
- $7s^{2}$ indicates that there are 2 electrons in the 7s orbital.

Q.17. Total number of electrons in π^* orbitals of O_2, O_2^+ and O_2^- is

A) 5 B) 6

C) 7 D) 8

Answer: 6

Solution:

$$O_2(16): \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \begin{cases} \pi 2p_x^2 \\ \pi 2p_y^2 \end{cases} \begin{cases} \pi^* 2p_x^1 \\ \pi^* 2p_y^1 \end{cases}$$

Two π^*electrons are present in O_2.

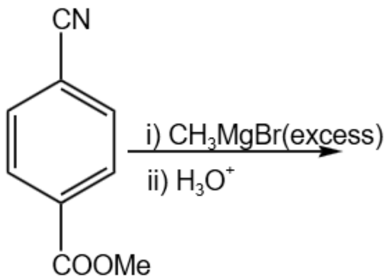
$$O_2^+(16): \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \begin{cases} \pi 2p_x^2 \\ \pi 2p_y^2 \end{cases} \begin{cases} \pi^* 2p_x^1 \\ \pi^* 2p_y^0 \end{cases}$$

One π^* electron is present in O_2^+.

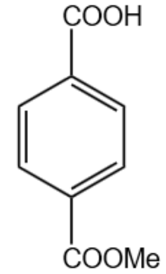
$$O_2^-(16): \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \begin{cases} \pi 2p_x^2 \\ \pi 2p_y^2 \end{cases} \begin{cases} \pi^* 2p_x^2 \\ \pi^* 2p_y^1 \end{cases}$$

Three π^* electrons present in O_2^-.

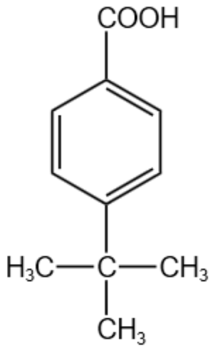
Q.18. The product of the following reaction is



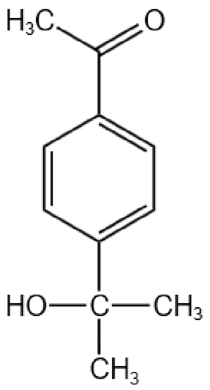
A)



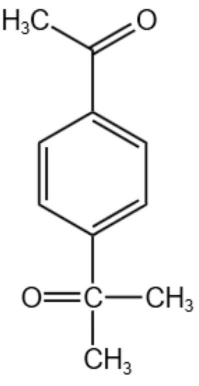
B)



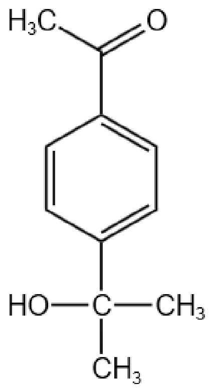
C)



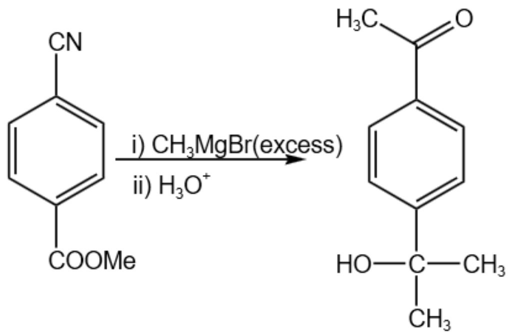
D)



Answer:



Solution: Grignard reagents will add once to nitriles to form imines. The imines can be treated with aqueous acid to give ketones. Grignard reagent is a strong nucleophile. It undergoes nucleophilic addition reaction when it reacts with ester and produces ketone which on further addition gives tertiary alcohol.



Q.19. Match the List-1 with List-II

List-I	Element	List-II	Electronic configuration
P	S	1	$1s^2\ 2s^2\ 2p^6 3s^2\ 3p^4$
Q	N	2	$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6$
R	Kr	3	$1s^2\ 2s^2\ 2p^3$
S	Ar	4	$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$

A) P→3; Q-1; R→4; S→2

B) P→1; Q→4; R→2; S→3

C) P→1; Q→3; R→4; S→2

D) P→1; Q→3; R→2; S→4

Answer: P→1; Q→3; R→2; S→4

Solution:

List-I	Element	List-II	Electronic configuration
P	S(16)	1	$1s^2\ 2s^2\ 2p^6 3s^2\ 3p^4$
Q	N(7)	2	$1s^2\ 2s^2\ 2p^3$
R	Kr(36)	3	$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6$
S	Ar(18)	4	$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$

Q.20. Match column I with column II

Column I	Column II
1. $[Ni(CO)_4]$	P. dsp^2
2. $[Co(NH_3)_6]Cl_3$	Q. sp^3
3. $K_2[Ni(CN)_4]$	R. d^2sp^3
4. $[CoF_6]^{3-}$	S. sp^3d^2

A) $1-Q,\ 2-R,\ 3-P,\ 4-S$

B) $1-R,\ 2-Q,\ 3-P,\ 4-S$

C) $1-Q,\ 2-R,\ 3-S,\ 4-P$

D) $1-Q,\ 2-S,\ 3-P,\ 4-R$

Answer: $1-Q,\ 2-R,\ 3-P,\ 4-S$

Solution: 1. The hybridisation of $Ni(CO)_4$ is sp^3 and it has tetrahedral geometry.

