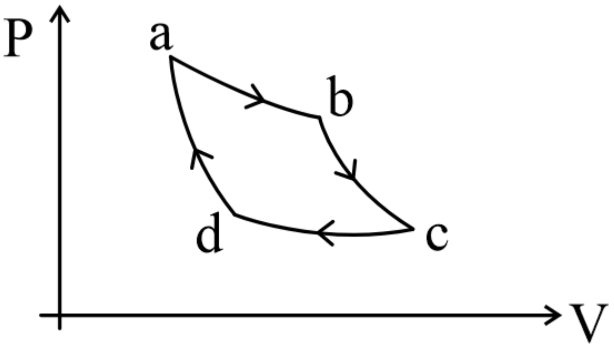
JEE Main 2024

8th April Session 1

Physics

Q.1. The PV curve shown in the diagram consists of two isothermal and two adiabatic curves.



A) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

B) $\frac{V_a}{V_d} = \left(\frac{V_c}{V_b}\right)^2$

C) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$

D) $\frac{V_a}{V_d} = \frac{V_c}{V_b}$

Answer: $\frac{V_a}{V_d} = \frac{V_b}{V_c}$

Solution: From the given diagram, the curves $a \to b$and $c \to d$ represent the isothermal processes, while the curves $b \to c$ and $d \to a$ represent the adiabatic processes.

Hence, for the complete processes, it can be written that

$T_a = T_b \quad \ldots(1)$
$T_c = T_d \quad \ldots(2)$
$T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1} \quad \ldots(3)$
$T_a V_a^{\gamma-1} = T_d V_d^{\gamma-1} \quad \ldots(4)$

Dividing equations (3) and (4), we have

$$\frac{T_b V_b^{\gamma-1}}{T_a V_a^{\gamma-1}} = \frac{T_c V_c^{\gamma-1}}{T_d V_d^{\gamma-1}}$$
$$\Rightarrow \frac{T_a V_b^{\gamma-1}}{T_a V_a^{\gamma-1}} = \frac{T_d V_c^{\gamma-1}}{T_d V_d^{\gamma-1}} \quad \text{[by equations (1) and (2)]}$$
$$\Rightarrow \frac{V_b^{\gamma-1}}{V_a^{\gamma-1}} = \frac{V_c^{\gamma-1}}{V_d^{\gamma-1}}$$
$$\Rightarrow \frac{V_b}{V_a} = \frac{V_c}{V_d}$$
$$\Rightarrow \frac{V_a}{V_d} = \frac{V_b}{V_c}$$

Q.2. The correct expression for Bernoulli's theorem is (the symbols have their usual meaning)

A) $P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

B) $P + \rho g h + \rho v^2 = \text{constant}$

C) $P + 2\rho g h + \rho v^2 = \text{constant}$

D) $P + \frac{1}{2}\rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

Answer: $P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

Solution: Bernoulli's equation is a fundamental principle in fluid dynamics that describes the behaviour of an ideal fluid moving along a streamline. It's named after the Swiss mathematician Daniel Bernoulli who formulated it in the 18th century. The equation states that in a steady, incompressible flow of a fluid, the sum of the pressure energy, kinetic energy, and potential energy per unit volume remains constant along any streamline. Mathematically, it can be expressed as:

$$P+\rho g h+\frac{1}{2} \rho v^2=\text{constant}$$

Bernoulli's equation is derived from the conservation of energy principle applied to a flowing fluid.

Q.3. Three masses $m_1=1.2\ \text{kg},\ m_2=1.6\ \text{kg},\ m_3=400\ \text{g}$ have the same kinetic energy. Find the ratio of their momentum.

A) $\sqrt{2}: 2: 1$

B) $\sqrt{3}: 2: 1$

C) $1: \sqrt{2}: 1$

D) $\sqrt{2}: \sqrt{2}: 1$

Answer: $\sqrt{3}: 2: 1$

Solution: The formula for kinetic energy is given by

$$K=\frac{p^2}{2 m} \quad \ldots(1)$$

Given that

$$K_1=K_2=K_3 \quad \ldots(2)$$

Equations (1) and (2) imply that

$$\frac{p_1^2}{2 m_1}=\frac{p_2^2}{2 m_2}=\frac{p_3^2}{2 m_3}$$

$$\Rightarrow \frac{p_1^2}{2 \times 1.2}=\frac{p_2^2}{2 \times 1.6}=\frac{p_3^2}{2 \times 0.4}$$

$$\Rightarrow \frac{p_1^2}{2.4}=\frac{p_2^2}{3.2}=\frac{p_3^2}{0.8}$$

$$\Rightarrow \frac{p_1^2}{3}=\frac{p_2^2}{4}=\frac{p_3^2}{1}=k \text{ (let)} \quad \ldots(3)$$

Equation (3) implies that

$$p_1^2: p_2^2: p_3^2=3 k: 4 k: k$$
$$\Rightarrow p_1: p_2: p_3=\sqrt{3}: 2: 1$$

Q.4. In a series LCR Circuit, the value of resistance as well as $\left(X_L-X_C\right)$ is halved, then the new current amplitude $\left(I_2\right)$ will satisfy: ($I_1$is old current amplitude)

A) $I_2=0$

B) $I_2=\frac{I_1}{2}$

C) $I_2=I_1$

D) $I_2=2 I_1$

Answer: $I_2=2 I_1$

Solution: Current amplitude is given by, $I_1=\frac{V_0}{\sqrt{R^2+\left(X_L-X_C\right)^2}}$

Now for the second case, we can write

$$I_2=\frac{V_0}{\sqrt{\left(\frac{R}{2}\right)^2+\left(\frac{X_L-X_C}{2}\right)^2}}=2 \times \frac{V_0}{\sqrt{R^2+\left(X_L-X_C\right)^2}}=2 I_1$$

Q.5. If a stationary particle is cut into two masses m_a and m_b, with velocities v_a and v_b, then the ratio of the kinetic energies of the two particles is.

A) $m_b v_b: m_a v_a$

B) $m_b: m_a$

C) $v_b: v_a$

D) $1: 1$

Answer: $m_b: m_a$

Solution: As the initial particle is at rest, the split parts will have opposite velocities.

Hence, from the conservation of momentum, it follows that

$$m_a v_a = m_b v_b$$
$$\Rightarrow p_a = p_b \quad \dots(1)$$

The ratio of the kinetic energies can be found as follows:

$$\frac{K_a}{K_b} = \frac{\frac{p_a^2}{2m_a}}{\frac{p_b^2}{2m_b}}$$
$$= \frac{m_b}{m_a} \quad [\text{by equation (1)}]$$

Q.6. If an electron and a proton have the same kinetic energy, find the ratio of their linear momentum. (mass of electron = 9.1×10^{-31} kg, mass of proton = 1.67×10^{-27} kg)

A) 1.33×10^{-2} B) 1.67×10^{-3}

C) 2.33×10^{-2} D) 1.23×10^{-2}

Answer: 2.33×10^{-2}

Solution: The formula for kinetic energy is given by

$$K = \frac{p^2}{2m} \quad \dots(1)$$

Equation (1) implies that

$$p = \sqrt{2mK} \quad \dots(2)$$

Given that,

$$K_e = K_p$$

Hence, using equation (2), the ratio of momentum of an electron to that of a proton is given by

$$\frac{p_e}{p_p} = \frac{\sqrt{2m_e K_e}}{\sqrt{2m_p K_p}}$$
$$= \sqrt{\frac{m_e}{m_p}}$$
$$= \sqrt{\frac{9.1 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}}}$$
$$\approx 2.33 \times 10^{-2}$$

Q.7. Two charged spheres have radii a, b then ratio of their charges when potential of both spheres is equal is

A) $\frac{a}{b}$ B) $\frac{a^2}{b^2}$

C) $\frac{b}{a}$ D) $\frac{b^2}{a^2}$

Answer: $\frac{a}{b}$

Solution: Potential of a charged sphere is given by, $V = \frac{kq}{r}$.

Therefore, we can write

$$\frac{kq_1}{a} = \frac{kq_2}{b}$$
$$\Rightarrow \frac{q_1}{q_2} = \frac{a}{b}$$

Q.8. Which of the following is incorrect for paramagnetic materials?

A) They align in the direction of magnetic field B) They are strongly attracted by magnetic field

C) Magnetic susceptibility is slightly more than zero.

D) None of the above

Answer: They are strongly attracted by magnetic field

Solution: Paramagnetic substances are materials that exhibit paramagnetism, a phenomenon where they are weakly attracted to an external magnetic field.

