JEE Main 2024

9th April Session 1

Physics

Q.1. A wire has a length of 20 cm, cross-sectional area of 2 cm^2 and Young's modulus of $2 \times 10^{11} \text{ N m}^{-2}$. Two persons pull the wire from its both ends, each with a force of 200 N. What is the increase in length of the wire?

A) 10^{-4} m B) 10^{-3} m

C) 10^{-6} m D) 10^{-2} m

Answer: 10^{-6} m

Solution: The formula to calculate the stress in the wire is

$$\frac{F}{A} = Y\frac{\Delta L}{L} \quad \ldots(1)$$

From equation (1), it follows that

$$\frac{200 \text{ N}}{2\times10^{-4} \text{ m}^2} = 2 \times 10^{11} \text{ N m}^{-2} \times \frac{\Delta L}{20\times10^{-2} \text{ m}}$$
$$\Rightarrow \Delta L = \frac{200 \text{ N}\times20\times10^{-2} \text{ m}}{2\times10^{-4} \text{ m}^2\times2\times10^{11} \text{ N m}^{-2}}$$
$$= 10^{-6} \text{ m}$$

Q.2. In an interference experiment, the intensity at a point on the screen was found to be one-fourth of that of the maximum intensity. Calculate the distance of the point from the central maximum. Given that the wavelength of the light used is λ, the distance between the source and screen is D and the slit width is d.

A) $\frac{\lambda D}{d}$ B) $\frac{2\lambda D}{3d}$

C) $\frac{\lambda D}{3d}$ D) $\frac{\lambda D}{2d}$

Answer: $\frac{\lambda D}{3d}$

Solution: Let's assume that the initial intensity of the interfering lights is the same, which is I_0.

So,

$$I_{\max} = 4I_0 \quad \ldots(1)$$

And, the intensity of the light at the given point will be

$$I=\frac{1}{4}I_{\max}$$
$$=I_0 \quad \ldots(2)$$

Thus, the phase difference between the light waves can be calculated as follows:

$$I_0 = I_0 + I_0 + 2I_0 \cos\phi$$
$$\Rightarrow I_0 = 2I_0 + 2I_0 \cos\phi$$
$$\Rightarrow \cos\phi = -\frac{1}{2}$$
$$= \cos\frac{2\pi}{3}$$
$$\Rightarrow \phi = \frac{2\pi}{3}$$

And, the corresponding path difference can be calculated as follows:

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$
$$\Rightarrow \Delta x = \frac{\lambda}{2\pi} \times \frac{2\pi}{3}$$
$$= \frac{\lambda}{3}$$

So, the required distance of the point from the central maxima can be calculated as follows:

$$y=\frac{\Delta x D}{d}$$
$$=\frac{\lambda D}{3d}$$

Q.3. The dimensional formula for latent heat is

A) $[\mathrm{M^0L^{-1}T^{-2}}]$ B) $[\mathrm{M^0L^2T^{-2}}]$

C) $[\mathrm{ML^{-2}T^{-2}}]$ D) $[\mathrm{M^2L^{-2}T^{-2}}]$

Answer: $[\mathrm{M^0L^2T^{-2}}]$

Solution: Latent heat is defined as the amount of heat required to change the state of a particular amount of substance. Mathematically, it can be expressed as

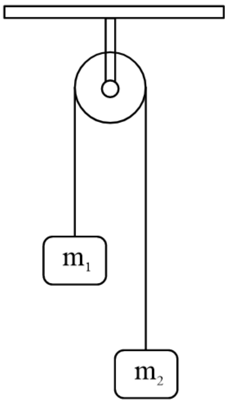
$$L = \frac{Q}{m} \quad \ldots(1)$$

Using dimensional analysis, from equation (1), it follows that

$$[L] = \frac{[Q]}{[m]} = \frac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{M}]} = [\mathrm{M^0L^2T^{-2}}]$$

Q.4. In the pulley-block system shown, the pulley and the block are ideal. If the acceleration of the block is $\frac{g}{8}$, find $m_1 : m_2$.

(Given $m_2 > m_1$)



A) $3:4$ B) $5:7$

C) $7:9$ D) $9:11$

Answer: $7:9$

Solution: Acceleration is given by,

$$a = \frac{\text{total pulling force}}{\text{total mass of the system}}$$

$$\Rightarrow \frac{g}{8} = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$\Rightarrow \frac{1}{8} = \frac{(m_2 - m_1)}{(m_1 + m_2)}$$

$$\Rightarrow \frac{1-8}{1+8} = \frac{(m_2 - m_1) - (m_1 + m_2)}{(m_2 - m_1) + (m_1 + m_2)}$$

$$\Rightarrow \frac{-7}{9} = \frac{-2m_1}{2m_2}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{7}{9}$$

Q.5. A particle oscillates in simple harmonic motion such that its speed and acceleration at distance $2\ \mathrm{m}$ from mean position are $4\ \mathrm{m\ s^{-1}}$ and $16\ \mathrm{m\ s^{-2}}$ respectively. Find the amplitude of oscillation of the particle.

A) $\sqrt{3}\ \mathrm{m}$ B) $\sqrt{6}\ \mathrm{m}$

C) $\sqrt{8}\ \mathrm{m}$ D) $\sqrt{10}\ \mathrm{m}$

Answer: $\sqrt{6}$ m

Solution: Velocity of the particle is given by,

$v=\omega\sqrt{A^2-y^2}$
$\Rightarrow 4=\omega\sqrt{A^2-2^2}$
$\Rightarrow 4=\omega\sqrt{A^2-4}$

Now, acceleration is given by

$a=\left|\omega^2 y\right|$
$\Rightarrow 16=\omega^2\times 2$
$\Rightarrow \omega^2=8$

Therefore,

$\Rightarrow 16=\omega^2\left(A^2-4\right)$
$\Rightarrow 16=8\left(A^2-4\right)$
$\Rightarrow A^2=6$
$\Rightarrow A=\sqrt{6}$ m

Q.6. The equivalent energy of 1 g mass is equal to:

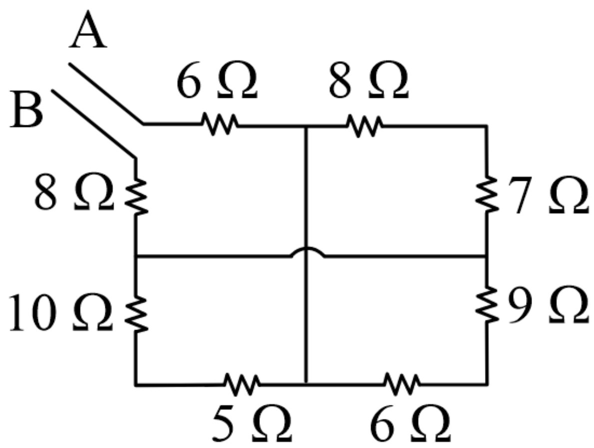
A) 5.6×10^{12} MeV
B) 8.3×10^{12} MeV
C) 5.6×10^{26} MeV
D) 8.3×10^{26} MeV

Answer: 5.6×10^{26} MeV

Solution: Using mass-energy equivalence we get,

$E=mc^2$
$=10^{-3}\times\left(3\times 10^8\right)^2$ J
$=10^{-3}\times\left(3\times 10^8\right)^2\times\frac{1}{1.6\times 10{-19}}$ eV
$=5.625\times 10^{32}$ eV
$=5.625\times 10^{26}$ MeV

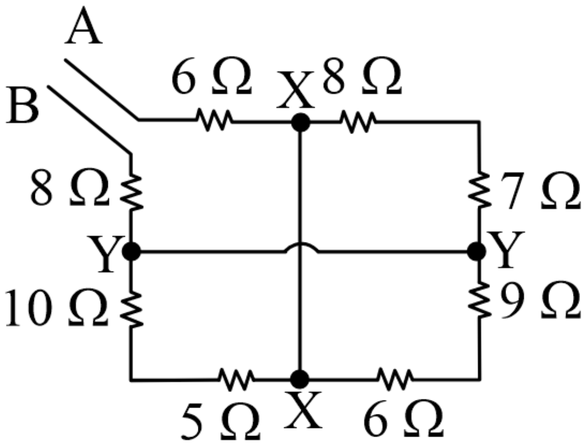
Q.7. Find the equivalent resistance between terminals A and B for the given network.