2. In $[Co(NH_3)_6]Cl_3$, Co is in $+3$ oxidation state with the configuration $3d^6$. In the presence of NH_3 a strong ligand, the $3d$ electrons pair up leaving two d-orbitals empty. Hence, the hybridisation is d^2sp^3 forming an inner orbital octahedral complex.

3. $K_2[Ni(CN)_4]$ involves dsp^2 hybridization and has square planar geometry.

4. Hybridisation of $[CoF_6]^{3-}$ is sp^3d^2 and shape is octahedral.

Q.21. $Sc,\ Ti,\ V,\ Cr,\ Mn$

Find magnetic moment of M^+ whose element having maximum second ionisation energy.

A) $5.9\ BM$

B) $3.87\ BM$

C) $4.9\ BM$

D) $2.83\ BM$

Answer: $5.9\ BM$

Solution: The second ionisation energy generally increases significantly after removing the first electron from an atom. Therefore, the element with the highest second ionisation energy among Sc, Ti, V, Cr, and Mn will likely be the one where removing the second electron requires the most energy.

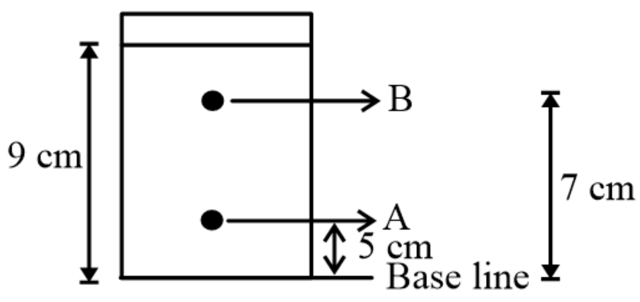
Chromium (Cr) in its neutral state has an atomic number of 24, with an electron configuration of $[Ar]3d^54s^1$

The electron configuration of Cr^+ will be $[Ar]3d^5$.

Among given atoms mono positive cations, chromium ion configuration is more stable. Hence, it has more ionisation energy.

The magnetic moment is $\sqrt{(n(n+2))}BM = \sqrt{35} = 5.90\,BM$

Q.22.



If $R_f(B) = XR_f(A)$. Then find the value of X.

A) $\frac{9}{7}$

B) $\frac{9}{5}$

C) $\frac{7}{5}$

D) $\frac{5}{7}$

Answer: $\frac{7}{5}$

Solution: The R_f (retardation factor) value is the ratio of the solute's distance travelled to the solvent's distance travelled.

R_f = Distance travelled by the substance from reference line (cm)/Distance travelled by the solvent front from reference line (cm)

$R_f(A) = \frac{5}{9}$ and $R_f(B) = \frac{7}{9}$

$R_f(B) = \frac{7}{5} R_f(A)$

Q.23. Which of the following will give positive iodoform test?

A) Butanal

B) isopropyl alcohol

C) 3-pentanone

D) Butanol

Answer: isopropyl alcohol

Solution: When iodine and sodium hydroxide are added to a compound that contains either a ketone with group $-COCH_3$ or a secondary alcohol with $-CH(OH)CH_3$ group, a pale yellow precipitate of iodoform or triiodomethane is formed. This reaction is known as iodoform reaction and it is used to identify the presence of ketone and alcohol, having this group.

So, only isopropyl alcohol will give iodoform test.

Q.24. Match column I with column II

Column I	Column II
P. Size	1. $Tl > In > Al > Ga > B$
Q. Ionization enthalpy	2. $B > Al > Tl > In > Ga$
R. Melting point	3. $B > Al > Tl > In > Ga$
S. Ionic radius	4. $B > Al < Ga > In < Tl$

A) $P-1,\ Q-4,\ R-2,\ S-3$

B) $P-4,\ Q-3,\ R-2,\ S-1$

C) $P-1,\ Q-3,\ R-4,\ S-2$

D) $P-2,\ Q-1,\ R-3,\ S-4$

Answer: $P-1,\ Q-4,\ R-2,\ S-3$

Solution: 1. Atomic and ionic radii of group 13 elements are lower than those of alkaline earth metals of group 2 primarily due to greater nuclear charge of group 13 elements as compared to group 2 elements. On moving down the group the atomic radius of Ga is slightly lower than that of Al. This is due to the presence of d - electrons in Ga which do not shield the nucleus effectively. As a result, the electrons in Ga experience greater force of attraction by the nucleus than in Al and hence the atomic radius of Ga 135 pm is slightly less than that of Al 143 pm. Thus, the increasing order of atomic radii of group 13 elements is $B < Ga < Al < In < Tl$

2. In Ga, there are ten d-electrons in the penultimate shell which screen the nuclear charge less effectively and thus, outer electron is held firmly. As a result, the ionisation energy of both Al and Ga is nearly the same. The increase in ionisation energy from In to Tl is due to poor screening effect of 14f electrons present in the inner shell.

