

JEE Main

1st Feb Shift 2



Answer: – COOH

Solution: A negative resonance effect occurs when the groups withdraw the electrons from other molecules through the process of delocalisation.

Oxygen is more electronegative than carbon, meaning it has a stronger pull on shared electrons. In the resonance structures, the double bond between the carbon and oxygen can shift such that the oxygen bears a negative formal charge. This delocalisation means that the oxygen atoms are effective at drawing electron density toward themselves and away from other parts of the molecule through the conjugated system.

Due to this effect, the carboxyl group can pull electron density away from adjacent parts of a molecule through the pi electron system (resonance), hence the description as having a resonance (-R) electron-withdrawing effect.

Q.7. $[\text{Co}(\text{NH}_3)_6]^{3+}$ and $[\text{CoF}_6]^{3-}$ are-

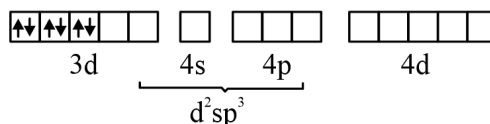
- A) Inner orbital complex and outer orbital complex respectively
- B) Outer orbital complex and inner orbital complex respectively
- C) Both are Inner orbital complex.
- D) Both are outer orbital complex.

Answer: Inner orbital complex and outer orbital complex respectively

Solution: In $[\text{Co}(\text{NH}_3)_6]^{3+}$ Oxidation state of cobalt = +3

Electronic configuration of $\text{Co}^{3+} = d^6$

NH_3 being a strong field ligand causes the pairing. Therefore, Ni can undergo d^2sp^3 hybridization.



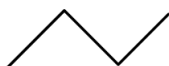
Hence, it is an inner orbital complex.

In $[\text{CoF}_6]^{3-}$ F is weak field ligand for Co^{3+} , thus pairing of d-orbital electrons does not happen, so it forms: outer orbital complex of hybridisation sp^3d^2

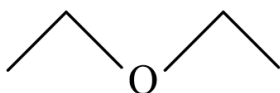
Hence, option A is the answer.

Q.8. Which of the following will have highest boiling point?

A)



B)



C)



D)





Answer:



Solution: The boiling points of aldehydes and ketones are higher than hydrocarbons and ethers of comparable molecular masses. This is due to weak molecular association in aldehydes and ketones arising out of dipole-dipole interactions. Also, their boiling points are lower than those of alcohols of similar molecular masses due to the absence of intermolecular hydrogen bonding.

Hence,



has highest boiling point.

Q.9. Statement I: Among Mn^{2+} and Cr^{2+} , Cr^{2+} is better reducing agent.

Statement II: Half filled electronic configuration is more stable.

- A) Both Statement I and Statement II are correct. B) Both Statement I and Statement II are wrong.
 C) Statement I is correct and Statement II is wrong. D) Statement I is wrong and Statement II is correct.

Answer: Both Statement I and Statement II are correct.

Solution: If an element is behaving as reducing agent that means, it itself is undergoing oxidation that is there is loss of electrons. while for reduction there is gain of electrons.

Cr^{2+} has the configuration $3d^4$. It can lose electron to form $3d^3$ which has stable configuration as it has half filled t_{2g} level. Hence it is better reducing agent than Mn^{2+} which has a stable half filled configuration.

Hence Statement I is correct.

The stability of exactly half-filled orbitals in degenerate orbitals is stronger than that of other partially filled configurations.

Hence, Statement II is correct.

Q.10. Solubility of $\text{Ca}_3(\text{PO}_4)_2$ in 100 mL of pure water is W gm. Find out K_{sp} of $\text{Ca}_3(\text{PO}_4)_2$ is:

(Molar mass of $\text{Ca}_3(\text{PO}_4)_2 = M$)

- A) $108 \times 10^5 \times \left(\frac{W}{M}\right)^5$ B) $108 \times \left(\frac{W}{M}\right)^5$
 C) $108 \times 10^4 \times \left(\frac{W}{M}\right)^5$ D) $108 \times 10^6 \times \left(\frac{W}{M}\right)^5$

Answer: $108 \times 10^5 \times \left(\frac{W}{M}\right)^5$

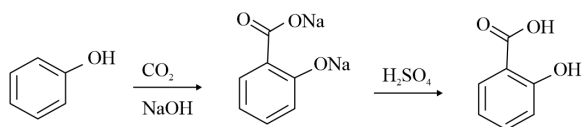


C) 1 – Q, 2 – S, 3 – P, 4 – R

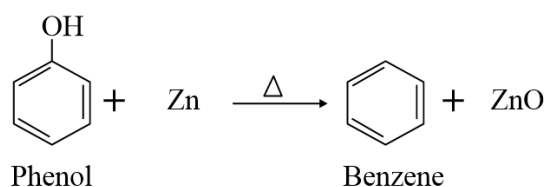
D) 1 – Q, 2 – R, 3 – S, 4 – P

Answer: 1 – Q, 2 – S, 3 – R, 4 – P

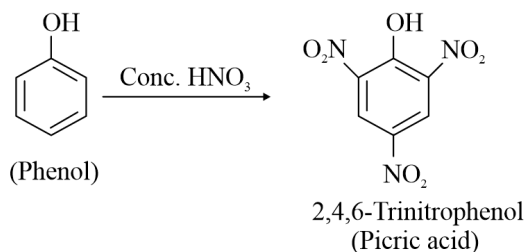
Solution: 1. Kolbe's reaction, also known as Kolbe Schmitt Reaction, is a type of addition reaction named after Hermann Kolbe and Rudolf Schmitt. When phenol is treated with sodium hydroxide, phenoxide ion is generated. The phenoxide ion generated is more reactive than phenol towards electrophilic aromatic substitution reaction. Hence, it undergoes an electrophilic substitution reaction with carbon dioxide, which is a weak electrophile. Ortho-hydroxybenzoic acid (salicylic acid) is formed as the primary product. This reaction is popularly known as Kolbe's reaction.



