JEE Main 2024

8th April Session 2

Physics

Q.1. A particle is projected at such an angle that its maximum height and range are same then find the angle of projection.

A) $\tan^{-1}(2)$

B) $\tan^{-1}(3)$

C) $\tan^{-1}(4)$

D) $\tan^{-1}(5)$

Answer: $\tan^{-1}(4)$

Solution: Equating the formulae for the maximum height and the range for a projectile, we have

$$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$
$$\Rightarrow \frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = 4$$
$$\Rightarrow \tan\theta = 4$$
$$\Rightarrow \theta = \tan^{-1}(4)$$

Q.2. If wavelength of electron and proton are same then find the ratio of their kinetic energies.

A) 1860

B) 1845

C) 1837

D) 1840

Answer: 1837

Solution: The kinetic energy of an electron is given by

$$Ke = \frac{p_e^2}{2me}$$
$$= \frac{1}{2me}\left(\frac{h}{\lambda}\right)^2 \quad \text{[By de Broglie's hypothesis]} \quad \ldots(1)$$

And the same for a proton is given by

$$Kp = \frac{p_p^2}{2mp}$$
$$= \frac{1}{2mp}\left(\frac{h}{\lambda}\right)^2 \quad \ldots(2)$$

Thus, equations (1) and (2) imply

$$\frac{Ke}{Kp} = \frac{\frac{1}{me}}{\frac{1}{mp}}$$
$$= \frac{mp}{me}$$
$$\approx 1837$$

Because we know that a proton is almost 1837 times heavier than an electron.

Q.3. A disc of mass m and radius R is rotating with angular speed ω. If another similar disc is placed gently on the rotating disc, then find out the new angular speed of the discs.

A) ω

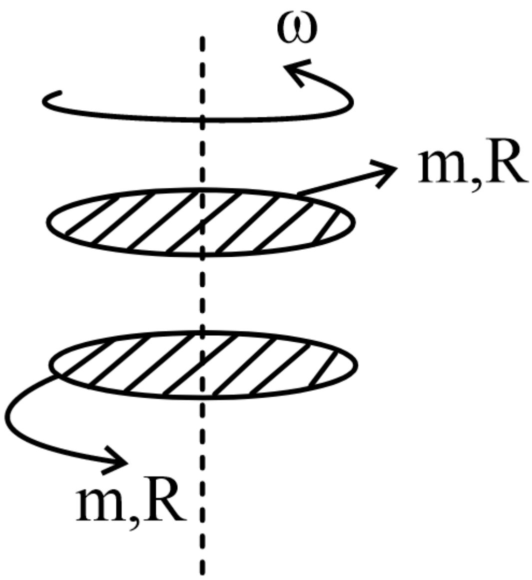
B) $\frac{\omega}{2}$

C) $\frac{\omega}{3}$

D) $\frac{\omega}{4}$

Answer: $\frac{\omega}{2}$

Solution: Let's consider the following diagram:



From the conservation of angular momentum, it follows that

$$L_i = L_f$$
$$\Rightarrow I_1\omega = I_2\omega' \quad \ldots(1)$$

From equation (1), it follows that

$$\frac{mR^2}{2}\omega = \left[\frac{mR^2}{2} + \frac{mR^2}{2}\right]\omega'$$
$$\Rightarrow \omega' = \frac{\omega}{2}$$

Q.4. Dimensional formula for $\varepsilon_0 E^2$, E being the electric field, is

A) $\left[ML^{-2}T^{-1}\right]$

B) $\left[ML^{-1}T^{-2}\right]$

C) $\left[M^{-1}L^{-2}T^{-1}\right]$

D) $\left[ML^{-3}T^{-2}\right]$

Answer: $\left[ML^{-1}T^{-2}\right]$

Solution: The energy density can be expressed as

$$\frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2 \quad \ldots(1)$$

From equation (1), it follows that

$$\frac{[\text{Energy}]}{[\text{Volume}]} = \left[\frac{1}{2}\varepsilon_0 E^2\right]$$
$$\Rightarrow \left[\varepsilon_0 E^2\right] = \frac{\left[ML^2T^{-2}\right]}{\left[L^3\right]}$$
$$= \left[ML^{-1}T^{-2}\right]$$

Q.5. The work done by a diatomic gas during an isobaric process is $100\ \text{J}$. Calculate the heat supplied.

A) $300\ \text{J}$

B) $350\ \text{J}$

C) $380\ \text{J}$

D) $400\ \text{J}$

Answer: $350\ \text{J}$

Solution: Given the work done in the isobaric process

$$dW = nR\,\Delta T$$
$$= 100 \text{ J}$$

Hence, the heat supplied can be calculated as follows:

$$dQ = nC_p\,\Delta T$$
$$= n\left[\frac{f}{2}+1\right]R\,\Delta T$$
$$= \left[\frac{f}{2}+1\right]nR\,\Delta T$$
$$= \left[\frac{5}{2}+1\right] \times 100 \text{ J} \quad (\text{as for diatomic gas, } f=5)$$
$$= 350 \text{ J}$$

Q.6. Two particles are projected horizontally from two different towers of heights H and $4H$ with velocity V and $\frac{V}{2}$ respectively. If horizontal range for first particle is 100 m then find horizontal range for other.

A) 100 m

B) 150 m

C) 200 m

D) 250 m

Answer: 100 m

Solution: First case:

Time of flight of the particle, $t_1 = \sqrt{\frac{2H}{g}}$

Therefore, horizontal range covered $R_1 = V \times \sqrt{\frac{2H}{g}}$

Second case::

Time of flight of the particle, $t_2 = \sqrt{\frac{2(4H)}{g}}$

Therefore, horizontal range covered $R_2 = \frac{V}{2} \times \sqrt{\frac{2(4H)}{g}} = V \times \sqrt{\frac{2H}{g}} = R_1 = 100 \text{ m}$

Q.7. A wave is given by the equation, $y = A\sin[\pi(330t - x)]$, then frequency of the wave is

A) 165 Hz

B) 330 Hz

C) 660 Hz

D) 200 Hz

Answer: 165 Hz

Solution: Standard equation of the wave is given by, $y = A\sin(\omega t - kx)$. By comparing it with the equation given in the question, we get

$$\omega t = 330\pi t$$
$$\Rightarrow \omega = 330\pi$$
$$\Rightarrow 2\pi f = 330\pi$$
$$\Rightarrow f = 165 \text{ Hz}$$

Q.8. An infinitely long current carrying wire of radius a carries uniform current. Find out the ratio of magnetic field at distance $\frac{a}{2}$ and $2a$.

