JEE Main

31st Jan Shift 2

Questions

Q.1. Which of the following is least ionic?

A) $BaCl_2$

B) KCl

C) $AgCl$

D) $CoCl_2$

Answer: $CoCl_2$

Solution: Cations here will be Ba^{2+}, K^+, Ag^+, Co^{2+}.

More positive charge, more will be the covalent character.

d-block are more covalent than s-block ions.

Hence, more covalent character means less ionic,

The order of ionic character will be:

$KCl > BaCl_2 > AgCl > CoCl_2$

Hence, the answer is D.

Q.2. The magnetic property of complexes $[NiCl_4]^{2-}$, $[Ni(CN)_4]^{2-}$ are

A) paramagnetic and diamagnetic respectively

B) diamagnetic and paramagnetic respectively

C) both paramagnetic

D) both diamagnetic

Answer: paramagnetic and diamagnetic respectively

Solution: In $[Ni(CN)_4]^{2-}$, Ni has $3d^8$ configuration but due to the strong ligand field, all the d-electrons are spin paired giving dsp^2 hybridization. Hence, it is diamagnetic.
In $[NiCl_4]^{2-}$, Ni has $3d^8$ configuration and there are two unpaired electrons (weak field chloride ligand do not pair up (d-electrons) hence, it is paramagnetic.

Q.3. The four quantum number for the outermost electron of K atom are given by:

A) $n = 4,\ l = 1,\ m = 0,\ s = \frac{1}{2}$

B) $n = 4,\ l = 0,\ m = 0,\ s = \frac{1}{2}$

C) $n = 3,\ l = 1,\ m = 0,\ s = \frac{1}{2}$

D) $n = 3,\ l = 0,\ m = 0,\ s = \frac{1}{2}$

Answer: $n = 4,\ l = 0,\ m = 0,\ s = \frac{1}{2}$

Solution: Electronic configuration of K is:

$K =\ \text{Atomic number} = 19$
$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$

The outermost electron is in $4s^1$

□ $n = 4,\ l = 0,\ m = 0,\ s = +\frac{1}{2}$

Hence, the answer is option B.

Q.4. Statement 1: S_8 disproportionates into $S_2O_3{}^{2-}$ and S^{2-} in alkaline medium.

Statement 2: ClO_4^- undergoes disproportionation in acidic medium.

A) Both statements are correct

B) Both statements are incorrect

C) Statement 1 is correct and 2 is incorrect

D) Statement 1 is incorrect and 2 is correct

Answer: Statement 1 is correct and 2 is incorrect

Solution: Disproportionation reaction, also called dismutation reaction, is basically a type of redox reaction involving simultaneous reduction and oxidation of atoms of the same element from one oxidation state (OS) to two different oxidation states.

$S_8 + 12\,OH^- \rightarrow 4S^{2-} + 2S_2O_3{}^{2-} + 6H_2O$

In the above reaction, the oxidation state of sulphur changes from 0 to $+2$ and -2 in $H_2S_2O_3$ and S^{2-}, respectivey.

Hence, statement 1 is correct.

ClO_4^- has $+7$ oxidation state, which is highest for Cl, hence, it can not undergo disproportionation reaction.

So, statement 2 is incorrect.

Q.5. If 5 moles of an ideal gas expands from 10 L to a volume of 100 L at 300K under isothermal and reversible condition then the work done is

(Given $R = 8.314\ J\ K^{-1}\ mol^{-1}$)

A) $-57.44\ kJ$

B) $-28.72\ kJ$

C) $-114.88\ kJ$

D) $-56.7\ kJ$

Answer: $-28.72\ kJ$

Solution: The work done in the isothermal reversible expansion process can be calculated using

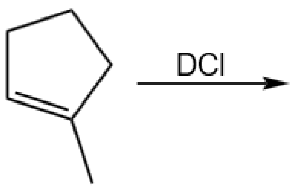
$W = -2.303\ nRT\log_{10}\left(\frac{V_2}{V_1}\right)$

$= -2.303 \times 5 \times 8.314 \times 300\ \log_{10}\left(\frac{100}{10}\right)$

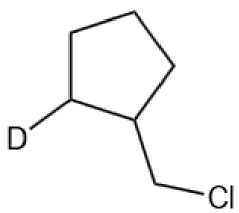
$== -2.303 \times 5 \times 8.314 \times 300 \times 1$

$= -28.72\ kJ$

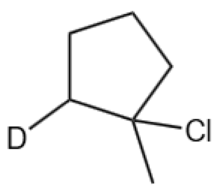
Q.6. What is the product of the following reaction



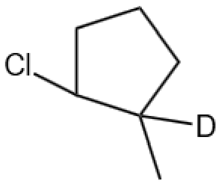
A)



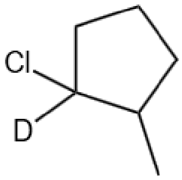
B)



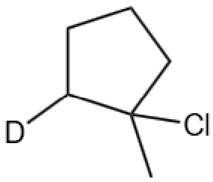
C)



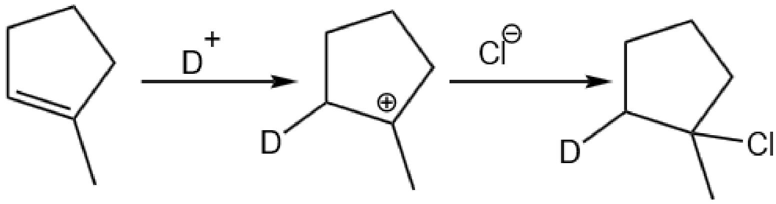
D)



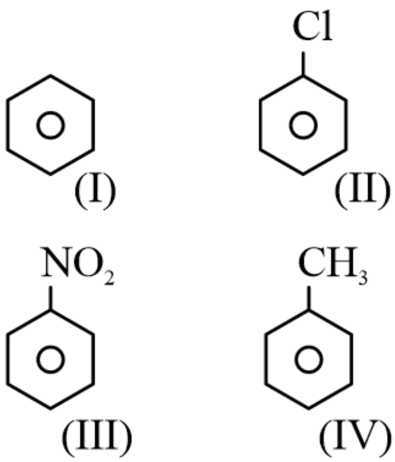
Answer:



Solution: The given alkene undergoes electrophilic addition reaction with DCl. The D^{+} electrophile adds to the alkene by Markonikov's passion. Then carbocation formed reacts with chloride. The reaction is shown below.



Q.7. The rate of electrophilic aromatic reaction substitution reaction in the given compounds in decreasing order is:



A) $IV > I > II > III$

B) $II > IV > I > III$

C) $II > IV > III > I$

D) None of the above

Answer: $IV > I > II > III$

Solution: Nitrobenzene shows $-\mathrm{M}$ effect.

In nitrobenzene, it is known that the nitro group is an electron-withdrawing species, and therefore, it comes under the category of deactivating species. Thus, it favours meta-position for the upcoming substitution in benzene ring since deactivating groups have higher electron density in meta position than ortho and para position.
Thus, the electrophilic substitution reaction for nitrobenzene is meta-directing.

Chlorobenzene shows $-\mathrm{I}$ effect.

Cl shows a negative inductive effect due to its high electronegativity means it withdraws the electron density from the benzene ring, so it is a deactivating group.

The activation effect on substitution reaction shown by toluene is due to hyperconjugation.

Hence, the answer is option A.