Here are some properties of paramagnetic substances:

a) Paramagnetic substances are weakly attracted to external magnetic fields.

b) The magnetisation of paramagnetic materials decreases with increasing temperature.

c) Unlike ferromagnetic materials, paramagnetic substances do not retain magnetisation in the absence of an external magnetic field.

d) Paramagnetic substances exhibit weak magnetisation in the presence of an external magnetic field.

From the above discussion, it can be concluded that this is the correct option.

Q.9. For a light ray incident the critical angle is $\theta = 45^{\circ}$, find the ratio of refractive index of desnser medium to rarer medium.

A) $\frac{\sqrt{2}}{1}$

B) $\frac{1}{\sqrt{2}}$

C) $\frac{1}{2}$

D) $\frac{2}{1}$

Answer: $\frac{\sqrt{2}}{1}$

Solution: The critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

Therefore, we can write

$$\mu_d \sin 45^{\circ} = \mu_r \sin 90^{\circ}$$
$$\Rightarrow \frac{\mu_d}{\mu_r} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} : 1$$

Q.10. If the radius of Earth is reduced by one fourth of its present value, then duration of day will be

A) $13 \text{ hrs } 20 \text{ mins}$

B) $18 \text{ hrs } 20 \text{ mins}$

C) $13 \text{ hrs } 30 \text{ mins}$

D) $16 \text{ hrs } 10 \text{ mins}$

Answer: $13 \text{ hrs } 30 \text{ mins}$

Solution: From the principle of conservation of angular momentum, it can be written that

$$I_1 \omega_1 = I_2 \omega_2$$
$$\Rightarrow \frac{2}{5} Me Re^2 \omega_1 = \frac{2}{5} Me \left(Re - \frac{Re}{4}\right)^2 \omega_2$$
$$\Rightarrow Re^2 \omega_1 = \frac{9}{16} Re^2 \omega_2$$
$$\Rightarrow \omega_1 = \frac{9}{16} \omega_2$$
$$\Rightarrow \frac{2\pi}{T_1} = \frac{9}{16} \frac{2\pi}{T_2}$$
$$\Rightarrow T_2 = \frac{9}{16} T_1 \quad \ldots (1)$$

From equation (1), it follows that

$$T_2 = \frac{9}{16} \times 24 \text{ hrs}$$
$$= 13 \text{ hrs } 30 \text{ mins}$$

Q.11. An electromagnetic radiation of intensity 360 W cm^{-2} is incident normally on a non-reflecting surface having area A. The average force on the surface is found to be $2.4 \times 10^{-4} \text{ N}$. Find the value of A.

A) 0.02 m^2

B) 2 m^2

C) 0.2 m^2

D) 20 m^2

Answer: 0.02 m^2

Solution: The formula for the radiation pressure for a non-reflecting surface is given by

$\frac{F}{A}=\frac{I}{c} \quad \ldots(1)$

From equation (1), it follows that

$\frac{2.4\times10^{-4}\,\mathrm{N}}{A}=\frac{360\times10^{4}\,\mathrm{W\,m^{-2}}}{3\times10^{8}\,\mathrm{m\,s^{-1}}}$

$\Rightarrow A=\frac{2.4\times10^{-4}\,\mathrm{N}\times3\times10^{8}\,\mathrm{m\,s^{-1}}}{360\times10^{4}\,\mathrm{W\,m^{-2}}}$

$=0.02\ \mathrm{m}^{2}$

Q.12. A solenoid of 10 turns, cross-section area $36\ \mathrm{cm}^{2}$ and of resistance $10\ \mathrm{m}\Omega$ is placed in magnetic field which is varying at a constant rate of $0.5\ \mathrm{T\ s}^{-1}$. Find the rate of heat dissipation.

A) $1.8\ \mathrm{W}$

B) $3.24\ \mathrm{W}$

C) $3.8\ \mathrm{W}$

D) $7.6\ \mathrm{W}$

Answer: $3.24\ \mathrm{W}$

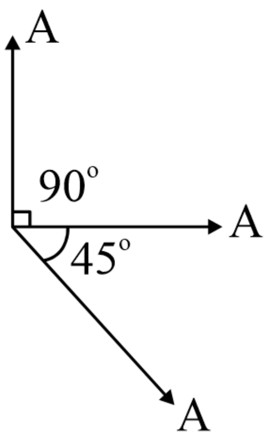
Solution: Magnetic flux is given by, $\phi=NBA$.

Therefore, EMF induced $\varepsilon=\left|\frac{d\phi}{dt}\right|=NA\left|\frac{dB}{dt}\right|=10\times\left(36\times10^{-4}\right)\times0.5=18\times10^{-2}\ \mathrm{V}$

Now, rate of heat dissipation will be

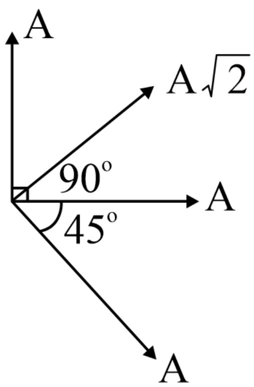
$P=\frac{\varepsilon^{2}}{R}=\frac{\left(18\times10^{-2}\right)^{2}}{10\times10^{-3}}=3.24\ \mathrm{W}$

Q.13. If the resultant of the vectors shown is $A\sqrt{x}$, then find x.



Answer: 3

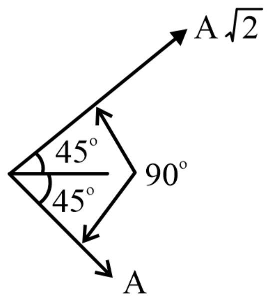
Solution: Let's consider the following diagram:



From the above figure, the resultant of the vectors marked is given by

$$R_1 = \sqrt{A^2 + A^2}$$
$$= \sqrt{2}A$$

Now look the figure below:



With respect to the above diagram, the net resultant vector is given by

$$R = \sqrt{\left(\sqrt{2}A\right)^2 + A^2}$$
$$= \sqrt{3}A$$

Hence, $x = 3$.

Q.14. In a clock, second hand and minute hand are of $75\ \text{cm}$ and $60\ \text{cm}$ respectively. After 30 minutes, ratio of distance travelled by the tip of second hand to that of minute hand is α. Find α.

Answer: 75

Solution: In one minute, the second arm travels a distance of $2\pi r$, where r is the length of the second hand.

Hence, at the given time, the distance travelled by the second hand is given by

$$D_s = 2\pi \times 75\ \text{cm} \times 30 \quad \ldots(1)$$

In the given time, the minute hand will cover half the full circular path. Hence, the distance travelled by the minute hand is given by

$$Dm = 2\pi \times \frac{60\ \text{cm}}{2}$$
$$= 2\pi \times 30\ \text{cm} \quad \ldots(2)$$

Thus, the required ratio can be found as

$$\frac{Ds}{Dm} = \frac{2\pi \times 75\ \text{cm} \times 30}{2\pi \times 30\ \text{cm}}$$
$$= 75$$

Hence, $\alpha = 75$.

Q.15. Find the force(in N) acting on the man, when it stops a ball($m = 150\ \text{g}$) moving with speed $20\ \text{m s}^{-1}$ in $0.1\ \text{s}$.

Answer: 30

Solution: When the ball stops, its velocity becomes 0 m s^{-1}.

Then, we can write

$$F = m\left|\frac{dv}{dt}\right| = \frac{150}{1000} \times \left|\frac{0-20}{0.1}\right| = 30 \text{ N}$$

Q.16. A closed pipe and an open pipe resonate with the same length. Then the ratio of frequencies in 7^{th} overtone is given by $\frac{\alpha-1}{\alpha}$. Find the value of α.

Answer: 16

Solution: For an open pipe, 7^{th} overtone correspond to the 8^{th} harmonic, while for a closed pipe the 7^{th} overtone corresponds to the 15^{th} harmonic.

The formula for the m^{th} harmonic in an open pipe is given by

$$\nu m = \frac{mv}{2L} \quad \ldots(1)$$

and the same for a closed pipe is given by

$$\nu m' = \frac{mv}{4L} \quad \ldots(2)$$

where, v is the velocity of sound and L is the length of the pipe.

Thus, the required ratio of the overtones can be found as follows:

$$\frac{\nu c}{\nu o} = \frac{\frac{15v}{4L}}{\frac{8v}{2L}} = \frac{15}{16} = \frac{16-1}{16}$$

Hence, $\alpha = 16$.