Thus, the correct order is, $B > Al < Ga > In < Tl$

3. Due to structure changes, melting point instead of decreasing increasing from Ga to Tl and as such Ga has the lowest melting point. Hence the actual order is $B > Al > Tl > In > Ga$

4. electrons of Ga are more attracted by nucleus. Thus, the increasing order of atomic radii of the group 13 elements is $B\ (85\ pm)\ <\ Ga\ (135\ pm)\ <\ Al\ (143\ pm)\ <\ In\ (167\ pm)\ <\ Tl\ (170\ pm)$.

Q.25. Match the list and choose the correct option.

	List I		List II
I	Ni-Cd cell	a	Rechargeable
II	Fuel cell	b	Used in flashlights
III	Mercury cell	c	Used in hearing aid
IV	Leclanche cell	d	Combustion energy into electrical energy

A) (I)-a; (II)-d; (III)-c; (IV)-b

B) (I)-b; (II)-a; (III)-c; (IV)-d

C) (I)-d; (II)-a; (III)-c; (IV)-b

D) (I)-a; (II)-b; (III)-c; (IV)-d

Answer: (I)-a; (II)-d; (III)-c; (IV)-b

Solution:

- **Ni-Cd cell (Nickel-Cadmium cell)**:
  - This is a rechargeable battery. It uses nickel oxide hydroxide and metallic cadmium as electrodes and an alkaline electrolyte. Therefore, Ni-Cd cell matches with option **a. Rechargeable**.
- **Fuel cell**:
  - Fuel cells convert the chemical energy from a fuel into electricity through a chemical reaction. They do not rely on combustion but rather on electrochemical processes. Hence, fuel cells match with option **d. Converts combustion energy into electrical energy** (although not involving traditional combustion).
- **Mercury cell**:
  - Mercury cells are known for their use of mercury oxide as one of the electrodes. They are often used in small devices like hearing aids due to their stable voltage output. Therefore, mercury cell matches with option **c. Used in hearing aid**.
- **Leclanche cell**:
  - Leclanche cells are primary (non-rechargeable) batteries that use a zinc anode, manganese dioxide cathode, and a paste of ammonium chloride in water as the electrolyte. They are commonly used in flashlights and other portable devices.

Q.26. What is the correct order of C-C bond length of ethane, ethene and ethyne?

A) Ethane > Ethene > Ethyne

B) Ethene > Ethane > Ethyne

C) Ethyne > Ethene > Ethane

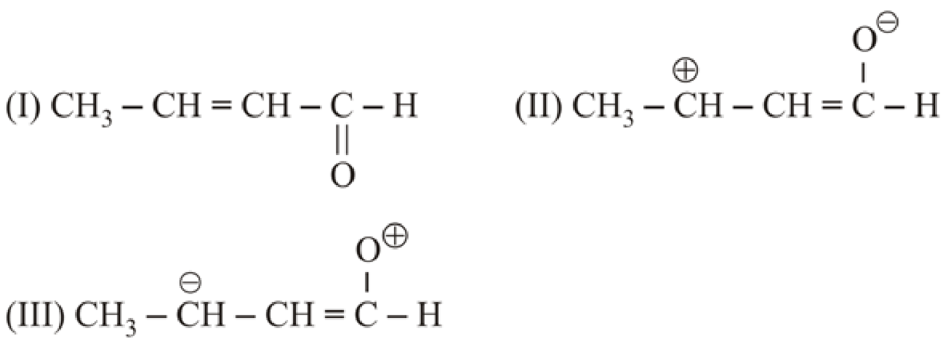
D) Ethyne > Ethane > Ethene

Answer: Ethane > Ethene > Ethyne

Solution:

1. **Ethane (C-C Single Bond)**: Ethane contains a single bond (sigma bond) between the two carbon atoms. The bond length in ethane is relatively longer due to the presence of a single bond and more freely rotating structure between the carbon atoms. The typical bond length of a C-C single bond in ethane is around 1.54 angstroms.
2. **Ethene (C=C Double Bond)**: Ethene contains a double bond (one sigma bond and one pi bond) between the two carbon atoms. The presence of a double bond results in a shorter bond length compared to a single bond because the electron density is concentrated between the two carbon atoms. The bond length of the C=C double bond in ethene is shorter than that of ethane and typically around 1.34 angstroms.
3. **Ethyne (C≡C Triple Bond)**: Ethyne), commonly known as acetylene, contains a triple bond (one sigma bond and two pi bonds) between the two carbon atoms. The presence of a triple bond results in an even shorter bond length compared to a double or single bond because the electron density is further concentrated between the carbon atoms. The bond length of the C≡C triple bond in ethyne is the shortest among these molecules and is typically around 1.20 angstroms.