A) a

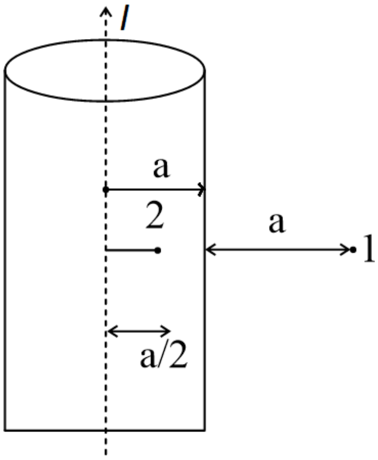
B) b

C) c

D) d

Answer: a

Solution:



We know that

$B_{inside} = B_2 = \frac{\mu_0 I r}{2\pi a^2}$

For $r = \frac{a}{2}$, we get

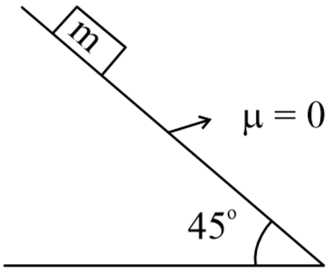
$B_{inside} = \frac{\mu_0 I a}{2\pi a^2 \times 2} = \frac{\mu_0 I}{4\pi a}$

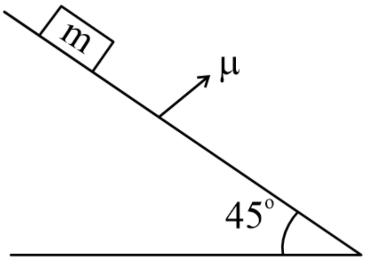
Now, at $r = 2a$

$B_{out} = B_1 = \frac{\mu_0 I}{2\pi(2a)} = \frac{\mu_0 I}{4\pi a}$

$\therefore \text{ratio} = \frac{B_{inside}}{B_{outside}} = \frac{B_2}{B_1} = 1:1$

Q.9. If the time taken by block on the frictionless incline is t and time taken on incline plane with friction is nt to reach at the bottom of the wedge. Find the value of μ in terms of n. (μ = Coefficient of friction)





A) n^2

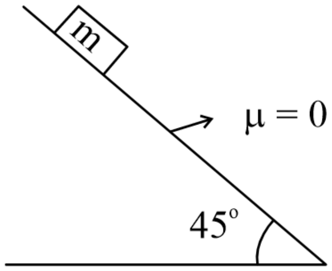
B) $1 - \frac{1}{n^2}$

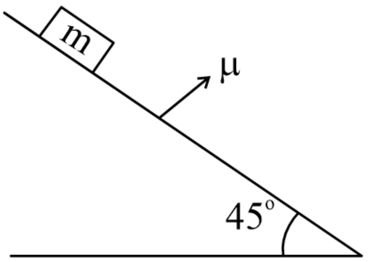
C) $\frac{1}{n}$

D) $n - 1$

Answer: $1 - \frac{1}{n^2}$

Solution:





Let the distance covered by the block be s.

For first case:

$$t=\sqrt{\frac{2s}{g\sin 45^\circ}}$$

For second case:

$$t'=\sqrt{\frac{2s}{g\sin 45^\circ-\mu g\cos 45^\circ}}$$
$$\Rightarrow nt=\sqrt{\frac{2s}{g\sin 45^\circ-\mu g\cos 45^\circ}}$$
$$\Rightarrow n\sqrt{\frac{2s}{g\sin 45^\circ}}=\sqrt{\frac{2s}{g\sin 45^\circ-\mu g\cos 45^\circ}}$$
$$\Rightarrow n^2=\frac{1}{1-\mu}$$
$$\Rightarrow \mu=1-\frac{1}{n^2}$$

Q.10. A water drop falls from sky and attends the terminal velocity of $6\ \text{cm s}^{-1}$. What will be the terminal velocity, in cm s^{-1} unit, if 8 similar drop condenses and falls from the sky?

A) 22 B) 23

C) 24 D) 25

Answer: 24

Solution: The formula for the terminal velocity of a spherical object is given by

$$v_T=\frac{2}{9}r^2g\frac{(\rho-\sigma)}{\eta}\quad \ldots(1)$$

Equation (1) implies that

$$v_T\propto r^2\quad \ldots(2)$$

If r be the radius of each tiny water drop which makes a bigger water drop of radius R before achieving the terminal velocity, it can be written that

$$8\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi R^3$$
$$\Rightarrow R=2r\quad \ldots(3)$$

So, using equations (2) and (3), the velocity of the bigger drop can be found as follows:

$$\frac{v_R}{v_r}=\frac{R^2}{r^2}$$
$$=\frac{(2r)^2}{r^2}$$
$$=4$$
$$\Rightarrow v_R=4v_r$$
$$=4\times 6\ \text{cm s}^{-1}$$
$$=24\ \text{cm s}^{-1}$$

Q.11. An isotope ${}^{12}_{5}B$ of mass m having proton(m_p) and neutron(m_n), then what will be the binding energy in terms of $m_p,\ m_n$ and m

A) $(2m-5m_p-7m_n)\times 931\ \text{MeV}$ B) $(m-7m_p-5m_n)\times 931\ \text{MeV}$

C) $\left(m-5m_p-7m_n\right)\times 931\ \text{MeV}$

D) $\left(m-5m_p-12m_n\right)\times 931\ \text{MeV}$

Answer: $\left(m-5m_p-7m_n\right)\times 931\ \text{MeV}$

Solution: Mass defect $\Delta M=\left(m-5m_p-7m_n\right)$

Therefore, binding energy will be $BE=\Delta M\times 931\ \text{MeV}=\left(m-5m_p-7m_n\right)\times 931\ \text{MeV}$

Q.12. An object of mass $0.2\ \text{kg}$ executes SHM along x-axis with frequency $\frac{25}{\pi}\ \text{Hz}$. At position $0.04\ \text{m}$. The body's K.E is $0.5\ \text{J}$ and P.E is $0.4\ \text{J}$. What is its amplitude in cm?.

A) 2

B) 6

C) 12

D) 15

Answer: 6

Solution: For any particle executing simple harmonic motion, the total mechanical energy is given by

$$E=K.E.+P.E.$$
$$=\frac{1}{2}m\omega^2A^2\quad \ldots(1)$$

From equation (1), it follows that

$$\frac{1}{2}\times 0.2\times\left(2\pi\times\frac{25}{\pi}\right)^2A^2=0.5+0.4$$
$$\Rightarrow A^2=\frac{0.9}{0.1\times 50^2}$$
$$\Rightarrow A=\sqrt{\frac{0.9}{0.1\times 50^2}}$$
$$\approx 0.06\ \text{m}$$
$$=6\ \text{cm}$$

Q.13. Two satellites are revolving around a planet at radius R and $4R$ respectively. If the speed of the first satellite is $6v$, then find the speed of the second satellite is βv. Find β.

Answer: 3

Solution: The orbital speed of a planet is given by

$$v=\sqrt{\frac{GM}{r}}\quad \ldots(1)$$

Using equation (1), it can be written that

$$v\propto\frac{1}{\sqrt{r}}\quad \ldots(2)$$

Using equation (2), the orbital speed of the second satellite can be calculated as follows:

$$\frac{v_1}{v_2}=\frac{\frac{1}{\sqrt{r_1}}}{\frac{1}{\sqrt{r_2}}}$$
$$=\sqrt{\frac{r_2}{r_1}}$$
$$=\sqrt{\frac{4R}{R}}$$
$$=2$$
$$\Rightarrow v_2=\frac{v_1}{2}$$
$$=\frac{6v}{2}$$
$$=3v$$

Hence, $\beta=3$.

Q.14. Some amount of water is heated using a constant supply source for 20 minutes. Now if we change the length L of heating element then same amount of water gets heated using same source in 15 minutes, the change in length is $\frac{L}{x}$. Find x .

Answer: 4

Solution: The amount of heat supplied by the source is given by

$$\Delta Q = P\,\Delta\,t \;\ldots(1)$$

Also, the power of the source is given by

$$P = \frac{V^2}{R} = \frac{V^2}{\frac{\rho L}{A}} = \frac{V^2 A}{\rho L} \quad \ldots(2)$$

Equations (1) and (2) imply that

$$\Delta Q = \frac{V^2 A}{\rho L}\,\Delta\,t$$
$$\Rightarrow \Delta t \propto L \quad \ldots(3)$$

Using equation (3), it can be written that

$$\frac{t_2}{t_1} = \frac{L_2}{L_1}$$
$$\Rightarrow \frac{L_2}{L_1} = \frac{15}{20}$$
$$\Rightarrow L_2 = \frac{3}{4}L_1$$
$$= \frac{3}{4}L$$

Hence, the change in length is given by

$$\Delta L = L_1 - L_2 = L - \frac{3}{4}L = \frac{L}{4}$$

Thus, $x = 4$.