Chemistry

Q.17. Three equilibrium reactions given below.

$\mathrm{X} \rightleftharpoons \mathrm{Y} \quad \mathrm{K}_1 = 1$

$\mathrm{Y} \rightleftharpoons \mathrm{Z} \quad \mathrm{K}_2 = 2$

$\mathrm{Z} \rightleftharpoons \mathrm{W} \quad \mathrm{K}_3 = 4$

Calculate the equilibrium constant for $\mathrm{X} \rightleftharpoons \mathrm{W}$.

A) $\frac{1}{4}$

B) 8

C) $\frac{1}{16}$

D) 4

Answer: 8

Solution: If two or more reactions are added to give another, the equilibrium constant for the reaction is the product of the equilibrium constants of the equations added. K_1 , K_2, K_3 represent the equilibrium constants for reactions being added together, and K represents the equilibrium constant for the desired reaction.

$$\mathrm{K} = \mathrm{K}_1\mathrm{K}_2\mathrm{K}_3$$

$$\mathrm{K} = 1 \times 2 \times 4 = 8$$

Q.18. Match column I with column II:

Column I	Column II
1.NH_3	P. Trigonal bipyramidal
2.BrF_5	Q. Pyramidal
3.PCl_5	R. Tetrahedral
4.CH_4	S. Square pyramidal

A) $1-Q,\ 2-S,\ 3-P,\ 4-R$

B) $1-Q,\ 2-S,\ 3-R,\ 4-P$

C) $1-S,\ 2-Q,\ 3-P,\ 4-R$

D) $1-Q,\ 2-P,\ 3-S,\ 4-R$

Answer: $1-Q,\ 2-S,\ 3-P,\ 4-R$

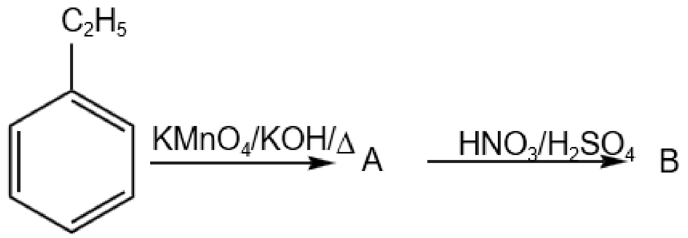
Solution: 1. NH_3 molecule is sp^3 hybridized and has a pyramidal shape.

2.BrF_5 molecule is surrounded by six electron pairs. Hybridisation of Br atom in this molecule is sp^3d^2. Five positions are occupied by F atoms forming sigma bonds with sp^3d^2 hybrid orbitals and one position occupied by lone pair. So, the molecule has a square pyramidal shape.

3. The hybridisation of PCl_5 is sp^3d and the shape of the molecule is trigonal bipyramidal.

4. In the molecule CH_4 the central atom C has 4 valence electrons where the C atom is forming 4 sigma bonds with H atoms and therefore the stearic number of C is 4 which imply that the hybridisation of the molecule is sp^3 where the geometry and the shape is tetrahedral.

Q.19.



The number of π bonds in product B is

A) 4

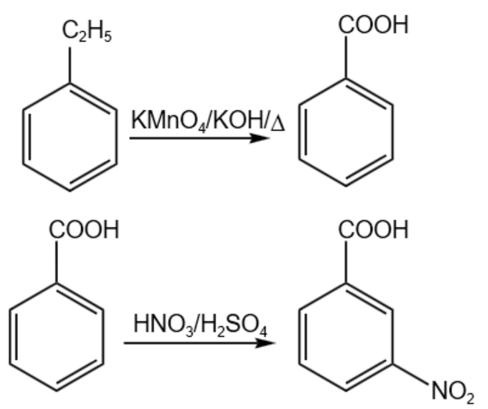
B) 5

C) 6

D) None of these

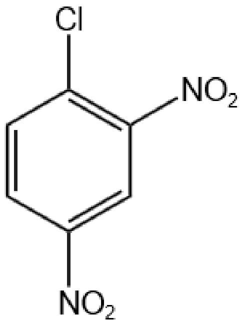
Answer: 5

Solution: Ethyl benzene undergoes oxidation to benzoic acid in the presence of hot potassium permanganate in a basic medium. Benzoic acid on nitration gives meta nitro benzoic acid as shown below.



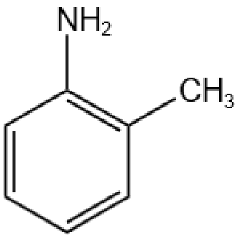
Q.20. Consider the following statements.

Statement-1 :



IUPAC name is 4-chloro-1,3-dinitrobenzene.

Statement-2:



IUPAC name is 2-methylaniline.

A) Both Statement-1 and Statement-2 are correct.

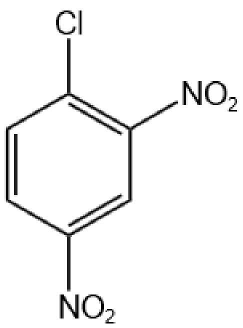
B) Both Statement-1 and Statement-2 are incorrect.

C) Statement-1 is incorrect and Statement-2 is correct.

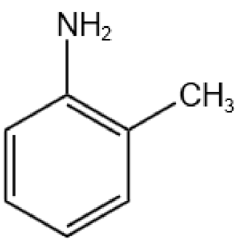
D) Statement-1 is correct and Statement-2 is incorrect.

Answer: Statement-1 is incorrect and Statement-2 is correct.

Solution:



In the above compound, parent chain name is benzene. Now the lowest locant rule must be followed to give numbering to substituents. So, the IUPAC name is 1-chloro-2,4-dinitrobenzene.



In the above compound methyl group is attached to aniline at second carbon. So, the IUPAC name is 2-methylaniline.

Q.21. Match column I with column II

Column I	Column II
1. Ammonium phospho Molybdate	P. Blue colour
2.$Fe_4[Fe(CN)_6]_3$	Q. Yellow colour
3.$K_3[Co(NO_2)_6]$	R. Brown colour
4.$[Fe(H_2O)_5(NO)]SO_4$	S. Canary yellow colour

A) $1-S,\ 2-P,\ 3-Q,\ 4-R$

B) $1-P,\ 2-S,\ 3-Q,\ 4-R$

C) $1-S,\ 2-P,\ 3-R,\ 4-Q$

D) None of the above

Answer: $1-S,\ 2-P,\ 3-Q,\ 4-R$

Solution:

Column I	Column II
1. Ammonium phospho Molybdate	P. Canary yellow colour
2.$Fe_4[Fe(CN)_6]_3$	Q. Blue colour
3.$K_3[Co(NO_2)_6]$	R. Yellow colour
4.$[Fe(H_2O)_5(NO)]SO_4$	S. Brown colour

Q.22. We have two complexes, $[Fe(H_2O)_6]^{2+}$ and $[Cu(H_2O)_6]^{2+}$. The magnetic properties respectively are

A) Diamagnetic and diamagnetic

B) Paramagnetic and Paramagnetic

C) Diamagnetic and Paramagnetic

D) Paramagnetic and Diamagnetic

Answer: Paramagnetic and Paramagnetic

Solution: Iron in the +2 oxidation state ($[Fe(H_2O)_6]^{2+}$) has an electronic configuration of $[Ar]3d^6$. Since there are four unpaired electrons out of six d-electrons, $[Fe(H_2O)_6]^{2+}$ exhibits paramagnetism. Paramagnetic substances are attracted to an external magnetic field due to the presence of unpaired electrons.

Copper in the +2 oxidation state ($[Cu(H_2O)_6]^{2+}$) has an electronic configuration of $[Ar]3d^9$. The copper ion has one unpaired electron, out of nine d-electrons, hence, it is paramagnetic.

Q.23. Statement 1: For 13th group element stability of oxidation state is: $Ga^+ < In^+ < Tl^+$

Statement 2: On moving down the group stability of lower oxidation state increases due to poor shielding of d and f electron.

A) Both statements 1 and 2 are false

B) Both statements 1 and 2 are true

C) Statement 1 is false and 2 is true

D) Statement 1 is true and 2 is false

Answer: Both statements 1 and 2 are true

Solution: Statement 1: Going down the group the stability of lower oxidation state increases due to inert pair effect. The inert-pair effect is the tendency of the two electrons in the outermost atomic s-orbital to remain unshared in compounds of post-transition metals.