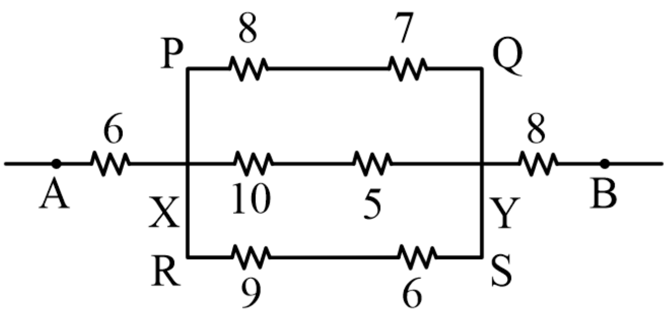
A) $12\ \Omega$
B) $15\ \Omega$
C) $17\ \Omega$
D) $19\ \Omega$

Answer: $19\ \Omega$

Solution: Let's redraw the diagram as follows:



The equivalent diagram can be drawn as below:



With respect to the above diagram, the equivalent resistance (R) between points X and Y can be calculated as follows:

$\frac{1}{R}=\frac{1}{8+7}+\frac{1}{10+5}+\frac{1}{9+6}$

$=\frac{1}{15}+\frac{1}{15}+\frac{1}{15}$

$=\frac{3}{15}$

$=\frac{1}{5}$

$\Rightarrow R=5$

Thus, the equivalent resistance between terminals A and B can be calculated as follows;

$R_{eq}=(6+R+8)\ \Omega$

$=19\ \Omega$

Q.8. What will be the order of de-Broglie wavelength of an alpha particle, proton and electron if their kinetic energies are the same?

A) $\lambda_e<\lambda_p<\lambda_\alpha$

B) $\lambda_p<\lambda_e<\lambda_\alpha$

C) $\lambda_p<\lambda_\alpha<\lambda_e$

D) $\lambda_\alpha<\lambda_p<\lambda_e$

Answer: $\lambda_\alpha<\lambda_p<\lambda_e$

Solution: We know that de-Broglie's wavelength is given by

$$\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{h}{\sqrt{2mKE}}$$

If KE is same then,

$$\lambda\propto\frac{1}{\sqrt{\mathrm{m}}},\ \mathrm{m}\uparrow\lambda\downarrow$$

The order of mass of electron, proton and alpha particle is $\alpha>\mathrm{p}>\mathrm{e}$. Hence, the order of de-broglie's wavelength is $\lambda_{\alpha}<\lambda_{p}<\lambda_{\mathrm{e}}$.

Q.9. A vehicle travels half of the distance with speed $3\ \mathrm{m\ s^{-1}}$ and other half of the distance in two equal time intervals with speed $6\ \mathrm{m\ s^{-1}}$ and $9\ \mathrm{m\ s^{-1}}$. The average speed of vehicle is:

A) $\frac{10}{7}\ \mathrm{m\ s^{-1}}$

B) $\frac{20}{7}\ \mathrm{m\ s^{-1}}$

C) $\frac{30}{7}\ \mathrm{m\ s^{-1}}$

D) $\frac{40}{7}\ \mathrm{m\ s^{-1}}$

Answer: $\frac{30}{7}\ \mathrm{m\ s^{-1}}$

Solution: Let half of the distance be $x\ \mathrm{m}$. Also, let the other half is covered in time $2t$.

Average speed $=\frac{\text{total distance}}{\text{total time}}$

$$=\frac{x+x}{\frac{x}{3}+t+t}\left(\mathrm{as},\ 6t+9t=x\right)$$

$$=\frac{x+x}{\frac{x}{3}+2\times\frac{x}{15}}$$

$$=\frac{90}{21}=\frac{30}{7}\ \mathrm{m\ s^{-1}}$$

Q.10. Find the ratio of initial to final pressure for a gas compressed adiabatically from 5 litres to 4 litres. Given $\gamma=\frac{3}{2}$.

A) $\frac{8}{\sqrt{5}}$

B) $\frac{8}{5\sqrt{5}}$

C) $\frac{8}{2\sqrt{5}}$

D) $\frac{8}{3\sqrt{5}}$

Answer: $\frac{8}{5\sqrt{5}}$

Solution: Using the equation of state for an adiabatic process, it follows that

$$PV^{\gamma}=\mathrm{Constant}$$

$$\Rightarrow P_1V_1^{\gamma}=P_2V_2^{\gamma}$$

$$\Rightarrow\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^{\gamma}$$

$$=\left(\frac{4}{5}\right)^{\frac{3}{2}}$$

$$=\frac{8}{5\sqrt{5}}$$

Q.11. An inductor when connected to a $20\ \mathrm{V}$ DC battery gives current of $5\ \mathrm{A}$ and when connected to a $(20\ \mathrm{V},\ 50\ \mathrm{Hz})$ AC supply the current through the inductor is $4\ \mathrm{A}$. Find the inductance of the loop.

(Take $\pi=3$)

A) $0.01\ \mathrm{H}$

B) $0.02\ \mathrm{H}$

C) $0.03\ \mathrm{H}$

D) $0.04\ \mathrm{H}$

Answer: $0.01\ \mathrm{H}$

Solution: Resistance of the inductor, $R=\frac{20}{5}=4\ \Omega$.

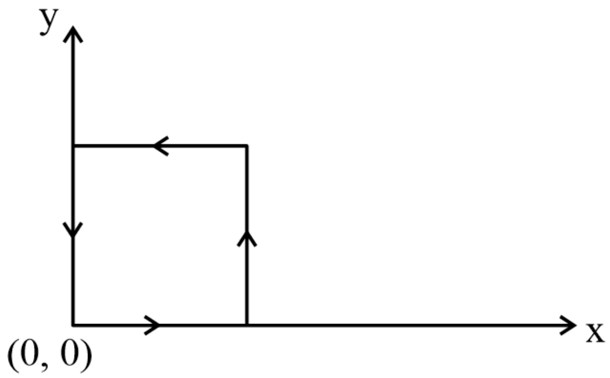
Now, for AC supply

$$Z=\sqrt{X_L{}^2+R^2}$$
$$=\sqrt{(\omega L)^2+4^2}$$

and in this case current is given by,

$$i=\frac{V}{Z}$$
$$\Rightarrow 4=\frac{20}{\sqrt{(\omega L)^2+4^2}}$$
$$\Rightarrow \sqrt{(\omega L)^2+4^2}=5$$
$$\Rightarrow \omega L=3$$
$$\Rightarrow 2\pi f L=3$$
$$\Rightarrow 2\times 3\times 50L=3$$
$$\Rightarrow L=0.01\ \text{H}$$

Q.12. A square loop of side $2\ \text{m}$ carrying current i is placed in a magnetic field $B=\left(1\ +\ 4x\right)\hat{k}$. Find the net force acting on the loop.



A) $10i$

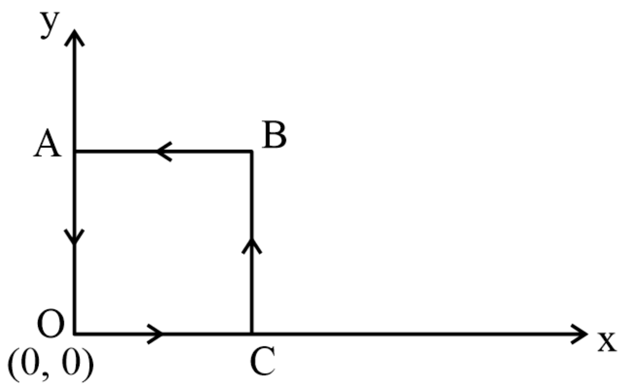
B) $16i$

C) $12i$

D) $14i$

Answer: $16i$

Solution: Let's consider the diagram:



The formula to calculate the force on a current carrying conductor in a magnetic field is given by

$\vec{F} = i\vec{l} \times \vec{B} \quad \ldots(1)$

From the diagram, it is clear that the forces on the arms AB and OC of the square loop will be zero.

The force on arm AO is given by

$$\vec{F}_{AO} = i \times 2 \times (1 + 4 \times 0)\left(-\hat{i}\right)$$
$$= -2i\left(\hat{i}\right) \quad \ldots(2)$$

The force on the arm BC is given by

$$\vec{F}_{BC} = i \times 2 \times (1 + 4 \times 2)\left(\hat{i}\right)$$
$$= 18i\left(\hat{i}\right) \quad \ldots(3)$$

Hence, the magnitude of the net force on the entire loop is given by

$$F_n = \left|\vec{F}_{BC} + \vec{F}_{AO}\right|$$
$$= 16i$$

Q.13. If particle A is on earth's surface and another particle B is revolving around the earth, $\frac{R}{20}$ above earth's surface. Then the difference in mechanical energies of A and B will be (both particles have the same mass m):

A) $\frac{10}{22}\frac{GMm}{R}$

B) $\frac{10}{21}\frac{GMm}{R}$

C) $\frac{11}{21}\frac{GMm}{R}$

D) $\frac{11}{23}\frac{GMm}{R}$

Answer: $\frac{11}{21}\frac{GMm}{R}$

Solution: For particle A, the total energy is given by

$$E_A = -\frac{GMm}{R} \quad \ldots(1)$$

For particle B, the total energy is given by

$$E_B = -\frac{GMm}{2\left(R+\frac{R}{20}\right)}$$
$$= -\frac{10}{21}\frac{GMm}{R} \quad \ldots(2)$$

Hence, the required difference in energies is given by

$$\Delta E = |E_A - E_B|$$
$$= \left|-\frac{GMm}{R} + \frac{10}{21}\frac{GMm}{R}\right|$$
$$= \frac{11}{21}\frac{GMm}{R}$$

Q.14. For a moving particle of mass m, the velocity and displacement are related by $v = \beta\sqrt{x}$. The work done in moving the particle to move from $x = 0$ to $x = d$ is found to be $\frac{m\beta^2 d}{p}$. Find the value of p.