Q.15. If least count of vernier calliper is $\frac{1}{20N}$ mm. If main scale division is 1 mm. The number of N division of vernier scale that coincide with main scale is given by $\left(N - \frac{1}{\beta}\right)$ mm. Find the value of β.

Answer: 20

Solution: The least count of the slide calliper can be calculated as follows:

$$L.C. = 1\ M.S.D. - 1\ V.S.D.$$
$$\Rightarrow \frac{1}{20N}\ \text{mm} = 1\ \text{mm} - 1\ V.S.D.$$
$$\Rightarrow 1\ V.S.D. = \left(1 - \frac{1}{20N}\right)\ \text{mm} \quad \ldots(1)$$

So, for the N divisions, from equation (1), it follows that

$$(N)\ V.S.D. = N\left(1 - \frac{1}{20N}\right)\ \text{mm} = \left(N - \frac{1}{20}\right)\ \text{mm}$$

Thus, $\beta = 20$.

Chemistry

Q.16. Molecular orbital σ^* represents:

A) $\psi_A + \psi_B$

B) $\psi_A - \psi_B$

C) $\psi_A - 2\psi_B$

D) $\psi_A + 2\psi_B$

Answer: $\psi_A - \psi_B$

Solution: Anti-bonding orbitals are formed where the atomic orbitals combine such that it leads to predominantly destructive interference i.e. subtraction of wave function. The most important feature of antibonding orbitals is that the molecular orbitals have higher energy than the corresponding atomic orbitals. Therefore, the molecule has higher energy than the isolated separate atoms.

Important characteristics of Anti-bonding Molecular Orbitals
-The probability of finding the electron in the internuclear region decreases in the antibonding molecular orbitals.
- The electrons present in the antibonding molecular orbital result in the repulsion between the two atoms.
-The anti-bonding molecular orbitals have higher energy because of the repulsive forces and lower stability.
They result from the subtractive effect of the atomic orbitals. The amplitude of the new wave is given by:

$\psi = \psi_A - \psi_B$

Q.17. Consider the given reaction:
$Cr_2O_7^{2-} \rightleftharpoons CrO_4^{2-}$
Above reaction shifts in forward direction in

A) Acidic Medium
B) Basic Medium
C) Neutral Medium
D) Slightly acidic medium

Answer: Basic Medium

Solution: The chromates and dichromates are interconvertible in aqueous solution depending upon pH of the solution. The oxidation state of chromium in chromate and dichromate is the same.

$2\ CrO_4^{2-} + 2H^+ \rightarrow Cr_2O_7^{2-} + H_2O$

$Cr_2O_7^{2-} + 2\ OH^- \rightarrow 2\ CrO_4^{2-} + H_2O$

Q.18. Select the correct options:
Statement 1: Benzene sulphonyl chloride reacts with 1°, 2° and 3° amines.
Statement 2: All products of the reaction above are soluble in NaOH.

A) Statement 1 is true and statement 2 is false.
B) Statement 1 is false and statement 2 is true.
C) Statement 1 and statement 2, both are true
D) Statement 1 and statement 2, both are false

Answer: Statement 1 and statement 2, both are false

Solution: Benzenesulphonyl chloride, which is also known as Hinsberg's reagent, reacts with primary and secondary amines to form sulphonamides.

(a) The reaction of benzenesulphonyl chloride with primary amine yields N-ethylbenzenesulphonyl amide. The hydrogen attached to nitrogen in sulphonamide is strongly acidic due to the presence of strong electron withdrawing sulphonyl group. Hence, it is soluble in alkali.

(b) In the reaction with secondary amine, N,N-diethylbenzenesulphonamide is formed. Since N, N-diethylbenzene sulphonamide does not contain any hydrogen atom attached to nitrogen atom, it is not acidic and hence insoluble in alkali.

(c) Tertiary amines do not react with benzenesulphonyl chloride. This property of amines reacting with benzenesulphonyl chloride in a different manner is used for the distinction of primary, secondary and tertiary amines and also for the separation of a mixture of amines.

Q.19. If de-broglie wavelength of the electron is equal to de-broglie wavelength of proton, then what is the realation between their kinetic energy

A) $KE_e > KE_p$
B) $KE_p > KE_e$
C) $KE_p = KE_e$
D) $2\,KE_e = KE_p$

Answer: $KE_e > KE_p$

Solution: de Broglie wavelength $\lambda = \frac{h}{\sqrt{2mKE}}$

We get $KE = \frac{h^2}{2m\lambda^2}$

$\Rightarrow KE \propto \frac{1}{m}$

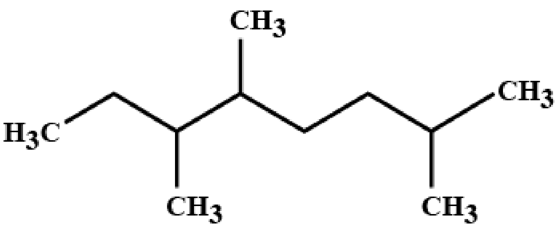
We know that $m_p > m_e$

$\therefore \frac{KE_e}{KE_p} = \frac{m_p}{m_e} > 1$

$\Rightarrow KE_e > KE_p$

Hence, the answer is option A.

Q.20. Consider the following compound:



What is the IUPAC nomenclature of the compound.

A) 2,5,6-trimethyl octane

B) 3,4,7-trimethyl octane

C) 2,4-ethyl, 3-methyl octane

D) Isopropyl hexane

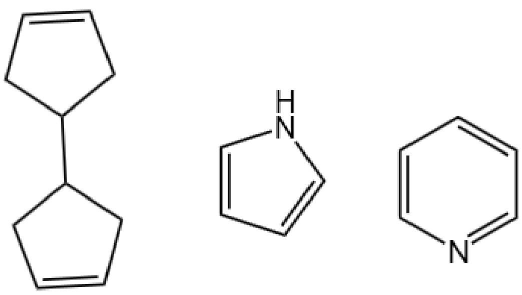
Answer: 2,5,6-trimethyl octane

Solution: The longest continuous chain of carbon atoms in the molecule consists of eight carbon atoms (oct- refers to eight).

Substituents within this parent chain are methyl Groups.
At carbon atom 2 of the octane chain, there is a methyl group attached. At carbon atom 5 of the octane chain, there is another methyl group attached. At carbon atom 6 of the octane chain, there is a third methyl group attached.

Therefore, 2,5,6-trimethyloctane is the IUPAC name of the given compound.

Q.21. Which of the following are aromatic compounds?



(i) (ii) (iii)

A) Only (i) and (ii)

B) Only (ii) and (iii)

C) Only (i) and (iii)

D) All are aromatic

Answer: Only (ii) and (iii)

Solution: Four Criteria for Aromaticity

The molecule is cyclic (a ring of atoms)

The molecule is planar (all atoms in the molecule lie in the same plane)

The molecule is fully conjugated (p orbitals at every atom in the ring)

The molecule has $4n+2$ π electrons ($n=0$ or any positive integer)

The structures pyrrole and pyridine are following above rules. Hence, they are both aromatic from the given compounds.