Q.27. Stability order of the given resonating structure is:



A) $I > II > III$

B) $III > II > I$

C) $I > III > II$

D) None of the above

Answer: $I > II > III$

Solution: In structure I, octet of all is complete and there is no formal charge. Hence, it is most stable.

In II case second, positive charge is on less electronegative and negative charge on more electronegative atom.

In case III, positive charge is present more electronegative atom and negative charge is present on less electronegative atom.

Hence, order of stability will be $I > II > III$

Q.28. How many oxygen atoms are present in fuming sulphuric acid?

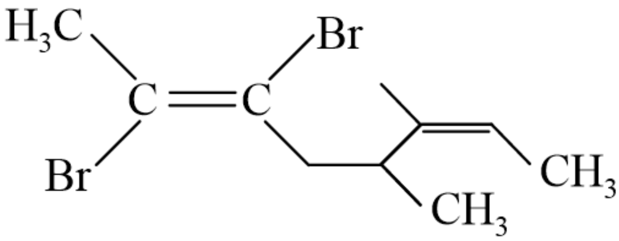
Answer: 7

Solution: The formula of fuming Sulphuric acid is $H_2S_2O_7$.

Fuming sulfuric acid, also known as oleum or fuming H_2SO_4, is a highly concentrated form of sulfuric acid (H_2SO_4) that contains excess sulfur trioxide (SO_3). The term "fuming" refers to the release of sulfur trioxide fumes when it's exposed to air.

Hence the answer is 7.

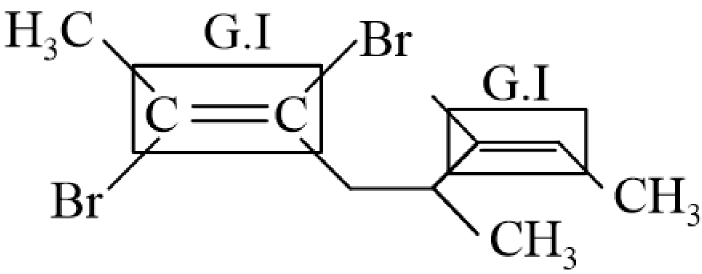
Q.29.



The total number of stereoisomers of given compound is:

Answer: 8

Solution:



As can be seen, it is an asymmetrical compound having to geometrical isomeric centres and one optical isomerism centre.

Hence,

Total number of stereoisomers$= 2^n$

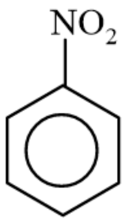
Here $n = 3$

Therefore,

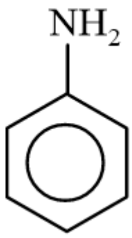
Total number of stereoisomers$= 2^3 = 8$

Q.30. How many of the following compounds will not give Friedel craft reaction?

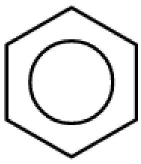
(a)



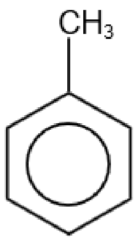
(b)



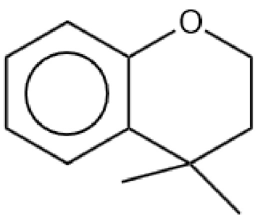
(c)



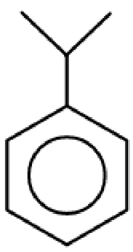
(d)



(e)



(f)



Answer: 2

Solution: A Friedel-Crafts reaction is an organic coupling reaction involving an electrophilic aromatic substitution that is used for the attachment of substituents to aromatic rings. The two primary types of Friedel-Crafts reactions are the alkylation and acylation reactions.

Friedel craft reaction is not given by compounds having electron withdrawing groups and aniline.

Examples of deactivating groups with relative order from highest to lowest deactivating groups,
$-NO_2 > CF_3 > -COR > -CN > -COOR > -SO_3H$

Hence, the answer is 2 as the structure a and b will not give Friedel craft reaction.

Q.31. Number of incorrect statements are:

1. Glucose is aldohexose

2. Glucose have many isomeric forms in aqueous medium

3. Glucose is soluble in water due to the presence of an aldehyde functional group

4. Glucose is a reducing sugar

Answer: 2

Solution: 1. Glucose (also known as dextrose) is a carbohydrate compound consisting of six carbon atoms and an aldehyde group and they are referred to as aldohexose.

2. In basic solution, glucose exists in three isomeric forms as given below. It exists as glucose, fructose and mannose in basic solution.

3. Glucose is soluble in water due to the presence of oxygen or the -OH groups (5 groups) which facilitates hydrogen bonding extensively.