Answer: 2

Solution: Given the velocity of the particle is

$$v = \beta\sqrt{x} \quad \ldots(1)$$

The acceleration of the particle is, then, given by

$$a = v\frac{dv}{dx}$$
$$= (\beta\sqrt{x})\frac{d}{dx}(\beta\sqrt{x})$$
$$= \beta^2\sqrt{x}\left[\frac{1}{2\sqrt{x}}\right]$$
$$= \frac{\beta^2}{2}$$

So, the particle is moving with constant acceleration, which implies that the force on the particle is given by

$$F = ma$$
$$= \frac{m\beta^2}{2}$$

which is also constant.

Thus, the work done by the particle is

$$W = F(d-0)$$
$$= \frac{m\beta^2}{2}d$$

Hence, $p = 2$.

Q.15. The work done by one mole of monatomic gas undergoing adiabatic expansion such that the volume changes from V to $2V$ is found to be $\frac{x}{2}PV\left(1-2^{1-\gamma}\right)$, where P is the pressure and γ is the ratio of specific heats. Find the value of x.

Answer: 3

Solution: The value of the ratio of specific heats for monatomic can be calculated as follows:

$$\gamma = 1 + \frac{f}{2}$$
$$= 1 + \frac{3}{2}$$
$$= \frac{5}{2}$$

The final pressure for the given process can be calculated as follows:

$$PV^{\gamma} = \text{constant}$$
$$\Rightarrow PV^{\gamma} = P'(2V)^{\gamma}$$
$$\Rightarrow P' = \frac{P}{2^{\gamma}}$$

The formula to calculate the required work done is given by

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} \quad \ldots(1)$$

Hence, from equation (1), it follows that

$$W = \frac{\frac{P}{2^{\gamma}}(2V) - PV}{1 - \frac{5}{3}}$$
$$= -\frac{3}{2}PV\left[\frac{1}{2^{\gamma-1}} - 1\right]$$
$$= \frac{3}{2}PV\left[1 - 2^{1-\gamma}\right]$$

Thus, $x = 3$.

Q.16. Angle between two vectors $\vec{A}$ and $\vec{B}$ is $\cos^{-1}\left(\frac{5}{9}\right)$. If $\left|\vec{A} + \vec{B}\right| = \sqrt{2}\left|\vec{A} - \vec{B}\right|$ and $\vec{A} = n\vec{B}$, n being an integer, find the value of n.

Answer: 3

Solution: Given,

$$\left|\vec{A} + \vec{B}\right| = \sqrt{2}\left|\vec{A} - \vec{B}\right| \quad \ldots(1)$$

By squaring of equation (1), we have

$$A^2 + B^2 + 2AB\cos\theta = 2\left[A^2 + B^2 - 2AB\cos\theta\right]$$
$$\Rightarrow A^2 + B^2 = 6AB \times \frac{5}{9}$$
$$\Rightarrow \left(n^2 + 1\right)B^2 = 6nB^2 \times \frac{5}{9}$$
$$\Rightarrow n^2 + 1 = \frac{10}{3}n$$
$$\Rightarrow 3n^2 - 10n + 3 = 0$$
$$\Rightarrow (n-3)(3n-1) = 0$$
$$\Rightarrow n = 3, \frac{1}{3}$$

Hence, the allowed value of $n = 3$.

Q.17. The wavelength of light emitted by the bulb which uses the LED having the band gap of $1.42\ \text{eV}$ is found to be $x\ \text{nm}$. Find the value of x to the nearest integer.

(Take $e = 1.6 \times 10^{-19}\ \text{C}$)

Answer: 875

Solution: The formula to calculate the energy of the emitted light can be written as

$$E=\frac{hc}{\lambda} \quad \ldots(1)$$

From equation (1), it follows that

$$1.42\text{ eV}\times\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}\times3\times10^{8}\text{ m s}^{-1}}{\lambda}$$
$$\Rightarrow\lambda=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}\times3\times10^{8}\text{ m s}^{-1}}{1.42\times1.6\times10^{-19}\text{ J}}$$
$$\approx 8.75\times10^{-7}\text{ m}$$
$$=875\text{ nm}$$

Hence, $x=875$.

Chemistry

Q.18. Which will not form when PbS react with dil. HNO_3?

A) NO B) NO_2

C) S D) None

Answer: NO_2

Solution: The reaction is as follows:
$PbS+\text{ dil. }HNO_3\rightarrow Pb(NO_3)_2+NO+S+H_2O$
Hence, the products are $Pb(NO_3)_2$, S and NO.

Hence, it is clear from the above reaction that NO_2 is not formed in the reaction, so the answer is option B.

Q.19. The chemical formula of the compound present in the tooth enamel is:

A) $Ca_{10}(PO_4)_6(OH)_2$ B) $Ca_8(PO_4)_4(OH)_2$

C) $Ca_6(PO_4)_2(OH)_2$ D) $Ca_8(PO_4)_6(OH)_2$

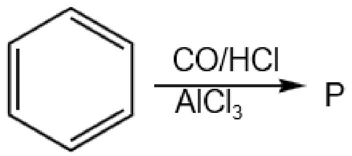
Answer: $Ca_{10}(PO_4)_6(OH)_2$

Solution: Tooth enamel:

- The chemical name and the formula of the compound that makes up the tooth enamel is Calcium hydroxyapatite $Ca_{10}(PO_4)_6(OH)_2$
- The hardest substance found in the body is tooth enamel.
- It is one of the 4 issues that make a tooth.
- The other three are dentin, cementum, and dental pulp.

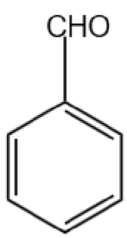
The chemical formula of the compound that makes up the tooth enamel is $Ca_{10}(PO_4)_6(OH)_2$(Calcium hydroxyapatite).

Q.20. For the reaction

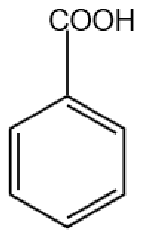


The product P is

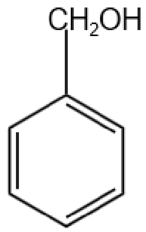
A)



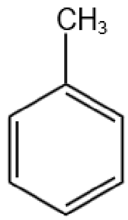
B)



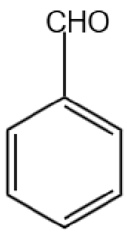
C)



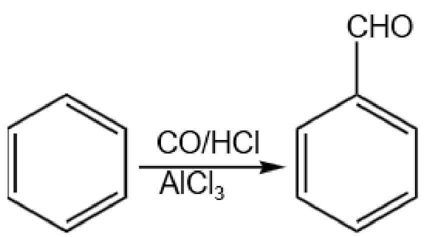
D)



Answer:



Solution: The given reaction is an example of Gatterman - Koch reaction. When benzene or its derivative is treated with carbon monoxide and hydrogen chloride in the presence of anhydrous aluminium chloride or cuprous chloride, it gives benzaldehyde or substituted benzaldehyde.



Q.21. Consider the following electronic configuration:

$Cu^{2+} = [Ar]\ 3d^9 4s^0$

$Cu^{+} = [Ar]\ 3d^{10} 4s^0$

Which option is correct?

A) Cu^{2+} is more stable in aqueous solution

B) Cu^{+} is more stable in aqueous solution

C) Cu^{+} and Cu^{2+} are equally stable in aqueous solution

D) Depends upon copper salt

Answer: Cu^{2+} is more stable in aqueous solution

Solution: The copper (I) compounds are unstable in aqueous solution and undergo disproportionation.

$2\,Cu^{+} \rightarrow Cu^{2+} + Cu$

The stability of $Cu^{2+}(aq)$ rather than $Cu^{+}(aq)$ is due to the much more negative ΔH_{hyd} of $Cu^{2+}(aq)$ than $Cu^{+}(aq)$, which more than compensates for the second ionisation enthalpy of Cu.

Q.22. Equal volume of $1\ M\ HCl$ and $1\ M\ H_2SO_4$ neutralized by $dil\ NaOH$ and heat released is x and $y\ kcal$ respectively, then which is correct?

A) $x = y$
B) $x = 0.5\,y$
C) $x = 0.4\,y$
D) $x = 2y$

Answer: $x = 0.5\,y$

Solution: H_2SO_4 has two removable hydrogen and HCl has one removable hydrogen. Thus, H_2SO_4 produces high enthalpy of neutralisation for 1 mole of H_2SO_4 compared to HCl which has one removable hydrogen.

$HCl + NaOH \longrightarrow NaCl + H_2O \quad \Delta H = x\ kcal$

$H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O \quad \Delta H = y\ kcal$

So, $y = 2x$

Q.23. Which of the following orbitals has highest energy?