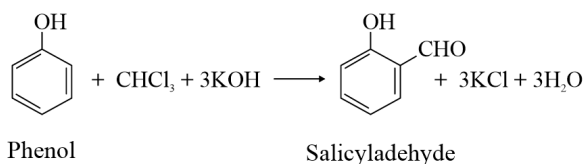
2. When phenol is heated with zinc dust, the product formed is benzene only.



3. With concentrated nitric acid, phenol is converted to 2,4,6-Trinitrophenol. The product is commonly known as picric acid.



4. When phenol reacts with chloroform in the presence of bases KOH / NaOH) an aldehyde group (– CHO) gets introduced in the ring at a position ortho to the phenol group and salicylaldehyde is formed. This reaction is called the Reimer-Tiemann reaction.



Hence, answer is A.

Q.14. Statement I: In p and d block both metal and non metal are present.

Statement II: Electronegativity and ionisation enthalpy of metal is greater than non-metals.

A) Both Statement I and II are correct.

B) Both Statement I and II are wrong.

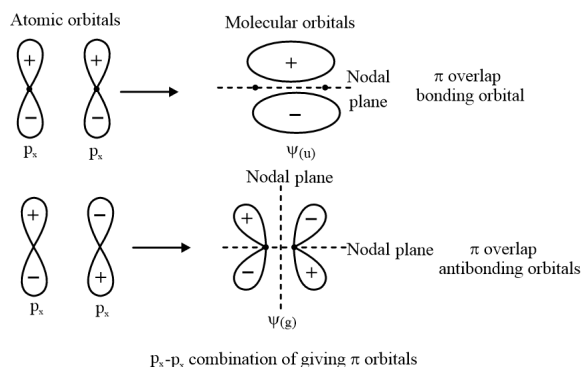
C) Statement I is correct and statement II is wrong.

D) Statement I is wrong and statement II is correct.

Answer: Both Statement I and II are wrong.



Solution:



Statement1: In π BMO electron density is present above and below the internuclear axis.

Hence, statement 1 is incorrect.

Statement 2: In π ABMO two nodal planes are present.

This statement is correct.

Hence, the answer is D.

Q.17. Match the following

Compound	Uses
(i) CCl_4	(p) Refrigerator and AC
(ii) CH_2Cl_2	(q) Non biodegradable insecticide
(iii) Freons	(r) Fire Extinguisher
(iv) DDT	(s) Paint Removal

A) (i)-r, (ii)-s, (iii)-p, (iv)-q

B) (i)-p, (ii)-s, (iii)-r, (iv)-q

C) (i)-s, (ii)-r, (iii)-p, (iv)-q

D) (i)-r, (ii)-s, (iii)-q, (iv)-p

Answer: (i)-r, (ii)-s, (iii)-p, (iv)-q

Solution: CCl_4 is used in the manufacturing of refrigerants, as a cleaning agent and was also used as a fire extinguisher.

CH_2Cl_2 is used as paint removal.

Freons are used as refrigerants in refrigerators and air conditioners.

DDT (p, p'-Dichlorodiphenyltrichloroethane) is a colourless, crystalline, tasteless and almost odourless organochloride known for its insecticidal properties.

Hence option A is the answer.

Q.18. Statement I: Ni^{2+} with DMG shows red colouration.

Statement II : Both Fe^{2+} and Fe^{3+} are blue coloured.

A) Statement I is correct Statement II is wrong.

B) Statement I is wrong Statement II is correct.

C) Both statements I and II are correct.

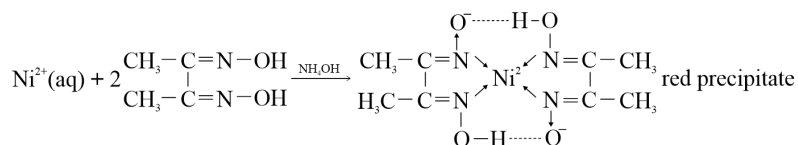
D) Both statements I and II are wrong.

Answer: Statement I is correct Statement II is wrong.



Solution: On addition of **dimethylglyoxime** (DMG) to nickel ion solution and on adding little bit of ammonia to make solution basic, it will give a red precipitate. This is a very specific test for nickel +2 cation.

The reaction is as follows,



Hence statement I is correct.

Ferrous ion is a divalent iron compound ion. It has +2 oxidation state and is light green in color while ferric ion is trivalent ion and is very pale violet or brown in colour.

Hence statement II is wrong.

Q.19. Number of radial nodes present in 3p are:

Answer: 1

Solution: 3p orbital the principal quantum number $n = 3$, placing it on the third energy level and the angular quantum number $l = 1$ gives the shape of p-orbital.