Q.22. Statement 1: Kjeldahl method fails for pyridine.

Statement 2: In Kjeldahl method pyridine is easily converted to N_2.

A) Both statements 1 and 2 are true

B) Both statements 1 and 2 are false

C) Statement 1 is true and 2 is false

D) Statement 1 is false and 2 is true

Answer: Statement 1 is true and 2 is false

Solution: Kjeldahl method is not applicable to compounds containing nitrogen in nitro and azo groups and nitrogen present in rings (e.g. pyridine, quinoline, isoquinoline) as nitrogen of these compounds does not convert to ammonium sulphate under the conditions of this method.

While the Kjeldahl's method is not suitable for compounds containing nitrogen in azo and nitro groups or in rings (quinoline, pyridine, etc.). In these cases, the nitrogen cannot be converted to ammonium sulphate by following the Kjeldahl method.

Hence, the answer is C.

Q.23. How many unpaired electrons present in $[NiCl_4]^{2-}$, $\left[Co(NH_3)_6\right]^{+3}$ respectively?

A) $0, 0$

B) $2, 4$

C) $2, 0$

D) $0, 4$

Answer: $2, 0$

Solution:

- The electronic configuration of Ni^{2+} in $\left[NiCl_4\right]^{2-}$ can be represented as $\left[Ar\right]\ 3d^8$.
- For a d^8 configuration, according to Hund's rule, all five d-orbitals will be singly occupied before any pairing occurs. Therefore, there are two unpaired electrons in the d orbitals of Ni^{2+} in $\left[NiCl_4\right]^{2-}$
- In the complex $\left[Co(NH_3)_6\right]^{+3}$, Cobalt (Co) in the $+3$ oxidation state loses three electrons compared to the neutral state, leading to the configuration $\left[Ar\right]3d^6$.
- The strong field ligands cause pairing of the d-electrons to maximise stability. So, there are no unpaired electrons present in it.

Q.24. Given

$$A \rightarrow B\ :\ k_1$$
$$B \rightarrow C\ :\ k_2$$

Rate of formation of B is zero. What is the concentration of B interms of A?

A) $\frac{k_1}{k_2}[A]$

B) $\frac{k_2}{k_1}[A]$

C) $k_1 k_2 [A]$

D) $k_1^2 [A]$

Answer: $\frac{k_1}{k_2}[A]$

Solution: The rate of consumption of A is

$$-\frac{d[A]}{dt} = k_1[A]$$

The rate of formation of B is

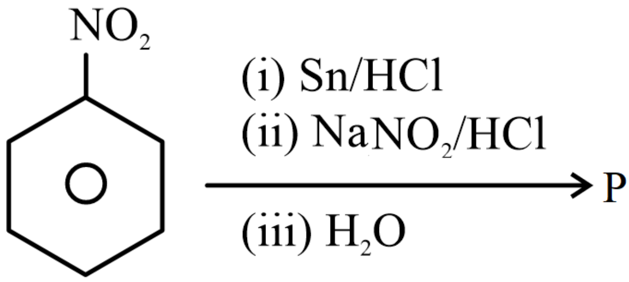
$$\frac{d[B]}{dt} = k_1[A] - k_2[B]$$

Given that, $\frac{d[B]}{dt} = 0$

$$\Rightarrow k_1[A] = k_2[B]$$

$$\Rightarrow \frac{k_1}{k_2}[A] = [B]$$

Q.25. The product P is:



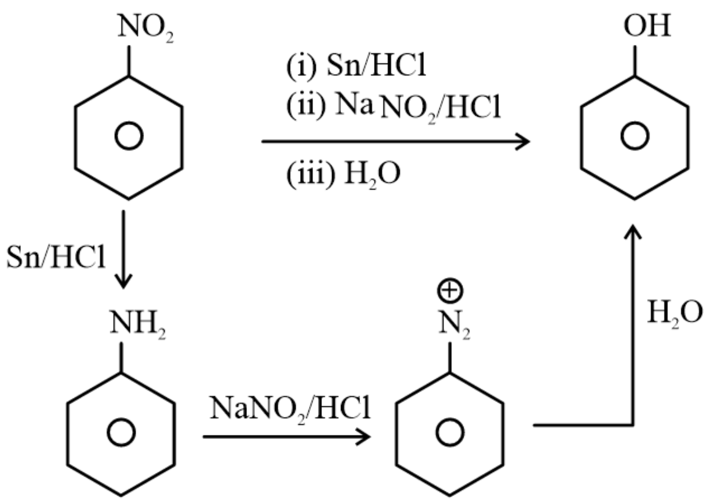
A) Phenol
B) Picric acid
C) Nitro benzene
D) Benzoic acid

Answer: Phenol

Solution: Reduction of Nitro Compounds: HCl / Sn can be used to reduce nitro compounds (compounds containing the $-NO_2$ group) to their corresponding amines. The reaction is known as the Clemmensen reduction. The nitro group is reduced to an amino group $(-NH_2)$ in the presence of HCl / Sn under reflux conditions.

Diazonium salt formation occurs when an aromatic primary amine reacts with nitrous acid or sodium nitrite in the presence of a strong acid such as hydrochloric acid. The process of converting a primary aromatic amine to a diazonium salt is referred to as diazotisation (or dissociation).

The diazonium ion reacts with the water in the solution and phenol is formed - either in solution or as a black oily liquid (depending on how much is formed). Nitrogen gas is evolved.

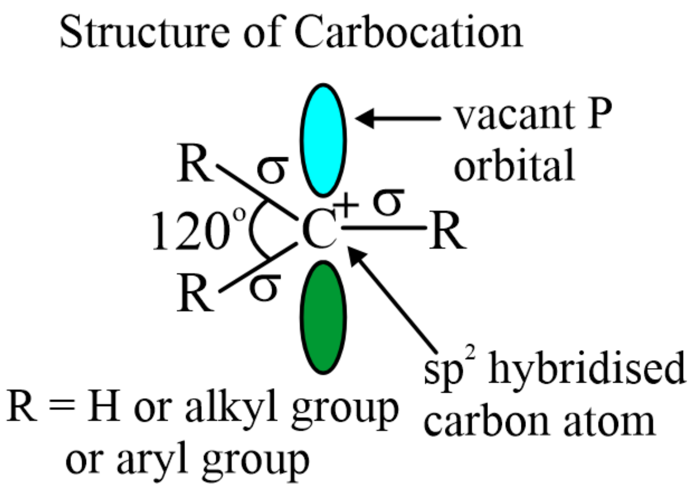


Q.26. Carbocation structure is:

A) Trigonal planar
B) Pyramidal
C) Tetrahedral
D) Sqaure planar

Answer: Trigonal planar

Solution: Carbocations typically have three substituents which makes the carbon sp^2hybridized and gives the overall molecule a trigonal planar geometry. The carbocation's substituents are all in the same plane and have a bond angle of $120°$ between them.



Q.27. For the reaction $Tl^{+}_{(0.001M)} + Cu_{(s)} \rightleftharpoons Tl_{(s)} + Cu^{2+}_{(0.01M)}$ $E^{o}_{cell} = 0.56V$. E_{cell} can be increased by

A) By increasing $\left[Cu^{2+}\right]$

B) By decreasing $\left[Cu^{2+}\right]$

C) By increasing $[Tl]$

D) By decreasing $[Tl]$

Answer: By decreasing $\left[Cu^{2+}\right]$

Solution: $Tl^{+}_{(0.001M)} + Cu_{(s)} \rightleftharpoons Tl_{(s)} + Cu^{2+}_{(0.01M)}$ $E^{o}_{cell} = 0.56V$

The Nernst equation for above reaction can be written as follows,

$$E_{cell} = E^{o}_{cell} - \frac{0.059}{2}\log\frac{\left[Cu^{2+}\right]}{\left[Tl^{+}\right]^2}$$

$Tl_{(s)}$ cannot effect the E_{cell} of the reaction. Hence, decrease of $\left[Cu^{2+}\right]$ can increase the E_{cell} of the reaction.