Statement 2 It is the consequence of poor screening effect of the intervening d and f orbital electrons. Due to the inert pair effect, the heavier members in the groups of p-block elements prefer to show lower oxidation state in their stable compounds.

Q.24. Which of the following molecules follow octet rule?

$BeF_2,\ BF_3,\ H_2SO_4,\ NO_2,\ PCl_5,\ BrF_5,\ CO_2,\ SiH_4,\ CH_4,\ NH_3,\ CCl_4,\ C_2H_6$

A) $CO_2,\ SiH_4,\ CH_4,\ NH_3,\ CCl_4,\ C_2H_6$

B) $BeF_2,\ BF_3$

C) $H_2SO_4,\ NO_2,\ PCl_5,\ BrF_5$

D) $CO_2,\ H_2SO_4$

Answer: $CO_2,\ SiH_4,\ CH_4,\ NH_3,\ CCl_4,\ C_2H_6$

Solution: The given compound is BeF_2 . In this molecule, there are total of 16 electrons. Beryllium does not have eight electrons around it. Therefore, it does not obey octet rule.

When boron forms covalent bonds with three fluorine atoms, it shares one electron with each fluorine atom. This results in boron having only six electrons in its valence shell, which is less than the eight electrons required by the octet rule.

In sulfuric acid (H_2SO_4), each oxygen has a full octet (eight valence electrons), whereas sulfur has an expanded octet (twelve valence electrons).

A free radical is a chemical species with an odd number of valance electrons—consequently, it violates the octet rule. One important example is nitrogen dioxide (NO_2).

In the PCl_5 molecule, the central phosphorus atom is bonded to five atoms, thus having 10 bonding electrons and violating the octet rule.

In BrF_5 the atom that violates the octet rule is bromine. The central bromine atom forms five covalent bonds to five fluorine atoms. Therefore, it is an expanded valence shell molecule. The atom of bromine expands its octet, hence the molecule BrF_5 violates the octet rule.

In carbon dioxide each oxygen shares four electrons with the central carbon, two (shown in red) from the oxygen itself and two (shown in black) from the carbon. All four of these electrons are counted in both the carbon octet and the oxygen octet, so that both atoms are considered to obey the octet rule.

Silicon in SiH_4 does indeed follow the octet rule. Silicon, like carbon, is a member of Group 14 and thus has four valence electrons.

methane (CH_4) follows the octet rule. Each hydrogen atom shares one electron with the carbon atom, and the carbon atom shares four electrons (one with each hydrogen), giving it a total of eight electrons in its outer shell.

The nitrogen atom in NH_3 has five valence electrons and it shares three of these with three hydrogen atoms to form three covalent bonds. The remaining two electrons on nitrogen form a lone pair, giving nitrogen a total of eight electrons around it, which satisfies the octet rule.

In case of CCl_4 , the octet is complete for both.

C_2H_6 (ethane) follows the octet rule, as each carbon atom shares 3 electrons with hydrogen atoms and 1 electron with the other carbon atom, completing its octet.

Q.25. A solution contains $100\ g$ water and $10\ g$ of AB_2. The boiling of the solution was found to be $100.52^{\circ}C$. The degree of dissociation of AB_2 is:
[MW of $AB_2 = 200\ gm/mol$; $K = 0.52\ Kkg/mol$]

A) 0.8 B) 0.5

C) 0.6 D) 0.3

Answer: 0.5

Solution: The molality of the solution$= \frac{n_{solute}}{W_{solvent}\ in\ g} \times 1000$

$\Rightarrow m = \frac{10}{200} \times \frac{1000}{100} = 0.5m$

$\Delta T_b = ik_b m$

$\Delta T_b = 100.52 - 100 = 0.52$

$\Rightarrow 0.52 = i \times 0.52 \times 0.5$

$i = 2$

$$\begin{array}{ccc} AB_2 \rightarrow & A^{2+} & + 2B^- \\ 1-\alpha & \alpha & 2\alpha \end{array}$$

Hence, $i = 1 + 2\alpha$

Now, $1 + 2\alpha = 2$

$\Rightarrow \alpha = 0.5$

Q.26. Match column I with column II

Column I	Column II
1. $F,\ O$	P. Having high electron gain enthalpy
2. $S,\ Cl$	Q. Most electronegative atom
3. $Rb < Cs$	R. Increasing order of the ionisation energy
4. $Al < Ga$	S. Increasing order of size

A) $1-Q,\ 2-P,\ 3-S,\ 4-R$

B) $1-Q,\ 2-P,\ 3-R,\ 4-S$

C) $1-Q,\ 2-S,\ 3-P,\ 4-R$

D) $1-P,\ 2-Q,\ 3-S,\ 4-R$

Answer: $1-Q,\ 2-P,\ 3-S,\ 4-R$

Solution: 1. $F,\ O$ are most electronegative atoms. Electronegativity is a chemical property that describes the tendency of an atom or a functional group to attract electrons toward itself.

2. For S and Cl, the electron gain enthalpy is high. Electron gain enthalpy is defined as the amount of energy released when an electron is added to an isolated gaseous atom.

3. $Rb < Cs$: Increasing order of the size, while going from top to bottom in a group, number of shells increases, hence size increases.

4. $Al < Ga$: In Ga $3d$ electrons are present in the penultimate shell. These 3d electrons shield the nucleus poorly to attract the outermost shell electrons. So, nucleus of Ga attracts the outermost shell electrons more strongly than that in case of Al. Hence, ionisation energy of Ga is slightly higher than that of Al.

Q.27. Which of the following compound will not give Hinsberg's test?

A) CH_3CONH_2

B) $H_2N-NHCONH_2$

C) $CH_3CH_2NH_2$

D) CH_3NHCH_3

Answer: CH_3CONH_2

Solution: Benzenesulphonyl chloride ($C_6H_5SO_2Cl$), which is also known as Hinsberg’s reagent, reacts with primary and secondary amines to form sulphonamides. The nitrogen part in the compound should be involved in the nucleophilic reaction. In case amide, the lone pair on the nitrogen participate in conjugation, hence, it cannot react with Hinsberg's reagent.

Q.28. Number of secondary carbon atoms in $2,4-$ dimethylpentane.

A) 2

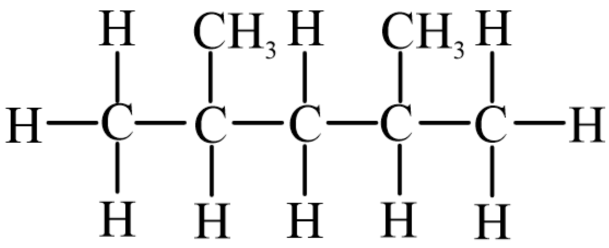
B) 1

C) 3

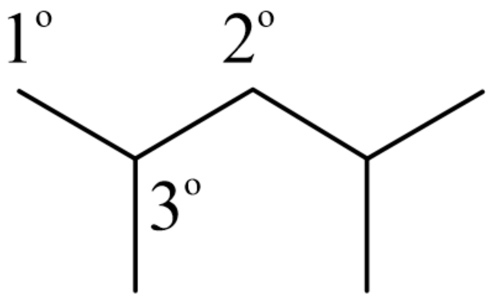
D) 0

Answer: 1

Solution:



2,4-dimethylpentane



2,4-dimethylpentane

The simplest hydrocarbon having two tertiary and one secondary carbon atom is $2,\ 4-$dimethyl pentane. It contains 4 primary carbon atoms.

Hence, the answer is option B.