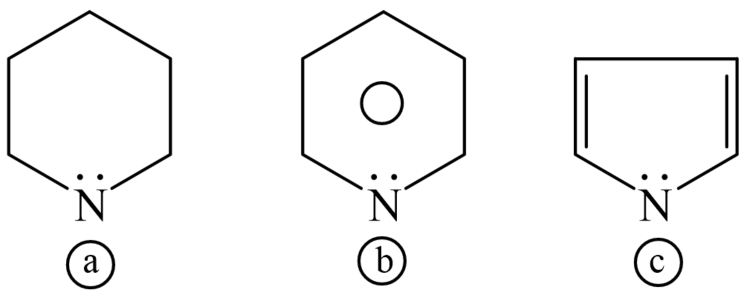
A) $n = 6,\ l = 0$
B) $n = 5,\ l = 2$
C) $n = 4,\ l = 2$
D) $n = 3,\ l = 1$

Answer: $n = 5,\ l = 2$

Solution: The higher value of $n + l$, higher is the energy of the orbital. The $n + l$ value is greatest for $5f$ orbital. Hence, it has the highest energy among the given options.

So, the answer is option B.

Q.24. Correct basic strength order for the following:



A) $a > b > c$
B) $b > a > c$
C) $c > b > a$
D) None

Answer: $a > b > c$

Solution: Basic strength depends on how easily electron can be donated by the compound.

In case a, Nitrogen is sp^3 hybridised and also lone paire is localised and easily available for donation, hence, it has highest basic strength.

In case b, Nitrogrn is sp^2 hybridised and also lone pair is localised but less basic than a.

In case c, lone pair of electrons are delocalised, hence, electrons are not available for donation, so it is least basic.

Hence, the answer is option A.

Q.25. Statement 1: $[Co(en)_2Cl_2]^{+1}$ have 3 geometrical isomers.

Statement 2: $[Co(en)_2Cl_2]^{+1}$ have octahedral geometry.

A) Both statements 1 and 2 are false B) Both statements 1 and 2 are true

C) Statement 1 is false and 2 is true D) Statement 1 is true and 2 is false

Answer: Statement 1 is false and 2 is true

Solution: Consider an octahedral complex of metal M with coordination number six and a bidentate ligand AA and monodentate ligand B having molecular formula $[\mathrm{M(AA)_2B_2}]$. Bidentate ligand AA has two identical coordinating atoms. Cis- isomer is obtained when two bidentate AA ligands as well as two 'B' ligands are at adjacent positions. Trans-isomer is obtained when two AA ligands and two B ligands are at opposite positions.

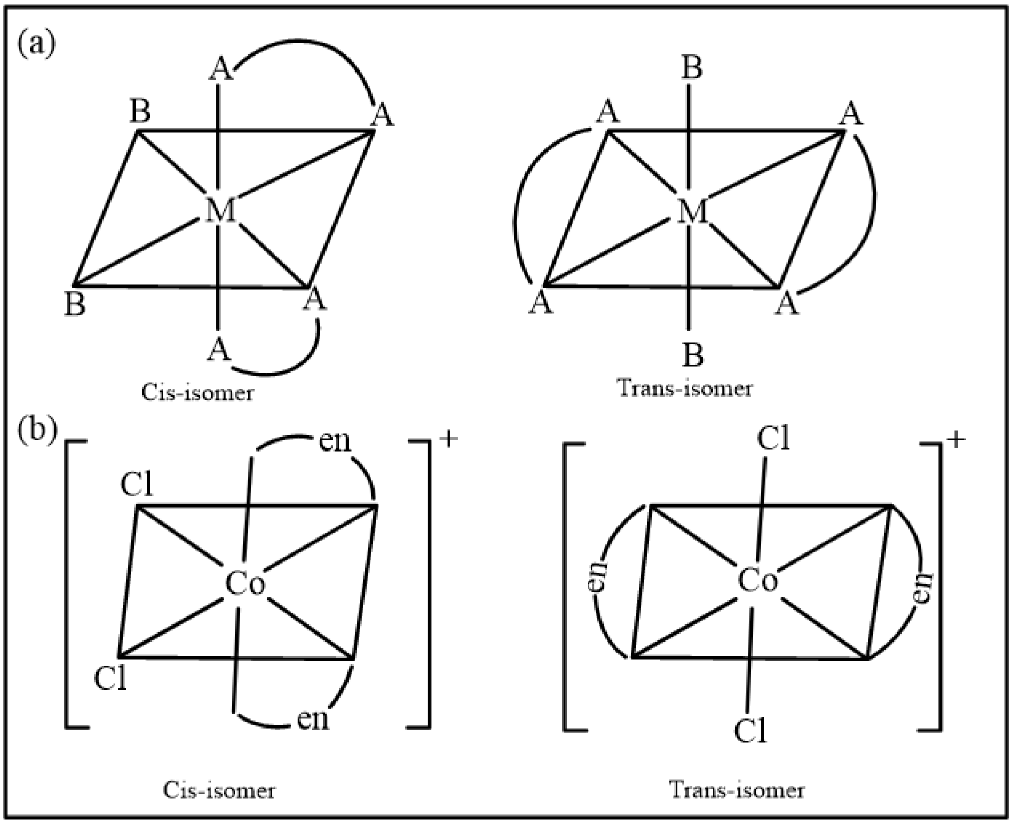
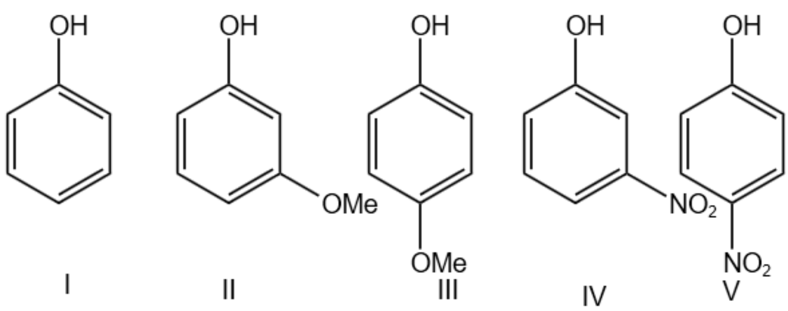


Fig. (a) and (b) : cis and trans-isomers of $[CoCl_2(en)_2]^+$

Hence, answer is option C.

Q.26. Arrange the following according to their acidic nature order.



A) V > IV > II > III > I B) V > III > IV > II > I

C) V > II > IV > III > I D) V > I > IV > II > III

Answer: V > IV > II > III > I

Solution: The order of acidic strength (from most acidic to least acidic) based on the stability of the phenoxide ion formed after deprotonation is:
5 (p-nitrophenol) > 3 (p-methoxyphenol) > 2 (m-methoxyphenol) > 4 (m-nitrophenol) > 1 (phenol)

The acidic nature increased by electron with drawing groups. Nitro group shows -M and -I effects at para position and -I effect only at meta position. Methoxy group shows +M and -I effects at para position and -I effect only at meta position.

Q.27. S-I: Sulphur exists as S_8, while oxygen exists as O_2.
S-II: In oxygen, $p\pi - p\pi$ bonding occurs while it is not effective in sulphur.

A) Both S-I and S-II are true

B) S-I is true and S-II is false

C) S-I is false and S-II is true

D) Both S-I and S-II are false

Answer: Both S-I and S-II are true

Solution: Sulphur typically exists as S_8 molecules, where eight sulfur atoms are bonded together in a ring-like structure. This form of sulfur is stable and commonly found in nature. On the other hand, oxygen exists as O_2 molecules, where two oxygen atoms are bonded together via a double bond.

Oxygen molecules exhibit $p\pi - p\pi$ bonding, referring to the overlapping of p-orbitals of adjacent oxygen atoms. Each oxygen atom in O_2 has two unpaired electrons in its p-orbitals, and when they come together, these orbitals overlap side-by-side to form a double bond. This $p\pi - p\pi$ bonding is a type of π (pi) bonding, characteristic of double bonds. In contrast, sulfur does not effectively engage in pπ−pπ bonding like oxygen.

Q.28. Which of the following statement is incorrect?

A) $KMnO_4$ and $NaOH$ can be used as secondary standard.