We can calculate the number of radial nodes using the formula,

$$N = n - l - 1$$

The value of n is 3 and the value of l is 1.

$$N = 3 - 1 - 1$$

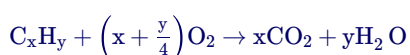
$$N = 1$$

The number of radial nodes in 3p orbitals is 1.

Q.20. A 10 mL hydrocarbon (C_xH_y) on combustion gives 40 mL CO_2 and 50 mL H_2O . Calculate the value of $x + y$.

Answer: 14

Solution: The combustion reaction can be represented as



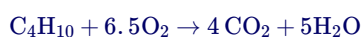
At STP, 1 mole of any gas occupies 22.4 L

10 mL of hydrocarbon, 50 mL of water and 40 mL of carbon dioxide corresponds to 0.45, 2.23, and 1.79 moles respectively.

Hence, the combustion of 1 mole of hydrocarbon will produce 5 moles of water and 4 moles of carbon dioxide.

Thus, the molecular formula contains 4 carbon atoms and 10 hydrogen atoms.

The hydrocarbon is C_4H_{10} and the combustion reaction is



$$x + y = 4 + 10 = 14$$

Q.21. In Kjeldahl's method of estimation of nitrogen, containing 1 gm organic compound nitrogen is converted to ammonia which is further treated with 10 mL, 2M H_2SO_4 . Find the percent of nitrogen?

Answer: 56



Solution: Equivalent of NH_3 = Equivalent of H_2SO_4

$$\begin{aligned} \text{moles of NH}_3 (n_{\text{NH}_3}) \times 1 &= \frac{2 \times 10 \times 2}{1000} \\ &= 4 \times 10^{-2} \text{ moles} \end{aligned}$$

Moles of nitrogen = 4×10^{-2} moles

$$\begin{aligned}\text{Weight of Nitrogen} &= 4 \times 10^{-2} \times 14 \\ &= 56 \times 10^{-2} \text{ gm}\end{aligned}$$

$$\begin{aligned} \text{Percent of nitrogen} &= \frac{56 \times 10^{-2}}{\text{weight of the compound}} \times 100 \\ &= \frac{56 \times 10^{-2}}{1} \times 100 = 56\% \end{aligned}$$

Q.22. Find $\Delta_r G^\circ$ (in kJ/mol), 300 K, if K_{eq} is 10.

Given $R = 8.314 \text{ J/molK}$

Give an answer to the nearest integer value.

Answer: 6

Solution: $\Delta_r G^\circ = -RT \ln K$

$$\begin{aligned} &= -2.303 RT \log K \\ &= -2.303 \times 8.314 \times 300 \log 10 \text{ J/mol} \\ &= \frac{-2.303 \times 8.314 \times 300}{1000} \text{ kJ/mol} \\ &= 5.77 \approx 6 \text{ kJ/mol} \end{aligned}$$

Q.23. For a given first order chemical reaction $A_{(g)} \rightarrow 2B_{(g)}$. If the initial pressure of A is 0.1 atm and total pressure of the system after 100 sec is 0.18 atm. Value of the rate constant (sec^{-1}) is $n \times 10^{-2}$, Give the value of n.

Answer: 16

Solution: $A_{(g)} \rightarrow 2B_{(g)}$

	(8)	(9)
at t = 0	0.1	—
at t = 100	0.1 - P	2P

$$P_{\text{Total}_{t-100}} = 0.1 - P + 2P = 0.18$$

$$0.1 + P = 0.18 \text{ atm}$$

$$P = 0.18 - 0.1 = 0.08 \text{ atm}$$

$$P_{\text{inial}} = 0.1 \text{ atm}$$

$$P_t = 0.1 - 0.08 = 0.02 \text{ atm}$$

$$K = \frac{1}{t} \ln \left(\frac{0.1}{0.1 - P} \right)$$

$$K = \frac{2.303}{100} \log \frac{0.1}{0.02} = 0.016 = 16 \times 10^{-2} = n \times 10^{-2}$$

$$\Rightarrow n = 16$$

Q.24. If α and β be the roots of equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q, r be the consecutive terms of a non-constant GP and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is:

- A) $\frac{80}{9}$
- B) $\frac{16}{9}$
- C) $\frac{9}{16}$
- D) $\frac{-9}{16}$

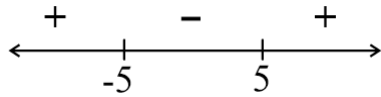
Answer: $\frac{80}{9}$



Solution: Given: $f(x) = \frac{\sqrt{x^2-25}}{4-x^2} + \log(x^2 + 2x - 15)$

$$\Rightarrow x^2 - 25 \geq 0$$

$$\Rightarrow (x-5)(x+5) \geq 0$$



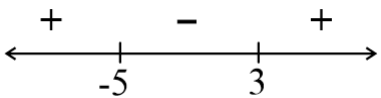
$$\Rightarrow x \in (-\infty, -5] \cup [5, \infty) \dots (i)$$

$$4 - x^2 \neq 0$$

$$\Rightarrow x \neq 2, -2 \dots (ii)$$

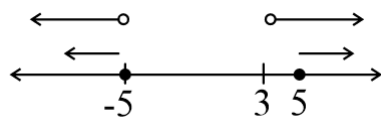
$$\Rightarrow x^2 + 2x - 15 > 0$$

$$\Rightarrow (x+5)(x-3) > 0$$



$$\Rightarrow x \in (-\infty, -5) \cup (3, \infty) \dots (iii)$$

Using (i), (ii) and (iii)



$$\Rightarrow x \in (-\infty, -5) \cup [5, \infty)$$

So, on comparing with $x \in (-\infty, \alpha) \cup [\beta, \infty)$ we get,

$$\Rightarrow \alpha^2 + \beta^2 = (-5)^2 + (5)^2$$

$$\Rightarrow \alpha^2 + \beta^2 = 50$$

Q.27. If $\int_0^{\frac{\pi}{3}} \cos^4 x \, dx = a\pi + b\sqrt{3}$ then find the value of $a^2 + b$