4. Glucose is a reducing sugar. In aqueous solution glucose exists as an equilibrium greatly favouring the glucopyranose form with traces of the acyclic form also present. The glucopyranose hemiacetal and acyclic glucose aldehyde are both shown in red.

Q.32. How many of the following will give a positive tollen's test?

1. Acetone.

2. Formaldehyde

3. Formic acid

4. Acetic acid

5.Benzaldehyde

Answer: 3

Solution: Tollens Test is a very useful method to distinguish between aldehydes and ketones. This qualitative lab test is also referred to as the silver mirror test. Tollens test is generally given by compounds having aldehydic group (aldehydes,alpha-hydroxy ketones and formic acid).

Hence, tollen's test will be given by formaldehyde, formic acid and benzaldehyde.

Mathematics

Q.33. Evaluate: $\lim_{x \to 0} \frac{e-(1+2x)^{\frac{1}{2x}}}{x}$

A) e

B) 1

C) 0

D) ∞

Answer: e

Solution:

Let, $y = \lim_{x \to 0} \frac{e-(1+2x)^{\frac{1}{2x}}}{x}$

$\Rightarrow y = \frac{e-(1+2x)^{\infty}}{0} = \frac{e-e}{0} = \frac{0}{0}$ form

We know that, $(1+x)^{\frac{1}{x}} = e\left(\frac{1}{1} - \frac{x}{2} + \frac{11x^2}{24} - \ldots\ldots\right)$

$\Rightarrow (1+2x)^{\frac{1}{x}} = e\left(1 - \frac{(2x)}{2} + \frac{11(2x)^2}{24} - \ldots\ldots\right)$

$\Rightarrow y = \lim_{x \to 0} \frac{e-e\left(1-x+\frac{11(2x)^2}{24}-\ldots\ldots\right)}{x}$

$\Rightarrow y = \lim_{x \to 0} \frac{e-e+ex}{x}$

$\Rightarrow y = \lim_{x \to 0} \frac{ex}{x}$

$\Rightarrow y = e$

Q.34.

In the expansion of $\left(x^{\frac{2}{3}} + \frac{1}{2}x^{\frac{-2}{5}}\right)^9$, find the sum of coefficients of $x^{\frac{2}{3}}$ and $x^{\frac{-2}{5}}$.

A) 5

B) $\frac{21}{4}$

C) 6

D) $\frac{33}{16}$

Answer: $\frac{21}{4}$

Solution:

The general term in the expansion of $\left(x^{\frac{2}{3}}+\frac{1}{2}x^{\frac{-2}{5}}\right)^{9}$ is given by,

$T_{r+1}={}^{9}C_{r}\left(x^{\frac{2}{3}}\right)^{9-r}\left(\frac{x^{\frac{-2}{5}}}{2}\right)^{r}$

$\Rightarrow T_{r+1}={}^{9}C_{r}\frac{x^{\frac{18-2r}{3}-\frac{2r}{5}}}{2^{r}}$

$\Rightarrow T_{r+1}={}^{9}C_{r}\frac{x^{\frac{90-10r-6r}{15}}}{2^{r}}$

$\Rightarrow T_{r+1}={}^{9}C_{r}\frac{x^{\frac{90-16r}{15}}}{2^{r}}$

Finding coefficient of $x^{\frac{2}{3}}$

$\Rightarrow \frac{90-16r}{15}=\frac{2}{3}$

$\Rightarrow 90-16r=10$

$\Rightarrow r=5$

So, the coefficient is $\frac{{}^{9}C_{5}}{2^{5}}=\frac{9\times8\times7\times6}{4\times3\times2\times32}=\frac{63}{16}$

Finding coefficient of $x^{\frac{-2}{5}}$

$\Rightarrow \frac{90-16r}{15}=\frac{-2}{5}$

$\Rightarrow 90-16r=-6$

$\Rightarrow r=6$

So, the coefficient is $\frac{{}^{9}C_{6}}{2^{6}}=\frac{9\times8\times7}{3\times2\times64}=\frac{3\times7}{16}=\frac{21}{16}$

Thus, the required sum is $\frac{63}{16}+\frac{21}{16}=\frac{84}{16}=\frac{21}{4}$

Q.35. Find the number of real solutions of the equation $2\sin^{-1}(x)+3\cos^{-1}(x)=\frac{7\pi}{5}$.

A) 3

B) 0

C) 1

D) 2

Answer: 1

Solution:

Given: $2\sin^{-1}(x)+3\cos^{-1}(x)=\frac{7\pi}{5}$

$\Rightarrow 2\left[\sin^{-1}(x)+\cos^{-1}(x)\right]+\cos^{-1}(x)=\frac{7\pi}{5}$

$\Rightarrow 2\times\frac{\pi}{2}+\cos^{-1}(x)=\frac{7\pi}{5}$

$\Rightarrow \cos^{-1}(x)=\frac{7\pi}{5}-\pi$

$\Rightarrow \cos^{-1}(x)=\frac{2\pi}{5}$

$\Rightarrow x=\cos\left(\frac{2\pi}{5}\right)$

So, the number of real solutions of the given equation is 1.