Q.8. Half life of a first order reaction is $36\ \mathrm{hr}$. Find out time (in hour) required for concentration of reactant to get reduced by 90% is

A) $60\ \mathrm{hr}$

B) $72\ \mathrm{hr}$

C) $120\ \mathrm{hr}$

D) $100\ \mathrm{hr}$

Answer: $120\ \mathrm{hr}$

Solution: For a first order reaction, the relation between half-life and rate constant is given by the reaction,

$$k=\frac{0.693}{t_{0.5}}=\frac{0.693}{36}\mathrm{hr}^{-1}$$

According to the first order rate equation,

$$k=\frac{2.303}{t}\log\frac{[R_0]}{[R]}$$

Where R_0 is the initial concentration of the reactant when $t=0$. Here, $[R_0]=100\%$ and $[R]=10\%$

On substituting the values in the above equation,

$$t=\frac{2.303}{\left(\frac{0.693}{36}\right)}\log\left(\frac{100}{10}\right)$$

$$t=\frac{36}{0.3}\mathrm{hr}$$

$$=360/3=120\,\mathrm{hr}.$$

Q.9. Statement 1: Among 15th group hydrides, reducing character decreases from NH_3 to BiH_3.

Statement 2: E_2O_3 and E_2O_5 are always basic (Where E is group 15 element)

A) Both statements are correct

B) Both statements are incorrect

C) Statement 1 is correct and 2 is incorrect

D) Statement 1 is incorrect and 2 is correct

Answer: Both statements are incorrect

Solution: As we move down a group, the atomic size increases and the stability of the hydrides of group 15 elements decreases. Since the stability of hydrides decreases on moving from NH_3 to BiH_3, the reducing character of the hydrides increases on moving from NH_3 to BiH_3.

The oxide in the higher oxidation state of the element is more acidic than that of lower oxidation state.

Hence, both statements are incorrect.

Q.10. Match the following and select the correct option.

List I	List II
(a) $[Cr(H2O)6]^{3+}$	(i) $t_{2g}{}^{2}e_{g}{}^{0}$
(b) $[Fe(H_2O)_6]^{3+}$	(ii) $t_{2g}{}^{3}e_{g}{}^{0}$
(c) $[Ni(H_2O)_6]^{2+}$	(iii) $t_{2g}{}^{3}e_{g}{}^{2}$
(d) $[V(H_2O)_6]^{3+}$	(iv) $t_{2g}{}^{6}e_{g}{}^{2}$

A) $a-(ii),\ b-(iii),\ c-(iv),\ d-(i)$

B) $a-(iii),\ b(ii),\ c-(iv),\ d-(i)$

C) $a-(ii),\ b-(iii),\ c-(i),\ d-(iv)$

D) $a-(ii),\ b-(iv),\ c-(iii),\ d-(i)$

Answer: $a-(ii),\ b-(iii),\ c-(iv),\ d-(i)$

Solution: The electronic configurations for central metal ions are:

$Cr^{3+}=[Ar]\ 3d^3 4s^0=3\Rightarrow t_{2g}{}^{3}e_{g}{}^{0}$

$Fe^{3+}=[Ar]\ 3d^5 4s^0=5\Rightarrow t_{2g}{}^{3}e_{g}{}^{2}$

$Ni^{2+}=[Ar]\ 3d^8 4s^0=8\Rightarrow t_{2g}{}^{6}e_{g}{}^{2}$

$V^{3+}=[Ar]\ 3d^2 4s^0=2\Rightarrow t_{2g}{}^{2}e_{g}{}^{0}$

Hence, the answer is A

Q.11. Which of the following statements are correct?

A) Mn_2O_7 is an oil at room temperature

B) V_2O_4 reacts with acid to give VO^{2+}

C) CrO is a basic oxide

D) V_2O_5 does not reacts with acids

A) A only

B) A and B only

C) A, B and C

D) All are correct

Answer: A, B and C

Solution: All the metals except scandium form MO oxides, which are ionic. Mn_2O_7 is a covalent green oil. In vanadium there is gradual change from the basic V_2O_3 to less basic V_2O_4 and to amphoteric V_2O_5 . V_2O_4 dissolves in acids to give VO^{2+} salts. Similarly, V_2O_5 reacts with alkalies as well as acids to give VO_4^{3-} and VO_4^{+} respectively. The well characterised CrO is basic but Cr_2O_3 is amphoteric.

Q.12. There is some fragrance oil found in flowers which is water insoluble but can be mixed in the vapour phase. What is the best method to extract this oil compound?

A) Distillation

B) Steam distillation

C) Crystallisation

D) Distillation in reduced pressure

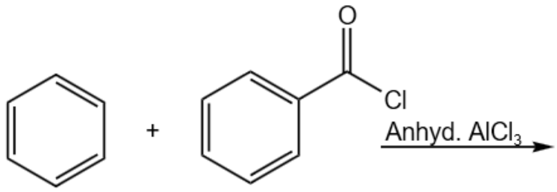
Answer: Steam distillation

Solution: The steam distillation process is used to separate organic compounds that are temperature-sensitive like aromatic substances. It also helps to extract oils from natural products like citrus oil, eucalyptus oil, and more natural substances that are derived from the organic matter.

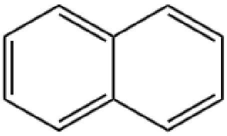
Also, it is used when compounds are insoluble in water.

Hence, the answer is B.

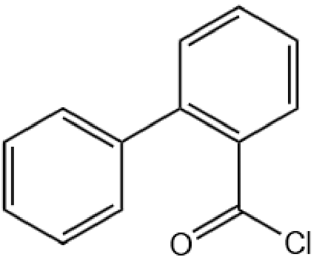
Q.13. Which is the major product formed in the following reaction?



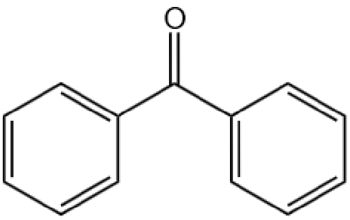
A)



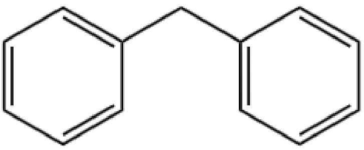
B)



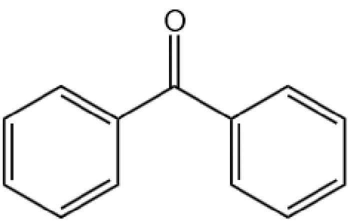
C)



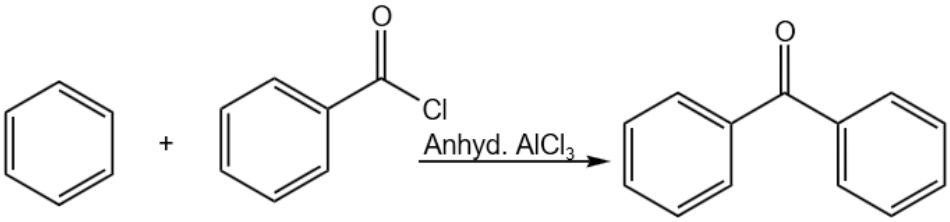
D)



Answer:

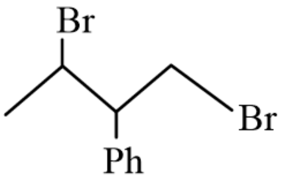


Solution: The electrophile produced in the reaction of benzene with benzoyl chloride in the presence of anhydrous $AlCl_3$ is benzoylinium cation. The product formed in this reaction is benzophenone. This reaction is called Friedel Craft's acylation reaction.

