JEE Main

## 1st Feb Shift 2

## Questions

Q.1. Which of the following have colour due to $d$-d transition?
A) $\mathrm{KMnO}_{4}$
B) $\mathrm{K}_{2} \mathrm{Cr}_{2} \mathrm{O}_{7}$
C) $\mathrm{K}_{2} \mathrm{CrO}_{4}$
D) $\mathrm{CuSO}_{4} .5 \mathrm{H}_{2} \mathrm{O}$

Answer: $\mathrm{CuSO}_{4} .5 \mathrm{H}_{2} \mathrm{O}$
Solution: Hydrated copper sulphate $\mathrm{CuSO}_{4} .5 \mathrm{H}_{2} \mathrm{O}$ is blue. In this complex, the oxidation state of Cu is +2 and valence shell electronic configuration is $3 \mathrm{~d}^{9}$. The ligand (water) molecules cause splitting of d orbitals. This facilitates d-d transition which leads to the colour of the complex. In the other compounds $\mathrm{KMnO}_{4}, \mathrm{~K}_{2} \mathrm{Cr}_{2} \mathrm{O}_{7}$ and $\mathrm{K}_{2} \mathrm{CrO}_{4}$ have d ${ }^{0}$ valence shell electronic configuration of central atoms.
Q.2. Which of the following has intramolecular hydrogen bonding in it?
A) $\mathrm{NH}_{3}$
B) $\mathrm{H}_{2} \mathrm{O}$
C)

D)


Answer:


Solution: Intramolecular hydrogen bonds are those which occur within one single molecule. This occurs when two functional groups of a molecule can form hydrogen bonds with each other. H-Bonding occurs in o-nitro phenol because the nitro group is present close to the -OH group enough for Hydrogen and Oxygen interaction.


Hence option C is the answer.
Q.3. Which of the following set of elements can be detected by Lassaigne's Test?
A) N, P and S only
B) N and S only
C) $\mathrm{N}, \mathrm{S}$ and halogens
D) P and halogens only

Answer: N, S and halogens

Solution: Nitrogen, sulphur, and halogens present in organic compounds are detected by Lassaigne's test. Here, a small piece of Na metal is heated in a fusion tube with the organic compound. The principle is that, in doing so, Na converts all the elements present into ionic form.
$\mathrm{Na}+\mathrm{C}+\mathrm{N} \rightarrow \mathrm{NaCN}$
$2 \mathrm{Na}+\mathrm{S} \rightarrow \mathrm{Na}_{2} \mathrm{~S}$
$\mathrm{Na}+\mathrm{X} \rightarrow \mathrm{NaX}(\mathrm{X}=\mathrm{Cl}, \mathrm{Br}$ or I$)$
The formed ionic salts are extracted from the fused mass by boiling it with distilled water. This is called sodium fusion extract.
Q.4. Which of the following has highest third ionisation energy?
A) V
B) Mn
C) Cr
D) Fe

Answer: Mn
Solution: The electronic configuration of these metals is:
$\mathrm{Cr}:[\mathrm{Ar}] 3 \mathrm{~d}^{5} 4 \mathrm{~s}^{1}$
$\mathrm{V}:[\mathrm{Ar}] 3 \mathrm{~d}^{3} 4 \mathrm{~s}^{2}$
$\mathrm{Mn}:[\mathrm{Ar}] 3 \mathrm{~d}^{5} 4 \mathrm{~s}^{2}$
$\mathrm{Fe}:[\mathrm{Ar}] 3 \mathrm{~d}^{6} 4 \mathrm{~s}^{2}$
The amount of energy required to take an electron from a dispositive cation is called the third ionisation enthalpy.
$\mathrm{Cr}^{2+}:[\mathrm{Ar}] 3 \mathrm{~d}^{4}$
$\mathrm{V}^{2+}:[\mathrm{Ar}] 3 \mathrm{~d}^{3}$
$\mathrm{Mn}^{2+}:[\mathrm{Ar}] 3 \mathrm{~d}^{5}$
$\mathrm{Fe}^{2+}:[\mathrm{Ar}] 3 \mathrm{~d}^{6}$
The third ionisation enthalpy of Mn means removal of the electron from the stable configuration of $3 \mathrm{~d}^{5}$. The metal having the highest third ionisation enthalpy is Mn .
Q.5. What is the order of reducing character for $\mathrm{AsH}_{3}, \mathrm{NH}_{3}, \mathrm{PH}_{3}$ (group 15 hydrides)?
A) $\mathrm{PH}_{3}>\mathrm{NH}_{3}>\mathrm{AsH}_{3}$
B) $\mathrm{NH}_{3}>\mathrm{PH}_{3}>\mathrm{AsH}_{3}$
C) $\mathrm{NH}_{3}>\mathrm{AsH}_{3}>\mathrm{PH}_{3}$
D) $\mathrm{AsH}_{3}>\mathrm{PH}_{3}>\mathrm{NH}_{3}$

Answer: $\quad \mathrm{AsH}_{3}>\mathrm{PH}_{3}>\mathrm{NH}_{3}$
Solution: Hydrides: Hydrides are those chemical compounds that are formed by the reaction of hydrogen with other elements.

## Reducing the character of hydrides in group 15:

- The reducing character of a compound depends upon the ease of production of the hydrogen atom.
- As we move down the group, the bond strength M-H bond of group 15 hydrides decreases due to an increase in the size of atoms.
- Hence, the reducing character of group 15 hydrides increases down the group.
- Therefore, the trend of the reducing strength is: $\mathrm{AsH}_{3}>\mathrm{PH}_{3}>\mathrm{NH}_{3}$

Hence, the reducing character of hydrides of group 15 elements increases from top to bottom.
Q.6. Among the following which shows negative resonance effect.
A) -COOH
B) $-\mathrm{CH}_{3}$
C) $-\mathrm{NH}_{2}$
D) -OH

Answer: -COOH
Solution: A negative resonance effect occurs when the groups withdraw the electrons from other molecules through the process of delocalisation.

Oxygen is more electronegative than carbon, meaning it has a stronger pull on shared electrons. In the resonance structures, the double bond between the carbon and oxygen can shift such that the oxygen bears a negative formal charge. This delocalisation means that the oxygen atoms are effective at drawing electron density toward themselves and away from other parts of the molecule through the conjugated system.

Due to this effect, the carboxyl group can pull electron density away from adjacent parts of a molecule through the pi electron system (resonance), hence the description as having a resonance (-R) electron-withdrawing effect.
Q.7. $\left[\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{6}\right]^{3+}$ and $\left[\mathrm{CoF}_{6}\right]^{3-}$ are-
A) Inner orbital complex and outer orbital complex respectively
B) Outer orbital complex and inner orbital complex respectively
C) Both are Inner orbital complex.
D) Both are outer orbital complex.

Answer: Inner orbital complex and outer orbital complex respectively
Solution: In $\left[\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{6}\right]^{3+}$ Oxidation state of cobalt $=+3$
Electronic configuration of $\mathrm{Co}^{3+}=\mathrm{d}^{6}$
$\mathrm{NH}_{3}$ being a strong field ligand causes the pairing. Therefore, Ni can undergo $\mathrm{d}^{2} \mathrm{sp}^{3}$ hybridization.


Hence, it is an inner orbital complex.
In $\left[\mathrm{CoF}_{6}\right]^{3-} \mathrm{F}$ is weak field ligand for $\mathrm{Co}^{3+}$, thus pairing of d-orbital electrons does not happen, so it forms: outer orbital complex of hybridisation $\mathrm{sp}^{3} \mathrm{~d}^{2}$

Hence, option A is the answer.
Q.8. Which of the following will have highest boiling point?
A)

B)

C)

CHO
D)


Answer:


Solution: The boiling points of aldehydes and ketones are higher than hydrocarbons and ethers of comparable molecular masses. This is due to weak molecular association in aldehydes and ketones arising out of dipole-dipole interactions. Also, their boiling points are lower than those of alcohols of similar molecular masses due to the absence of intermolecular hydrogen bonding.