B) Primary standard should not undergo change in air

C) Reaction of primary standard with another substance should not be instantaneous

D) Primary standard should be soluble in H_2O

Answer: Reaction of primary standard with another substance should not be instantaneous

Solution: A solution of secondary standard is the one which may be used for standardisation after finding out its exact concentration by titration against a standard solution of primary standard. A secondary standard cannot be used for preparing standard solution by direct weighing. Sodium hydroxide and potassium permanganate are examples of secondary standards.
primary standard should also satisfy the following requirements:
1. It must be easily available in pure and dry form.
2. It should not undergo change in air i.e. it should not be hygroscopic, oxidised by air or affected by gases such as carbon dioxide present in the atmosphere or lose water of crystallisation, so that it can be stored safely.
3. It should be easy to detect the impurities present in it.
4. It should have high relative molecular mass so that weighing errors are negligible.
5. Its reaction with another substance should be instantaneous and stoichiometry.
6. The substance should be readily soluble in water

Q.29. The purification method of organic compounds does not depend on :

A) Nature of compound

B) Shape of compound

C) Density of compound

D) Solubility of compound

Answer: Shape of compound

Solution: a. **Nature of compound**: The nature of the organic compound, including its chemical properties, solubility characteristics, and reactivity, plays a crucial role in determining the appropriate purification method. Different purification techniques are employed based on whether the compound is acidic, basic, polar, non-polar, etc.

b. **Shape of compound**: The shape of the compound generally does not directly influence the choice of purification method. Purification methods are primarily based on the physical and chemical properties of the compound, such as solubility, volatility, or reactivity, rather than its specific shape or structure.

c. **Density of compound**: Density affects purification by enabling separation based on settling, centrifugation, or gradient methods.

d. **Solubility of compound**: The solubility of a compound is one of the most important factors determining the choice of purification method. Different purification techniques, such as recrystallisation, extraction, or chromatography, rely heavily on the differences in solubility between the compound and its impurities.

Q.30. How many of the following are essential amino acids?

1. Histidine
2. Arginine
3. Lysine
4. Valine
5. Proline
6. Glutamic acid
7. Phenyl alanine
8. Tryptophan
9. Glycine

A) 6 B) 5

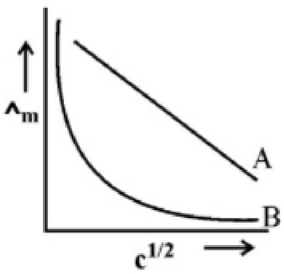
C) 4 D) 1

Answer: 6

Solution: Essential amino acids, also known as indispensable amino acids, are amino acids that humans and other vertebrates cannot synthesise from metabolic intermediates. These amino acids must be supplied from an exogenous diet because the human body lacks the metabolic pathways required to synthesise these amino acids.

Histidine, Arginine. Lysine, Valine, Phenyl alanine and Tryptophan are essential amino acids.

Q.31. In the plot of molar conductivity$(\wedge_m)$ vs square root of concentration $\left(c^{1/2}\right)$, following curves are obtained for two electrolytes A and B :



The nature of electrolytes A and B.

A) A → Strong electrolyte; B → Strong electrolyte B) A → Weak electrolyte; B → Strong electrolyte

C) A → Strong electrolyte; B → Weak electrolyte D) A → Weak electrolyte: B → Weak electrolyte

Answer: A → Strong electrolyte; B → Weak electrolyte

Solution: From the given graph we can say that:

Electrolyte A : Strong electrolyte

For strong electrolytes, the molar conductivity increases slowly with the dilution. The plot between the molar conductivity and $c^{1/2}$ is a straight line

Electrolyte B : Weak electrolyte

For weak electrolytes, the graph plotted between molar conductivity and $c^{1/2}$ (where c is the concentration) is not a straight line. Weak electrolytes have lower molar conductivities and lower degree of dissociation at higher concentrations which increases steeply at lower concentrations.

Q.32. Identify the correct order of de-broglie wavelength of proton, electron and alpha particles moving with same kinetic energy.

A) $\lambda_p < \lambda_\alpha < \lambda_e$ B) $\lambda_p < \lambda_e < \lambda_\alpha$

C) $\lambda_\alpha < \lambda_p < \lambda_e$ D) $\lambda_\alpha < \lambda_e < \lambda_p$

Answer: $\lambda_\alpha < \lambda_p < \lambda_e$

Solution: We know that de-broglie's wavelength is given by

$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mKE}}$

if KE same then

$\lambda \propto \frac{1}{\sqrt{m}}$ m increases λ decreases

The order of mass of the electron, proton and alpha particle is $\alpha > p > e$

Hence, the order of de-broglie's wavelength is $\lambda_\alpha < \lambda_p < \lambda_e$

Q.33. Rate of a reaction is given as: $rate = k[A][B]^2$

If concentration of both reactants is doubled then rate becomes x times the previous and the order of reaction is y, then what is the value of (x+y).

A) 4 B) 6

C) 8 D) 2

Answer: 6

Solution: Rate law for the reaction is given as : Rate = $k[A][B]^2$.

When the concentration of reactant B is doubled, rate can be expressed as :

Rate = $k[A][2B]^2 = 4k[A][B]^2$.

Hence, when the concentration of reactant B is doubled, the rate of reaction becomes four times its initial value. When the concentration of reactant A is doubled, the rate of reaction becomes two times its initial value. Hence, x+y = 6.

Q.34. How many of the given compounds have sp^2 hybridisation?

BF_3, H_2SO_4, NH_4^+, NH_3

Answer: 1

Solution: The type of hybridization seen in a BF_3 molecule is sp^2. In this molecule, all the bonds formed are sigma bonds. There are lone pair of electrons in the Boron atom. In this molecule, three sp^2 hybrid orbitals of Boron form bonds with three p orbitals of three Fluorine atoms.

The hybridization of sulphur in sulphuric acid is sp^3 and it is tetrahedral shaped. The structure is often drawn with two double bonds, with double bond formed from d-orbitals on sulphur and p-orbitals on oxygen.

The hybridization of the nitrogen atom in the compound $NH_4^+$is sp^3.

During the formation of NH_3, one $2s$ orbital and three $2p$ orbitals of nitrogen combine to create four equivalent energy hybrid orbitals, which is known as sp^3 hybridization.

Q.35. Number of ambidentate nucleophiles among the following is:

CN^-, SCN^-, NO_2^-, CH_3COO^-, $C_2O_4^{2-}$, NH_2^-, SO_4^{2-}

Answer: 3

Solution: Ambidentate ligands are those ligands that can attach themselves to the other central metal atoms through two different atoms.

CN^-/NC^-, SCN^-/NCS^-, NO_2^-/ONO^-, these 3 are ambidentate ligands.

Q.36. $O_2^-, N_2, O_2^{2-}, He_2^+, H_2^+, CN^-$

How many of them have unpaired electrons?

Answer: 3

Solution: The molecular species with odd number of electrons and molecules with 10,16 electrons have unpaired electrons after filling the respective molecular orbitals.

$O_2^- - 17e^-$
$N_2 - 14\ e^-$
$O_2^{2-} - 18\ e^-$
$He_2^+ - 3e^-$
$H_2^+ - 1e^-$
$CN^- - 14e^-$

Q.37. Given $E^o = 1.33V$, for $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

How many of the following will undergo oxidation by dichromate?

Fe, Ni, Cr, Cu, Ag, Au

Answer: 5

Solution: Given that the standard electrode potential of $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$ is 1.33V.

For spontaneous reaction E^0_{cell} must be positive.

$\Rightarrow E^0_{cell} = 1.33 - \text{SRP value of respective anode} = +\text{Ve}$

So, the metal couple have less SRP value gives the positive electrode potential.

$E^0_{Fe^{2+}/Fe} = -0.44\ V,\ E^0_{Ni^{2+}/Ni} = -0.26\ V, E^0_{Cr^{3+}/Cr} = -0.74\ V$

$E^0_{Cu^{2+}/Cu} = 0.34\ V,\ E^0_{Ag^{+}/Ag} = 0.80\ V, E^0_{Au^{3+}/Au} = 1.5\ V$

So, except, Gold , remaining metals can be oxidised by dichromate.

Q.38. How many of the following are colourless?

$La^{3+},\ Eu^{3+},\ Sm^{3+},\ Nd^{2+},\ Lu^{3+}$

Answer: 2

Solution: Colour in f-block elements is because of f-f transition which is possible if the electronic configuration is from f^1 to f^{13}, but if f^0 or f^{14} then there will be no f-f transition and it will be colourless.

La^{3+} and Lu^{3+} have 0 and 14 electrons in f-subshell, hence, these will be colourless.

Hence, the answer is 2.