Q.28. The mole of H_2O and C_2H_5OH are $9\ \text{mol}\ \text{and}\ 1\ \text{mol}$ respectively. The mass percent of C_2H_5OH will be

A) 22.11%

B) 11.22%

C) 12%

D) 10%

Answer: 22.11%

Solution: Given that

$n_{H_2O} = 9,\ n_{C_2H_5OH} = 1$

So the mass of each can be written as follows,

$w_{H_2O} = 162\ \text{grams},\ w_{C_2H_5OH} = 46\ \text{grams}.$

The mass percentage of $C_2H_5OH = \frac{w_{C_2H_5OH}}{w_{C_2H_5OH} + w_{H_2O}} \times 100$

$\Rightarrow \frac{w}{w}\% \left(C_2H_5OH\right) = \frac{46}{46+162} \times 100 = 22.11\%$

Q.29. What is the formula of canary yellow precipitate?

A) $(NH_4)_3\left[P(Mo_3O_{10})_4\right]$

B) $(NH_4)_2\left[P(Mo_3O_{10})_2\right]$

C) $(NH_4)\left[P(Mo_3O_{10})_3\right]$

D) $(NH_4)_4\left[P(Mo_3O_{10})_3\right]$

Answer: $(NH_4)_3\left[P(Mo_3O_{10})_4\right]$

Solution: To check the presence of phosphorus in an organic compound , the organic compound is first treated with sodium peroxide, such that all the phosphorus gets converted to sodium phosphate.
The aqueous extract is then heated with concentrated nitric acid and ammonium molybdate. A canary yellow precipitate of ammonium phosphomolybdate is obtained. The formation of this yellow precipitate confirms the presence of phosphorus.
$Na_3PO_4 + 3\,HNO_3 \rightarrow H_3PO_4 + 3\,NaNO_3$

$$H_3PO_4 + 12(NH_4)_2MoO_4 + 21\,HNO_3 \rightarrow \underset{\text{Canary yellow ppt}}{(NH_4)_3PO_4.\,12\,MoO_3} + 21\,NH_4NO_3 + 12H_2O$$

Hence, the answer is option A.

Q.30. The total number of compounds having bond order 2 among the following are:

$F_2,\ N_2,\ Ne_2,\ O_2,\ Be_2$

Answer: 1

Solution: Bond order is the number of chemical bonds between a pair of atoms and indicates the stability of a bond.

Bond order $= \frac{N_b - N_a}{2}$

Fluorine molecule,F_2, has 18 electrons in total..it has 10 bonding electrons and 8 anti bonding electrons. therefore its bonding order is 1.

N_2 has 14 electrons, then it has bond order 3.

According to molecular orbital theory, the Ne_2 molecule does not exist because it has the same number of bonding and antibonding molecules, resulting in a bond order of zero.

The number of bonding electrons $= 10$

The number of anti-bonding electrons $= 6$

Hence, bond order is $= \frac{10-6}{2} = 2$

Bond order $= \frac{N_b - N_a}{2} = \frac{4-4}{2} = 0$

So, Be_2 does not exist.

Q.31. Given $\Delta H_{vap} = 40\ \mathrm{kJ/mol}$ for H_2O at temperature $273K$ and the pressure $1\ \mathrm{bar}$, Find ΔU_{vap}

Answer: 18

Solution: $H_2O_{(l)} \rightarrow H_2O_{(g)}$

$\Delta H = \Delta U + \Delta n_g RT.....(1)$
$\Delta\, n_g = 1$
$R = 8.314$
$T = 273\ K$

On substituting values in equation (1) we get

$40 \times 10^3 = \Delta U_{vap} + 1 \times 8.314 \times 273$
$\Delta\, U_{vap} = 17.6 \approx 18$

Q.32. Find the total number of correct statements:

1. N_2 behaves as inert gas at room temperature.

2. Oxides of metals are basic generally

3. Oxides of non metals are acidic generally

4. As we move down the group in group 15 then stability of +5 oxidation state decreases

5. General oxidation state of group 15 are +3, +5, -3

Answer: 5

Solution: 1. Due to the presence of triple bond between the two nitrogen atoms the bond dissociation energy of N_2 $(941.1\ \mathrm{kJ/mol})$) is very high therefore nitrogen is less reactive at room temperature.

2. Metallic oxides are basic in nature because they react with dilute acids to form salt and water. They also react with water to form metal hydroxides which are alkaline in nature because these metal hydroxides release OH^- ions in solution.

3. When non-metal oxides are dissolved in water it forms an acidic solution because these non-metal hydroxide release ions. It can be checked by dipping blue litmus paper into the formed solution. The blue litmus paper will turn red, showing the solution is acidic in nature.

4. Due to the inert pair effect, the stability of the +5 state decreases and the +3 state increases as we move down the group in the periodic table.

5. General oxidation state of group 15 are +3, +5, -3

All statements are correct. Hence, the answer is 5

Q.33. Given wavelength of wave is 15800Å . If its wave number is $X \times 10^{-1}\ \mathrm{cm}^{-1}$. The value of X is (Nearest integer)

Answer: 63291

Solution: Given that the wavelength of wave $= 15800\text{Å}\ = 15800 \times 10^{-8}\mathrm{cm}$

The relation between wavelength and wave number is

$\bar{\nu} = \frac{1}{\lambda}$

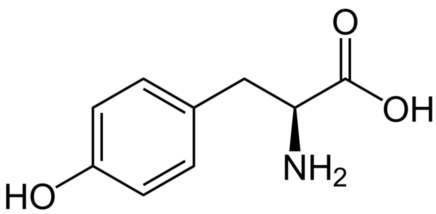
$\Rightarrow \bar{\nu} = \frac{1}{15800 \times 10^{-8}} = 6329.11\ \mathrm{cm}^{-1} = 63291.1 \times 10^{-1}\mathrm{cm}^{-1}$

Q.34. Total number of carbon in Tyrosine is

Answer: 9

Solution:

- **Chemical Formula of Tyrosine**: $C_9H_{11}NO_3$
- **Structure**: Tyrosine is an aromatic amino acid with a benzene ring attached to an amino group $(-NH_2)$ and a carboxyl group $(-COOH)$. It has a side chain consisting of a hydroxyl group $(-OH)$ attached to the benzene ring.



Mathematics

Q.35. The term independent of x in the expansion of $\left(2\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$ is 105, then a^2 is

A) $\frac{1}{16}$

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) $\frac{1}{8}$

Answer: $\frac{1}{4}$

Solution: Given expansion is $\left(2\sqrt{a}x^2+\frac{1}{2x^3}\right)^{10}$.

General term is given by,

$T_{r+1}={}^{10}C_r\left(2\sqrt{a}x^2\right)^{10-r}\left(\frac{1}{2x^3}\right)^r$

$\Rightarrow T_{r+1}={}^{10}C_r\left(2\sqrt{a}\right)^{10-r}\left(\frac{1}{2}\right)^r x^{20-5r}$

For term to be independent of x, $20-5r=0$.