Hence,

has highest boiling point.
Q.9. Statement I: Among $\mathrm{Mn}^{2+}$ and $\mathrm{Cr}^{2+}, \mathrm{Cr}^{2+}$ is better reducing agent.

Statement II: Half filled electronic configuration is more stable.
A) Both Statement I and Statement II are correct.
B) Both Statement I and Statement II are wrong.
C) Statement I is correct and Statement II is wrong.
D) Statement I is wrong and Statement II is correct.

Answer: Both Statement I and Statement II are correct.
Solution: If an element is behaving as reducing agent that means, it itself is undergoing oxidation that is there is loss of electrons. while for reduction there is gain of electrons.
$\mathrm{Cr}^{2+}$ has the configuration $3 \mathrm{~d}^{4}$. It can loose electron to form $3 \mathrm{~d}^{3}$ which has stable configuration as it has half filled $\mathrm{t}_{2} \mathrm{~g}$ level. Hence it is better reducing agent than $\mathrm{Mn}^{2+}$ which has a stable half filled configuration.

Hence Statement I is correct.
The stability of exactly half-filled orbitals in degenerate orbitals is stronger than that of other partially filled configurations.
Hence, Statement II is correct.
Q.10. Solubility of $\mathrm{Ca}_{3}\left(\mathrm{PO}_{4}\right)_{2}$ in 100 mL of pure water is W gm . Find out $\mathrm{K}_{\text {sp }}$ of $\mathrm{Ca}_{3}\left(\mathrm{PO}_{4}\right)_{2}$ is:
(Molar mass of $\mathrm{Ca}_{3}\left(\mathrm{PO}_{4}\right)_{2}=\mathrm{M}$ )
A) $108 \times 10^{5} \times\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^{5}$
B) $108 \times\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^{5}$
C) $108 \times 10^{4} \times\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^{5}$
D) $108 \times 10^{6} \times\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^{5}$
Answer: $108 \times 10^{5} \times\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^{5}$

Solution: Given the solubility of $\mathrm{Ca}_{3}\left(\mathrm{PO}_{4}\right)_{2}$ is $\mathrm{Wg} / 100 \mathrm{~mL}$. The solubility in $\mathrm{mol} / \mathrm{L}$ is $\mathrm{S}=\frac{\mathrm{W}}{\mathrm{M}} \times \frac{1000}{100} \mathrm{~mol} / \mathrm{L}$.
The equilibrium reaction of calcium phosphate is

$$
\begin{array}{cc}
\mathrm{Ca}_{3}\left(\mathrm{PO}_{4}\right)_{2} \rightleftharpoons 3 \mathrm{Ca}^{2+} & +2 \mathrm{PO}_{4}^{3-} \\
3 \mathrm{~S} & 2 \mathrm{~S}
\end{array}
$$

Now, $\mathrm{K}_{\text {sp }}=\left[\mathrm{Ca}^{2+}\right]^{3}\left[\mathrm{PO}_{4}^{3-}\right]^{2}$
$\Rightarrow \mathrm{K}_{\text {sp }}=(3 \mathrm{~S})^{3}(2 \mathrm{~S})^{2}=108 \times \mathrm{S}^{5}$
$\Rightarrow \mathrm{K}_{\text {sp }}=108 \times\left(\frac{\mathrm{W}}{\mathrm{M}} \times 10\right)^{5}$
$\Rightarrow \mathrm{K}_{\mathrm{sp}}=108 \times 10^{5} \times\left(\frac{\mathrm{W}}{\mathrm{M}}\right)^{5}$
Q.11.
$\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{Br} \xrightarrow{\text { alc. } \mathrm{KOH} / \Delta} \mathrm{A} \xrightarrow{\mathrm{Br}_{2} / \mathrm{CCl}_{4}} \mathrm{~B} \xrightarrow{\mathrm{KCN} \text { Excess }} \mathrm{C} \xrightarrow{\mathrm{H}_{3} \mathrm{O}^{+}} \mathrm{D}$
The final product D in the above reaction is
A) Succinic acid
B) Malonic acid
C) Gluconic acid
D) Oxalic acid

## Answer: Malonic acid

Solution: The ethyl bromide undergo elimination reaction with alc. KOH gives ethene. Ethene gives dibromoethane with bromine in carbon tetra chloride. The dibromoethane converts to dicyanoethane with excess KCN. The dicyanoethane on hydrolysis gives malonic acid.
$\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{Br} \xrightarrow{\text { alc. } \mathrm{KOH} / \Delta} \mathrm{CH}_{2}=\mathrm{CH}_{2}$
$\mathrm{CH}_{2}=\mathrm{CH}_{2} \xrightarrow{\mathrm{Br}_{2} / \mathrm{CCl}_{4}} \mathrm{Br}-\mathrm{CH}_{2}-\mathrm{CH}_{2}-\mathrm{Br}$
$\mathrm{Br}-\mathrm{CH}_{2}-\mathrm{CH}_{2}-\mathrm{Br} \xrightarrow{\mathrm{KCN} \text { Excess }} \mathrm{NC}-\mathrm{CH}_{2}-\mathrm{CH}_{2}-\mathrm{CN}$
$\mathrm{NC}-\mathrm{CH}_{2}-\mathrm{CH}_{2}-\mathrm{CN} \xrightarrow{\mathrm{H}_{3} \mathrm{O}^{+}} \mathrm{HOOC}-\mathrm{CH}_{2}-\mathrm{CH}_{2}-\mathrm{COOH}($ malonic acid $)$
Q.12. Which one of the following has meta directing group?
A) $\mathrm{NO}_{2}, \mathrm{CHO}, \mathrm{CN}, \mathrm{SO}_{3} \mathrm{H}$
B) $\mathrm{NO}_{2}, \mathrm{NH}_{2}, \mathrm{CN}, \mathrm{OH}$
C) $\mathrm{NO}_{2}, \mathrm{CN}, \mathrm{OR}, \mathrm{COOH}$
D) $\mathrm{NH}_{2}, \mathrm{CH}_{3}, \mathrm{CN}, \mathrm{SO}_{3} \mathrm{H}$

Answer: $\mathrm{NO}_{2}, \mathrm{CHO}, \mathrm{CN}, \mathrm{SO}_{3} \mathrm{H}$
Solution: Meta-directing deactivating groups are those that either do not donate electrons by resonance (sulfonic acid groups and ammonium ion groups) or actually withdraw electrons by resonance (carbonyl, nitrile, and nitro groups).