A) 1

B) $\frac{1}{2}$

C) $\frac{1}{9}$

D) $\frac{1}{8}$

Answer: $\frac{1}{8}$

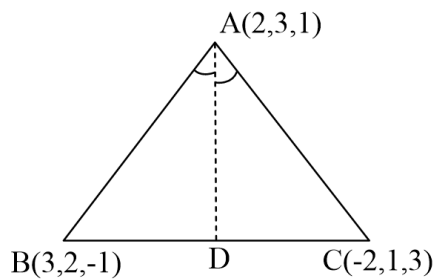


- A) $\frac{2}{\sqrt{3}}$ B) $\frac{\sqrt{2}}{3}$
 C) $\sqrt{\frac{2}{3}}$ D) $\frac{1}{\sqrt{3}}$

Answer: $\sqrt{\frac{2}{3}}$

Solution: Given,

$A(2, 3, 1)$, $B(3, 2, -1)$, $C(-2, 1, 3)$ are vertices of $\triangle ABC$ and AD is angle bisector,



Since AD is angle bisector, so $\frac{AB}{BC} = \frac{BD}{DC}$

$$\Rightarrow \frac{\sqrt{6}}{2\sqrt{6}} = \frac{BD}{DC}$$

$$\Rightarrow \frac{BD}{DC} = \frac{1}{2}$$

Now, using the section formula, the coordinates of $D \equiv \left(\frac{6-2}{3}, \frac{4+1}{3}, \frac{-2+3}{3}\right) \equiv \left(\frac{4}{3}, \frac{5}{3}, \frac{1}{3}\right)$

So, $\vec{AD} = \frac{-2}{3}\hat{i} - \frac{4}{3}\hat{j} - \frac{2}{3}\hat{k}$ and $\vec{AC} = -4\hat{i} - 2\hat{j} + 2\hat{k}$

Now, projection of \vec{AD} on \vec{AC} is given by,

$$\frac{\vec{AD} \cdot \vec{AC}}{|\vec{AC}|} = \frac{\frac{8}{3} + \frac{8}{3} - \frac{4}{3}}{\sqrt{16+4+4}} = \frac{4}{\sqrt{24}} = \frac{4}{2\sqrt{6}} = \sqrt{\frac{2}{3}}$$

Q.30. If m and n be the coefficient of 7th and 13th terms in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{5}}}\right)^{18}$, then the value of $\left(\frac{m}{n}\right)^{\frac{1}{3}}$ is:

- A) $\frac{4}{9}$ B) $\frac{9}{4}$
 C) $\frac{2}{3}$ D) $\frac{3}{2}$

Answer: $\frac{4}{9}$



Solution:

Given expansion is $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{5}{5}}}\right)^{18}$.

$$\Rightarrow T_7 = {}^{18}C_6 \left(\frac{1}{3}x^{\frac{1}{3}}\right)^{12} \left(\frac{1}{2x^{\frac{5}{5}}}\right)^6$$

$$\Rightarrow m = {}^{18}C_6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{2}\right)^6$$

$$\Rightarrow T_{13} = {}^{18}C_{12} \left(\frac{1}{3}x^{\frac{1}{3}}\right)^6 \left(\frac{1}{2x^{\frac{5}{5}}}\right)^{12}$$

$$\Rightarrow n = {}^{18}C_{12} \left(\frac{1}{3}\right)^6 \left(\frac{1}{2}\right)^{12}$$

$$\Rightarrow \frac{m}{n} = \frac{{}^{18}C_6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{2}\right)^6}{{}^{18}C_{12} \left(\frac{1}{3}\right)^6 \left(\frac{1}{2}\right)^{12}}$$

$$\Rightarrow \frac{m}{n} = \frac{\left(\frac{1}{3}\right)^6}{\left(\frac{1}{2}\right)^6}$$

$$\Rightarrow \frac{m}{n} = \left(\frac{2}{3}\right)^6$$

$$\Rightarrow \left(\frac{m}{n}\right)^{\frac{1}{3}} = \frac{4}{9}$$

Q.31. Find the minimum value of $\left|z + \frac{3+4i}{2}\right|$ such that $|z| \leq 1$

A) $\frac{3}{2}$

B) 5

C) 3

D) $\frac{5}{2}$

Answer: $\frac{3}{2}$

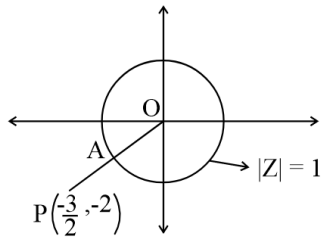


Solution: Given: $\left|z + \frac{3+4i}{2}\right|$; $|z| \leq 1$

Let, $z = x + iy$

$\Rightarrow |z| = 1$ gives $x^2 + y^2 = 1$, which represents a circle having centre at $(0, 0)$ and radius as 1 unit.