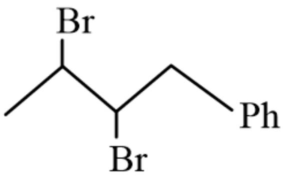


Q.14. Correct structure of $2,\ 3-\text{dibromo}-1-\text{phenyl}\ \ \text{butane}$ is:

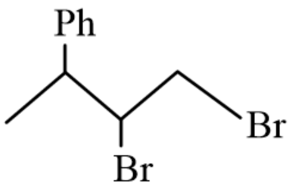
A)



B)

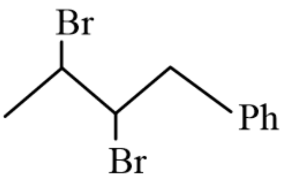


C)



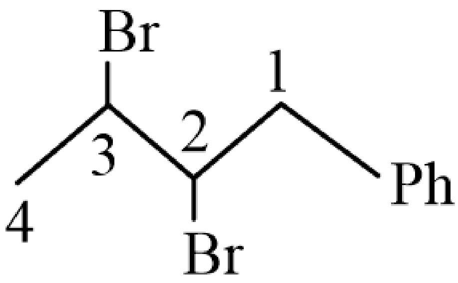
D) None of the above

Answer:



Solution: Rules for IUPAC nomenclature:

- Locate the longest carbon chain in our compound.
- Name that parent chain (find the root word)
- Figure out the ending.
- Number your carbon atoms.
- Name the side groups.
- Put the side groups in alphabetical order.
- In the given compound, 4 carbon atoms chain is present with Bromine at carbon number 2 and 3, and the phenyl group is present at the carbon number 1.
- Hence, the name of the compound is $2,\ 3-\text{dibromo}-1-\text{phenyl butane}.$



Hence, the answer is option B.

Q.15. Find the molarity of $70\%\frac{\text{W}}{\text{W}}\ \text{H}_2\text{SO}_4$ solution having density $1.54\ \text{g/mL}$.

A) 10

B) 11

C) 9.8

D) 12

Answer: 11

Solution: Given here is:

$70\ \text{g of H}_2\text{SO}_4$
$100\ \text{g of solution}$

$$\text{Molarity}=\frac{\frac{70}{98}}{\frac{100}{1.54}}\times 1000$$
$$=10.99\approx 11\ \text{M}$$

Q.16. The number of vitamins that can be stored in our body out of the given are:

Vitamins A, B1, B6, B12, C, D, E and K

Answer: 4

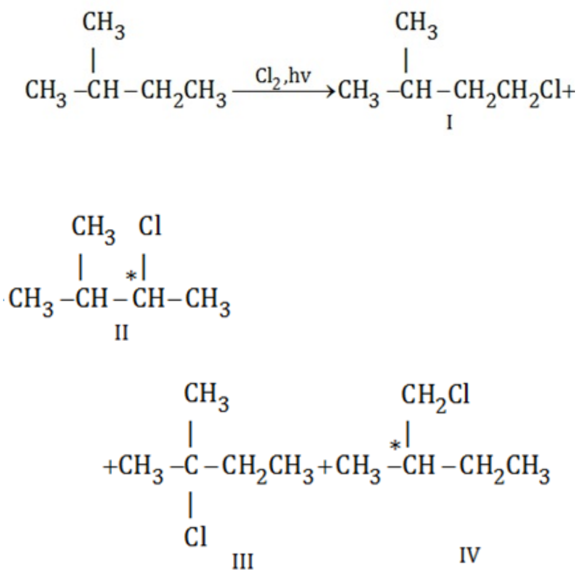
Solution: Fat-soluble vitamins are stored in the body's liver, fatty tissue, and muscles. The four fat-soluble vitamins are vitamins A, D, E, and K. These vitamins are absorbed more easily by the body in the presence of dietary fat. Water-soluble vitamins are not stored in the body.

Hence, the answer is 4.

Q.17. How many isomeric products are formed by monochlorination of 2-methylbutane in the presence of sunlight?

Answer: 6

Solution: Mono chlorination of 2-methyl butane gives four structural isomers (I, II, III, IV)



Here only two, i.e., II and IV are optically active. Since each optically active compound has two enantiomers, therefore, total 6 isomeric compounds are possible.

Q.18. A compound (X) with molar mass 108 g/mol undergoes acetylation to give a product with molar mass 192 g/mol. The number of acetyl groups present in the product is

Answer: 2

Solution: The compounds like amines and alcohols undergo acetylation with acetyl chloride. During this reaction, one proton of amine or alcohol is replaced with one acetyl group(CH_3CO-). The molar mass of the acetyl group is 43 g/mol. The increase in molar mass due to one acetyl group is $43 - 1 = 42 \text{g/mol}$.

The molar mass of reactant is 108 g/mol. The molar mass of product is 192 g/mol. The change in mass is 84.

Hence, the number of acetyl groups $= \frac{84}{42} = 2$

Q.19. The number of moles of H^+ ion required by one mole of permanganate ion to oxidise oxalate to carbon dioxide is

Answer: 8

Solution: The reaction between MnO_4^- with $C_2O_4^{2-}$ is as given below:

$2MnO_4^- + 16H^+ + 5C_2O_4^{2-} \rightarrow 2Mn^{+2} + 8H_2O + 10CO_2$

From this, we can see that 16 moles of H^+react with 2 moles of MnO_4^-. Hence, one mole of permanganate requires eight moles of H^+ ions

Q.20. The potassium chloride is heated with potassium dichromate and concentrated sulphuring acid to give products. The oxidation state of the chromium in the product is

Answer: 3

Solution: The potassium chloride is heated with potassium dichromate and concentrated sulphuring acid gives chromium sulphate and chlorine gas as products.

$K_2Cr_2O_7 + 7H_2SO_4 + 6KCl \longrightarrow 2Cr_2(SO_4)_3 + 7H_2O + 3Cl_2 + 4K_2SO_4$

$Cr_2O_7^{2-}$is reduced to Cr^{3+}.

Thus, the final oxidation state of Cr is +3.

Q.21. Let $f : R \rightarrow (0, \infty)$ be increasing function such that $\lim_{x\rightarrow\infty} \frac{f(7x)}{f(x)} = 1$ then $\lim_{x\rightarrow\infty} \left\{ \frac{f(5x)}{f(x)} - 1 \right\}$ is equal to

A) 4

B) 0

C) $\frac{4}{5}$

D) 1

Answer: 0

Solution: Let, x be any poistive real number.

$\Rightarrow x < 5x < 7x$

$\Rightarrow f(x) < f(5x) < f(7x)$

$\Rightarrow \frac{f(x)}{f(x)} < \frac{f(5x)}{f(x)} < \frac{f(7x)}{f(x)}$

$\Rightarrow 1 < \frac{f(5x)}{f(x)} < \frac{f(7x)}{f(x)}$

$\Rightarrow \lim_{x\to\infty} 1 < \lim_{x\to\infty} \frac{f(5x)}{f(x)} < \lim_{x\to\infty} \frac{f(7x)}{f(x)}$

It is given that, $\lim_{x\to\infty} \frac{f(7x)}{f(x)} = 1$

So, by using Sandwich theorem,

$\Rightarrow \lim_{x\to\infty} \frac{f(5x)}{f(x)} = 1$

$\Rightarrow \lim_{x\to\infty} \left\{\frac{f(5x)}{f(x)} - 1\right\} = 0$

Q.22. If $a = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$ then $a^2 + b^2 =$