They are meta-directing not because they decrease the electron density at ortho-para positions.
Hence, these are $\mathrm{NO}_{2}, \mathrm{CHO}, \mathrm{CN}, \mathrm{SO}_{3} \mathrm{H}$ meta directing group.
Q.13. Match the following:

| Column I | Column II |
| :--- | :--- |
| 1. Phenol $+\mathrm{CO}_{2}+\mathrm{NaOH}$ <br> $\rightarrow$ | P. Salicylaldehyde |
| 2. Phenol $+\mathrm{Zn} \rightarrow$ | Q. Salicylic acid |
| 3. Phenol + conc. $\mathrm{HNO}_{3}$ | R. Picric acid |
| 4. Phenol $+\mathrm{KOH}+\mathrm{CHCl}_{3}$ | S. Benzene |

A) $1-\mathrm{Q}, 2-\mathrm{S}, 3-\mathrm{R}, 4-\mathrm{P}$
B) $1-\mathrm{S}, 2-\mathrm{Q}, 3-\mathrm{R}, 4-\mathrm{P}$
C) $1-\mathrm{Q}, 2-\mathrm{S}, 3-\mathrm{P}, 4-\mathrm{R}$
D) $1-\mathrm{Q}, 2-\mathrm{R}, 3-\mathrm{S}, 4-\mathrm{P}$

Answer: $1-\mathrm{Q}, 2-\mathrm{S}, 3-\mathrm{R}, 4-\mathrm{P}$
Solution: 1.Kolbe's reaction, also known as Kolbe Schmitt Reaction, is a type of addition reaction named after Hermann Kolbe and Rudolf Schmitt. When phenol is treated with sodium hydroxide, phenoxide ion is generated. The phenoxide ion generated is more reactive than phenol towards electrophilic aromatic substitution reaction. Hence, it undergoes an electrophilic substitution reaction with carbon dioxide, which is a weak electrophile. Ortho-hydroxybenzoic acid (salicylic acid) is formed as the primary product. This reaction is popularly known as Kolbe's reaction.

2. When phenol is heated with zinc dust, the product formed is benzene only.

3. With concentrated nitric acid, phenol is converted to $2,4,6$-Trinitrophenol. The product is commonly known as picric acid.

4. When phenol reacts with chloroform in the presence of bases $\mathrm{KOH} / \mathrm{NaOH}$ ) an aldehyde group ( -CHO ) ) gets introduced in the ring at a position ortho to the phenol group and salicylaldehyde is formed. This reaction is called the Reimer-Tiemann reaction.


Hence, answer is A .
Q.14. Statement I: In p and d block both metal and non metal are present.

Statement II: Electronegativity and ionisation enthalpy of metal is greater than non-metals.
A) Both Statement I and II are correct.
B) Both Statement I and II are wrong.
C) Statement I is correct and statement II is wrong.
D) Statement I is wrong and statement II is correct.

Answer: Both Statement I and II are wrong.

Solution: In p block both metals and non metals are present whereas in d block only metals are present. Hence statement I is wrong.

Non-Metals are electronegative elements and tend to share or accept electrons in a chemical reaction. Due to the electrons accepting nature of non-metals, they gains more electrons to complete their octet. So, non-metals have high ionisation enthalpy due to their small size and high electronegativity as compared to metal.

Hence statement II is wrong.
Q.15. The total number of monochlorination products of isopentane is
A) 4
B) 6
C) 10
D) 12

Answer: 6
Solution: The structure of isopentane is


The above molecule contain two types of primary hydrogen, one type of secondary hydrogen and one type of tertiary hydrogen. Hence, a total of 4 structural isomers are possible.

1


3


Among the four, the products 2,3 are having chiral centers. Hence, the total number of isomeric monochlorinated products for isopentane is 6 .
Q.16. Statement1: In $\pi$ BMO electron density is not present above and below the internuclear axis.

Statement 2: In $\pi$ ABMO two nodal planes are present.
A) Both statements are correct
B) both statement are incorrect
C) Statement 1 is correct and 2 is incorrect
D) Statement 1 is incorrect and 2 is correct

Answer: Statement 1 is incorrect and 2 is correct

## Solution:



Statement1: $\ln \pi$ BMO electron density is present above and below the internuclear axis.
Hence, statement 1 is incorrect.
Statement 2: In $\pi$ ABMO two nodal planes are present.
This statement is correct.
Hence, the answer is D.
Q.17. Match the following

| Compound | Uses |
| :--- | :--- |
| (i) $\mathrm{CCl}_{4}$ | (p) Refrigerator and AC |
| (ii) $\mathrm{CH}_{2} \mathrm{Cl}_{2}$ | (q) Non biodegradable <br> insecticide |
| (iii) Freons | (r) Fire Extinguisher |
| (iv) DDT | (s) Paint Removal |

A) (i)-r, (ii)-s, (iii)-p ,(iv)-q
B) (i)-p, (ii)-s, (iii)-r ,(iv)-q
C) (i)-s, (ii)-r, (iii)-p ,(iv)-q
D) (i)-r, (ii)-s, (iii)-q ,(iv)-p

Answer: (i)-r, (ii)-s, (iii)-p ,(iv)-q
Solution: $\mathrm{CCl}_{4}$ is used in the manufacturing of refrigerants, as a cleaning agent and was also used as a fire extinguisher.
$\mathrm{CH}_{2} \mathrm{Cl}_{2}$ is used as paint removal.
Freons are used as refrigerants in refrigerators and air conditioners.
DDT ( $p$, p'-Dichlorodiphenyltrichloroethane) is a colourless, crystalline, tasteless and almost odourless organochloride known for its insecticidal properties.

Hence option $A$ is the answer.
Q.18. Statement I: $\mathrm{Ni}^{2+}$ with DMG shows red colouration.

Statement II : Both $\mathrm{Fe}^{2+}$ and $\mathrm{Fe}^{3+}$ are blue coloured.
A) Statement I is correct Statement II is wrong.
B) Statement I is wrong Statement II is correct.
C) Both statements I and II are correct.
D) Both statements I and II are wrong.

Answer: Statement I is correct Statement II is wrong.

Solution: On addition of dimethylglyoxime (DMG) to nickel ion solution and on adding little bit of ammonia to make solution basic, it will give a red precipitate. This is a very specific test for nickel +2 cation.

The reaction is as follows,


Hence statement I is correct.
Ferrous ion is a divalent iron compound ion. It has +2 oxidation state and is light green in color while ferric ion is trivalent ion and is very pale violet or brown in colour.

Hence statement II is wrong.
Q.19. Number of radial nodes present in 3 p are:

Answer: 1
Solution: 3 p orbital the principal quantum number $\mathrm{n}=3$, placing it on the third energy level and the angular quantum number $\mathrm{l}=1$ gives the shape of $p$-orbital.
We can calculate the number of radial nodes using the formula,
$\mathrm{N}=\mathrm{n}-\mathrm{l}-1$
The value of $n$ is 3 and the value of 1 is 1 .
$\mathrm{N}=3-1-1$
$\mathrm{N}=1$
The number of radial nodes in 3 p orbitals is 1 .
Q.20. A 10 mL hydrocarbon $\left(\mathrm{C}_{\mathrm{x}} \mathrm{H}_{\mathrm{y}}\right)$ on combustion gives $40 \mathrm{~mL} \mathrm{CO}_{2}$ and $50 \mathrm{~mL} \mathrm{H} \mathrm{H}_{2} \mathrm{O}$. Calculate the value of $\mathrm{x}+\mathrm{y}$.

Answer: 14
Solution: The combustion reaction can be represented as
$\mathrm{C}_{\mathrm{x}} \mathrm{H}_{\mathrm{y}}+\left(\mathrm{x}+\frac{\mathrm{y}}{4}\right) \mathrm{O}_{2} \rightarrow \mathrm{xCO}_{2}+\mathrm{yH}_{2} \mathrm{O}$
At STP, 1 mole of any gas occupies 22.4 L
10 mL of hydrocarbon, 50 mL of water and 40 mL of carbon dioxide corresponds to $0.45,2.23$, and 1.79 moles respectively.