$\Rightarrow r=4$

$\Rightarrow {}^{10}C_4\left(2\sqrt{a}\right)^6\left(\frac{1}{2}\right)^4=105$

$\Rightarrow \frac{10\times9\times8\times7\times4\times a^3}{4\times3\times2}=105$

$\Rightarrow 10\times3\times7\times4\times a^3=105$

$\Rightarrow 2\times4\times a^3=1$

$\Rightarrow a=\frac{1}{2}$

$\Rightarrow a^2=\frac{1}{4}$

Q.36. The line segment joining the points $(5,2)$ and $(2,a)$ subtends an angle $\frac{\pi}{4}$ at the origin then the absolute value of the product of all possible values of a is

A) 4

B) -4

C) 3

D) 5

Answer: -4

Solution: Slope of OP, $m_1=\frac{2-0}{5-0}=\frac{2}{5}$.

Slope of OQ, $m_2=\frac{a-0}{2-0}=\frac{a}{2}$

$\Rightarrow \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\Rightarrow \tan\frac{\pi}{4}=\left|\frac{\frac{2}{5}-\frac{a}{2}}{1+\frac{2}{5}\times\frac{a}{2}}\right|$

$\Rightarrow \tan\frac{\pi}{4}=\left|\frac{\frac{4-5a}{10}}{\frac{10+2a}{10}}\right|$

$\Rightarrow |10+2a|=|4-5a|$

$\Rightarrow 10+2a=4-5a,\ 10+2a=-(4-5a)$

$\Rightarrow 6=-7a,\ 14=3a$

$\Rightarrow a=\frac{-6}{7},\ \frac{14}{3}$

So, the required product is given by, $P=\frac{-6}{7}\times\frac{14}{3}=-4$

Q.37. If value of $\frac{3\cos36^\circ+5\sin18^\circ}{5\cos36^\circ-3\sin18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$ where a,b,c are natural numbers and $\gcd(a,c)=1$ then $a+b+c$ is

A) 50

B) 32

C) 100

D) 52

Answer: 52

Solution: Let, $y=\frac{3\cos 36^{\circ}+5\sin 18^{\circ}}{5\cos 36^{\circ}-3\sin 18^{\circ}}$

$\Rightarrow y=\frac{3\left(\frac{\sqrt{5}+1}{4}\right)+5\left(\frac{\sqrt{5}-1}{4}\right)}{5\left(\frac{\sqrt{5}+1}{4}\right)-3\left(\frac{\sqrt{5}-1}{4}\right)}$

$\Rightarrow y=\frac{3\sqrt{5}+3+5\sqrt{5}-5}{5\sqrt{5}+5-3\sqrt{5}+3}$

$\Rightarrow y=\frac{8\sqrt{5}-2}{2\sqrt{5}+8}$

$\Rightarrow y=\frac{4\sqrt{5}-1}{\sqrt{5}+4}\times\frac{\sqrt{5}-4}{\sqrt{5}-4}$

$\Rightarrow y=\frac{20-16\sqrt{5}-\sqrt{5}+4}{5-16}$

$\Rightarrow y=\frac{24-17\sqrt{5}}{-11}$

$y=\frac{17\sqrt{5}-24}{11}=\frac{a\sqrt{5}-b}{c}$

$\Rightarrow a=17,\ b=24,\ c=11$

$\Rightarrow a+b+c=52$

Q.38. If mean, mean deviation about mean and variance of 5 observations $a, 25, a, b, c$ are $18, 4$ and $\frac{136}{5}$ respectively, then

A) 83

B) 2

C) 1

D) 3

Answer: 1

Solution: Mean, $\frac{a+25+a+b+c}{5}=18$

$\Rightarrow 2a+25+b+c=90$

$\Rightarrow 2a+b+c=65$

Mean deviation about mean is given by, $\frac{|a-18|+7+|a-18|+|b-18|+|c-18|}{5}=4$

$\Rightarrow 2|a-18|+7+|b-18|+|c-18|=13$

Variance, $\frac{a^2+25^2+a^2+b^2+c^2}{5}-18^2=\frac{136}{5}$

$\Rightarrow \frac{2a^2+b^2+c^2+625}{5}=\frac{136}{5}+324$

$\Rightarrow 2a^2+b^2+c^2=1131$

Q.39. Find the value of $\int \frac{1}{\sqrt{1-e^x}}dx$.

A) $\log\left|\frac{\sqrt{1-e^x}+1}{\sqrt{1-e^x}-1}\right|+C$

B) $\log\left|\frac{\sqrt{1-e^x}-1}{\sqrt{1-e^x}+1}\right|+C$

C) $\log\left|\frac{\sqrt{1-e^x}+2}{\sqrt{1-e^x}-2}\right|+C$

D) $\log\left|\frac{\sqrt{1-e^x}-2}{\sqrt{1-e^x}+2}\right|+C$

Answer: $\log\left|\frac{\sqrt{1-e^x}-1}{\sqrt{1-e^x}+1}\right|+C$

Solution: Let, $I=\int \frac{1}{\sqrt{1-e^x}} dx$

Putting, $1-e^x=t^2$

$\Rightarrow -e^x dx=2t dt$

$\Rightarrow I=\int \frac{-2t}{e^x \times t} dt$

$\Rightarrow I=\int \frac{-2}{\left(1-t^2\right)} dt$

$\Rightarrow I=\int \frac{2}{t^2-1} dt$

$\Rightarrow I=\frac{2}{2} \log \left|\frac{t-1}{t+1}\right|+C$

$\Rightarrow I=\log \left|\frac{\sqrt{1-e^x}-1}{\sqrt{1-e^x}+1}\right|+C$

Q.40. If $Re\left(\frac{1+i \cos \theta}{1-i \cos \theta}\right)=0, \text{ where } \theta \in[-\pi, 2\pi]$, then find the sum of all values value of θ.

A) 3π

B) π

C) 2π

D) 4π

Answer: 2π

Solution: Given: $Re\left(\frac{1+i \cos \theta}{1-i \cos \theta}\right)=0$

$\Rightarrow Re\left(\frac{1+i \cos \theta}{1-i \cos \theta} \times \frac{1+i \cos \theta}{1+i \cos \theta}\right)=0$

$\Rightarrow Re\left(\frac{1+i^2 \cos^2 \theta+2i \cos \theta}{1-i^2 \cos^2 \theta}\right)=0$

$\Rightarrow Re\left(\frac{1-\cos^2 \theta+2i \cos \theta}{1+\cos^2 \theta}\right)=0$

$\Rightarrow \frac{1-\cos^2 \theta}{1+\cos^2 \theta}=0$

$\Rightarrow \cos^2 \theta=1$

$\Rightarrow \theta=-\pi, 0, \pi, 2\pi$

So, the required sum is 2π.