Q.29. Find out the magnitude of work done on the gas when 1 mole of an ideal gas undergoes compression form 9 litre to 1 litre through a reversible isothermal process at $25^{\circ}\mathrm{C}$. (in Joule)

A) $11412\ \mathrm{J}$

B) $5448\ \mathrm{J}$

C) $5705\ \mathrm{J}$

D) $4765\ \mathrm{J}$

Answer: $5448\ \mathrm{J}$

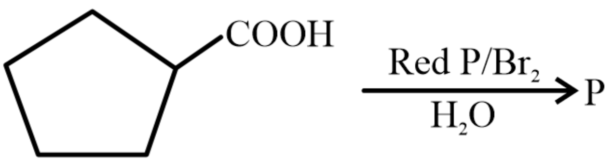
Solution: According to the available data : n=1 mol , V_1=9L ,V_2=1L ,T=298K , R=8.314JK $^{-1}$ mol $^{-1}$

The amount of work done in reversible isothermal process ,

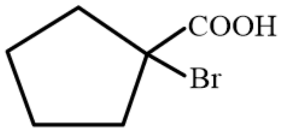
$$\mathrm{W} = -2.303\,\mathrm{nRT}\ \ \log\frac{\mathrm{V_2}}{\mathrm{V_1}}$$

$$\mathrm{W} = -2.303 \times 1 \times 8.314 \times 298 \log\frac{1}{9} = 5448\ \mathrm{J}$$

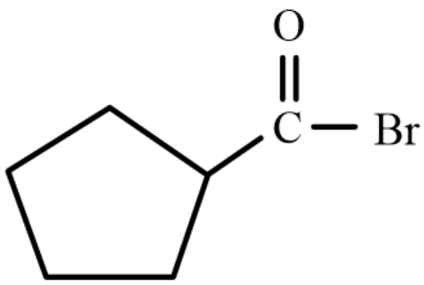
Q.30. Give the product P in the following reaction:



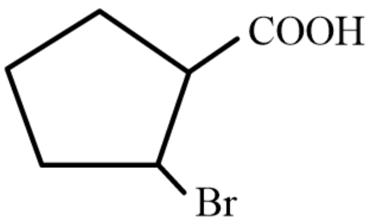
A)



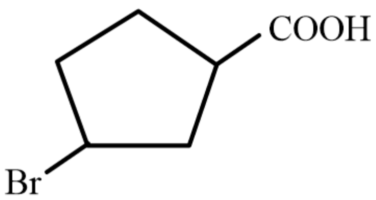
B)



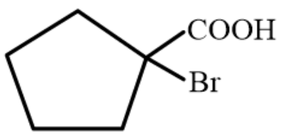
C)



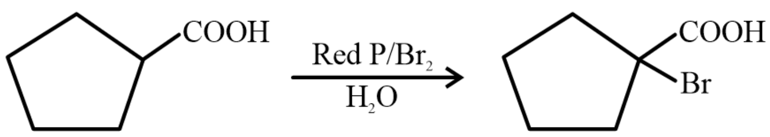
D)



Answer:



Solution: The Hell–Volhard–Zelinsky halogenation reaction is a chemical transformation that involves the halogenation of a carboxylic acid at the α carbon. For this reaction to occur the α carbon must bear at least one proton.



Q.31. A + B → C.
The time taken for 1/4th reaction to occur is twice the time taken from next 1/4th reaction. Find order of reaction

A) 1

B) 2

C) 3

D) 4

Answer: 2

Solution: The integrated rate equation for zero order reaction is

$A_0 - A_t = kt$

The integrated rate equation for first order reaction is

$kt = \ln \frac{A_0}{A_t}$

The integrated rate equation for second order reaction is

$kt = \left(\frac{1}{A_t} - \frac{1}{A_0}\right)$

The given data matches with the second order reaction.

$t_1 = \frac{1}{k}\left(\frac{4}{3A_0} - \frac{1}{A_0}\right) = \frac{1}{3kA_0}$

$t_2 = \frac{1}{k}\left(\frac{2}{A_0} - \frac{4}{3A_0}\right) = \frac{2}{3kA_0}$

Q.32. Which of the following statements regarding D-glucose is incorrect?

A) It does not give Schiff's test

B) It has asymmetrical C-atom

C) It forms a dicarboxylic acid on reaction with Br_2 water

D) In aqueous solution it exists as an equilibrium of two anomeric forms

Answer: It forms a dicarboxylic acid on reaction with Br_2 water

Solution: Despite having the aldehyde group, glucose does not give Schiff's test. It is evidence of cyclic form of glucose. Glucose is found to exist in two different crystalline forms which are named as α and β. These forms are known as anomers. Glucose gets oxidised to six carbon carboxylic acid (gluconic acid) on reaction with a mild oxidising agent like bromine water. This indicates that the carbonyl group is present as an aldehydic group.

Q.33. How many out of the following undergo disproportionation reaction?

$I_2,\ F_2,\ Cl_2,\ Br_2$

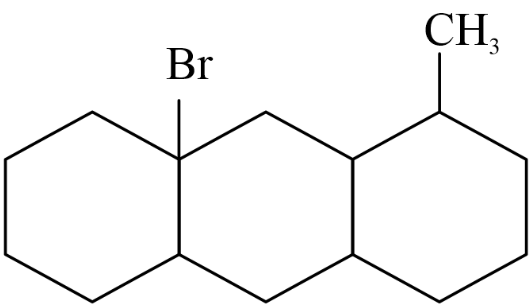
Answer: 1

Solution: A disproportionation reaction is a reaction in which, the same element is simultaneously oxidised and reduced. In this type of reaction, a single substance gives two products, one is oxidised while the other is reduced.

Fluorine can never give electron and can not acquire positive charge. Hence, Fluorine can not undergo disproportionation reaction.

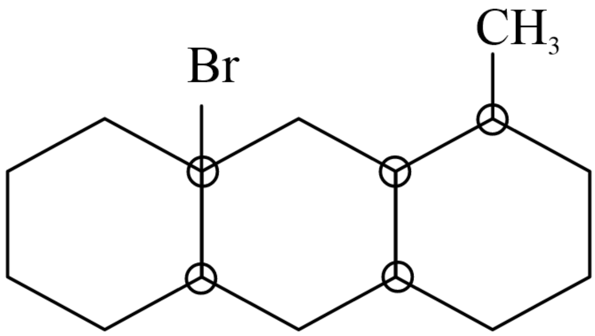
The rest of the species undergoes disproportionation reaction.

Q.34. The total number of optical isomers in the given compound are:



Answer: 32

Solution: Optical isomers are two compounds which contain the same number and kinds of atoms, and bonds (i.e., the connectivity between atoms is the same), and different spatial arrangements of the atoms, but which have non-superimposable mirror images.



As can be seen in the above diagram there are 5 chiral centres.

2^n, where n is the number of chiral carbon atoms
$= 2^5 = 32$

Q.35. 900 gm glucose requires how many grams of O_2 to complete combustion into CO_2 and H_2O?

Answer: 960

Solution: Moles of $C_6H_{12}O_6 = \frac{900}{180} = 5$ moles

$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$

According to the reaction, one mole of glucose requires six moles of oxygen gas for combustion.

$\therefore$ moles of Oxygen $= 5 \times 6 = 30$ moles

Mass of O_2 required$= 30 \times 32 = 960$ gms

Q.36. $CoCl_3.xNH_3$ upon treatment with silver nitrate gives 2 moles of silver chloride.

Find the sum of n and x, if $+n$ is the oxidation state of cobalt.

Answer: 8

Solution: 2 moles of $AgCl$ is formed so the reaction will be:

$CoCl_3.5NH_3 + 2AgNO_3 \rightarrow 2$ mole $AgCl$ ppt

The coordination compound will be

$[Co(NH_3)_5Cl]Cl_2$

Oxidation state of $Co = +3$

Hence, the sum $= n + x = 3 + 5 = 8$

Q.37. Find among the spin only magnetic moment (nearest integer) of M in MO_4^{2-}, M being the atom having least atomic radii among
Sc, Ti, V, Cr, Mn, Zn.

Answer: 0

Solution: Atomic radii of the first transition series decrease from Sc to Cr, then remains almost constant till Ni and then increases from Cu to Zn. The reason of this variation in atomic radii has been attributed to the increase in nuclear charge in the beginning of the series. But as the electrons continue to be filled in d-orbitals, they screen the outer $4s$-electrons from the influence of nuclear charge. When the increased nuclear charge and the increased screening effect balance each other in the middle of transition series, the atomic radii become almost constant . Towards the end of the series, the repulsive interaction between electrons in orbitals become very dominant. As a result there is an expansion of the electron cloud; consequently, the atomic size increases. Hence, least atomic radius element is chromium. Hence, the compound is CrO_4^{2-}. In this compound Cr has d^0 configuration, hence, spin only magnetic moment is 0.