Hence, the combustion of 1 mole of hydrocarbon will produce 5 moles of water and 4 moles of carbon dioxide.
Thus, the molecular formula contains 4 carbon atoms and 10 hydrogen atoms.
The hydrocarbon is $\mathrm{C}_{4} \mathrm{H}_{10}$ and the combustion reaction is

$$
\begin{aligned}
& \mathrm{C}_{4} \mathrm{H}_{10}+6.5 \mathrm{O}_{2} \rightarrow 4 \mathrm{CO}_{2}+5 \mathrm{H}_{2} \mathrm{O} \\
& \mathrm{x}+\mathrm{y}=4+10=14
\end{aligned}
$$

Q.21. In Kjeldahl's method of estimation of nitrogen, containing 1 gm organic compound nitrogen is converted to ammonia which is further treated with $10 \mathrm{~mL}, 2 \mathrm{M} \mathrm{H}_{2} \mathrm{SO}_{4}$. Find the percent of nitrogen?

Answer: 56

Solution: Equivalent of $\mathrm{NH}_{3}=$ Equivalent of $\mathrm{H}_{2} \mathrm{SO}_{4}$
moles of $\mathrm{NH}_{3}\left(\mathrm{n}_{\mathrm{NH}_{3}}\right) \times 1=\frac{2 \times 10 \times 2}{1000}$

$$
=4 \times 10^{-2} \text { moles }
$$

Moles of nitrogen $=4 \times 10^{-2}$ moles
Weight of Nitrogen
$=4 \times 10^{-2} \times 14$
$=56 \times 10^{-2} \mathrm{gm}$
Percent of nitrogen
$=\frac{56 \times 10^{-2}}{\text { weight of the compound }} \times 100$
$=\frac{56 \times 10^{-2}}{1} \times 100=56 \%$
Q.22. Find $\Delta_{\mathrm{r}} \mathrm{G}^{\circ}$ (in $\mathrm{kJ} / \mathrm{mol}$ ), 300 K , if $\mathrm{K}_{\mathrm{eq}}$ is 10 .

Given $\mathrm{R}=8.314 \mathrm{~J} / \mathrm{molK}$
Give an answer to the nearest integer value.
Answer: 6
Solution:

$$
\begin{aligned}
\Delta_{\mathrm{r}} \mathrm{G}^{\circ}= & -\mathrm{RT} \ln \mathrm{~K} \\
& =-2.303 \mathrm{RT} \operatorname{logK} \\
& =-2.303 \times 8.314 \times 300 \log 10 \mathrm{~J} / \mathrm{mol} \\
& =\frac{-2.303 \times 8.314 \times 300}{1000} \mathrm{~kJ} / \mathrm{mol} \\
& =5.77 \approx 6 \mathrm{~kJ} / \mathrm{mol}
\end{aligned}
$$

Q.23. For a given first order chemical reaction $\mathrm{A}_{(\mathrm{g})} \rightarrow 2 \mathrm{~B}_{\mathrm{g}}$. If the initial pressure of A is 0.1 atm and total pressure of the system after 100 sec is 0.18 atm . Value of the rate constant $\left(\mathrm{sec}^{-1}\right)$ is $\mathrm{n} \times 10^{-2}$, Give the value of n .

Answer: 16
Solution: $\quad \mathrm{A}_{(\mathrm{g})} \rightarrow 2 \mathrm{~B}_{(\mathrm{g})}$
$\begin{array}{llll}\text { at } \mathrm{t} & =0 & 0.1 & - \\ \text { at } \mathrm{t} & =100 & 0.1-\mathrm{P} & 2 \mathrm{P}\end{array}$
$\mathrm{P}_{\text {Totalal }_{\mathrm{t}}=100}=0.1-\mathrm{P}+2 \mathrm{P}=0.18$
$0.1+\mathrm{P}=0.18 \mathrm{~atm}$
$\mathrm{P}=0.18-0.1=0.08 \mathrm{~atm}$
$P_{\text {inial }}=0.1 \mathrm{~atm}$
$P_{t}=0.1-0.08=0.02 \mathrm{~atm}$
$\mathrm{K}=\frac{1}{\mathrm{t}} \ln \left(\frac{0.1}{0.1-\mathrm{P}}\right)$
$\mathrm{K}=\frac{2.303}{100} \log \frac{0.1}{0.02}=0.016=16 \times 10^{-2}=\mathrm{n} \times 10^{-2}$
$\Rightarrow \mathrm{n}=16$
Q.24. If $\alpha$ and $\beta$ be the roots of equation $p x^{2}+q x-r=0$, where $p \neq 0$. If $p, q, r$ be the consecutive terms of a non-constant GP and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then the value of $(\alpha-\beta)^{2}$ is:
A) $\frac{80}{9}$
B) $\frac{16}{9}$
C) $\frac{9}{16}$
D) $\frac{-9}{16}$

Answer: $\frac{80}{9}$

Solution: Given,
$\alpha$ and $\beta$ be the roots of equation $p x^{2}+q x-r=0$
So, $\alpha+\beta=-\frac{q}{p}, \alpha \beta=-\frac{r}{p}$
It is given that, $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$
$\Rightarrow \frac{\alpha+\beta}{\alpha \beta}=\frac{3}{4}$
$\Rightarrow \frac{q}{r}=\frac{3}{4}$
Now, $p, q, r$ are in GP
So, common ratio of this GP will be $\frac{q}{p}=\frac{r}{q}=\frac{4}{3}$
$\Rightarrow p x^{2}+q x-r=0$
$\Rightarrow x^{2}+\frac{q}{p} x-\frac{r}{p}=0$
$\Rightarrow x^{2}+\frac{4}{3} x-\left(\frac{4}{3}\right)^{2}=0$
$\Rightarrow 9 x^{2}+12 x-16=0$
$\Rightarrow(\alpha-\beta)^{2}=(\alpha+\beta)^{2}-4 \alpha \beta$
$\Rightarrow(\alpha-\beta)^{2}=\left(\frac{-12}{9}\right)^{2}-4\left(\frac{-16}{9}\right)$
$\Rightarrow(\alpha-\beta)^{2}=\frac{16}{9}+\frac{64}{9}$
$\Rightarrow(\alpha-\beta)^{2}=\frac{80}{9}$
Q.25. The number of solutions of the equation $4 \sin ^{2} x-4 \cos ^{3} x+9-4 \cos x=0, x \in[-2 \pi, 2 \pi]$ is:
A) 1
B) 0
C) 2
D) 3

Answer: 0
Solution: Given: $4 \sin ^{2} x-4 \cos ^{3} x+9-4 \cos x=0$
$\Rightarrow 4-4 \cos ^{2} x-4 \cos ^{3} x+9-4 \cos x=0$
$\Rightarrow 4 \cos ^{3} x+4 \cos ^{2} x+4 \cos x-13=0$
$\Rightarrow 4 \cos ^{3} x+4 \cos ^{2} x+4 \cos x=13$
We know that, $-1 \leq \cos x \leq 1$
So, the maximum value of LHS is $(4+4+4)=12$
But, RHS is 13.
So, no solutions are possible for the given equation.
Q.26.