Q.41. Area bounded by $x^2+y^2=8 \text{ and } y^2=2x$ in 1^{st} quadrant is

A) $\frac{2}{3}+\pi$

B) $2+3\pi$

C) $\frac{3}{2}+\pi$

D) $\frac{3}{2}+2\pi$

Answer: $\frac{2}{3}+\pi$

Solution: Given: $x^2+y^2=8$ and $y^2=2x$

$\Rightarrow x^2+2x=8$

$\Rightarrow x^2+2x-8=0$

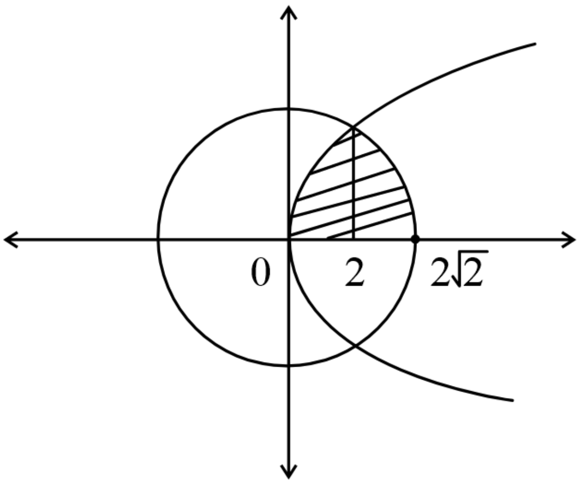
$\Rightarrow x^2+4x-2x-8=0$

$\Rightarrow x(x+4)-2(x+4)=0$

$\Rightarrow (x+4)(x-2)=0$

$\Rightarrow x=2,-4\ \left(\text{not possible as radius is } 2\sqrt{2}\right)$

$\Rightarrow y=\pm 2$



So, the points of intersection are $(2,2)$ and $(2,-2)$.

Now, the required area is given by,

$A=\int_0^2 \sqrt{2}x^{\frac{1}{2}}dx+\int_2^{2\sqrt{2}}\sqrt{8-x^2}dx$

$\Rightarrow A=\left[\frac{2\sqrt{2}x^{\frac{3}{2}}}{3}\right]_0^2+\left[\frac{x}{2}\sqrt{8-x^2}+4\sin^{-1}\frac{x}{2\sqrt{2}}\right]_2^{2\sqrt{2}}$

$\Rightarrow A=\frac{2\sqrt{2}\times 2\sqrt{2}}{3}+\frac{2\sqrt{2}}{2}\sqrt{8-8}+4\sin^{-1}\frac{2\sqrt{2}}{2\sqrt{2}}-\frac{2}{2}\sqrt{8-4}-4\sin^{-1}\frac{2}{2\sqrt{2}}$

$\Rightarrow A=\frac{8}{3}+2\pi-2-4\left(\frac{\pi}{4}\right)$

$\Rightarrow A=\left(\frac{2}{3}+\pi\right)$ square units.

Q.42. If $y=y(x)$ be the solution of the differential equation $\sec y\frac{dy}{dx}+2x\sin y=x^3\cos y,\ y(1)=0$ then find the value of $y\left(\sqrt{3}\right)$

A) $\frac{\pi}{3}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{8}$

D) $\frac{2\pi}{3}$

Answer: $\frac{\pi}{4}$

Solution: Given,

$$\sec y\frac{dy}{dx}+2x\sin y=x^3\cos y,\ y(1)=0$$

$$\Rightarrow \sec^2 y\frac{dy}{dx}+2x\tan y=x^3$$

Now, let $\tan y=t\Rightarrow \sec^2 y\frac{dy}{dx}=\frac{dt}{dx}$

$$\frac{dt}{dx}+2xt=x^3$$

Now, finding $IF=e^{\int 2x dx}=e^{x^2}$

Now, the solution is given by,

$$t\cdot e^{x^2}=\int x^3\cdot e^{x^2}dx$$

Now, let $x^2=z\Rightarrow 2xdx=dz$

$$\Rightarrow t\cdot e^{x^2}=\frac{1}{2}\int z\cdot e^z dz$$

$$\Rightarrow t\cdot e^{x^2}=\frac{1}{2}[ze^z-e^z]+c$$

$$\Rightarrow \tan y\cdot e^{x^2}=\frac{1}{2}e^{x^2}\left[x^2-1\right]+c$$

Now, using $y(1)=0$ we get,

$$\Rightarrow 0\cdot e^1=\frac{1}{2}e^1[0]+c\Rightarrow c=0$$

So, $\tan y=\frac{1}{2}\left[x^2-1\right]$

$$y=\tan^{-1}\left(\frac{1}{2}\left[x^2-1\right]\right)$$

Hence, $y\left(\sqrt{3}\right)=\tan^{-1}\left(\left(\frac{1}{2}[3-1]\right)\right)=\frac{\pi}{4}$

Q.43. If $\alpha=\lim_{x\to 0}\frac{e^{\sqrt{\tan x}}-e^{\sqrt{x}}}{\sqrt{\tan x}-\sqrt{x}}$ and $\beta=\lim_{x\to 0}(1+\sin x)^{\frac{1}{2\operatorname{cosec} x}}$, then $\alpha+\beta$ is

A) $\sqrt{e}-1$

B) $1-\sqrt{e}$

C) $1+\sqrt{e}$

D) $-1-\sqrt{e}$

Answer: $1+\sqrt{e}$

Solution: Given: $\alpha = \lim_{x\to 0} \frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}$

$\Rightarrow \alpha = \lim_{x\to 0} \frac{e^{\sqrt{x}}\left(e^{\sqrt{\tan x}-\sqrt{x}} - 1\right)}{\sqrt{\tan x} - \sqrt{x}}$

We know that, $\lim_{f(x)\to 0} \frac{\left(e^{f(x)} - 1\right)}{f(x)} = 1.$

$\Rightarrow \alpha = \lim_{x\to 0} e^{\sqrt{x}}$

$\Rightarrow \alpha = 1$

Now, $\beta = \lim_{x\to 0} (1 + \sin x)^{\frac{1}{2}\operatorname{cosec} x}$

$\Rightarrow \beta = \lim_{x\to 0} (1 + \sin x)^{\frac{1}{2\sin x}}$, which is 1^{∞} form

$\Rightarrow \beta = \lim_{x\to 0} e^{(1+\sin x-1)\frac{1}{2\sin x}} = e^{\frac{1}{2}}$

$\Rightarrow \alpha + \beta = 1 + \sqrt{e}$

Q.44. If the shortest distance between the lines $\frac{x-\lambda}{2} = \frac{y-4}{4} = \frac{z-7}{8}$ & $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{4}$ is $\frac{13}{\sqrt{21}}$, then the sum of values of λ is

A) $\frac{21}{5}$

B) $\frac{27}{5}$

C) $\frac{29}{5}$

D) $\frac{24}{5}$

Answer: $\frac{27}{5}$

Solution: Given,

$\frac{x-\lambda}{2} = \frac{y-4}{4} = \frac{z-7}{8}$ & $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{4}$

$\Rightarrow \frac{x-\lambda}{1} = \frac{y-4}{2} = \frac{z-7}{4}$ & $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{4}$

Now, direction ratio are equal so the lines are parallel,

Now, using the formula of shortest distance between the parallel lines we get,

$\mathrm{S.D} = \left|\frac{\hat{i}+2\hat{j}+4\hat{k}\times\left((\lambda-2)\hat{i}+\hat{j}+3\hat{k}\right)}{\left|\hat{i}+2\hat{j}+4\hat{k}\right|}\right|$

$\Rightarrow \frac{13}{\sqrt{21}} = \left|\frac{\hat{i}+2\hat{j}+4\hat{k}\times\left((\lambda-2)\hat{i}+\hat{j}+3\hat{k}\right)}{\sqrt{21}}\right|$

$\Rightarrow \left|\hat{i} + 2\hat{j} + 4\hat{k} \times \left((\lambda-2)\hat{i} + \hat{j} + 3\hat{k}\right)\right| = 13$

$\Rightarrow \left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ \lambda-2 & 1 & 3 \end{matrix}\right\| = 13$

$\Rightarrow \left|2\hat{i} + \hat{j}\,(4\lambda - 11) + \hat{k}\,(5 - 2\lambda)\right| = 13$

$\Rightarrow 2^2 + (4\lambda - 11)^2 + (5 - 2\lambda)^2 = 13^2$

$\Rightarrow 20\lambda^2 - 108\lambda + 150 = 169$

$\Rightarrow 20\lambda^2 - 108\lambda - 19 = 0$

So, the sum of roots will be $\frac{108}{20} = \frac{27}{5}$

Q.45. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=3\hat{i}-\hat{j}+\lambda\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}-5\hat{k}$. If $\vec{r}$, which is a unit vector, is parallel to $\vec{b}+\vec{c}$ and $\vec{r}.\vec{a}=3$, then find λ.