Now, $\left|z + \frac{3+4i}{2}\right|$ gives the distance of z from $\left(\frac{3}{2}, -2\right)$



So, we need to find the minimum distance of AP .

$$\Rightarrow AP = OP - OA$$

$$\Rightarrow AP = \sqrt{\left(-\frac{3}{2} - 0\right)^2 + (0 + 2)^2} - 1$$

$$\Rightarrow AP = \sqrt{\frac{9}{4} + 4} - 1$$

$$\Rightarrow AP = \frac{5}{2} - 1$$

$$\Rightarrow AP = \frac{3}{2}$$

Q.32. If $\frac{dx}{dy} = \frac{1+x-y^2}{y}$ and $x(1) = 1$, then $5x(2)$ is equal to _____.

A) 2

B) 3

C) 4

D) 5

Answer: 5



Solution: Given: $\frac{dx}{dy} = \frac{1+x-y^2}{y}$

$$\Rightarrow \frac{dx}{dy} = \frac{1+x}{y} - y$$

$$\Rightarrow \frac{dx}{dy} - \frac{1+x}{y} = -y$$

Now let $1+x=t \Rightarrow \frac{dx}{dy} = \frac{dt}{dy}$

$$\Rightarrow \frac{dt}{dy} - \frac{t}{y} = -y$$

$$\Rightarrow \text{IF} = e^{\int -\frac{1}{y} dy}$$

$$\Rightarrow \text{IF} = e^{-\log y}$$

$$\Rightarrow \text{IF} = e^{\log \frac{1}{y}}$$

$$\Rightarrow \text{IF} = \frac{1}{y}$$

So, the solution of the given differential equation is

$$\frac{t}{y} = \int \frac{1}{y} \times (-y) dy$$

$$\Rightarrow \frac{(1+x)}{y} = \int \frac{1}{y} \times (-y) dy$$

$$\Rightarrow \frac{(1+x)}{y} = -y + c$$

Also, $x(1) = 1$

$$\Rightarrow \frac{2}{1} = -1 + c$$

$$\Rightarrow c = 3$$

$$\Rightarrow \frac{(1+x)}{y} = -y + 3$$

Putting $y = 2$

$$\Rightarrow \frac{(1+x)}{2} = -2 + 3$$

$$\Rightarrow (1+x) = 2$$

$$\Rightarrow x = 1$$

$$\Rightarrow 5x = 5$$

Q.33. If S_n be the sum of first n terms of an AP and if $S_{10} = 390$ and the ratio of the 10^{th} term to 5^{th} term is $15 : 7$, then find the value of $S_{15} - S_5$

A) 885

B) 790

C) 800

D) 700

Answer: 790



Solution: Given: $\frac{a_{10}}{a_5} = \frac{15}{7}$

$$\Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7}$$

$$\Rightarrow 7a + 63d = 15a + 60d$$

$$\Rightarrow 3d = 8a$$

Also, $S_{10} = 390$

$$\Rightarrow \frac{10}{2} [2a + 9d] = 390$$

$$\Rightarrow 2a + 24d = 78$$

$$\Rightarrow a = 3$$

$$\Rightarrow d = 8$$

$$\Rightarrow S_{15} - S_5 = \frac{15}{2} (6 + 14 \times 8) - \frac{5}{2} (6 + 4 \times 8)$$

$$\Rightarrow S_{15} - S_5 = \frac{15}{2} \times 118 - \frac{5}{2} \times 38$$

$$\Rightarrow S_{15} - S_5 = 885 - 95$$

$$\Rightarrow S_{15} - S_5 = 790$$

Q.34. If the mirror image of the point $P(3, 4, 9)$ in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ) then find the value of $14(\alpha + \beta + \gamma)$

Answer: 108

Solution: The given point is $P(3, 4, 9)$

Let, $Q(\alpha, \beta, \gamma)$ be the image of $P(3, 4, 9)$ and 'O' be the mid-point of PQ which is on the given line

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda \text{ (say)}$$

$$\Rightarrow x = 3\lambda + 1, y = 2\lambda - 1, z = \lambda + 2$$

let $(3\lambda + 1, 2\lambda - 1, \lambda + 2)$ be the co-ordinates of O .

Direction ratios of the line is $a_1 = 3, b_1 = 2, c_1 = 1$

Also, direction ratios of PO are

$$(3\lambda - 2, 2\lambda - 5, \lambda - 7)$$

As, PO is perpendicular to the line

$$3(3\lambda - 2) + 2(2\lambda - 5) + \lambda - 7 = 0$$

$$\Rightarrow 14\lambda - 23 = 0$$

$$\Rightarrow \lambda = \frac{23}{14}$$

$$\therefore O \text{ point is } \left(3 \cdot \frac{23}{14} + 1, 2 \cdot \frac{23}{14} - 1, \frac{23}{14} + 2 \right)$$

Since, O is mid-point of PQ

$$\therefore \left(3 \cdot \frac{23}{14} + 1, 2 \cdot \frac{23}{14} - 1, \frac{23}{14} + 2 \right) = \left(\frac{3+\alpha}{2}, \frac{4+\beta}{2}, \frac{9+\gamma}{2} \right)$$