If the domain of the function $f(x)=\frac{\sqrt{x^{2}-25}}{4-x^{2}}+\log \left(x^{2}+2 x-15\right)$ is $(-\infty, \alpha) \cup[\beta, \infty)$, then $\alpha^{2}+\beta^{2}$ is equal to
A) 50
B) 100
C) 20
D) 10

Solution:
Given: $f(x)=\frac{\sqrt{x^{2}-25}}{4-x^{2}}+\log \left(x^{2}+2 x-15\right)$
$\Rightarrow x^{2}-25 \geq 0$
$\Rightarrow(x-5)(x+5) \geq 0$

$\Rightarrow x \in(-\infty,-5] \cup[5, \infty) \ldots(i)$
$4-x^{2} \neq 0$
$\Rightarrow x \neq 2,-2$
$\Rightarrow x^{2}+2 x-15>0$
$\Rightarrow(x+5)(x-3)>0$

$\Rightarrow x \in(-\infty,-5) \cup(3, \infty) \ldots(i i i)$
Using (i), (ii) and (iii)

$\Rightarrow x \in(-\infty,-5) \cup[5, \infty)$
So, on comparing with $x \in(-\infty, \alpha) \cup[\beta, \infty)$ we get,

$$
\begin{aligned}
& \Rightarrow \alpha^{2}+\beta^{2}=(-5)^{2}+(5)^{2} \\
& \Rightarrow \alpha^{2}+\beta^{2}=50
\end{aligned}
$$

Q.27. If $\int_{0}^{\frac{\pi}{3}} \cos ^{4} x \mathrm{~d} x=a \pi+\mathrm{b} \sqrt{3}$ then find the value of $a^{2}+b$
A) 1
B) $\frac{1}{2}$
C) $\frac{1}{9}$
D) $\frac{1}{8}$

Answer: $\frac{1}{8}$

Solution: Let,

$$
\begin{aligned}
& I=\int_{0}^{\frac{\pi}{3}} \cos ^{4} x \mathrm{~d} x \\
& \Rightarrow I=\frac{1}{4} \int_{0}^{\frac{\pi}{3}}\left(2 \cos ^{2} x\right)^{2} \mathrm{~d} x \\
& \Rightarrow I=\frac{1}{4} \int_{0}^{\frac{\pi}{3}}(1+\cos 2 x)^{2} \mathrm{~d} x \\
& \Rightarrow I=\frac{1}{4} \int_{0}^{\frac{\pi}{3}}\left(1+\cos ^{2} 2 x+2 \cos 2 x\right) \mathrm{d} x \\
& \Rightarrow I=\frac{1}{4} \int_{0}^{\frac{\pi}{3}}(1+2 \cos 2 x) \mathrm{d} x+\frac{1}{8} \int_{0}^{\frac{\pi}{3}} 2 \cos ^{2} 2 x d x \\
& \Rightarrow I=\frac{1}{4} \int_{0}^{\frac{\pi}{3}}(1+2 \cos 2 x) \mathrm{d} x+\frac{1}{8} \int_{0}^{\frac{\pi}{3}}(1+\cos 4 x) d x \\
& \Rightarrow I=\left[\frac{1}{4}(x+\sin 2 x)+\frac{1}{8}\left(x+\frac{\sin 4 x}{4}\right)\right]_{0}^{\frac{\pi}{3}} \\
& \Rightarrow I=\frac{1}{4}\left(\frac{\pi}{3}+\sin \frac{2 \pi}{3}\right)+\frac{1}{8}\left(\frac{\pi}{3}+\frac{\sin \frac{4 \pi}{3}}{4}\right) \\
& \Rightarrow I=\left(\frac{\pi}{12}+\frac{\sqrt{3}}{8}\right)+\left(\frac{\pi}{24}-\frac{\sqrt{3}}{64}\right) \\
& \Rightarrow I=\frac{\pi}{8}+\frac{7 \sqrt{3}}{64}
\end{aligned}
$$

So, on comparing with given value we get,
$a^{2}+b=\frac{1}{64}+\frac{7}{64}$
$\Rightarrow a^{2}+b=\frac{8}{64}=\frac{1}{8}$
Q.28. The probability that Ajay will not go to office is $\frac{1}{5}$ and probability that Ajay and Vijay will go to office is $\frac{2}{7}$, if their visits to office is independent of each other, then find the probability that Ajay will go to the office, but Vijay will not go, is
A) $\frac{18}{35}$
B) $\frac{12}{28}$
C) $\frac{24}{35}$
D) $\frac{13}{35}$

Answer: $\frac{18}{35}$
Solution: The probability that Ajay will not go to office is, $P\left(A^{\prime}\right)=\frac{1}{5}$

$$
\Rightarrow P(A)=\frac{4}{5}
$$

The probability that Ajay and Vijay will go to office is, $P(A \cap V)=\frac{2}{7}$

$$
\begin{aligned}
& \Rightarrow P(A) \times P(V)=\frac{2}{7} \\
& \Rightarrow P(V)=\frac{2}{7} \times \frac{5}{4} \\
& \Rightarrow P(V)=\frac{5}{14}
\end{aligned}
$$

The probability that Ajay will go and Vijay will not go is,

$$
\begin{aligned}
& \Rightarrow P\left(A \cap V^{\prime}\right)=\frac{4}{5} \times \frac{9}{14} \\
& \Rightarrow P\left(A \cap V^{\prime}\right)=\frac{18}{35}
\end{aligned}
$$

Q. 2 If $A(2,3,1), B(3,2,-1), C(-2,1,3)$ are vertex of $\triangle A B C$ and $A D$ is angle bisector of angle $A$, then projection of $\overrightarrow{A D}$ on $\overrightarrow{A C}$ is
A) $\frac{2}{\sqrt{3}}$
B) $\frac{\sqrt{2}}{3}$
C) $\sqrt{\frac{2}{3}}$
D) $\frac{1}{\sqrt{3}}$

Answer: $\sqrt{\frac{2}{3}}$
Solution: Given,
$A(2,3,1), B(3,2,-1), C(-2,1,3)$ are vertices of $\triangle A B C$ and $A D$ is angle bisector,


Since $A D$ is angle bisector, so $\frac{A B}{B C}=\frac{B D}{D C}$
$\Rightarrow \frac{\sqrt{6}}{2 \sqrt{6}}=\frac{B D}{D C}$
$\Rightarrow \frac{B D}{D C}=\frac{1}{2}$
Now, using the section formula, the coordinates of $D \equiv\left(\frac{6-2}{3}, \frac{4+1}{3}, \frac{-2+3}{3}\right) \equiv\left(\frac{4}{3}, \frac{5}{3}, \frac{1}{3}\right)$
So, $\overrightarrow{A D}=\frac{-2}{3} \hat{i}-\frac{4}{3} \hat{j}-\frac{2}{3} \hat{k}$ and $\overrightarrow{A C}=-4 \hat{i}-2 \hat{j}+2 \hat{k}$
Now, projection of $\overrightarrow{A D}$ on $\overrightarrow{A C}$ is given by,
$\frac{\overrightarrow{A D} \cdot \overrightarrow{A C}}{|\overrightarrow{A C}|}=\frac{\frac{8}{3}+\frac{8}{3}-\frac{4}{3}}{\sqrt{16+4+4}}=\frac{4}{\sqrt{24}}=\frac{4}{2 \sqrt{6}}=\sqrt{\frac{2}{3}}$
Q. 30 .

If $m$ and $n$ be the coefficient of $7^{\text {th }}$ and $13^{\text {th }}$ terms in the expansion of $\left(\frac{1}{3} x^{\frac{1}{3}}+\frac{1}{2^{\frac{2}{5}}}\right)^{18}$, then the value of $\left(\frac{m}{n}\right)^{\frac{1}{3}}$ is:
A) $\frac{4}{9}$
B) $\frac{9}{4}$
C) $\frac{2}{3}$
D) $\frac{3}{2}$