Mathematics

Q.39. If $\int \frac{2-\tan x}{3+\tan x}dx = \alpha x + \beta \log(3\cos x + \sin x) + \gamma$, where γ is constant of integration, then find $\alpha + \beta$

A) 1
B) 2
C) 3
D) 4

Answer: 1

Solution: Let, $I=\int\frac{2-\tan x}{3+\tan x}dx$

$\Rightarrow I=\int\frac{2\cos x-\sin x}{3\cos x+\sin x}dx$

Now, let $2\cos x-\sin x=a\left(3\cos x+\sin x\right)+b(3\cos x+\sin x)'$

$\Rightarrow 2\cos x-\sin x=a\left(3\cos x+\sin x\right)+b\left(\cos x-3\sin x\right)$

$\Rightarrow 2\cos x-\sin x=\cos x\left(3a+b\right)+\sin x\left(a-3b\right)$

$\Rightarrow 3a+b=2,\ \ a-3b=-1$

$\Rightarrow a=\frac{1}{2},\ b=\frac{1}{2}$

$\Rightarrow I=\int\frac{2\cos x-\sin x}{3\cos x+\sin x}dx=\frac{1}{2}\int\frac{3\cos x+\sin x}{\sin x+3\cos x}dx+\frac{1}{2}\int\frac{\cos x-3\sin x}{\sin x+3\cos x}dx$

We know that, $\int\frac{f'(x)}{f(x)}=\log\left|f\left(x\right)\right|+C$

$\Rightarrow I=\int\frac{2\cos x-\sin x}{3\cos x+\sin x}dx=\frac{1}{2}\int dx+\frac{1}{2}\log\left|\sin x+3\cos x\right|+\gamma$

$\Rightarrow I=\int\frac{2\cos x-\sin x}{3\cos x+\sin x}dx=\frac{x}{2}+\frac{1}{2}\log\left|\sin x+3\cos x\right|+\gamma$

$\Rightarrow \alpha=\beta=\frac{1}{2}$

$\Rightarrow \alpha+\beta=1$

Q.40. The remainder when $(428)^{2024}$ is divided by 21 is

A) 3

B) 1

C) 2

D) 4

Answer: 1

Solution: Let, $y=(428)^{2024}$

$\Rightarrow y=(420+8)^{2024}$

$\Rightarrow y={}^{2024}C_0(420)^{2024}+{}^{2024}C_1(420)^{2023}(8)+{}^{2024}C_2(420)^{2022}(8)^2+\ldots\ldots+{}^{2024}C_{2024}(8)^{2024}$

$\Rightarrow y=21\lambda+8^{2024}$

$\Rightarrow y=21\lambda+64^{1012}$

$\Rightarrow y=21\lambda+(63+1)^{1012}$

$\Rightarrow y=21\lambda+{}^{1012}C_1(63)^{1012}+{}^{1012}C_2(63)^{1011}+{}^{1012}C_3(63)^{1010}+\ldots\ldots+{}^{1012}C_{1012}$

$\Rightarrow y=21\lambda+21k+1$

Q.41. If the domain of the function $f\left(x\right)=\sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $R-\left(\alpha,\beta\right)$ then find $\alpha,\ \beta$

A) $-3,\frac{-2}{3}$

B) $-4,\frac{-2}{3}$

C) $-4,\frac{-3}{2}$

D) $-2,\frac{-2}{3}$

Answer: $-4,\frac{-2}{3}$

Solution: Given,

$f(x)=\sin^{-1}\left(\frac{x-1}{2x+3}\right)$

Now, we know that, for $\sin^{-1}t,\ -1\le t\le 1$

So, $-1\le\left(\frac{x-1}{2x+3}\right)\le 1$

Now, taking $\left(\frac{x-1}{2x+3}\right)\le 1$ we get,

$\frac{x-1}{2x+3}-1\le 0$

$\Rightarrow\frac{x-1-2x-3}{2x+3}\le 0$

$\Rightarrow\frac{-x-4}{2x+3}\le 0$

$\Rightarrow\frac{x+4}{2x+3}\ge 0$

$x\in\left(-\infty,-4\right]\cup\left(\frac{-3}{2},\infty\right)\ \ldots.(i)$

Now, solving $\left(\frac{x-1}{2x+3}\right)\ge -1$

$\Rightarrow\frac{x-1}{2x+3}+1\ge 0$

$\Rightarrow\frac{x-1+2x+3}{2x+3}\ge 0$

$\Rightarrow\frac{3x+2}{2x+3}\ge 0$

$\Rightarrow x\in\left(-\infty,\frac{-3}{2}\right)\cup\left[\frac{-2}{3},\infty\right)\ \ldots\ldots(ii)$

Now, taking the intersection of both the equations we get,

$x\in R-\left(-4,\frac{-2}{3}\right)$

Q.42. If $f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{b}{a}|\tan x|}, & x>\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, then a^2+b^2 is

A) 3

B) 9

C) 81

D) 24

Answer: 81

Solution:

Given: $f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{b}{a}|\tan x|}, & x>\frac{\pi}{2}\end{cases}$

$\Rightarrow LHL=\lim_{x\to\frac{\pi}{2}}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}$

$\Rightarrow LHL=\left(\frac{8}{7}\right)^{0}=1$

$\Rightarrow 1=a-8$

$\Rightarrow a=9$

$\Rightarrow RHL=\lim_{x\to\frac{\pi}{2}}|1+|\cot x||^{\frac{b}{a}|\tan x|}$

$\Rightarrow RHL=e^{\frac{b}{a}|\tan x|(1+|\cot x|-1)}$

$\Rightarrow RHL=e^{\frac{b}{a}}=1$

$\Rightarrow b=0$

$\Rightarrow a^2+b^2=81$

Q.43. If $\cos\theta\cos(60-\theta)\cos(60+\theta)\le\frac{1}{8}$, $\theta\in[0,2\pi]$, then find the sum of values of θ for which $\cos 3\theta$ is maximum.

A) $\frac{\pi}{2}$

B) π

C) 3π

D) 6π

Answer: 6π

Solution: Given: $\cos\theta\cos(60-\theta)\cos(60+\theta)\le\frac{1}{8}$

$\Rightarrow \frac{1}{4}\cos 3\theta\le\frac{1}{8}$

$\Rightarrow \cos 3\theta\le\frac{1}{2}$

Now, $\cos 3\theta$ is maximum.

$\Rightarrow \cos 3\theta=\frac{1}{2}, \theta\in[0,\ 2\pi]$

$\Rightarrow 3\theta\in\left[\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{11\pi}{3},\frac{13\pi}{3},\frac{17\pi}{3}\right]$

$\Rightarrow \theta\in\left[\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\frac{11\pi}{9},\frac{13\pi}{9},\frac{17\pi}{9}\right]$

So, the required sum is $\frac{\pi}{9}(1+5+7+11+13+17)=\frac{\pi}{9}(54)=6\pi$.

Q.44. A variable line passing through $(3,5)$ cuts x and y axis. Find the minimum area made between axis and the line.

A) 30

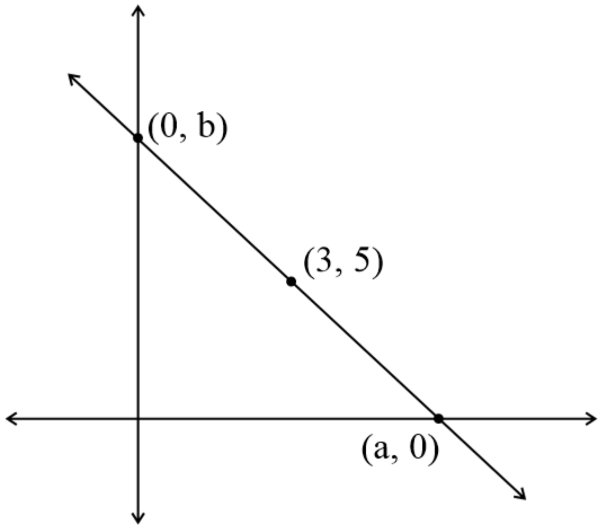
B) 60

C) 15

D) 45

Answer: 30

Solution:



From the figure above, the equation of line is given by $\frac{x}{a}+\frac{y}{b}=1$.

$\Rightarrow \frac{3}{a}+\frac{5}{b}=1$

We know that, $AM \geq GM$.

$\Rightarrow \frac{\frac{3}{a}+\frac{5}{b}}{2} \geq \sqrt{\frac{3}{a}\times\frac{5}{b}}$

$\Rightarrow \frac{1}{2} \geq \sqrt{\frac{15}{ab}}$

$\Rightarrow ab \geq 60$

Now, area of triangle is given by,

$A=\frac{1}{2}ab$

$\Rightarrow \frac{1}{2}ab \geq 30$

So, the required minimum area is 30 square units.