A) $2\pi^2 - 20\pi + 50$

B) $4\pi^2 - 40\pi + 50$

C) $8\pi^2 - 40\pi + 50$

D) $2\pi^2 - 40\pi + 100$

Answer: $8\pi^2 - 40\pi + 50$

Solution: Given: $a = \sin^{-1}(\sin(5)) \text{ and } b = \cos^{-1}(\cos(5))$

$\Rightarrow a = 5 - 2\pi \text{ and } b = 2\pi - 5$

$\Rightarrow a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$

$\Rightarrow a^2 + b^2 = 25 + 4\pi^2 - 20\pi + 4\pi^2 + 25 - 20\pi$

$\Rightarrow a^2 + b^2 = 8\pi^2 - 40\pi + 50$

Q.23. If $z_1 + z_2 = 5$ & $z_1{}^3 + z_2{}^3 = 20 + 15i$ then the value of $\left|z_1{}^4 + z_2{}^4\right|$ will be

A) $15\sqrt{15}$

B) 75

C) $30\sqrt{3}$

D) $25\sqrt{3}$

Answer: 75

Solution: Given,

$z_1 + z_2 = 5$ & $z_1^3 + z_2^3 = 20 + 15i$

Now, solving $z_1^3 + z_2^3 = 20 + 15i$

$\Rightarrow (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2) = 20 + 15i$

$\Rightarrow (5)^3 - 3z_1z_2(5) = 20 + 15i$

$\Rightarrow 125 - 15z_1z_2 = 20 + 15i$

$\Rightarrow z_1z_2 = 7 - i$

Now, finding $z_1^4 + z_2^4 = \left(z_1^2 + z_2^2\right)^2 - 2(z_1z_2)^2$

$\Rightarrow z_1^4 + z_2^4 = \left((z_1 + z_2)^2 - 2z_1z_2\right)^2 - 2(z_1z_2)^2$

$\Rightarrow z_1^4 + z_2^4 = (25 - 2(7 - i))^2 - 2(7 - i)^2$

$\Rightarrow z_1^4 + z_2^4 = (11 + 2i)^2 - 2(48 - 14i)$

$\Rightarrow z_1^4 + z_2^4 = (121 - 4 + 44i) - 96 + 28i$

$\Rightarrow z_1^4 + z_2^4 = 21 + 72i$

$\Rightarrow |z_1^4 + z_2^4| = \sqrt{21^2 + 72^2} = \sqrt{5625} = 75$

Q.24. If 2^{nd}, 8^{th}, 44^{th} terms of $A.P$ are 1^{st}, 2^{nd} & 3^{rd} terms respectively of $G.P$ and first term of $A.P$ is 1 then the sum of first 20 terms of $A.P$ is

A) 850 B) 900

C) 910 D) 970

Answer: 970

Solution: Given,

2^{nd}, 8^{th}, 44^{th} terms of $A.P$ are 1^{st}, 2^{nd} & 3^{rd} terms respectively of $G.P$

And first term of $A.P$ is 1 and let d be common difference

So, $1 + d$, $1 + 7d$, $1 + 43d$ are 1^{st}, 2^{nd} & 3^{rd} term of $G.P$

So, $\frac{1+7d}{1+d} = \frac{1+43d}{1+7d}$

$\Rightarrow (1 + 7d)^2 = (1 + 43d)(1 + d)$

$\Rightarrow 1 + 49d^2 + 14d = 1 + 43d^2 + 44d$

$\Rightarrow 6d^2 = 30d$

$\Rightarrow d = 5$

Now finding the sum of first 20 terms of $A.P$ we get,

$S_{20} = 10[2 \times 1 + 19 \times 5] = 970$

Q.25. A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is

A) $\frac{2}{9}$ B) $\frac{2}{27}$

C) $\frac{1}{27}$ D) $\frac{1}{9}$

Answer: $\frac{2}{9}$

Solution: Given: $P(H)=2P(T)$

We know that, $P(H)+P(T)=1$

$\Rightarrow 2P(T)+P(T)=1$

$\Rightarrow P(T)=\frac{1}{3}$ and $P(H)=\frac{2}{3}$

$\Rightarrow P(2 \text{ tails and } 1 \text{ head})=\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}\times\frac{3!}{2!}(\text{for arrangements})$

$\Rightarrow P(2 \text{ tails and } 1 \text{ head})=\frac{2}{9}$

Q.26. If a line of negative slope passing through the centre of circle $x^2+y^2-16x-4y=0$ intersects positive x & y axis at A & B respectively, then find the minimum value of $OA+OB$ {where O is origin}

A) 16

B) 18

C) 9

D) 8

Answer: 18

Solution: Given,

If a line of negative slope passing through the centre of circle $x^2+y^2-16x-4y=0$ which is $(8,2)$ intersects positive x & y axis at A & B respectively,

So, let the line be $\frac{x}{a}+\frac{y}{b}=1$

Now, given line passes through the centre,

So, $\frac{8}{a}+\frac{2}{b}=1$

Now, using $A.M\geq H.M$ we get,

$$\frac{\frac{a}{2}+\frac{a}{2}+\frac{a}{2}+\frac{a}{2}+\frac{b}{1}+\frac{b}{1}}{6}\geq\frac{6}{\frac{2}{a}+\frac{2}{a}+\frac{2}{a}+\frac{2}{a}+\frac{1}{b}+\frac{1}{b}}$$

$\Rightarrow \frac{2a+2b}{6}\geq\frac{6}{1}$

$\Rightarrow 2a+2b\geq 36$

$\Rightarrow a+b\geq 18$

$\Rightarrow OA+OB\geq 18$

Hence, the minimum value of $OA+OB=18$

Q.27. The number of solutions of equation $e^{\sin x}-2e^{-\sin x}=2$ is

A) 0

B) 1

C) 2

D) More than 2

Answer: 0

Solution: Let, $e^{\sin x} = t$

$\Rightarrow t - \frac{2}{t} = 2$

$\Rightarrow t^2 - 2t - 2 = 0$

$\Rightarrow t^2 - 2t + 1 = 3$

$\Rightarrow (t-1)^2 = 3$

$\Rightarrow t = 1 \pm \sqrt{3}$

$\Rightarrow t \approx 2.73, \ -0.73\,(\text{not possible})$

$\Rightarrow t \approx 2.73$

$\Rightarrow e^{\sin x} \approx 2.73$

$\Rightarrow \sin x \approx \log(2.73)$

$\Rightarrow \sin x > 1 \rightarrow \text{Not possible}$

So no solutions will be possible for the given equation.

Q.28. A is a square matrix of order 3. If $|A| = 2$ then find the value of $det\underbrace{(adjadj......adjA)}_{2024\,times}$.

A) 2^{2024}

B) $2^{2^{2024}}$

C) 2024

D) 2024^{2024}

Answer: $2^{2^{2024}}$

Solution: Given,

A is a square matrix of order 3

And $|A| = 2$

Now, we know that, $det\underbrace{(adjadj......adjA)}_{n\,times} = |A|^{(m-1)^n}$ where m is the order of matrix.

$\Rightarrow det\underbrace{(adjadj......adjA)}_{2024\,times} = 2^{2^{2024}}$.

Q.29. If reflection of $(2,3,4)$ in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is (α, β, γ) then find the value of $|2\alpha + 3\beta + 4\gamma|$

A) 20

B) 29

C) 21

D) 22

Answer: 29

Solution: Given:

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

Let, $P \equiv (2,3,4)$ and $Q \equiv (\alpha, \beta, \gamma)$.