Q.36. Evaluate: $\int_{-1}^{2} \log\left(x+\sqrt{1+x^2}\right)dx$

A) $-2\log\left(2+\sqrt{5}\right)+\log\left(1+\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)$

B) $2\log\left(2+\sqrt{5}\right)+\log\left(1+\sqrt{2}\right)+\left(\sqrt{5}-\sqrt{2}\right)$

C) $2\log\left(2+\sqrt{5}\right)+\log\left(1+\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)$

D) $2\log\left(2+\sqrt{5}\right)-\log\left(1+\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)$

Answer: $2\log\left(2+\sqrt{5}\right)-\log\left(1+\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)$

Solution: Let, $I=\int_{-1}^{2} \log\left(x+\sqrt{1+x^2}\right)dx$

$\Rightarrow I=\int_{-1}^{1} \log\left(x+\sqrt{1+x^2}\right)dx+\int_{1}^{2} \log\left(x+\sqrt{1+x^2}\right)dx$

Here, $\int_{-1}^{1} \log\left(x+\sqrt{1+x^2}\right)dx$ is an odd function and hence, its value is 0.

$\Rightarrow I=\int_{1}^{2} \log\left(x+\sqrt{1+x^2}\right)dx$

$$\Rightarrow I=\left[\log\left(x+\sqrt{1+x^2}\right)\times x\right]_1^2-\int_1^2\left(\frac{1+\frac{2x}{2\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}\times x\right)dx$$

$$\Rightarrow I=\left[2\log\left(2+\sqrt{1+4}\right)-\log\left(1+\sqrt{1+1}\right)\right]-\int_1^2\frac{x\left(\sqrt{1+x^2}+x\right)}{\left(\sqrt{1+x^2}\right)\left(x+\sqrt{1+x^2}\right)}dx$$

$$\Rightarrow I=2\log\left(2+\sqrt{5}\right)-\log\left(1+\sqrt{2}\right)-\int_1^2\frac{x}{\left(\sqrt{1+x^2}\right)}dx$$

Putting, $1+x^2=t^2$

$\Rightarrow xdx=tdt$

$$\Rightarrow I=2\log\left(2+\sqrt{5}\right)-\log\left(1+\sqrt{2}\right)-\int_{\sqrt{2}}^{\sqrt{5}}\frac{tdt}{t}$$

$$\Rightarrow I=2\log\left(2+\sqrt{5}\right)-\log\left(1+\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)$$

Q.37. If $f(x)=\frac{1}{2+\sin 3x+\cos 3x}$ and range of $f(x)$ is $[a,b]$ then the ratio of $A.M.$ of $a,\ b$ and $G.M.$ of $a,\ b$ will be

A) $\sqrt{3}$

B) 2

C) $\sqrt{2}$

D) $\sqrt{5}$

Answer: $\sqrt{2}$

Solution: Given,

$$f(x)=\frac{1}{2+\sin 3x+\cos 3x}$$

Now, we know that,

$$2+\sin 3x+\cos 3x\in\left[\left(2-\sqrt{2}\right),\left(2+\sqrt{2}\right)\right]$$

So, $f(x)\in\left[\frac{1}{2-\sqrt{2}},\frac{1}{2+\sqrt{2}}\right]$

Now, ratio of $A.M$ & $G.M$ is given by,

$$\frac{\frac{1}{2}\left[\frac{1}{2-\sqrt{2}}+\frac{1}{2+\sqrt{2}}\right]}{\sqrt{\left[\frac{1}{2-\sqrt{2}}\cdot\frac{1}{2+\sqrt{2}}\right]}}$$

$$=\frac{\frac{1}{2}\left[\frac{4}{4-2}\right]}{\sqrt{\frac{1}{4-2}}}=\sqrt{2}$$

Q.38. If $\log y=\sin^{-1}(x)$, then find the value of $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$ at $x=\frac{1}{2}$

A) e^{π}

B) e

C) $e^{\frac{\pi}{2}}$

D) $e^{\frac{\pi}{6}}$

Answer: $e^{\frac{\pi}{6}}$

Solution: Given: $\log y=\sin^{-1}(x)$

Putting $x=\frac{1}{2}$, $\log y=\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$

$\Rightarrow y=e^{\frac{\pi}{6}} \quad \ldots(i)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}} \quad \ldots(ii)$

Putting, $x=\frac{1}{2}$.