Answer: $\frac{4}{9}$

Solution:

$$
\begin{aligned}
& \text { Given expansion is }\left(\frac{1}{3} x^{\frac{1}{3}}+\frac{1}{2^{\frac{2}{5}}}\right)^{18} \\
& \Rightarrow T_{7}={ }^{18} C_{6}\left(\frac{1}{3} x^{\frac{1}{3}}\right)^{12}\left(\frac{1}{2 x^{\frac{2}{5}}}\right)^{6} \\
& \Rightarrow m={ }^{18} C_{6}\left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^{6} \\
& \Rightarrow T_{13}={ }^{18} C_{12}\left(\frac{1}{3} x^{\frac{1}{3}}\right)^{6}\left(\frac{1}{2 x^{\frac{2}{5}}}\right)^{12} \\
& \Rightarrow n={ }^{18} C_{12}\left(\frac{1}{3}\right)^{6}\left(\frac{1}{2}\right)^{12} \\
& \Rightarrow \frac{m}{n}=\frac{{ }^{18} C_{6}\left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^{6}}{{ }^{18} C_{12}\left(\frac{1}{3}\right)^{6}\left(\frac{1}{2}\right)^{12}} \\
& \Rightarrow \frac{m}{n}=\frac{\left(\frac{1}{3}\right)^{6}}{\left(\frac{1}{2}\right)^{6}} \\
& \Rightarrow \frac{m}{n}=\left(\frac{2}{3}\right)^{6} \\
& \Rightarrow\left(\frac{m}{n}\right)^{\frac{1}{3}}=\frac{4}{9} \\
& \Rightarrow
\end{aligned}
$$

Q.31. Find the minimum value of $\left|z+\frac{3+4 i}{2}\right|$ such that $|z| \leq 1$
A) $\frac{3}{2}$
B) 5
C) 3
D) $\frac{5}{2}$

Answer: $\frac{3}{2}$

Solution: Given: $\left|z+\frac{3+4 i}{2}\right| ;|z| \leq 1$
Let, $z=x+i y$
$\Rightarrow|z|=1$ gives $x^{2}+y^{2}=1$, which represents a circle having centre at $(0,0)$ and radius as 1 unit.
Now, $\left|z+\frac{3+4 i}{2}\right|$ gives the distance of $z$ from $\left(\frac{3}{2},-2\right)$


So, we need to find the minimum distance of $A P$.

$$
\begin{aligned}
& \Rightarrow A P=O P-O A \\
& \Rightarrow A P=\sqrt{\left(-\frac{3}{2}-0\right)^{2}+(0+2)^{2}}-1 \\
& \Rightarrow A P=\sqrt{\frac{9}{4}+4}-1 \\
& \Rightarrow A P=\frac{5}{2}-1 \\
& \Rightarrow A P=\frac{3}{2}
\end{aligned}
$$

Q.32. If $\frac{d x}{d y}=\frac{1+x-y^{2}}{y}$ and $x(1)=1$, then $5 x(2)$ is equal to $\qquad$ -
A) 2
B) 3
C) 4
D) 5

Answer: 5

Solution:
Given: $\frac{d x}{d y}=\frac{1+x-y^{2}}{y}$
$\Rightarrow \frac{d x}{d y}=\frac{1+x}{y}-y$
$\Rightarrow \frac{d x}{d y}-\frac{1+x}{y}=-y$
Now let $1+x=t \Rightarrow \frac{d x}{d y}=\frac{d t}{d y}$
$\Rightarrow \frac{d t}{d y}-\frac{t}{y}=-y$
$\Rightarrow \mathrm{IF}=e^{\int-\frac{1}{y} d y}$
$\Rightarrow \mathrm{IF}=e^{-\log y}$
$\Rightarrow \mathrm{IF}=e^{\log \frac{1}{y}}$
$\Rightarrow \mathrm{IF}=\frac{1}{y}$
So, the solution of the given differential equation is
$\frac{t}{y}=\int \frac{1}{y} \times(-y) d y$
$\Rightarrow \frac{(1+x)}{y}=\int \frac{1}{y} \times(-y) d y$
$\Rightarrow \frac{(1+x)}{y}=-y+c$
Also, $x(1)=1$
$\Rightarrow \frac{2}{1}=-1+c$
$\Rightarrow c=3$
$\Rightarrow \frac{(1+x)}{y}=-y+3$
Putting $y=2$
$\Rightarrow \frac{(1+x)}{2}=-2+3$
$\Rightarrow(1+x)=2$
$\Rightarrow x=1$
$\Rightarrow 5 x=5$
Q.33. If $S_{n}$ be the sum of first $n$ terms of an AP and if $S_{10}=390$ and the ratio of the $10^{t h}$ term to $5^{t h}$ term is $15: 7$, then find the value of $S_{15}-S_{5}$
A) 885
B) 790
C) 800
D) 700

Answer: 790

Solution: Given: $\frac{a_{10}}{a_{5}}=\frac{15}{7}$

$$
\begin{aligned}
& \Rightarrow \frac{a+9 d}{a+4 d}=\frac{15}{7} \\
& \Rightarrow 7 a+63 d=15 a+60 d \\
& \Rightarrow 3 d=8 a
\end{aligned}
$$

Also, $S_{10}=390$
$\Rightarrow \frac{10}{2}[2 a+9 d]=390$
$\Rightarrow 2 a+24 a=78$
$\Rightarrow a=3$
$\Rightarrow d=8$
$\Rightarrow S_{15}-S_{5}=\frac{15}{2}(6+14 \times 8)-\frac{5}{2}(6+4 \times 8)$
$\Rightarrow S_{15}-S_{5}=\frac{15}{2} \times 118-\frac{5}{2} \times 38$
$\Rightarrow S_{15}-S_{5}=885-95$
$\Rightarrow S_{15}-S_{5}=790$
Q.34. If the mirror image of the point $P(3,4,9)$ in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is $(\alpha, \beta, \gamma)$ then find the value of $14(\alpha+\beta+\gamma)$

Answer: 108
Solution: The given point is $P(3,4,9)$
Let, $Q(\alpha, \beta, \gamma)$ be the image of $P(3,4,9)$ and ' $O$ ' be the mid-point of $P Q$ which is on the given line
$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}=\lambda$ (say)
$\Rightarrow x=3 \lambda+1, y=2 \lambda-1, z=\lambda+2$
let $(3 \lambda+1,2 \lambda-1, \lambda+2)$ be the co-ordinates of $O$.
Direction ratios of the line is $a_{1}=3, b_{1}=2, c_{1}=1$
Also, direction ratios of $P O$ are
$(3 \lambda-2,2 \lambda-5, \lambda-7)$
As, $P O$ is perpendicular to the line
$3(3 \lambda-2)+2(2 \lambda-5)+\lambda-7=0$
$\Rightarrow 14 \lambda-23=0$
$\Rightarrow \lambda=\frac{23}{14}$
$\therefore O$ point is $\left(3 \cdot \frac{23}{14}+1,2 \cdot \frac{23}{14}-1, \frac{23}{14}+2\right)$
Since, $O$ is mid-point of $P Q$
$\therefore\left(3 \cdot \frac{23}{14}+1,2 \cdot \frac{23}{14}-1, \frac{23}{14}+2\right)=\left(\frac{3+\alpha}{2}, \frac{4+\beta}{2}, \frac{9+\gamma}{2}\right)$
So, on comparing we get,
$\alpha=\frac{62}{7}, \beta=\frac{4}{7}, \gamma=\frac{-12}{7}$
Hence, $14(\alpha+\beta+\gamma)=108$
Q.35. If the system of the equation $x+2 y+3 z=5,2 x+3 y+z=9 \& 4 x+3 y+\lambda z=\mu$ have infinite number of solutions then find the value of $\lambda+2 \mu$