Now, direction ratio of $PQ = \alpha - 2, \beta - 3, \gamma - 4$

And the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ will be perpendicular,

So, using perpendicular condition we get,

$\Rightarrow (\alpha - 2) \times 2 + (\beta - 3) \times 3 + (\gamma - 4) \times 4 = 0$

$\Rightarrow 2\alpha - 4 + 3\beta - 9 + 4\gamma - 16 = 0$

$\Rightarrow |2\alpha + 3\beta + 4\gamma| = 29$

Q.30. If ${}^{6}C_m + 2\left({}^{6}C_{m+1}\right) + {}^{6}C_{m+2} = {}^{8}C_3$, $m \neq 1$ and $\frac{{}^{n-1}P_3}{{}^{n}P_4} = \frac{1}{8}$, then ${}^{n-1}C_m + {}^{n}P_m = ?$

A) 456

B) 371

C) 91

D) 20

Answer: 371

Solution: Given:

$${}^{6}C_m + 2\left({}^{6}C_{m+1}\right) + {}^{6}C_{m+2} = {}^{8}C_3$$

$$\Rightarrow {}^{6}C_m + {}^{6}C_{m+1} + {}^{6}C_{m+1} + {}^{6}C_{m+2} = {}^{8}C_3$$

Now, using ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$ we get,

$$\Rightarrow {}^{7}C_{m+1} + {}^{7}C_{m+2} = {}^{8}C_3$$

$$\Rightarrow {}^{8}C_{m+2} = {}^{8}C_3$$

$$\Rightarrow m = 1,\ 3$$

But, $m \neq 1$

$$\Rightarrow m = 3$$

Now, $\frac{{}^{n-1}P_3}{{}^{n}P_4} = \frac{1}{8}$

$$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{8}$$

$$\Rightarrow \frac{(n-1)!}{n(n-1)!} = \frac{1}{8}$$

$$\Rightarrow n = 8$$

$$\Rightarrow {}^{n-1}C_m + {}^{n}P_m = {}^{7}C_3 + {}^{8}P_3$$

$$\Rightarrow {}^{n-1}C_m + {}^{n}P_m = \frac{7\times6\times5}{6} + 8 \times 7 \times 6$$

$$\Rightarrow {}^{n-1}C_m + {}^{n}P_m = 35 + 336$$

$$\Rightarrow {}^{n-1}C_m + {}^{n}P_m = 371$$

Q.31. If $L_1 : \vec{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} - 14\hat{j} + 5\hat{k}\right)$ and $L_2 : \vec{r} = \left(\hat{j} + \hat{k}\right) + \mu\left(-2\hat{i} - 4\hat{j} + 7\hat{k}\right)$ then find the shortest distance between the lines L_1 and L_2.

A) $\frac{5}{\sqrt{221}}$

B) $\frac{1}{\sqrt{221}}$

C) $\frac{6}{\sqrt{221}}$

D) $\frac{30}{\sqrt{221}}$

Answer: $\frac{5}{\sqrt{221}}$

Solution: The shortest distance will be given by $d=\frac{|(a_1-a_2).(r_1\times r_2)|}{|r_1\times r_2|}$

$$\Rightarrow d=\frac{\begin{vmatrix}1 & -2 & 0\\ 2 & -14 & 5\\ -2 & -4 & 7\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -14 & 5\\ -2 & -4 & 7\end{vmatrix}}$$

$$\Rightarrow d=\frac{|-78+48+0|}{|78\hat{i}+24\hat{j}-36\hat{k}|}$$

$$\Rightarrow d=\frac{30}{6\sqrt{13^2+4^2+6^2}}$$

$$\Rightarrow d=\frac{5}{\sqrt{221}}$$

Q.32. Find the value of the integral $\frac{120}{\pi^3}\left|\int_0^\pi \frac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x\right|$

Answer: 15

Solution: Let, $I=\int_0^\pi \frac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$

Now using the property $\int_0^{2a} f(x)\,\mathrm{d}x=\int_0^a f(x)\,\mathrm{d}x+\int_0^a f(2a-x)\,\mathrm{d}x$ we get,

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\left(x^2-(\pi-\mathrm{x})^2\right)\mathrm{d}x$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\left(2\pi\mathrm{x}-\pi^2\right)\mathrm{d}x$$

$$\Rightarrow I=2\pi\underbrace{\int_0^{\frac{\pi}{2}} \frac{x\cdot\sin x\cos x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x}_{I1}-\pi^2\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$$

Now, solving $I_1=\int_0^{\frac{\pi}{2}} \frac{x\cdot\sin x\cos x}{\sin^4 x+\cos^4 x}dx\quad....(1)$

$$\Rightarrow I_1=\int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\cdot\sin x\cos x}{\sin^4 x+\cos^4 x}dx\quad....(2)$$

Adding both equations we get,

$$\Rightarrow 2I_1=\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

$$\Rightarrow I_1=\frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

Now, putting the value in I we get,

$$\Rightarrow I=2\pi\cdot\frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{x\cdot\sin x\cos x}{\sin^4 x+\cos^4 x}dx-\pi^2\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$$

$$\Rightarrow I=-\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}\,\mathrm{d}x$$

$$\Rightarrow I=-\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1-2\sin^2 x\cos^2 x}\,\mathrm{d}x$$

$$\Rightarrow I = -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\frac{1}{2}\sin 2x}{1-\frac{1}{2}\sin^2 2x} \mathrm{d}\, x$$

$$\Rightarrow I = -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{2-\sin^2 2x} \mathrm{d}\, x$$

$$\Rightarrow I = -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1+\cos^2 2x} \mathrm{d}\, x$$

Now, let $\cos 2x = t \Rightarrow -2\sin 2x dx = dt$

$$\Rightarrow I = -\frac{\pi^2}{2}\int_1^{-1} \frac{-\frac{1}{2}}{1+t^2} \mathrm{d}\, t$$

$$\Rightarrow I = -\frac{\pi^2}{4}\int_{-1}^{1} \frac{1}{1+t^2} \mathrm{d}\, t$$

$$\Rightarrow I = -\frac{\pi^2}{4} \cdot \frac{\pi}{2} = -\frac{\pi^3}{8}$$

Hence, $\frac{120}{\pi^3}\left|\int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x+\cos^4 x} \mathrm{d}\, x\right| = \frac{120}{\pi^3} \times \frac{\pi^3}{8} = 15$

Q.33. The number of ways to distribute 21 identical apples to three children so that each child gets atleast 2 apples is

Answer: 136

Solution: Let us initially distribute 2 apples to each children.

So, the remaining 15 apples are to be distributed such that each children can get any number of apples.