Q.45. The parabola $y^2=4x$ divides the area of the circle $x^2+y^2=5$ in 2 parts. The area of smaller part is

A) $\frac{4}{3}+\frac{5\pi}{2}$

B) $\frac{8}{3}+\frac{5\pi}{2}$

C) $\frac{8}{3}+\frac{\pi}{2}$

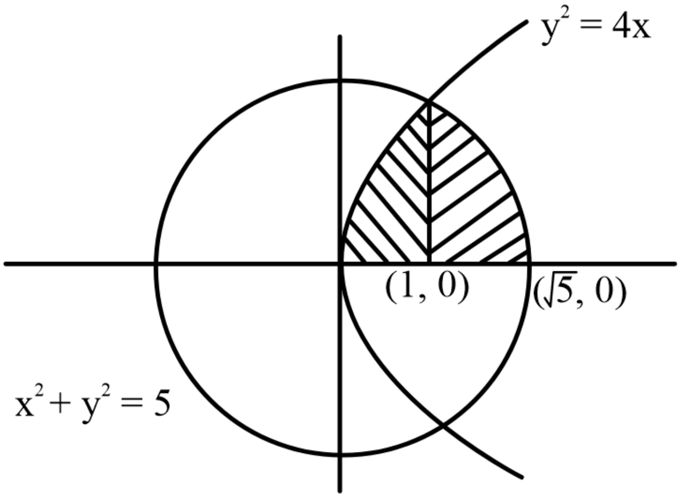
D) $\frac{4}{3}+\frac{\pi}{2}$

Answer: $\frac{8}{3}+\frac{5\pi}{2}$

Solution: Given,

The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in 2 parts,

Now, plotting the diagram of the parabola and circle we get,



Now, from the above diagram,

The area of smaller part will be $= 2\left[\int_0^1 2\sqrt{x}\,\mathrm{d}\,x + \int_1^{\sqrt{5}} \sqrt{5 - x^2}\,\mathrm{d}\,x\right]$

$= 2\left[\frac{4}{3}\left[x^{\frac{3}{2}}\right]_0^1 + \frac{1}{2}\left[x\sqrt{5 - x^2} + 5\sin^{-1}\frac{x}{\sqrt{5}}\right]_0^{\sqrt{5}}\right]$

$= 2\left[\frac{4}{3} + \frac{1}{2}\left[0 + 5 \cdot \frac{\pi}{2}\right]\right]$

$= \frac{8}{3} + \frac{5\pi}{2}$

Q.46. If the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$ are α and β, then find the equation whose roots are $\alpha^4 + \beta^4$ and $\frac{\alpha^6+\beta^6}{10}$.

A) $x^2 - 196x + 956 = 0$

B) $x^2 - 195x + 9506 = 0$

C) $x^2 - 99x + 198 = 0$

D) $x^2 - 197x + 950 = 0$

Answer: $x^2 - 195x + 9506 = 0$

Solution: Given: $x^2+2\sqrt{2}x-1=0$

$\Rightarrow \alpha+\beta=-2\sqrt{2},\ \alpha\beta=-1$

$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow \alpha^2+\beta^2=\left(-2\sqrt{2}\right)^2-2(-1)$

$\Rightarrow \alpha^2+\beta^2=10$

$\Rightarrow \alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2$

$\Rightarrow \alpha^4+\beta^4=(10)^2-2(-1)^2$

$\Rightarrow \alpha^4+\beta^4=98$

$\Rightarrow \left(\alpha^2+\beta^2\right)^3=\alpha^6+\beta^6+3\alpha^2\beta^2\left(\alpha^2+\beta^2\right)$

$\Rightarrow (10)^3=\alpha^6+\beta^6+3(10)$

$\Rightarrow 970=\alpha^6+\beta^6$

$\Rightarrow \frac{\alpha^6+\beta^6}{10}=97$

So, the required equation is $x^2-195x+9506=0$.

Q.47. The solution of the differential equation $\left(x^2+y^2\right)dx-5xydy=0,\ y(1)=0$ is

A) $\left|x^2-4y^2\right|^5=x^4$

B) $\left|x^2-4y^2\right|^5=x^2$

C) $\left|x^2-4y^2\right|^6=x$

D) $\left|x^2-4y^2\right|^6=x^4$

Answer: $\left|x^2-4y^2\right|^5=x^2$

Solution: Given,

$\left(x^2+y^2\right)dx-5xydy=0$

$\Rightarrow 5\frac{dy}{dx}=\frac{x^2+y^2}{xy}$

Now, let $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow v+x\frac{dv}{dx}=\frac{1+v^2}{5v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1-4v^2}{5v}$

$\Rightarrow\int\frac{5v}{1-4v^2}dv=\int\frac{dx}{x}$

$1-4v^2=t\Rightarrow -8vdv=dt$

$\Rightarrow\frac{5}{8}\int\frac{-1}{t}dt=\int\frac{dx}{x}$

$\Rightarrow\frac{-5}{8}\ln t=\ln x+\ln c$

$\Rightarrow\frac{-5}{8}\ln\left(1-4v^2\right)=\ln x+\ln c$

$\Rightarrow\frac{-5}{8}\ln\left(1-\frac{4y^2}{x^2}\right)=\ln x+\ln c$

Now, using $y(1)=0$ we get, $\ln c=0$

$\Rightarrow -5\ln\left(1-\frac{4y^2}{x^2}\right)=8\ln x$

$\Rightarrow -5\ln\left(x^2-4y^2\right)+5\ln x^2=8\ln x$

$\Rightarrow -5\ln\left(x^2-4y^2\right)=-2\ln x$

$\Rightarrow\left|x^2-4y^2\right|^5=x^2$

Q.48. Given a system of equations,

$3x+4y+\lambda z=4$
$5x+7y+2z=8$
$97x+197y+83z=\mu.$

Find λ, if the system has infinite solutions.

A) $\frac{323}{300}$

B) $\frac{300}{323}$

C) $\frac{323}{306}$

D) $\frac{306}{323}4$

Answer: $\frac{323}{306}$

Solution: Given:

$3x+4y+\lambda z=4$
$5x+7y+2z=8$
$97x+197y+83z=\mu$

$$\Rightarrow \triangle=\begin{vmatrix} 3 & 4 & \lambda \\ 5 & 7 & 2 \\ 97 & 197 & 83 \end{vmatrix}=0$$

Applying $R_3 \rightarrow R_3-19R_2$

$$\Rightarrow \triangle=\begin{vmatrix} 3 & 4 & \lambda \\ 5 & 7 & 2 \\ 2 & 64 & 45 \end{vmatrix}=0$$

$$\Rightarrow 3(315-128)-4(225-4)+\lambda(320-14)=0$$

$$\Rightarrow 3(187)-4(221)+\lambda(306)=0$$

$$\Rightarrow -323+\lambda(306)=0$$

$$\Rightarrow \lambda=\frac{323}{306}$$

Q.49. A circle with centre (α, β) passes through point $(0,0)$ and $(0,1)$ and touches the circle $x^2+y^2=9$ for all possible values of (α, β). Find the value of $4\left(\alpha^4+\beta^4\right)$.

A) 65

B) $\frac{65}{4}$

C) 16

D) 8

Answer: $\frac{65}{4}$

Solution: Let the circle smaller circle touch $y-$axis at $P \equiv (0, 1)$.

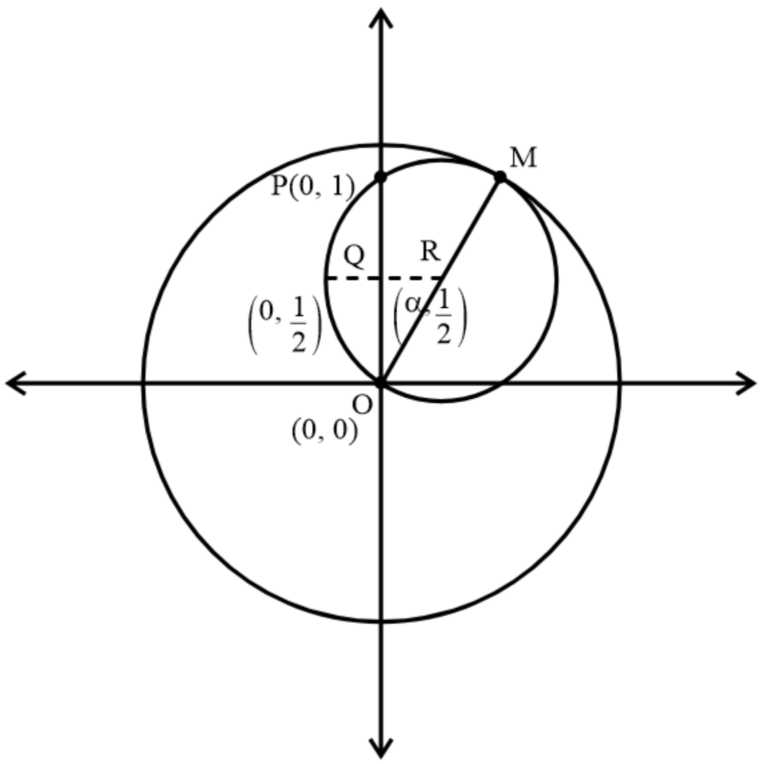
So, the mid point of chord OP is $Q \equiv \left(0, \frac{1}{2}\right)$.

We know that perpendicular bisector of chord of a circle passes through its centre.

So, centre of circle is $R \equiv \left(\alpha, \frac{1}{2}\right)$.

$\Rightarrow \beta = \frac{1}{2}$

Now, centre and radius of the circle $x^2 + y^2 = 9$ are $(0, 0)$ and 3 respectively.



$\Rightarrow OM = 2OR$

$\Rightarrow 3 = 2\sqrt{\alpha^2 + \frac{1}{4}}$

$\Rightarrow \frac{9}{4} = \alpha^2 + \frac{1}{4}$

$\Rightarrow \alpha^2 = 2$

$\Rightarrow 4\left(\alpha^4 + \beta^4\right) = 4\left(4 + \frac{1}{16}\right)$

$\Rightarrow 4\left(\alpha^2 + \beta^2\right) = \frac{65}{4}$

Q.50. If $f(x) = x^2 + 9$ and $g(x) = \frac{x}{x-9}$ and a curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a = fog(10),\ b = gof(3)$, then find $\frac{8e^2}{l}$, (where $e \rightarrow$ eccentricity and $l \rightarrow$ latus rectum length).