Mathematics

Q.38. Find the sum of diagonal elements of A^{13}, where $A=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$

A) 729

B) 27

C) 1458

D) 9

Answer: 729

Solution: Given: $A=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}4-1 & -2-1\\ 2+1 & -1+1\end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix}3 & -3\\ 3 & 0\end{bmatrix}$

$\Rightarrow A^4=\begin{bmatrix}3 & -3\\ 3 & 0\end{bmatrix}\begin{bmatrix}3 & -3\\ 3 & 0\end{bmatrix}=\begin{bmatrix}9-9 & -9+0\\ 9+0 & -9+0\end{bmatrix}$

$\Rightarrow A^4=\begin{bmatrix}0 & -9\\ 9 & -9\end{bmatrix}$

$\Rightarrow A^8=\begin{bmatrix}0 & -9\\ 9 & -9\end{bmatrix}\begin{bmatrix}0 & -9\\ 9 & -9\end{bmatrix}=\begin{bmatrix}0-81 & 0+81\\ 0+81 & -81+81\end{bmatrix}$

$\Rightarrow A^8=\begin{bmatrix}-81 & 81\\ 81 & 0\end{bmatrix}$

$\Rightarrow A^8\times A^4=\begin{bmatrix}-81 & 81\\ 81 & 0\end{bmatrix}\begin{bmatrix}0 & -9\\ 9 & -9\end{bmatrix}$

$\Rightarrow A^{12}=\begin{bmatrix}0+729 & 729-729\\ 0 & -729+0\end{bmatrix}$

$\Rightarrow A^{12}=\begin{bmatrix}729 & 0\\ 0 & -729\end{bmatrix}$

$\Rightarrow A^{13}=\begin{bmatrix}729 & 0\\ 0 & -729\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}$

$\Rightarrow A^{13}=\begin{bmatrix}1458 & -729\\ -729 & -729\end{bmatrix}$

So, the sum of diagonal elements is 729.

Q.39. If $\sin x=\frac{-4}{5}$, where $x\in\left(\pi,\frac{3\pi}{2}\right)$, then find the value of $3\tan^2 x-\cos x$.

A) $\frac{88}{15}$

B) $\frac{89}{15}$

C) $\frac{82}{15}$

D) $\frac{86}{15}$

Answer: $\frac{89}{15}$

Solution: Given: $\sin x = \frac{-4}{5}$

We know that, $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \frac{16}{25} = \frac{9}{25}$

$\Rightarrow \cos x = \frac{-3}{5}$

$\Rightarrow \tan x = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{4}{3}$

$\Rightarrow 3\tan^2 x - \cos x = 3\left(\frac{16}{9}\right) + \frac{3}{5} = \frac{16}{3} + \frac{3}{5}$

$\Rightarrow 3\tan^2 x - \cos x = \frac{89}{15}$

Q.40. Find the value of the integral $\int \frac{6}{\sin^2 x\left(1-\cot^2 x\right)} dx$

A) $\log\left|\frac{\sin x-1}{\sin x+1}\right| + C$

B) $3\log\left|\frac{\cos x-1}{\cos x+1}\right| + C$

C) $3\log\left|\frac{\cot x-1}{\cot x+1}\right| + C$

D) $3\log\left|\frac{\cot x+1}{\cot x-1}\right| + C$

Answer: $3\log\left|\frac{\cot x-1}{\cot x+1}\right| + C$

Solution: Let, $I = \int \frac{6}{\sin^2 x\left(1-\cot^2 x\right)} dx$

$\Rightarrow I = \int \frac{6\,\text{cosec}^2 x}{1-\cot^2 x} dx$

Putting, $\cot x = t$

$\Rightarrow -\text{cosec}^2 x dx = dt$

$\Rightarrow I = \int \frac{-6}{1-t^2} dt$

$\Rightarrow I = 6\int \frac{1}{t^2-1} dt$

$\Rightarrow I = 6 \times \frac{1}{2}\log\left|\frac{t-1}{t+1}\right| + C$

$\Rightarrow I = 3\log\left|\frac{\cot x-1}{\cot x+1}\right| + C$

Q.41. If $f(x) = \cos x - x + 1,\ x \in [0, \pi]$. Let M and m be the maximum and minimum values of $f(x)$, then find $(M - m)$

A) $-\pi - 2$

B) $\pi - 2$

C) $2 - \pi$

D) $2 + \pi$

Answer: $2 + \pi$

Solution: Given: $f(x) = \cos x - x + 1$

$\Rightarrow f'(x) = -\sin x - 1$

$\Rightarrow f'(x) = -(\sin x + 1)$

When $x \in [0, \pi]$, $\sin x > 0$

$\Rightarrow f'(x) \leq 0$

So, $f(x)$ is a decreasing function.

$\Rightarrow f(0) = \cos 0 - 0 + 1,\ f(\pi) = \cos \pi - \pi + 1$

$\Rightarrow f(0) = 1 + 1,\ f(\pi) = -1 - \pi + 1$

$\Rightarrow f(0) = 2,\ f(\pi) = -\pi$

$\Rightarrow M = 2,\ m = -\pi$

$\Rightarrow M + m = 2 + \pi$

Q.42. Let z be a complex number then $|z+2| = 1$ and imaginary part of $\frac{z+1}{z+2} = \frac{1}{5}$ then find the value of real part of $z+2$.

A) $\frac{\sqrt{6}}{5}$

B) $\frac{2}{5}$

C) $\pm\frac{2\sqrt{6}}{5}$

D) $-\frac{2\sqrt{6}}{5}$

Answer: $\pm\frac{2\sqrt{6}}{5}$

Solution: Let, $z = x + iy$

$\Rightarrow \sqrt{(x+2)^2 + y^2} = 1$

$\Rightarrow (x+2)^2 + y^2 = 1 \quad \ldots(i)$

Also, $\operatorname{Im}\left(\frac{x+iy+1}{x+iy+2}\right) = \frac{1}{5}$

$\Rightarrow \operatorname{Im}\left(\frac{x+iy+1}{x+iy+2} \times \frac{x+2-iy}{x+2-iy}\right) = \frac{1}{5}$

$\Rightarrow \frac{-(x+1)y + y(x+2)}{(x+2)^2 + y^2} = \frac{1}{5}$

$\Rightarrow \frac{-xy - y + xy + 2y}{(x+2)^2 + y^2} = \frac{1}{5}$

$\Rightarrow y = \frac{1}{5}\left\{\text{as } (x+2)^2 + y^2 = 1\right\}$

$\Rightarrow (x+2)^2 + \frac{1}{25} = 1$

$\Rightarrow (x+2)^2 = \frac{24}{25}$

$\Rightarrow x + 2 = \pm\frac{2\sqrt{6}}{5}$

Hence, real part of $z+2$ will be $= \pm\frac{2\sqrt{6}}{5}$

Q.43. Find area of $f(x) = \min\{\sin x, \cos x\}$ with $x-$axis in $x \in [-\pi, \pi]$.

A) $\frac{3}{\sqrt{2}}$

B) $3 - \sqrt{2}$

C) 4

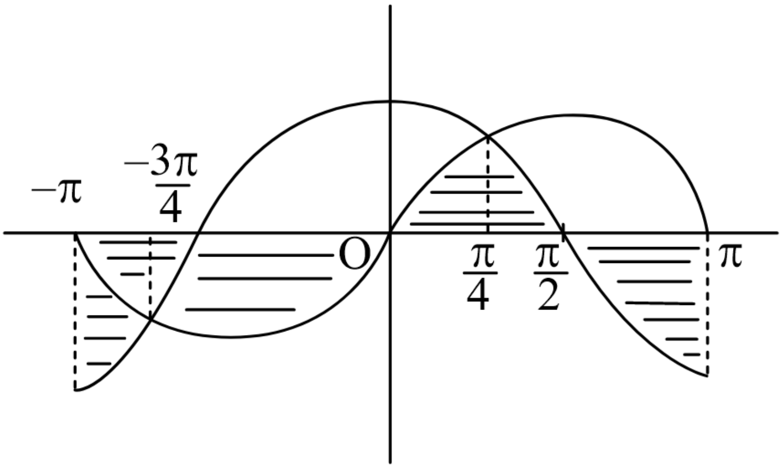
D) $3\sqrt{2}$

Answer: 4

Solution: Given,

$f(x) = \min\{\sin x, \cos x\}$

Now, plotting the diagram we get,



From the graph above, the required area is given by,

$$A = \left|\int_{-\pi}^{-\frac{3\pi}{4}} \cos x dx\right| + \left|\int_{-\frac{3\pi}{4}}^{0} \sin x dx\right| + \left|\int_{0}^{\frac{\pi}{4}} \sin x dx\right| + \left|\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx\right| + \left|\int_{\frac{\pi}{2}}^{\pi} \cos x dx\right|$$

$$\Rightarrow A = [-\sin x]_{-\pi}^{-\frac{3\pi}{4}} + \left|[\cos x]_{-\frac{3\pi}{4}}^{0}\right| + \left|[\cos x]_{0}^{\frac{\pi}{4}}\right| + \left|[-\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right| + \left|[-\sin x]_{\frac{\pi}{2}}^{\pi}\right|$$

$$\Rightarrow A = \left|-\sin\left(\frac{-3\pi}{4}\right) + \sin(-\pi)\right| + \left|1 - \cos\left(\frac{-3\pi}{4}\right)\right| + \left|\frac{1}{\sqrt{2}} - 1\right| + \left|-1 + \frac{1}{\sqrt{2}}\right| + |-\sin\pi + 1|$$

$$\Rightarrow A = \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} + 1$$

$\Rightarrow A = 4$ square units

Q.44. A differential equation is given as $\left(1+y^2\right)e^{\tan x}dx + \left(1+e^{2\tan x}\right)\cos^2 x dy = 0$ and $y(0) = 1$ then find the value of $y\left(\frac{\pi}{4}\right)$