So, on comparing we get,

$$\alpha = \frac{62}{7}, \beta = \frac{4}{7}, \gamma = \frac{-12}{7}$$

Hence, $14(\alpha + \beta + \gamma) = 108$



Q.35. If the system of the equation $x + 2y + 3z = 5$, $2x + 3y + z = 9$ & $4x + 3y + \lambda z = \mu$ have infinite number of solutions then find the value of $\lambda + 2\mu$

Answer: 17

Solution: Given,

The system of equations,

$$x + 2y + 3z = 5$$

$$2x + 3y + z = 9$$

$$4x + 3y + \lambda z = \mu \text{ have many solutions,}$$

$$\text{So, } \Delta = \Delta_1 = \Delta_2 = \Delta_3$$

$$\text{Now, finding } \Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(3\lambda - 3) - 2(2\lambda - 4) + 3(6 - 12) = 0$$

$$\Rightarrow \lambda = -13$$

$$\text{Now, finding } \Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix} = 0$$

$$\Rightarrow 3\mu - 27 - 4\mu + 72 - 30 = 0$$

$$\Rightarrow \mu = 15$$

$$\text{Hence, the value of } \lambda + 2\mu = -13 + 30 = 17$$

Q.36. If $y = \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x)$ then find the value of $96y' \left(\frac{\pi}{6} \right)$

Answer: 105



Solution: Given,

$$\begin{aligned}
 y &= \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x) \\
 \Rightarrow y &= \frac{(\sqrt{x}+1)(x^2-\sqrt{x})(\sqrt{x}-1)}{(x\sqrt{x}+x+\sqrt{x})(\sqrt{x}-1)} + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x) \\
 \Rightarrow y &= \frac{(\sqrt{x}+1)(x^2-\sqrt{x})(\sqrt{x}-1)}{\sqrt{x}(x+\sqrt{x}+1)(\sqrt{x}-1)} + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x) \\
 \Rightarrow y &= \frac{(\sqrt{x}+1)(x^2-\sqrt{x})(\sqrt{x}-1)}{\sqrt{x}((\sqrt{x})^3-1)} + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x) \\
 \Rightarrow y &= \frac{(\sqrt{x}+1)\sqrt{x}((\sqrt{x})^3-1)(\sqrt{x}-1)}{\sqrt{x}((\sqrt{x})^3-1)} + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x) \\
 \Rightarrow y &= (\sqrt{x}+1)(\sqrt{x}-1) + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x) \\
 \Rightarrow y &= x - 1 + \frac{1}{15} (3 \cos^5 x - 5 \cos^3 x) \\
 \Rightarrow \frac{dy}{dx} &= 1 + \frac{1}{15} (15 \cos^4 x (-\sin x) + 15 \cos^2 x \sin x) \\
 \Rightarrow \frac{dy}{dx} &= 1 + (\cos^4 x (-\sin x) + \cos^2 x \sin x) \\
 \Rightarrow y' \left(\frac{\pi}{6} \right) &= 1 + \left(\cos^4 \left(\frac{\pi}{6} \right) \left(-\sin \left(\frac{\pi}{6} \right) \right) + \cos^2 \left(\frac{\pi}{6} \right) \sin \left(\frac{\pi}{6} \right) \right) \\
 \Rightarrow y' \left(\frac{\pi}{6} \right) &= 1 + \left(\frac{(\sqrt{3})^4}{2^4} \left(-\frac{1}{2} \right) + \frac{(\sqrt{3})^2}{2^2} \times \frac{1}{2} \right) \\
 \Rightarrow y' \left(\frac{\pi}{6} \right) &= 1 - \frac{9}{32} + \frac{3}{8} = \frac{32-9+12}{32} = \frac{35}{32} \\
 \Rightarrow 96y' \left(\frac{\pi}{6} \right) &= 35 \times 3 = 105
 \end{aligned}$$

Q.37. If there are 20 lines numbered as 1, 2, 3, ..., 20 and the odd numbered lines intersect at a point and all the even numbered lines are parallel, then find the maximum number of points of intersection.

Answer: 101

Solution: Given:

Out of 20 numbered lines, odd numbered lines are concurrent, even number lines are parallel.

So, the maximum number of intersection points will be given by,

$$N = (1 \rightarrow \text{point of intersection when both lines are odd}) + (0 \rightarrow \text{point of intersection when both lines are parallel}) + ({}^{10}C_1 \times {}^{10}C_1)$$

$$\Rightarrow N = 1 + 0 + 100$$

$$\Rightarrow N = 101$$

Q.38. In a isobaric process work done by gas is 200 J. Adiabatic constant for the gas is 1.4, then find the heat supplied to the gas during the process.

A) 600 J

B) 400 J

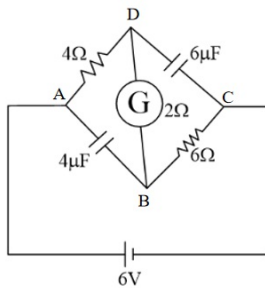
C) 500 J

D) 700 J

Answer: 700 J



Solution: Let's consider the following diagram:



When the capacitors are fully charged, there will be no current through them. The direction of the current in the steady state is depicted in the above diagram.