So, the required number of ways will be given by $N = {}^{n+r-1}C_{r-1}$.

$$\Rightarrow N = {}^{15+3-1}C_2$$

$$\Rightarrow N = {}^{17}C_2$$

$$\Rightarrow N = \frac{17\times 16}{2}$$

$$\Rightarrow N = 136$$

Q.34. If $\lim_{x\to 0} \frac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$ then find the value of $16\left(a^2 + b^2 + c^2\right)$

Answer: 81

Solution:

Given: $\lim_{x \to 0} \frac{ax^2 e^x - b \log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$

$\Rightarrow \lim_{x \to 0} \frac{ax^2 e^x - b \log_e(1+x) + cxe^{-x}}{x^2 \cdot x \cdot \frac{\sin x}{x}} = 1$

$\Rightarrow \lim_{x \to 0} \frac{ax^2\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\right) - b\left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right) + cx\left(1 - \frac{x}{1!} + \frac{x^2}{2!} + \ldots\right)}{x^2 \times x} = 1$

Coefficient of $x = 0$

$\Rightarrow -b + c = 0 \quad \ldots(iii)$

Coefficient of $x^2 = 0$

$\Rightarrow a + \frac{b}{2} - c = 0 \quad \ldots(ii)$

$\Rightarrow a - \frac{c}{2} = 0$

$\Rightarrow a = \frac{c}{2}$

Coefficient of $x^3 = 1$

$\Rightarrow a - \frac{b}{3} + \frac{c}{2} = 1 \quad \ldots(iii)$

$\Rightarrow \frac{c}{2} - \frac{c}{3} + \frac{c}{2} = 1$

$\Rightarrow \frac{2c}{3} = 1$

$\Rightarrow c = \frac{3}{2} = b$

$\Rightarrow a = \frac{3}{4}$

$\Rightarrow 16\left(a^2 + b^2 + c^2\right) = 16\left(\frac{9}{16} + \frac{9}{4} \times 2\right)$

$\Rightarrow 16\left(a^2 + b^2 + c^2\right) = 81$

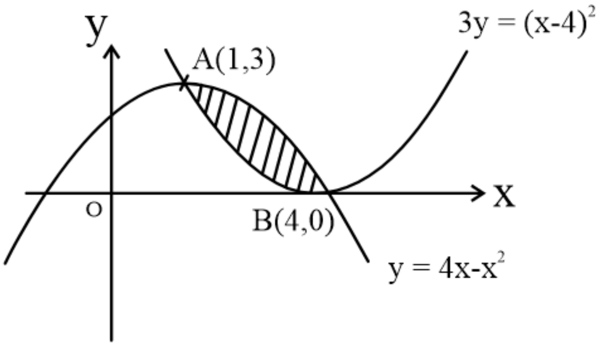
Q.35. If Area of the region enclosed by the parabola $y = 4x - x^2$ & $3y = (x-4)^2$ is A sqaure units, then the value of A is

Answer: 12

Solution: Given,

Equation of parabola $y=4x-x^2$ & $3y=(x-4)^2$

Now, plotting the diagram we get,



Now, from the above diagram area closed is given by,

$A=\int_1^4\left(4x-x^2\right)-\frac{(x-4)^2}{3}\,\mathrm{d}\,x$

$\Rightarrow A=\left[\left(2x^2-\frac{x^3}{3}\right)-\frac{(x-4)^3}{9}\right]_1^4$

$\Rightarrow A=\left[\left(32-\frac{64}{3}\right)-0\right]-\left[\left(2-\frac{1}{3}\right)-\frac{(3)^3}{9}\right]$

$\Rightarrow A=\frac{32-5+9}{3}=12$ square units.

Q.36. Let the mean and variance of 6 observations $a, b, 68, 44, 48, 60$ be 55 and 194 respectively. If $a>b$ then $a+3b$ is

Answer: 180

Solution: Mean, $\overline{x}=\frac{a+b+68+44+48+60}{6}=55$

$\Rightarrow a+b+220=330$

$\Rightarrow a+b=110 \quad \ldots(i)$

Variance, $\sigma^2=\frac{\sum\left(x_i-\overline{x}\right)^2}{n}$

$\Rightarrow \sigma^2=\frac{(a-55)^2+(b-55)^2+13^2+11^2+7^2+5^2}{6}=194$

$\Rightarrow (a-55)^2+(b-55)^2+364=1164$

$\Rightarrow (110-b-55)^2+(b-55)^2=800$

$\Rightarrow 2(55-b)^2=800$

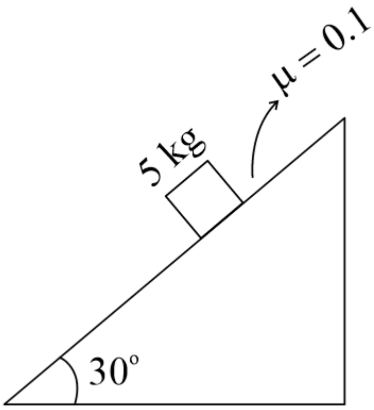
$\Rightarrow (55-b)=20$

$\Rightarrow b=35$

$\Rightarrow a=75$

$\Rightarrow a+3b=75+105=180$

Q.37. For the block shown, F_1 is the minimum force required to move the block upward and F_2 is the minimum force required to prevent it from slipping. Find $\left|\vec{F}_1 - \vec{F}_2\right|$ in N. Given that $g = 10\ \mathrm{m\ s}^{-2}$.



A) $5\sqrt{3}$

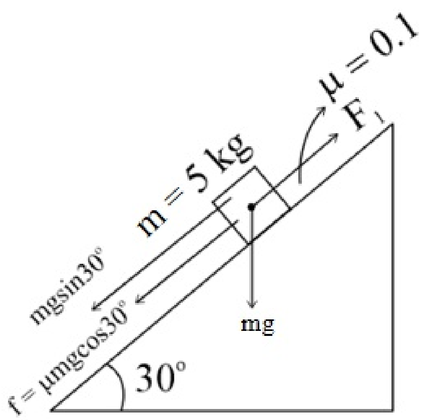
B) $50\sqrt{3}$

C) $\frac{5\sqrt{3}}{2}$

D) $25\sqrt{3}$

Answer: $5\sqrt{3}$

Solution: Let's consider the following FBD:



With refrence of the above figure, the minimum force (F_1) required to move the block upward is given by

$$F_1 = mg\sin 30^\circ + \mu mg\cos 30^\circ$$
$$= \left(5 \times 10 \times \frac{1}{2} + 0.1 \times 5 \times 10 \times \frac{\sqrt{3}}{2}\right)\ \mathrm{N}$$
$$= \left(25 + 2.5\sqrt{3}\right)\ \mathrm{N}$$

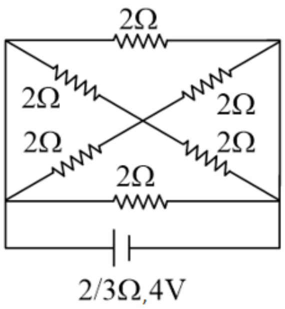
Also, the minimum force (F_2) required to prevent the block from slipping is given by(friction in this case will be acting up the incline plane)

$$F_2 = mg\sin 30^\circ - \mu mg\cos 30^\circ$$
$$= \left(50 \times \frac{1}{2} - 0.1 \times 50 \times \frac{\sqrt{3}}{2}\right)\ \mathrm{N}$$
$$= \left(25 - 2.5\sqrt{3}\right)\ \mathrm{N}$$

Hence,

$$\left|\vec{F}_1 - \vec{F}_2\right| = \left|\left(25 + 2.5\sqrt{3}\right) - \left(25 - 2.5\sqrt{3}\right)\right|$$
$$= 5\sqrt{3}\ \mathrm{N}$$

Q.38. Calculate the power dissipated in the following circuit.



A) 13

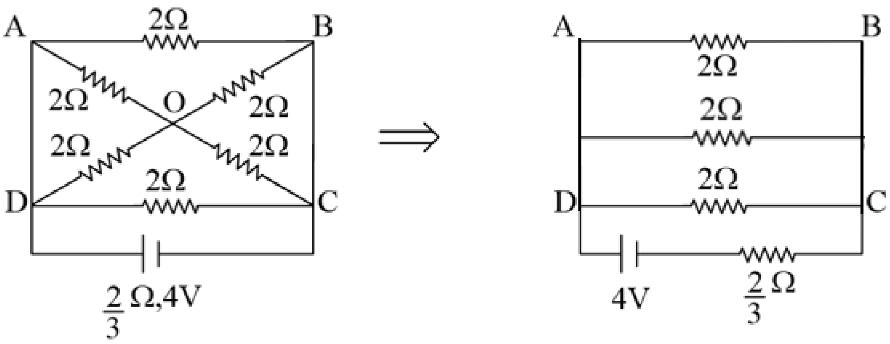
B) 15

C) 12

D) 7

Answer: 12

Solution: The equivalent circuit can be drawn as follows:



With reference to the above diagram, the resistances for the square ABCD can be simplified as follows:

$$\frac{1}{r}=\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}$$
$$=\frac{3}{2\,\Omega}$$
$$\Rightarrow r=\frac{2}{3}\,\Omega$$

where, r is the equivalent resistance for the square ABCD.