A) $\frac{1}{e}$

B) e

C) e^2

D) e^3

Answer: $\frac{1}{e}$

Solution: Given: $\left(1+y^2\right)e^{\tan x}dx+\left(1+e^{2\tan x}\right)\cos^2 x dy=0$

$\Rightarrow \frac{e^{\tan x}\sec^2 x}{\left(1+e^{2\tan x}\right)}dx=\frac{-1}{\left(1+y^2\right)}dy$

$\Rightarrow \int\frac{e^{\tan x}\sec^2 x}{\left(1+e^{2\tan x}\right)}dx=\int\frac{-1}{\left(1+y^2\right)}dy$

Putting $e^{\tan x}=t$

$\Rightarrow e^{\tan x}\sec^2 x dx=dt$

$\Rightarrow \int\frac{dt}{1+t^2}dt=\int\frac{-1}{1+y^2}dy$

$\Rightarrow \tan^{-1}\left(e^{\tan x}\right)=-\tan^{-1}(y)+C$

It is given that, $y(0)=1$.

$\Rightarrow \tan^{-1}\left(e^0\right)=-\frac{\pi}{4}+C$

$\Rightarrow \frac{\pi}{2}=C$

$\Rightarrow \tan^{-1}\left(e^{\tan x}\right)=-\tan^{-1}(y)+\frac{\pi}{2}$

Putting $x=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}(e)=-\tan^{-1}(y)+\frac{\pi}{2}$

$\Rightarrow \tan^{-1}(y)=\frac{\pi}{2}-\tan^{-1}(e)$

$\Rightarrow \tan^{-1}(y)=\cot^{-1}(e)$

$\Rightarrow \tan^{-1}(\mathrm{y})=\tan^{-1}\left(\frac{1}{\mathrm{e}}\right)$

$\Rightarrow \mathrm{y}=\frac{1}{\mathrm{e}}$

Q.45. Find the number of three digit numbers that can be formed using the digits $\{2,3,4,5,7\}$, which are not divisible by 3 where repetition is not allowed.

A) 20 B) 44

C) 36 D) 52

Answer: 36

Solution: Total numbers that can be formed using $\{2,3,4,5,7\}$ are ${}^5C_3\times 3!=60$

Forming numbers divisible by 3,

$(2,3,4\rightarrow 3!),\ (2,3,7\rightarrow 3!),\ (3,4,5\rightarrow 3!)$ and $(3,5,7\rightarrow 3!)$

So, total ways for forming numbers divisible by 3 will be $6\times 4=24$

So, the numbers that are not divisible by 3 are $60-24=36$

Q.46. Find the sum of the solution of the equation $8^{2x}-16\cdot 8^x+48=0$

A) $\frac{\log 3}{2\log 2}$ B) $\frac{4\log 2+\log 3}{2\log 2}$

C) $\frac{4\log 3}{2\log 2}$ D) $\frac{8\log 2+\log 3}{2\log 2}$

Answer: $\frac{4\log 2+\log 3}{2\log 2}$

Solution: Given,

$8^{2x} - 16 \cdot 8^{x} + 48 = 0$

Let $8^{x} = t$ we get,

$t^2 - 16t + 48 = 0$

$\Rightarrow (t - 12)(t - 4) = 0$

$\Rightarrow t = 12 \text{ or } t = 4$

$\Rightarrow x = \log_8 12 \text{ or } \log_8 4$

So, the sum will be $\log_8 12 + \log_8 4$

$= \frac{\log 12 + \log 4}{\log 8} = \frac{2\log 2 + \log 3 + 2\log 2}{2\log 2} = \frac{4\log 2 + \log 3}{2\log 2}$

Q.47. The set of all α, for which the vector $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle for all $t \in R$

A) $\left(\frac{-4}{3}, 0\right)$

B) $(-2, 0]$

C) $\left(\frac{-4}{3}, 1\right)$

D) $[0, 1)$

Answer: $\left(\frac{-4}{3}, 0\right)$

Solution: Given,

Vector $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$ are inclined at an obtuse angle,

Now, we know that for obtuse angle $\vec{a} \cdot \vec{b} < 0$

$\Rightarrow \alpha t^2 - 12 + 6\alpha t < 0$

$\Rightarrow \alpha t^2 + 6\alpha t - 12 < 0$

Now, for $\alpha < 0$, $D < 0$ we get,

$(6\alpha)^2 - 4 \cdot \alpha \cdot (-12) < 0$

$\Rightarrow 36\alpha^2 + 48\alpha < 0$

$\Rightarrow 12\alpha(3\alpha + 4) < 0$

$\Rightarrow \frac{-4}{3} < \alpha < 0$

Hence, $\alpha \in \left(\frac{-4}{3}, 0\right)$

Q.48. Let $\alpha = \sum_{r=0}^{n} \left(4r^2 + 2r + 1\right) \times {}^nC_r$ and $\beta = \sum_{n=0}^{n} \frac{{}^nC_r}{r+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is

A) 8

B) 7

C) 6

D) 5

Answer: 5

Solution: Given: $\alpha=\sum_{r=0}^{n}\left(4r^{2}+2r+1\right)\times{}^{n}C_{r}$

$\Rightarrow\alpha=4\sum_{r=0}^{n}\left(r^{2}\times{}^{n}C_{r}\right)+2\sum_{r=0}^{n}(r\times{}^{n}C_{r})+\sum_{r=0}^{n}{}^{n}C_{r}$

$\Rightarrow\alpha=4\sum_{r=0}^{n}\left(r^{2}\times\frac{n}{r}\times{}^{n-1}C_{r-1}\right)+2n\times2^{n-1}+2^{n}$

$\Rightarrow\alpha=4n\sum_{r=0}^{n}\left(r\times{}^{n-1}C_{r-1}\right)+2n\times2^{n-1}+2^{n}$

$\Rightarrow\alpha=4n\left\{\sum_{r=0}^{n}\left((r-1)\times{}^{n-1}C_{r-1}\right)+\sum_{r=0}^{n}{}^{n-1}C_{r-1}\right\}+2n\times2^{n-1}+2^{n}$

$\Rightarrow\alpha=4n\left\{(n-1)2^{n-2}+2^{n-1}\right\}+2^{n}(n+1)$

$\Rightarrow\alpha=4n\times2^{n-2}(n-1+2)+2^{n}(n+1)$

$\Rightarrow\alpha=4n(n+1)\times2^{n-2}+2^{n}(n+1)$

$\Rightarrow\alpha=2^{n}(n+1)^{2}$

Now, solving $\beta=\sum_{n=0}^{n}\frac{{}^{n}C_{r}}{r+1}$

$\Rightarrow\beta=\frac{1}{n+1}\sum_{n=0}^{n}\frac{n+1}{r+1}{}^{n}C_{r}$

$\Rightarrow\beta=\frac{1}{n+1}\sum_{n=0}^{n}{}^{n+1}C_{r+1}$

$\Rightarrow\beta=\frac{2^{n+1}}{n+1}$

$\Rightarrow\frac{2\alpha}{\beta}=\frac{2^{n+1}(n+1)^{2}}{\frac{2^{n+1}}{n+1}}$

$\Rightarrow\frac{2\alpha}{\beta}=(n+1)^{3}$

$\Rightarrow140<(n+1)^{3}<281$

$\Rightarrow(n+1)^{3}=216$

$\Rightarrow n+1=6$

$\Rightarrow n=5$

Q.49. If the circles $(x-\alpha)^{2}+(y-\beta)^{2}=(r_{1})^{2}$ and $(x-8)^{2}+\left(y-\frac{15}{2}\right)^{2}=(r_{2})^{2}$ touches at $(6,6)$ internally and the ratio of radii is $2:1$, then $\alpha+\beta+4\left[(r_{1})^{2}+(r_{2})^{2}\right]=$