The current in the circuit in the steady state is given by

$$i = \frac{6 \text{ V}}{4 \Omega + 2 \Omega + 6 \Omega} \\ = 0.5 \text{ A}$$

With reference to the above diagram, the potentials at points A, B, C and D are given by

$V_C = 0 \text{ V}$ (Let), then

$V_B = 6 \Omega \times 0.5 \text{ A} = 3 \text{ V}$,

$V_D = 3 \text{ V} + (2 \Omega \times 0.5 \text{ A}) = 4 \text{ V}$ and

$V_A = 6 \text{ V}$

Thus, the charge (q_1) across the $4 \mu\text{F}$ capacitor is

$$q_1 = (4 \times 10^{-6}) (V_A - V_B) \\ = 12 \mu\text{C}$$

And, the charge (q_2) across the $6 \mu\text{F}$ capacitor is

$$q_2 = (6 \times 10^{-6}) (V_D - V_C) \\ = 24 \mu\text{C}$$

Hence, the required ratio is given by

$$\frac{q_1}{q_2} = \frac{12}{24} = \frac{1}{2}$$

Q.41. A solid sphere of mass M and radius R , is rolling with velocity of center of mass v . If it starts moving upwards on an inclined plane, then find the height achieved by the sphere before coming to rest.

A) $\frac{5v^2}{7g}$

B) $\frac{7v^2}{10g}$

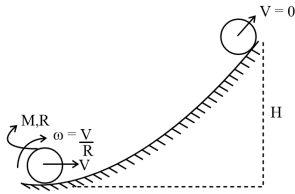
C) $\frac{3v^2}{10g}$

D) $\frac{7v^2}{5g}$

Answer: $\frac{7v^2}{10g}$



Solution: Let's consider the following diagram:



Let's consider that the angular velocity of the sphere is ω and the sphere will rise up to a vertical height H along the inclined plane before coming to rest.

The initial kinetic energy of the sphere can be written as

$$\begin{aligned} K &= \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 \\ &= \frac{1}{2} Mv^2 + \frac{1}{2} \left(\frac{2}{5} MR^2 \right) \left(\frac{v}{R} \right)^2 \\ &= \frac{7}{10} Mv^2 \quad \dots (1) \end{aligned}$$

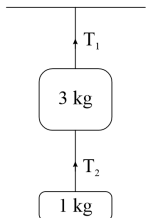
The final potential energy of the sphere can be written as

$$U = Mgh \quad \dots (2)$$

From the conservation of mechanical energy, equations (1) and (2) imply that

$$\begin{aligned} \frac{7}{10} Mv^2 &= Mgh \\ \Rightarrow H &= \frac{7v^2}{10g} \end{aligned}$$

Q.42. In the figure shown, find the ratio of the tensions in the strings $\frac{T_1}{T_2}$.

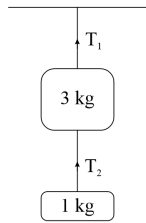


- A) $\frac{1}{2}$ B) $\frac{1}{4}$
C) $\frac{4}{1}$ D) $\frac{1}{3}$

Answer: $\frac{4}{1}$



Solution:



From the given figure, it is clear that the tension T_2 will be equal to the weight of the lower block, whereas the tension T_1 will have a combined weight from both the blocks.

Thus, from the given figure, it follows that

$$T_2 = 1 \text{ kg} \times g$$

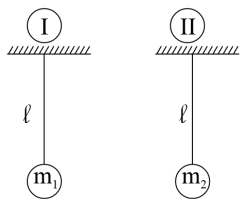
And,

$$T_1 = (3 \text{ kg} + 1 \text{ kg}) \times g$$

Hence, the required ratio is given by

$$\frac{T_1}{T_2} = \frac{4}{1}$$

Q.43. Find the ratio of $\frac{T_I}{T_{II}}$ of time periods of the two pendulums shown in the figure below. Given that, $m_1 > m_2$.



- A) 2 B) $\frac{1}{3}$
C) $\frac{1}{2}$ D) 1

Answer: 1

Solution: The formula to calculate the time period of a simple pendulum is given by

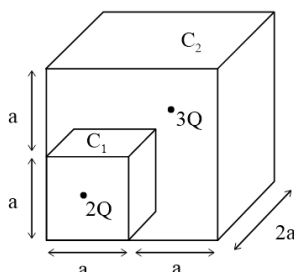
$$T = 2\pi\sqrt{\frac{l}{g}}$$

where, l , g are the effective length of the pendulum and the acceleration due to gravity respectively.

From the above equation, it can be concluded that the time period of oscillation of a simple pendulum is independent of the mass of the pendulum bob.

Hence, for the given scenario, the ratio of the time periods of two pendulums will be 1.

Q.44. There are two cubical Gaussian surfaces carrying charges as shown(C_1 is inside C_2). Find the ratio of the flux through surfaces C_1 and C_2 .





- A) 2 : 5 B) 1 : 1
C) 2 : 3 D) 5 : 2

Answer: 2 : 5

Solution: From the given figure, it is clear that the total charge enclosed by the cubical surface C_1 is $2Q$, whereas, the total charge enclosed by the surface C_2 is $(2Q + 3Q) = 5Q$.