Thus, the equivalent resistance (R_{eq}) of the entire circuit can be calculated as follows:

$$R_{eq}=\left(\frac{2}{3}+\frac{2}{3}\right)\,\Omega$$
$$=\frac{4}{3}\,\Omega$$

The current through the circuit is, then, given by

$$i=\frac{4\text{ V}}{\frac{4}{3}\,\Omega}$$
$$=3\text{ A}$$

Hence, the power delivered to the circuit is

$$P=i^2R_{eq}$$
$$=(3\text{ A})^2\times\frac{4}{3}\,\Omega$$
$$=12\text{ W}$$

Q.39. Mass of the moon is $\frac{1}{144}$ times the mass of a planet. Its diameter is $\frac{1}{16}$ time the diameter of the planet. If the escape velocity of the planet is V, then the escape velocity of the moon will be

A) $\frac{V}{3}$

B) $\frac{V}{4}$

C) $3V$

D) $4V$

Answer: $\frac{V}{3}$

Solution: The formula to calculate the escape velocity on the moon is given by

$$v_m = \sqrt{\frac{2GM_m}{R_m}} \quad \ldots(1)$$

The formula to calculate the escape velocity on the planet is given by

$$v_p = \sqrt{\frac{2GM_p}{R_p}} \quad \ldots(2)$$

From equations (1) and (2), it follows that

$$\frac{v_m}{V} = \frac{\sqrt{\frac{2GM_m}{R_m}}}{\sqrt{\frac{2GM_P}{R_P}}}$$

$$= \sqrt{\frac{R_P M_m}{R_m M_P}}$$

$$= \sqrt{16 \times \frac{1}{144}}$$

$$= \frac{1}{3}$$

$$\Rightarrow v_m = \frac{V}{3}$$

Q.40. If in the given expression, $E = \frac{b - x^2}{at}$, E represents the energy, x represents the length and t represents time, then find dimension of $\left[\frac{a}{b}\right]$, where $a,\ b$ are constants.

A) $\left[\mathrm{ML^2T^{-1}}\right]$

B) $\left[\mathrm{M^2L^{-2}T^{-1}}\right]$

C) $\left[\mathrm{M^{-1}L^{-2}T^1}\right]$

D) $\left[\mathrm{ML^0T^1}\right]$

Answer: $\left[\mathrm{M^{-1}L^{-2}T^1}\right]$

Solution: The given equation can be simplified as

$$E = \frac{b}{at} - \frac{x^2}{at} \quad \ldots(1)$$

From equation (1), it follows that

$$[E] = \left[\frac{b}{at}\right]$$

$$\Rightarrow \left[\frac{a}{b}\right] = \left[\frac{1}{Et}\right]$$

$$= \frac{1}{\left[\mathrm{ML^2T^{-2}}\right][\mathrm{T}]}$$

$$= \left[\mathrm{M^{-1}L^{-2}T^1}\right]$$

Q.41. A point object is placed at $100\ \mathrm{cm}$ from a convex spherical refractive surface having radius of curvature $200\ \mathrm{cm}$ and refractive index of the refractive surface is 1.5. Find image distance.

A) $50\ \mathrm{cm}$

B) $100\ \mathrm{cm}$

C) $200\ \mathrm{cm}$

D) $400\ \mathrm{cm}$

Answer: $200\ \mathrm{cm}$

Solution: For the convex surface:

Radius of curvature, $R = 200\ \text{cm}$

Refractive index of lens, $\mu_2 = 1.5$

Object distance, $u = -100\ \text{cm}$

Therefore,

$$\frac{\mu_2}{\nu} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$
$$\Rightarrow \frac{1.5}{v} - \frac{1}{-100} = \frac{1.5-1}{200}$$
$$\Rightarrow \frac{1.5}{v} = \frac{1}{400} - \frac{1}{100}$$
$$\Rightarrow v = 1.5 \times \frac{-400}{3} = -200\ \text{cm}$$

So, image is formed at $200\ \text{cm}$.

Q.42. Magnetic flux passing through a loop of resistance $8\ \Omega$ is given by $\phi = \left(5t^2 - 36t + 5\right)\ \text{Wb}$. Find current in the loop at $t = 2$ sec.

A) $1\ \text{A}$

B) $2\ \text{A}$

C) $3\ \text{A}$

D) $4\ \text{A}$

Answer: $2\ \text{A}$

Solution: Given, the variation of magnetic flux $\phi = \left(5t^2 - 36t + 5\right)\ \text{Wb}$, Resistance of coil $R = 8\ \Omega$

As we know, emf induced in the coil, $E = \left|-\frac{d\phi}{dt}\right| = \left|\frac{d\left(5t^2-36t+5\right)}{dt}\right| = |10t - 36|$ (Differentiate with respect to t).

Emf induced at time $t = 2\ \text{s}$

$\therefore E = |10 \times 2 - 36| = 16\ \text{V}$

Current in coil $i = \frac{E}{R} = \frac{16}{8} = 2\ \text{A}$

Q.43. At temperature 300 kelvin, 5 moles of gas are expanded from 10 liters to 100 liters. Find the work done(in J) by the gas.

A) $500R\ln(5)$

B) $500R\ln(10)$

C) $1500R\ln(5)$

D) $1500R\ln(10)$

Answer: $1500R\ln(10)$

Solution: As expansion happens at a constant temperature, therefore the process is isothermal. Work done in isothermal process is given by,

$$W = nRT\ln\left(\frac{V_{final}}{V_{initial}}\right)$$
$$= 5 \times R \times 300 \times \ln\left(\frac{100}{10}\right)$$
$$= 1500R\ln(10)$$

Q.44. A particle of mass m is projected from ground with speed v at an angle of 45° with the horizontal. Find its angular momentum about the point of projection when it reaches its maximum height.

A) $\frac{mv^3}{16g}$

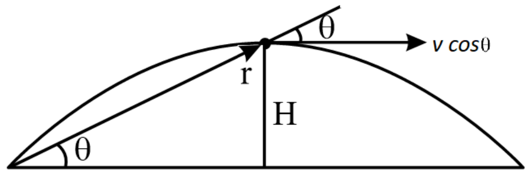
B) $\frac{mv^3}{4\sqrt{2}g}$

C) $\frac{mv^3}{3g}$

D) $\frac{\sqrt{3}mv^3}{8g}$

Answer: $\frac{mv^3}{4\sqrt{2}g}$

Solution:



Let range be R.

The angular momentum is given by,

$\vec{L}=\vec{r}\times\vec{p}$

$\Rightarrow\vec{L}=\left(\frac{H}{\sin\theta}\right)(mv\ \cos\theta)\ \sin\theta\ \left(-\hat{\mathrm{k}}\right)$, where $\left(-\hat{\mathrm{k}}\right)$ is directed inside the plane of the paper.