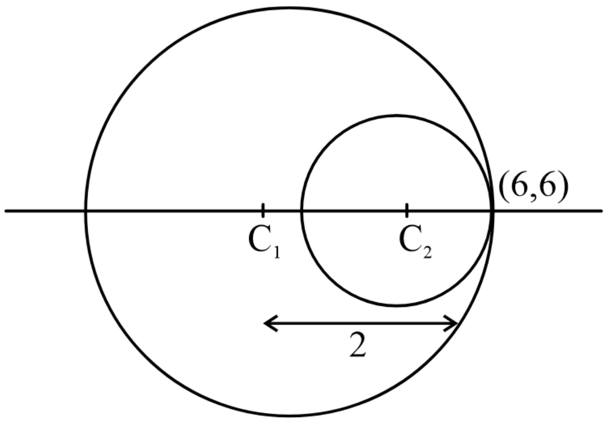
A) 100 B) 165

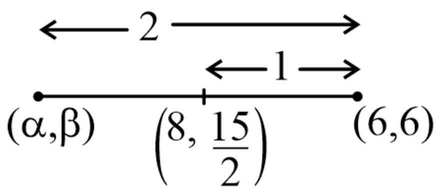
C) 144 D) 155

Answer: 144

Solution: Let, $C_1: (x-\alpha)^2+(y-\beta)^2=(r_1)^2$ and $C_2: (x-8)^2+\left(y-\frac{15}{2}\right)^2=(r_2)^2$.

$\Rightarrow C_1 \rightarrow \text{Centre } (\alpha, \beta) \text{ and radius} = r_1$ and $C_2 \rightarrow \text{Centre } \left(8, \frac{15}{2}\right) \text{ and radius} = r_2$





Now, using the section formula of external division,

$\Rightarrow \frac{16-\alpha}{2-1}=6,\ \frac{15-\beta}{2-1}=6$

$\Rightarrow \alpha=10,\ \beta=9$

$\Rightarrow r_1=\sqrt{(10-6)^2+(9-6)^2}$

$\Rightarrow r_1=\sqrt{16+9}$

$\Rightarrow r_1=5$

Also, $\frac{r_1}{r_2}=\frac{2}{1}$

$\Rightarrow r_2=\frac{5}{2}$

$\Rightarrow \alpha+\beta+4\left[(r_1)^2+(r_2)^2\right]=19+4\left(25+\frac{25}{4}\right)$

$\Rightarrow \alpha+\beta+4\left[(r_1)^2+(r_2)^2\right]=144$

Q.50. If $\lim_{x\to 0} 2\left[\frac{1-\cos x\sqrt{\cos 2x}\cdot\sqrt[3]{\cos 3x}\ \ldots\sqrt[10]{\cos 10x}}{x^2}\right]=k$, then find the value of k

Answer: 55

Solution: Given,

$$\lim_{x\to0}2\left[\frac{1-\cos x\sqrt{\cos 2x}\cdot\sqrt[3]{\cos 3x}\ \ldots\sqrt[10]{\cos 10x}}{x^2}\right]\left\{\frac{0}{0}\text{ form}\right\}$$

Now, using L-hospital in the above limit we get,

$$=\lim_{x\to0}2\left[\frac{-\frac{dz}{dz}}{2x}\right]$$

Where $z=\cos x\sqrt{\cos 2x}\cdot\sqrt[3]{\cos 3x}\ \ldots\sqrt[10]{\cos 10x}$

Now, taking $\log$ both side we get,

$$\log z=\log\cos x+\frac{1}{2}\log\cos 2x+\ldots\ldots\ldots\ldots\frac{1}{10}\log\cos 10x$$

Now, taking derivative both side we get,

$$\frac{1}{z}\frac{dz}{dx}=-\tan x-\tan 2x-\tan 3x\ \ldots\ldots\ldots\ldots-\tan 10x$$

$$\Rightarrow\frac{dz}{dx}=z(-\tan x-\tan 2x-\tan 3x\ \ldots\ldots\ldots\ldots-\tan 10x)$$

So, the limit will be,

$$=\lim_{x\to0}2\left[\frac{z(\tan x+\tan 2x+\tan 3x\ \ldots\ldots\ldots\ldots+\tan 10x)}{2x}\right]$$

$$=\lim_{x\to0}\left[\frac{(\tan x+\tan 2x+\tan 3x\ \ldots\ldots\ldots\ldots+\tan 10x)}{x}\right]\{\text{as } z=1\text{ at } x=0\}$$

$$=1+2+3+4+\ldots\ldots10\left\{\text{as }\lim_{x\to0}\frac{\tan x}{x}=1\right\}$$

$$=55$$

Q.51. In a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$, eccentricity is $\sqrt{3}$ and length of latusrectum is $4\sqrt{3}$ and if $(\alpha,6)$ lies on hyperbola and if product of focal distance from $(\alpha,6)$ is β then find the value of $\alpha^2+\beta$

Answer: 171

Solution: Given,

In hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Eccentricity $e = \sqrt{3}$

$\Rightarrow 1 + \frac{a^2}{b^2} = 3$

$\Rightarrow a^2 = 2b^2 \quad \ldots.(i)$

And length of latusrectum $\frac{2a^2}{b} = 4\sqrt{3} \quad \ldots.(ii)$

Now, solving both equations we get, $a = \sqrt{6}$ & $b = \sqrt{3}$

So, the equation of hyperbola will be,

$\frac{x^2}{6} - \frac{y^2}{3} = -1$

Now, $(\alpha, 6)$ lies on hyperbola so, we get,

$\frac{\alpha^2}{6} - \frac{6^2}{3} = -1 \Rightarrow \alpha^2 = 66$

Now, the product of focal distance will be, $PF_1 \cdot PF_2 = (ey_1 + b)(ey_1 - b) = e^2 \times 36 - b^2 \ \{\text{as } y_1 = 6\}$

$\Rightarrow \beta = 3 \times 36 - 3 = 105$

Hence, $\alpha^2 + \beta = 66 + 105 = 171$

Q.52. If $f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 1$ then find the number of points of maxima in $[0, 2\pi]$

Answer: 2

Solution: Given,

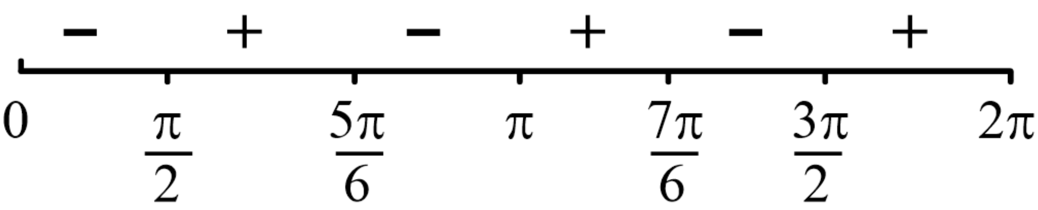
$f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x - 1$

Now, differentiating the above function we get,

$f'(x) = -12\cos^2 x \sin x - 6\sqrt{3}\cos x \sin x$

$\Rightarrow f'(x) = -6\sin x \cos x \left(2\cos x + \sqrt{3}\right)$

Now, using first derivative test in interval $[0, 2\pi]$ we get,



Now, from above first derivative test we can see that there are 2 points of local maxima at $x = \frac{5\pi}{6}$ & $x = \frac{7\pi}{6}$

Q.53. The number of critical points of the function $f(x) = (x-2)^{\frac{2}{3}}(2x+1)$ will be

Answer: 2

Solution: Given,

$f(x)=(x-2)^{\frac{2}{3}}(2x+1)$

Now, differentiating the function we get,

$f'(x)=\frac{2}{3}(x-2)^{\frac{2}{3}-1}(2x+1)+(x-2)^{\frac{2}{3}}2$

$\Rightarrow f'(x)=\frac{2}{3}(x-2)^{\frac{-1}{3}}(2x+1)+2(x-2)^{\frac{2}{3}}$

$\Rightarrow f'(x)=\frac{2(2x+1)+6(x-2)}{3(x-2)^{\frac{1}{3}}}$

$\Rightarrow f'(x)=\frac{10x-10}{3(x-2)^{\frac{1}{3}}}$

$\Rightarrow f'(x)=\frac{10(x-1)}{3(x-2)^{\frac{1}{3}}}$

So, critical points will be 1 & 2