Thus, using Gauss's law, the flux through surface C_1 is

$$\phi_1 = \frac{2Q}{\epsilon_0}$$

And, the flux through surface C_2 is

$$\phi_2 = \frac{5Q}{\epsilon_0}$$

Hence, the required ratio is given by

$$\frac{\phi_1}{\phi_2} = \frac{\frac{2Q}{\epsilon_0}}{\frac{5Q}{\epsilon_0}} = \frac{2}{5}$$

Q.45. A source produced electromagnetic wave of frequency 60 MHz. Find wavelength of this wave in air.

- A) 3 m B) 4 m
C) 5 m D) 6 m

Answer: 5 m

Solution: Given here: $\nu = 60 \times 10^6$ Hz

Using the relation between wavelength and frequency, we have

$$\text{Wavelength } \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{60 \times 10^6} = 5 \text{ m.}$$

Q.46. If a bulb of 40 W is producing a light of wavelength $\lambda = 4000 \text{ \AA}$. Then find the number of photons emitted by the bulb per second.

- A) 8×10^{19} B) 7×10^{19}
C) 6×10^{19} D) 5×10^{19}

Answer: 8×10^{19}

Solution: Given that,
Power of monochromatic light is, $P = 40 \text{ W}$
Wavelength of monochromatic light is, $\lambda = 4000 \text{ \AA}$
Expression for energy of a photon is given by,

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4000 \times 10^{-10}} = 5 \times 10^{-19} \text{ J}$$

Energy emitted by 40 W source per second, $T_E = 40 \text{ J}$

Number of photons emitted per second,

$$n = \frac{T_E}{E} = \frac{40}{5 \times 10^{-19}} = 8 \times 10^{19}$$

Q.47. 1000 drops of total surface energy E coalesce to form a bigger drop of surface energy E' . Find the value of E' .

- A) $\frac{E}{10}$ B) E
C) $10E$
D) $100E$



Solution: The relation between the angular velocity and the linear velocity for the given particle is

$$v = \omega r \dots (1)$$

Also, the time period of revolution of the particle is

$$T = \frac{2\pi}{\omega} \dots (2)$$

Equations (1) and (2) imply that

$$T = \frac{2\pi r}{v} \dots (3)$$

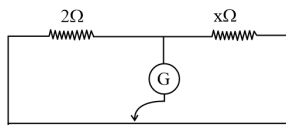
Since, $v \propto r^{-\frac{3}{2}}$, from equation (3), it can be written that

$$\begin{aligned} T &= \frac{2\pi r}{cr^{-\frac{3}{2}}} \text{ where } c \text{ is constant} \\ &= kr^{\frac{5}{2}} \\ \Rightarrow T &\propto r^{\frac{5}{2}} \end{aligned}$$

Here, c, k are proportionality constants.

Hence, $n = \frac{5}{2}$.

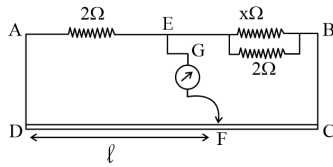
Q.50. Initially the balance point of the meter bridge is at 40 cm. Now if unknown resistance (X) is shunt by $2\ \Omega$ resistance then the new balance point is found to be at $\beta \times 10^{-1}$ cm. Find the value of β .



Answer: 625



Solution: Let's consider the following diagram:



For the first case, from the balanced condition of the meter bridge, it can be written that

$$\begin{aligned}\frac{2}{40} &= \frac{X}{100-40} \\ \Rightarrow X &= 60 \times \frac{2}{40} \Omega \\ &= 3 \Omega\end{aligned}$$

When the shunt is used, the equivalent resistance for the EB part of the above diagram becomes,

$$\begin{aligned}R_{eq} &= \frac{1}{\frac{1}{3\Omega} + \frac{1}{2\Omega}} \\ &= \frac{6}{5} \Omega\end{aligned}$$

Thus, for the second case, if l be the required balanced length in the meter bridge, it follows that

$$\begin{aligned}\frac{2}{l} &= \frac{\frac{6}{5}}{100-l} \\ \Rightarrow 2 \times \frac{5}{6} (100-l) &= l \\ \Rightarrow 500 - 5l &= 3l \\ \Rightarrow 8l &= 500 \\ \Rightarrow l &= \frac{500}{8} \\ &= 62.5 \\ &= 625 \times 10^{-1} \text{ cm}\end{aligned}$$

Hence, $\beta = 625$.

Q.51. Find the number of significant digits in the value 10.05.

Answer: 4

Solution: Rule for significant digits:

1. All non-zero numbers are significant.
2. Zeros between two non-zero digits are significant.
3. Leading zeros are not significant.
4. Trailing zeros to the right of the decimal are significant.

Therefore, number of significant digits are 4.

Q.52. A ball of mass 120 g moving with initial velocity 25 m s^{-1} is stopped by an external force F in 0.15 s. Find the magnitude of F in newton :

Answer: 20



Solution: Given that,

Mass of the ball, $m = 120 \text{ g} = 0.12 \text{ kg}$

Initial velocity of the ball, $u = 25 \text{ m s}^{-1}$

Final velocity of the ball, $v = 0 \text{ m s}^{-1}$

Time taken for the velocity change, $t = 0.15 \text{ s}$

We know that,

Force = mass \times acceleration

$$\Rightarrow F = m \left(\frac{v-u}{t} \right) = \frac{0.12(0-25)}{0.15} = -20 \text{ N} \text{ (} - \text{ve sign represents opposite direction)}$$

Therefore, required magnitude is 20.