Answer: 17
Solution: Given,
The system of equations,
$x+2 y+3 z=5$
$2 x+3 y+z=9$
$4 x+3 y+\lambda z=\mu$ have many solutions,
So, $\triangle=\triangle_{1}=\triangle_{2}=\triangle_{3}$
Now, finding $\triangle=\left|\begin{array}{lll}1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda\end{array}\right|=0$
$\Rightarrow 1(3 \lambda-3)-2(2 \lambda-4)+3(6-12)=0$
$\Rightarrow \lambda=-13$
Now, finding $\triangle_{3}=\left|\begin{array}{lll}1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & \mu\end{array}\right|=0$
$\Rightarrow 3 \mu-27-4 \mu+72-30=0$
$\Rightarrow \mu=15$
Hence, the value of $\lambda+2 \mu=-13+30=17$
Q.36. If $y=\frac{(\sqrt{x}+1)\left(x^{2}-\sqrt{x}\right)}{x \sqrt{x}+x+\sqrt{x}}+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right)$ then find the value of $96 y^{\prime}\left(\frac{\pi}{6}\right)$

Answer: 105

Solution: Given,

$$
\begin{aligned}
& y=\frac{(\sqrt{x}+1)\left(x^{2}-\sqrt{x}\right)}{x \sqrt{x}+x+\sqrt{x}}+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right) \\
& \Rightarrow y=\frac{(\sqrt{x}+1)\left(x^{2}-\sqrt{x}\right)(\sqrt{x}-1)}{(x \sqrt{x}+x+\sqrt{x})(\sqrt{x}-1)}+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right) \\
& \Rightarrow y=\frac{(\sqrt{x}+1)\left(x^{2}-\sqrt{x}\right)(\sqrt{x}-1)}{\sqrt{x}(x+\sqrt{x}+1)(\sqrt{x}-1)}+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right) \\
& \Rightarrow y=\frac{(\sqrt{x}+1)\left(x^{2}-\sqrt{x}\right)(\sqrt{x}-1)}{\sqrt{x}\left((\sqrt{x})^{3}-1\right)}+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right) \\
& \Rightarrow y=\frac{(\sqrt{x}+1) \sqrt{x}\left((\sqrt{x})^{3}-1\right)(\sqrt{x}-1)}{\sqrt{x}\left((\sqrt{x})^{3}-1\right)}+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right) \\
& \Rightarrow y=(\sqrt{x}+1)(\sqrt{x}-1)+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right) \\
& \Rightarrow y=x-1+\frac{1}{15}\left(3 \cos ^{5} x-5 \cos ^{3} x\right) \\
& \Rightarrow \frac{d y}{d x}=1+\frac{1}{15}\left(15 \cos ^{4} x(-\sin x)+15 \cos ^{2} x \sin ^{5} x\right) \\
& \Rightarrow \frac{d y}{d x}=1+\left(\cos ^{4} x(-\sin x)+\cos ^{2} x \sin ^{2} x\right) \\
& \Rightarrow y^{\prime}\left(\frac{\pi}{6}\right)=1+\left(\cos ^{4}\left(\frac{\pi}{6}\right)\left(-\sin \left(\frac{\pi}{6}\right)\right)+\cos ^{2}\left(\frac{\pi}{6}\right) \sin \left(\frac{\pi}{6}\right)\right) \\
& \Rightarrow y^{\prime}\left(\frac{\pi}{6}\right)=1+\left(\frac{(\sqrt{3})^{4}}{2^{4}}\left(-\frac{1}{2}\right)+\frac{(\sqrt{3})^{2}}{2^{2}} \times \frac{1}{2}\right) \\
& \Rightarrow y^{\prime}\left(\frac{\pi}{6}\right)=1-\frac{9}{32}+\frac{3}{8}=\frac{32-9+12}{32}=\frac{35}{32} \\
& \Rightarrow 96 y^{\prime}\left(\frac{\pi}{6}\right)=35 \times 3=105 \\
& \Rightarrow y^{2} \\
& \Rightarrow
\end{aligned}
$$

Q.37. If there are 20 lines numbered as $1,2,3, \ldots, 20$ and the odd numbered lines intersect at a point and all the even numbered lines are parallel, then find the maximum number of points of intersection.

Answer: 101
Solution: Given:
Out of 20 numbered lines, odd numbered lines are concurrent, even number lines are parallel.
So, the maximum number of intersection points will be given by,
$N=(1 \rightarrow$ point of intersection when both lines are odd $)+(0 \rightarrow$ point of intersection when both lines are parallel $)+\left({ }^{10} C_{1} \times{ }^{10} C_{1}\right.$
$\Rightarrow N=1+0+100$
$\Rightarrow N=101$
Q.38. In a isobaric process work done by gas is 200 J . Adiabatic constant for the gas is 1.4 , then find the heat supplied to the gas during the process.
A) 600 J
B) 400 J
C) 500 J
D) 700 J

Answer: 700 J

Solution: The work done for the given process can be written as

$$
\begin{equation*}
W=P \Delta V=n R \Delta T \tag{1}
\end{equation*}
$$

For an isobaric process, the formula to calculate the heat supplied is given by

$$
\begin{equation*}
Q=n C_{P} \Delta T \tag{2}
\end{equation*}
$$

Since the adiabatic gas constant is 1.4 , it indicates that the gas is a diatomic gas.
Hence, the expression for $C_{P}$ for the gas can be written as

$$
\begin{align*}
C_{P} & =\frac{1}{2} f R+R \quad\left(\text { as } C_{P}-C_{V}=R\right) \\
& =\frac{5}{2} R+R \\
& =\frac{7}{2} R \ldots(3) \tag{3}
\end{align*}
$$

Equations (1), (2) and (3) imply that

$$
\begin{aligned}
Q & =n\left(\frac{7}{2} R\right) \Delta T \\
& =\frac{7}{2} n R \Delta T \\
& =\frac{7}{2} W \\
& =\frac{7}{2} \times 200 \mathrm{~J} \\
& =700 \mathrm{~J}
\end{aligned}
$$

Q.39. Train A moves with velocity $20 \mathrm{~m} \mathrm{~s}^{-1}$ towards North and train B moves with velocity $30 \mathrm{~m} \mathrm{~s}^{-1}$ towards South. Then find the velocity of train $B$ with respect to train $A$.
A) $10 \mathrm{~m} \mathrm{~s}^{-1}$
B) $50 \mathrm{~m} \mathrm{~s}^{-1}$
C) $60 \mathrm{~m} \mathrm{~s}^{-1}$
D) $0 \mathrm{~m} \mathrm{~s}^{-1}$

Answer: $50 \mathrm{~m} \mathrm{~s}^{-1}$
Solution: The formula to calculate the velocity of train $B$ with respect to train $A$ is given by

$$
\vec{V}_{B A}=\vec{V}_{B}-\vec{V}_{A} \ldots(1)
$$

Since the trains are approaching from the opposite directions, from equation (1), it follows that

$$
\begin{aligned}
\left|\vec{V}_{B A}\right| & =30 \mathrm{~m} \mathrm{~s}^{-1}-\left(-20 \mathrm{~m} \mathrm{~s}^{-1}\right) \\
& =50 \mathrm{~m} \mathrm{~s}^{-1}
\end{aligned}
$$

Q.40. In the given circuit, the galvanometer has $2 \Omega$ resistance. Find the ratio of charge stored in $4 \mu \mathrm{~F}$ and $6 \mu \mathrm{Fc}$ capacitors.

A) $\frac{1}{3}$
B) $\frac{3}{5}$
C) $\frac{1}{2}$
D) $\frac{5}{9}$

Answer: $\frac{1}{2}$

Solution: Let's consider the following diagram:


When the capacitors are fully charged, there will be no current through them. The direction of the current in the steady state is depicted in the above diagram.