A) 12 B) 10

C) 9 D) 214

Answer: 214

Solution: Given: $f(x) = x^2 + 9$ and $g(x) = \frac{x}{x-9}$

$\Rightarrow fog(10) = f(g(10))$

$\Rightarrow fog(10) = f(10)$

$\Rightarrow fog(10) = 109 = a$

$\Rightarrow gof(3) = g(f(3))$

$\Rightarrow gof(3) = g(18)$

$\Rightarrow gof(3) = 2 = b$

So, the given equation of curve will be $\frac{x^2}{109} + \frac{y^2}{2} = 1$.

So, eccentricity will be given by,

$e = \sqrt{1 - \frac{2}{109}}$

$\Rightarrow e = \sqrt{\frac{107}{109}}$

And, the latus rectum will be given by,

$l = \frac{2\times2}{109} = \frac{4}{109}$

$\Rightarrow \frac{8e^2}{l} = \frac{8\times\frac{107}{109}}{\frac{4}{109}}$

$\Rightarrow \frac{8e^2}{l} = 214$

Q.51. A tetrahedral dice written $1, 2, 3, 4$ on their faces is thrown, find the probability such that quadratic equation $ax^2 + bx + c = 0$ has real roots.

A) $\frac{9}{64}$

B) $\frac{1}{4}$

C) $\frac{3}{16}$

D) $\frac{13}{64}$

Answer: $\frac{3}{16}$

Solution: Given: $ax^2 + bx + c = 0$

For real roots, $b^2 \geq 4ac$

So, the possible values of (b, a, c) could be,

$(2,1,1), (3,1,2), (3,2,1), (3,1,1), (4,1,1), (4,1,2), (4,2,1), (4,1,3), (4,3,1), (4,2,2), (4,1,4), (4,4,1)$

Thus, the number of favourable outcomes is 12.

Total number of outcomes is 64.

So, the required probability is given by,

$P(E) = \frac{12}{64}$

$\Rightarrow P(E) = \frac{3}{16}$

Q.52. If $f(x) = 3ax^3 + bx^2 + cx + 41,\ f(1) = 41,\ f'(1) = 2\ \&\ f''(1) = 4$ then find the value of $a^2 + b^2 + c^2$

Answer: 8

Solution: Given,

$f(x) = 3ax^3 + bx^2 + cx + 41$

$\Rightarrow f(1) = 3a + b + c + 41 = 41$

$\Rightarrow 3a + b + c = 0 \ .....(i)$

Now, solving $f'(x) = 9ax^2 + 2bx + c$

$\Rightarrow f'(1) = 9a + 2b + c = 2 \ .....(ii)$

And $f''(x) = 18a + 2b$

$\Rightarrow f''(1) = 18a + 2b = 4 \ ....(iii)$

Now, on solving the equation $(i), \ (ii) \ \& \ (iii)$ we get,

$a = 0, \ b = 2 \ \& \ c = -2$

Hence, $a^2 + b^2 + c^2 = 8$

Q.53. If $\frac{1}{1(1+d)} + \frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + ... \frac{1}{(1+9d)(1+10d)} = 5$ then find the value of $50d$

Answer: 5

Solution: Given,

$$\frac{1}{1(1+d)} + \frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + ... \frac{1}{(1+9d)(1+10d)} = 5$$

$$\Rightarrow \frac{1}{d}\left[\frac{d}{1(1+d)} + \frac{d}{(1+d)(1+2d)} + \frac{d}{(1+2d)(1+3d)} + ... \frac{d}{(1+9d)(1+10d)}\right] = 5$$

$$\Rightarrow \frac{1}{d}\left[\frac{1+d-1}{1(1+d)} + \frac{1+2d-(1+d)}{(1+d)(1+2d)} + ... \frac{(1+10d)-(1+9d)}{(1+9d)(1+10d)}\right] = 5$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{1} - \frac{1}{1+d} + \frac{1}{1+d} - \frac{1}{1+2d} + ... - \frac{1}{1+10d}\right] = 5$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{1} - \frac{1}{1+10d}\right] = 5$$

$$\Rightarrow \frac{1}{d}\left[\frac{1+10d-1}{1+10d}\right] = 5$$

$$\Rightarrow \left[\frac{10}{1+10d}\right] = 5$$

$\Rightarrow 1 + 10d = 2$

$\Rightarrow 10d = 1$

$\Rightarrow 50d = 5$

Q.54. If the coefficient of x^{70} in $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + ... x^{54}(1+x)^{46}$ is ${}^{99}C_p - {}^{46}C_q$ then the possible value of $p + q$ will be

Answer: 83

Solution: Given,

$x^2(1+x)^{98}+x^3(1+x)^{97}+x^4(1+x)^{96}+\ldots\ldots x^{54}(1+x)^{46}$

$=x^2(1+x)^{98}\frac{\left[1-\left(\frac{x}{x+1}\right)^{53}\right]}{1-\left(\frac{x}{x+1}\right)}$

$=x^2(1+x)^{98}\frac{\left[1-\left(\frac{x}{x+1}\right)^{53}\right]}{\frac{1}{x+1}}$

$=x^2(1+x)^{99}\left[\frac{(x+1)^{53}-x^{53}}{(x+1)^{53}}\right]$

$=x^2(1+x)^{46}\left[(x+1)^{53}-x^{53}\right]$

$=x^2(1+x)^{99}-x^{55}(1+x)^{46}$

Now, the coefficient of x^{70} in $x^2(1+x)^{99}-x^{55}(1+x)^{46}$ will be, ${}^{99}C_{68}-{}^{46}C_{15}$

Hence, $p+q=68+15=83$

Q.55. If a function f satisfies $f(m+n)=f(m)+f(n)$ for all $m,n\in N$ and $f(1)=1$ then find the largest natural number λ such that $\sum_{k=1}^{2022}f(\lambda+k)\leq 2022^2$

Answer: 1010

Solution: Given,

$f(m+n)=f(m)+f(n)$ and $f(1)=1$

$\Rightarrow f(x)=\alpha x$

Now by using $f(1)=1$ we get, $f(x)=x$

Now, solving

$\sum_{k=1}^{2022}f(\lambda+k)\leq 2022^2$

$\Rightarrow \sum_{k=1}^{2022}[f(\lambda)+f(k)]\leq 2022^2$

$\Rightarrow \lambda\sum_{k=1}^{2022}1+\sum_{k=1}^{2022}k\leq 2022^2$

$\Rightarrow \lambda\cdot 2022+\frac{2022\times 2023}{2}\leq 2022^2$

$\Rightarrow \lambda+\frac{2023}{2}\leq 2022$

$\Rightarrow \lambda\leq\frac{2021}{2}$

$\Rightarrow \lambda=1010$

Q.56. The sum of the squares of the modulus of the elements in set $\{z=a+ib;\ a,b\in Z,\ z\in C,\ |z-1|\leq 1,\ |z-5|\leq|z-5i|\}$ will be

Answer: 9

Solution: Given,

$|z-1| \leq 1$ & $|z-5| \leq |z-5i|$

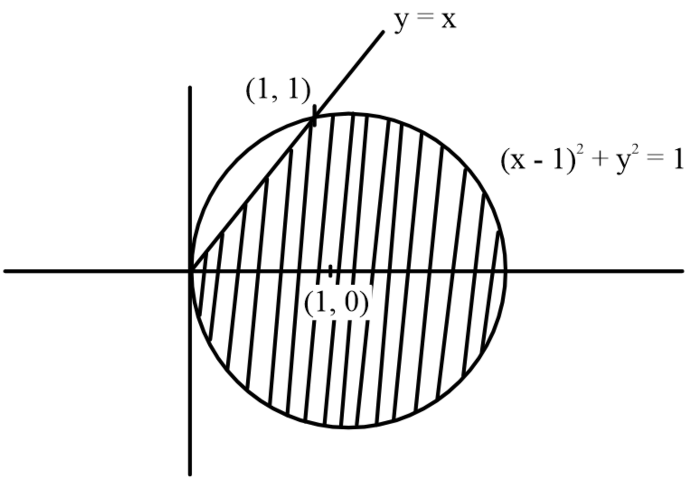
Now, solving $|z-5| \leq |z-5i|$

$\Rightarrow (x-5)^2 + y^2 \leq x^2 + (y-5)^2$

$\Rightarrow 25 - 10x \leq 25 - 10y$

$\Rightarrow y \leq x$

Now, plotting the diagram of $|z-1| \leq 1$ & $y \leq x$ we get,



So, from the above diagram the integral points will be $\{(0,0),(1,0),(1,1),(2,0),(1,-1)\}$

So, the complex numbers will be $z=0,\ z=1,\ z=1+i,\ z=1-i,\ z=2$

Hence, the sum of modulus of the elements will be $0+1+2+2+4=9$