A) 4

B) 3

C) 2

D) 1

Answer: 1

Solution: Given: $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=3\hat{i}-\hat{j}+\lambda\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}-5\hat{k}$

$\Rightarrow \vec{r}=\vec{b}+\vec{c}=5\hat{i}+2\hat{j}+(\lambda-5)\hat{k}$

$\Rightarrow \hat{r}=\frac{5\hat{i}+2\hat{j}+(\lambda-5)\hat{k}}{\sqrt{25+4+(\lambda-5)^2}}$

Now, $\vec{r}.\vec{a}=3$

$\Rightarrow \frac{5+4+3(\lambda-5)}{\sqrt{29+(\lambda-5)^2}}=3$

$\Rightarrow \frac{3+(\lambda-5)}{\sqrt{29+(\lambda-5)^2}}=1$

$\Rightarrow (\lambda-2)=\sqrt{29+(\lambda-5)^2}$

$\Rightarrow \lambda^2+4-4\lambda=29+\lambda^2+25-10\lambda$

$\Rightarrow 6\lambda=50$

$\Rightarrow \lambda=\frac{25}{3}$

Q.46. If the system of equations,

$x+y-z=\lambda$

$7x+9y+\mu z=-3$

$5x+y+2z=-1$ has infinity many solutions then $|2\mu+19\lambda|$

Answer: 28

Solution: Given,

The system of linear equations,

$x + y - z = \lambda$

$7x + 9y + \mu z = -3$

$5x + y + 2z = -1$

The above equations has infinity many solutions,

So, $\triangle = \triangle_1 = \triangle_2 = \triangle_3 = 0$

Now, finding $\triangle = 0$

$\Rightarrow \begin{vmatrix} 1 & 1 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow 18 - \mu - (14 - 5\mu) - (7 - 45) = 0$

$\Rightarrow 4\mu = -42$

$\Rightarrow \mu = -\frac{21}{2} \ldots\ldots (i)$

Now, finding $\triangle_3 = \begin{vmatrix} 1 & 1 & \lambda \\ 7 & 9 & -3 \\ 5 & 1 & -1 \end{vmatrix} = 0$ we get,

$\Rightarrow -9 + 3 - (-7 + 15) + \lambda(7 - 45) = 0$

$\Rightarrow -14 + \lambda(-38) = 0$

$\Rightarrow \lambda = \frac{-7}{19} \ldots\ldots\ldots (ii)$

Hence, the value of $|2\mu + 19\lambda| = 28$

Q.47. If the image of the point $(-4, 5)$ in the line $x + 2y = 2$ lies on the circle $(x+4)^2 + (y-3)^2 = r^2$ then the value of r^2 is

Answer: 4

Solution: Given,

The image of the point $(-4, 5)$ in the line $x + 2y = 2$ lies on the circle $(x+4)^2 + (y-3)^2 = r^2$

So, finding the image of point $(-4, 5)$ in line $x + 2y = 2$ we get,

$\frac{x+4}{1} = \frac{y-5}{2} = -2\left[\frac{-4+10-2}{1^2+2^2}\right]$

$\Rightarrow \frac{x+4}{1} = \frac{y-5}{2} = \left[\frac{-8}{5}\right]$

$\Rightarrow (x, y) \equiv \left(\frac{-28}{5}, \frac{9}{5}\right)$

Now, the point $\left(\frac{-28}{5}, \frac{9}{5}\right)$ is lying on the circle,

Hence, using the distance formula between the point and centre of the circle we get,

$r = \sqrt{\left(\frac{-28}{5} + 4\right)^2 + \left(\frac{9}{5} - 3\right)^2} = \frac{\sqrt{8^2+6^2}}{5}$

$\Rightarrow r = \frac{10}{5} = 2$

$\Rightarrow r^2 = 4$

Q.48. If $\alpha \neq a,\ \beta \neq b\ \&\ \gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$ then find the value of $\frac{a}{a-\alpha} + \frac{b}{b-\beta} + \frac{c}{c-\gamma}$

Answer: 1

Solution: Given,

$$\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$$

Now, using operation $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$ we get,

$$\Rightarrow \begin{vmatrix} \alpha & b & c \\ a-\alpha & \beta - b & 0 \\ a-\alpha & 0 & \gamma - c \end{vmatrix} = 0$$

$$\Rightarrow (a-\alpha)(\beta-b)(\gamma-c) \begin{vmatrix} \frac{\alpha}{a-\alpha} & \frac{b}{\beta-b} & \frac{c}{(\gamma-c)} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (a-\alpha)(\beta-b)(\gamma-c)\left(\frac{\alpha}{a-\alpha} - \frac{b}{\beta-b} - \frac{c}{(\gamma-c)}\right) = 0$$

$$\Rightarrow \left(\frac{\alpha-a+a}{a-\alpha} - \frac{b}{\beta-b} - \frac{c}{(\gamma-c)}\right) = 0$$

$$\Rightarrow -1 + \frac{a}{a-\alpha} + \frac{b}{b-\beta} + \frac{c}{(c-\gamma)} = 0$$

$$\Rightarrow \frac{a}{a-\alpha} + \frac{b}{b-\beta} + \frac{c}{(c-\gamma)} = 1$$

Q.49. How many different words can be formed from the letters of the word $MATHEMATICS$?

Answer: 4989600

Solution: Total number of letters are 11 out of which $M \to 2,\ A \to 2,\ T \to 2$.So, the required number of words are given by,

$$N = \frac{11!}{2! \times 2! \times 2!}$$

$$\Rightarrow N = \frac{39916800}{8}$$

$$\Rightarrow N = 4989600$$

Q.50. The number of distinct real roots of the equation $|x+1||x+3| - 4|x+2| + 5 = 0$ is/are

Answer: 2

Solution: Given,

$|x+1|\,|x+3|-4\,|x+2|+5=0$

Now, taking the case when $x<-3$ we get,

$x^2+4x+3+4x+8+5=0$

$\Rightarrow x^2+8x+16=0$

$\Rightarrow x=-4\ldots\ldots(i)$

Now, taking the case when $-3\leq x<-2$ we get,

$-x^2-4x-3+4x+8+5=0$

$\Rightarrow -x^2+10=0$

$\Rightarrow x=\pm\sqrt{10}\ \{\text{No, solution as }-3\leq x<-2\}$

Now, taking the case when $-2\leq x<-1$ we get,

$-x^2-4x-4-4x-8+5=0$

$\Rightarrow -x^2-8x-6=0$

$\Rightarrow x^2+8x+6=0$

$\Rightarrow x=\frac{-8\pm\sqrt{40}}{2}\ \{\text{No, solution as }-2\leq x<-1\}$

Now, taking the case when $x\geq-1$ we get,

$x^2+4x+3-4x-8+5=0$

$\Rightarrow x^2=0$

$\Rightarrow x=0\ldots.(ii)$

Hence, from above equations only two roots are possible.