$\Rightarrow \frac{1}{e^{\frac{\pi}{6}}}\frac{dy}{dx}=\frac{1}{\sqrt{1-\frac{1}{4}}}$

$\Rightarrow \frac{dy}{dx}=\frac{2e^{\frac{\pi}{6}}}{\sqrt{3}} \quad \ldots(iii)$

Now, $\sqrt{1-x^2}y'=y$

$\Rightarrow \frac{-2x}{2\sqrt{1-x^2}}y'+\sqrt{1-x^2}y''=y'$

$\Rightarrow -xy'+\left(1-x^2\right)y''=y'\sqrt{1-x^2}$

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=y'\sqrt{1-x^2}$

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=\frac{2e^{\frac{\pi}{6}}}{\sqrt{3}}\sqrt{1-\frac{1}{4}}$

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=\frac{2e^{\frac{\pi}{6}}}{2}$

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=e^{\frac{\pi}{6}}$

Q.39. If $f'(x)=3f(x)+\alpha$. If $f(0)=7$ and $\lim\limits_{x\to-\infty}f(x)=0$, then find $f\left(\frac{1}{3}\right)$.

A) e^7

B) 7

C) $7e$

D) 7^e

Answer: $7e$

Solution: Given: $f'(x) = 3f(x) + \alpha$

$\Rightarrow \frac{dy}{dx} = 3y + \alpha$

$\Rightarrow \frac{dy}{dx} = 3\left(y + \frac{\alpha}{3}\right)$

$\Rightarrow \int \frac{dy}{\left(y+\frac{\alpha}{3}\right)} = \int 3dx$

$\Rightarrow \log\left|y + \frac{\alpha}{3}\right| = 3x + c$

$\Rightarrow y + \frac{\alpha}{3} = e^{3x} \times e^{c}$

Now, $f(0) = 7$

$\Rightarrow 7 + \frac{\alpha}{3} = e^{c}$

Also, at $x = -\infty,\ y = 0$

$\Rightarrow 0 + \frac{\alpha}{3} = 0$

$\Rightarrow \alpha = 0$

$\Rightarrow e^{c} = 7$

$\Rightarrow y = 7e^{3x}$

$\Rightarrow f\left(\frac{1}{3}\right) = 7e^{3\times\frac{1}{3}}$

$\Rightarrow f\left(\frac{1}{3}\right) = 7e$

Q.40. Find the value of integral $\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right) \mathrm{d}\,x$

A) $\frac{-2}{3}$

B) $\frac{-1}{2}$

C) $\frac{-5}{3}$

D) $\frac{-1}{4}$

Answer: $\frac{-1}{4}$

Solution:

Let, $I=\int_{\frac{1}{4}}^{\frac{3}{4}}\cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\mathrm{d}x$

$\Rightarrow I=\int_{\frac{1}{4}}^{\frac{3}{4}}\cos\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)\mathrm{d}x$

Now, let $\theta=\tan^{-1}\sqrt{\frac{1+x}{1-x}}$

$\Rightarrow\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow\cos 2\theta=\frac{1-\frac{1+x}{1-x}}{1+\frac{1+x}{1-x}}$

$\Rightarrow\cos 2\theta=\frac{1-x-(1+x)}{1-x+1+x}=\frac{-2x}{2}=-x$

So, $I=\int_{\frac{1}{4}}^{\frac{3}{4}}-x\,\mathrm{d}x$

$\Rightarrow I=-\left[\frac{x^2}{2}\right]_{\frac{1}{4}}^{\frac{3}{4}}$

$\Rightarrow I=-\frac{1}{4}$

Q.41. A dice is thrown three times such that the outcomes are x_1, x_2, x_3 respectively. Find the probability of getting the outcomes such that $x_1<x_2<x_3$.

A) $\frac{5}{216}$

B) $\frac{1}{27}$

C) $\frac{5}{54}$

D) $\frac{7}{54}$

Answer: $\frac{5}{54}$

Solution: $x_1, x_2, x_3\in\{1,2,3,4,5,6\}$

Total number of outcomes are $6^3=216$

The number of ways of choosing 3 numbers out of $\{1,2,3,4,5,6\}$ are ${}^6C_3=\frac{6\times5\times4}{6}=20$.

So, the required probability is given by,

$P(E)=\frac{20}{216}=\frac{5}{54}$

Q.42. Let $\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}=\frac{1}{1\times2}+\frac{1}{3\times4}+\frac{1}{5\times6}+\ldots+\frac{1}{2023\times2024}$. Find α.

A) 2023

B) 1012

C) 1013

D) 2024

Answer: 1012

Solution: Given: $\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}=\frac{1}{1\times2}+\frac{1}{3\times4}+\frac{1}{5\times6}+\ldots+\frac{1}{2023\times2024}$

$\Rightarrow \frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}=\left(1-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{2023}-\frac{1}{2024}\right)$

$\Rightarrow \frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{2023}+\frac{1}{2024}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2024}\right)$

$\Rightarrow \frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{2023}+\frac{1}{2024}\right)-\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{1012}\right)$

$\Rightarrow \frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\ldots\ldots+\frac{1}{\alpha+1012}=\frac{1}{1013}+\frac{1}{1014}+\ldots+\frac{1}{2024}$