$\Rightarrow\vec{L}=\left(\frac{\frac{v^2\sin^2\theta}{2g}}{\sin\theta}\right)(mv\ \cos\theta)\ \sin\theta\ \left(-\hat{\mathrm{k}}\right)$

$=\frac{mv^3\cos\theta\sin^2\theta}{2g}\left(-\hat{\mathrm{k}}\right)$

But, $\theta=45^\circ$

$\therefore\quad\left|\vec{L}\right|=\frac{mv^3}{4\sqrt{2}g}$

Q.45. Force on a $2\ \mathrm{kg}$ particle varies with time as, $\vec{F}=\left(6t\hat{\mathrm{i}}-6t^2\hat{\mathrm{j}}\right)\ \mathrm{N}$. Find power delivered at $t=2\ \mathrm{s}$.

A) $64\ \mathrm{W}$ B) $90\ \mathrm{W}$

C) $264\ \mathrm{W}$ D) $150\ \mathrm{W}$

Answer: $264\ \mathrm{W}$

Solution: As we know, $\vec{a}=\frac{\vec{F}}{m}=\frac{6t\hat{\mathrm{i}}-6t^2\hat{\mathrm{j}}}{2}=\left(3t\hat{\mathrm{i}}-3t^2\hat{\mathrm{j}}\right)\ \mathrm{m\ s^{-2}}$

Now,

$\vec{v}=\int\vec{a}\,dt$

$\Rightarrow\vec{v}=\int_0^t\left(3t\hat{\mathrm{i}}-3t^2\hat{\mathrm{j}}\right)dt$

$=\left[\frac{3t^2}{2}\hat{\mathrm{i}}-\frac{3t^3}{3}\hat{\mathrm{j}}\right]_0^t$

$=\frac{3t^2}{2}\hat{\mathrm{i}}-t^3\hat{\mathrm{j}}$

Now, power is given by

$P=\vec{F}\cdot\vec{v}$

$=\left(6t\hat{\mathrm{i}}-6t^2\hat{\mathrm{j}}\right)\cdot\left(\frac{3t^2}{2}\hat{\mathrm{i}}-t^3\hat{\mathrm{j}}\right)$

$=9t^3+6t^5$

Power at $t=2\ \mathrm{s}$, $P=(9\times8)+(6\times32)=264\ \mathrm{W}$

Q.46. An unpolarised light is incident on a transparent glass at incident angle 60°. If the reflected ray is completely polarised, then the angle of refraction is

A) 60° B) 45°

C) 37° D) 30°

Answer: 30°

Solution: The refractive index of the glass plate (μ) for the polarised light can be found, using Brewster's law, as

$$\tan 60^\circ = \mu$$
$$\Rightarrow \mu = \sqrt{3}$$

If r is the angle of refraction, then by Snell's law, it follows that

$$\frac{\sin 60^\circ}{\sin r} = \mu = \sqrt{3}$$
$$\Rightarrow \sin r = \frac{\sin 60^\circ}{\sqrt{3}} = \frac{1}{2} = \sin 30^\circ$$
$$\Rightarrow r = 30^\circ$$

Q.47. What is the speed(in $\mathrm{m\ s^{-1}}$) of sound in oxygen gas at STP? Given that, $R = 8.3\ \mathrm{J\ mol^{-1}\ K^{-1}}$, $T = 27\ \mathrm{K}$ and $\gamma = 1.4$. Write the value to the nearest integer.

Answer: 330

Solution: The formula to calculate the speed of sound is given by

$$v = \sqrt{\frac{\gamma R T}{M}} \quad \ldots (1)$$

where, M is the molecular weight.

From equation (1), it follows that

$$v = \sqrt{\frac{1.4 \times 8.3 \times (27+273) \times 1000}{32}}\ \mathrm{m\ s^{-1}}$$
$$= 330\ \mathrm{m\ s^{-1}}$$

Q.48. The percentage by which the illumination of a lamp decreases if the current drops by 20% is $x\%$. Find the value of x.

Answer: 36

Solution: Given the value of current (i') in the second case is $i' = 0.8i$, where i is the current through the lamp in the first case.

In first case, the power is given by

$$P = i^2 R \quad \ldots (1)$$

where, R is the resistance.

In the second case, the power is given by

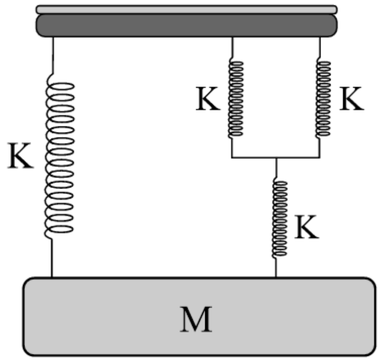
$$P' = (i')^2 R$$
$$= (0.8i)^2 R$$
$$= 0.64 i^2 R$$
$$= 0.64 P \quad \ldots (2)$$

Hence, the percentage drop of power can be calculated as follows:

$$\frac{P - P'}{P} \times 100\% = \frac{P - 0.64P}{P} \times 100\%$$
$$= 36\%$$

Hence, $x = 36$.

Q.49. The period of oscillation of the system shown in the figure is $\pi\sqrt{\frac{\alpha M}{5K}}$. Find the value of α.



Answer: 12

Solution: The equivalent spring constant of the given system can be found out as follows:

$$K e q=K+\frac{1}{\frac{1}{K}+\frac{1}{K+K}}$$
$$=K+\frac{2K}{3}$$
$$=\frac{5K}{3}$$

Thus, the time period of oscillation of the given system can be written as

$$T=2\pi\sqrt{\frac{M}{K e q}}$$
$$=2\pi\sqrt{\frac{M}{\frac{5K}{3}}}$$
$$=\pi\sqrt{\frac{12M}{5K}}$$

Hence, $\alpha=12$.

Q.50. Nucleus X has mass number 192. A second nucleus Y has radius half of X, then find mass number of nucleus Y.

Answer: 24

Solution: We know that $R=R_0A^{1/3}$

Given :

$$A_X=192$$
$$R_X=R$$
$$R_Y=\frac{R}{2}$$

Using the above equation, we can write

$$\frac{\mathrm{R}_Y}{\mathrm{R}_X}=\left(\frac{\mathrm{A}_2}{192}\right)^{1/3}$$
$$\Rightarrow\left(\frac{1}{2}\right)^3=\frac{\mathrm{A}_2}{192}$$
$$\Rightarrow A_2=\frac{192}{8}=24$$

Q.51. 3 moles of oxygen gas and 2 moles of argon gas are mixed together. If the total energy of the mixture is $\frac{x}{2}RT$, find the value of x.

Answer: 21

Solution: The formula to calculate the degrees of freedom of the mixture can be written as

$$F_{\text{mix}} = \frac{n_O F_O + n_{Ar} F_{Ar}}{n_O + n_{Ar}} \quad \ldots(1)$$

where, n represents the number of moles and F represents the degrees of freedom of the respective gases.

Since oxygen is a diatomic gas, $F_O = 5$ and argon, being a monatomic gas, has the degrees of freedom $F_{Ar} = 3$.

Thus, from equation (1), it follows that

$$\begin{aligned} F_{\text{mix}} &= \frac{3\times5+2\times3}{3+2} \\ &= \frac{21}{5} \end{aligned}$$

Thus, the total energy of the mixture can be calculated as follows:

$$\begin{aligned} E &= \frac{1}{2}\left(n_O + n_{Ar}\right)F_{\text{mix}}RT \\ &= \frac{1}{2} \times (3+2) \times \frac{21}{5}RT \\ &= \frac{21}{2}RT \end{aligned}$$

Hence, $x = 21$.