$\Rightarrow \alpha=1012$

Q.43. If $\frac{z-2i}{z+2i}$ is purely imaginary then find the maximum of value of $|z+8+6i|$

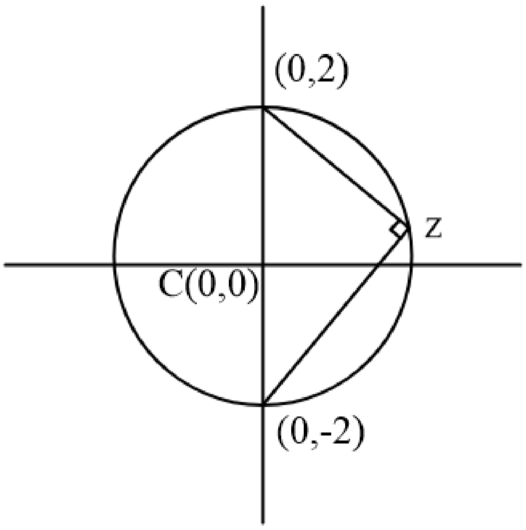
Answer: 12

Solution: Given,

$\frac{z-2i}{z+2i}$ is purely imaginary,

So, $\arg\left(\frac{z-2i}{z+2i}\right)=\pm\frac{\pi}{2}$

Now, plotting the diagram of the above complex number z we get,



Now, $|z-(-8-6i)|$ represents distance of z from $(-8,-6)$

Hence, maximum distance will be, $\sqrt{8^2+6^2}+2=12$

Q.44. If the variance of the observations below is 160 then find the value of $|c|$

x	c	$2c$	$3c$	$4c$	$5c$	$6c$
f	2	1	1	1	1	1

Answer: 7

Solution: Given,

x	c	$2c$	$3c$	$4c$	$5c$	$6c$
f	2	1	1	1	1	1

Now, we know that,

The variance of the observations is given by,

$$\frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - \left(\frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}\right)^2 = 160$$

$$\Rightarrow \frac{2c^2+4c^2+9c^2+16c^2+25c^2+36c^2}{7} - \left(\frac{2c+2c+3c+4c+5c+6c}{7}\right)^2 = 160$$

$$\Rightarrow \frac{92c^2}{7} - \left(\frac{22c}{7}\right)^2 = 160$$

$$\Rightarrow \frac{92c^2}{7} - \left(\frac{484c^2}{49}\right) = 160$$

$$\Rightarrow \frac{644c^2-484c^2}{49} = 160$$

$$\Rightarrow \frac{160c^2}{49} = 160$$

$$\Rightarrow |c| = 7$$

Q.45. If $\sum_{n=0}^{\infty} ar^n = 57$ & $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$ then find the value of $a + 18r$

Answer: 31

Solution: Given,

$\sum_{n=0}^{\infty} ar^n = 57$

$\Rightarrow \frac{a}{1-r} = 57$

$\Rightarrow a = 57(1-r) \ldots\ldots (i)$

And $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$

$\Rightarrow \frac{a^3}{1-r^3} = 9747$

$\Rightarrow \frac{(57(1-r))^3}{1-r^3} = 9747$

$\Rightarrow \frac{19(1-r)^2}{1+r+r^2} = 1$

$\Rightarrow 18r^2 - 39r + 18 = 0$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow 6r^2 - 9r - 4r + 6 = 0$

$\Rightarrow r = \frac{2}{3}$ or $\frac{3}{2}$ {rejected}

So, $a = 57\left(1 - \frac{2}{3}\right) = 19$

Hence, the value of $a + 18r = 19 + 12 = 31$

Q.46. The number of integers between 100 to 1000 whose sum of digits is 14

Answer: 70

Solution: Let, the three digit number be $x_1x_2x_3$

Now, according to the question $x_1 + x_2 + x_3 = 14$ { where $1 \le x_1 \le 9$ & $0 \le x_2, x_3 \le 9\}$

Now, finding the coefficient of x^{14} in the expansion of $\left(x + x^2 + \ldots . x^9\right)\left(1 + x + x^2 + \ldots . x^9\right)^2$ we get,

$=$ coefficient of x^{14} in $\frac{x\left(1-x^9\right)}{1-x} \times \left(\frac{\left(1-x^{10}\right)}{1-x}\right)^2$

$=$ coefficient of x^{14} in $x\left(1 - x^9\right)\left(1 - 2x^{10} + x^{20}\right)(1 - x)^{-3}$

$=$coefficient of x^{13} in $\left(1 - x^9 - 2x^{10}\right)(1 - x)^{-3}$

= coefficient of x^{13} in $\left(1 - x^9 - 2x^{10}\right)\left(1 + {}^3C_1 x + {}^4C_2 x^2 + {}^5C_3 x^3 + \ldots \ldots .\right)$

$= {}^{15}C_{13} - {}^6C_4 - 2 \cdot {}^5C_3$

$= 70$