The current in the circuit in the steady state is given by

$$
\begin{aligned}
i & =\frac{6 \mathrm{~V}}{4 \Omega+2 \Omega+6 \Omega} \\
& =0.5 \mathrm{~A}
\end{aligned}
$$

With reference to the above diagram, the potentials at points $A, B, C$ and $D$ are given by
$V_{C}=0 \mathrm{~V}$ (Let), then
$V_{B}=6 \Omega \times 0.5 \mathrm{~A}=3 \mathrm{~V}$,
$V_{D}=3 \mathrm{~V}+(2 \Omega \times 0.5 \mathrm{~A})=4 \mathrm{~V}$ and
$\mathrm{V}_{\mathrm{A}}=6 \mathrm{~V}$
Thus, the charge $\left(q_{1}\right)$ across the $4 \mu \mathrm{~F}$ capacitor is

$$
\begin{aligned}
q_{1} & =\left(4 \times 10^{-6}\right)\left(V_{A}-V_{B}\right) \\
& =12 \mu \mathrm{C}
\end{aligned}
$$

And, the charge $\left(q_{2}\right)$ across the $6 \mu \mathrm{~F}$ capacitor is

$$
\begin{aligned}
\mathrm{q}_{2} & =\left(6 \times 10^{-6}\right)\left(V_{D}-V_{C}\right) \\
& =24 \mu \mathrm{C}
\end{aligned}
$$

Hence, the required ratio is given by

$$
\frac{q_{1}}{q_{2}}=\frac{12}{24}=\frac{1}{2}
$$

Q.41. A solid sphere of mass $M$ and radius $R$, is rolling with velocity of center of mass $v$. If it starts moving upwards on an inclined plane, then find the height achieved by the sphere before coming to rest.
A) $\frac{5 v^{2}}{7 g}$
B) $\frac{7 v^{2}}{10 g}$
C) $\frac{3 v^{2}}{10 g}$
D) $\frac{7 v^{2}}{5 g}$

Answer: $\frac{7 v^{2}}{10 g}$


Let's consider that the angular velocity of the sphere is $\omega$ and the sphere will rise up to a vertical height $H$ along the inclined plane before coming to rest.

The initial kinetic energy of the sphere can be written as

$$
\begin{aligned}
K & =\frac{1}{2} M v^{2}+\frac{1}{2} I \omega^{2} \\
& =\frac{1}{2} M v^{2}+\frac{1}{2}\left(\frac{2}{5} M R^{2}\right)\left(\frac{v}{R}\right)^{2} \\
& =\frac{7}{10} M v^{2} \ldots(1)
\end{aligned}
$$

The final potential energy of the sphere can be written as
$U=M g H$
From the conservation of mechanical energy, equations (1) and (2) imply that
$\frac{7}{10} M v^{2}=M g H$
$\Rightarrow H=\frac{7 v^{2}}{10 g}$
Q.42. In the figure shown, find the ratio of the tensions in the strings $\frac{T_{1}}{T_{2}}$.

A) $\frac{1}{2}$
B) $\frac{1}{4}$
C) $\frac{4}{1}$
D) $\frac{1}{3}$

Answer: $\frac{4}{1}$

## Solution:



From the given figure, it is clear that the tension $T_{2}$ will be equal to the weight of the lower block, whereas the tension $T_{1}$ will have a combined weight from both the blocks.

Thus, from the given figure, it follows that
$T_{2}=1 \mathrm{~kg} \times g$
And,
$T_{1}=(3 \mathrm{~kg}+1 \mathrm{~kg}) \times g$
Hence, the required ratio is given by
$\frac{T_{1}}{T_{2}}=\frac{4}{1}$
Q.43. Find the ratio of $\frac{T_{I}}{T_{I I}}$ of time periods of the two pendulums shown in the figure below. Given that, $m_{1}>m_{2}$.

A) 2
B) $\frac{1}{3}$
C) $\frac{1}{2}$
D) 1

Answer: 1
Solution: The formula to calculate the time period of a simple pendulum is given by
$T=2 \pi \sqrt{\frac{l}{g}}$
where, $l, g$ are the effective length of the pendulum and the acceleration due to gravity respectively.
From the above equation, it can be concluded that the time period of oscillation of a simple pendulum is independent of the mass of the pendulum bob.

Hence, for the given scenario, the ratio of the time periods of two pendulums will be 1 .
Q.44. There are two cubical Gaussian surfaces carrying charges as shown $\left(C_{1}\right.$ is inside $\left.C_{2}\right)$. Find the ratio of the flux through surfaces $C_{1}$ and $C_{2}$.

A) $2: 5$
B) $1: 1$
C) $2: 3$
D) $5: 2$

Answer: 2:5
Solution: From the given figure, it is clear that the total charge enclosed by the cubical surface $C_{1}$ is $2 Q$, whereas, the total charge enclosed by the surface $C_{2}$ is $(2 Q+3 Q)=5 Q$.

Thus, using Gauss's law, the flux through surface $C_{1}$ is
$\phi_{1}=\frac{2 Q}{\varepsilon_{0}}$
And, the flux through surface $C_{2}$ is
$\phi_{2}=\frac{5 Q}{\varepsilon_{0}}$
Hence, the required ratio is given by

$$
\begin{aligned}
\frac{\phi_{1}}{\phi_{2}} & =\frac{\frac{2 Q}{\varepsilon_{0}}}{\frac{5 Q}{\varepsilon_{0}}} \\
& =\frac{2}{5}
\end{aligned}
$$

Q.45. A source produced electromagnetic wave of frequency 60 MHz . Find wavelength of this wave in air.
A) 3 m
B) 4 m
C) 5 m
D) 6 m

Answer: 5 m
Solution: Given here: $\nu=60 \times 10^{6} \mathrm{~Hz}$
Using the relation between wavelength and frequency, we have
Wavelength $\lambda=\frac{c}{\nu}=\frac{3 \times 10^{8}}{60 \times 10^{6}}=5 \mathrm{~m}$.
Q.46. If a bulb of 40 W is producing a light of wavelength $\lambda=4000 \AA$. Then find the number of photons emitted by the bulb per second.
A) $8 \times 10^{19}$
B) $7 \times 10^{19}$
C) $6 \times 10^{19}$
D) $5 \times 10^{19}$

Answer: $8 \times 10^{19}$
Solution: Given that,
Power of monochromatic light is, $P=40 \mathrm{~W}$
Wavelength of monochromatic light is, $\lambda=4000 \AA$
Expression for energy of a photon is given by,
$E=\frac{h c}{\lambda}$
$\Rightarrow E=\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{4000 \times 10^{-10}}=5 \times 10^{-19} \mathrm{~J}$
Energy emitted by 40 W source per second, $T_{E}=40 \mathrm{~J}$
Number of photons emitted per second,
$\mathrm{n}=\frac{T_{E}}{E}=\frac{40}{5 \times 10^{-19}}=8 \times 10^{19}$
Q.47. 1000 drops of total surface energy $E$ coalesce to form a bigger drop of surface energy $E^{\prime}$. Find the value of $E^{\prime}$.
A) $\frac{E}{10}$
B) $E$
C) $10 E$
D) 100 E

Answer: $\frac{E}{10}$
Solution: As we know, surface energy $E=T A$, where $T=$ surface tension and $A=$ surface area.
Now, total volume of the drops will remain same. Therefore,

$$
\begin{aligned}
& 1000\left(\frac{4}{3} \pi r^{3}\right)=\frac{4}{3} \pi R^{3} \\
& \Rightarrow R=10 r
\end{aligned}
$$

Now,

$$
\begin{aligned}
& \frac{E^{\prime}}{E}=\frac{T\left(4 \pi R^{2}\right)}{T\left(1000 \times 4 \pi r^{2}\right)}=\frac{1}{1000} \times\left(\frac{R}{r}\right)^{2}=\frac{1}{10} \\
& \Rightarrow E^{\prime}=\frac{E}{10}
\end{aligned}
$$

Q.48. A body of mass $m=4 \mathrm{~kg}$ is acted upon by two forces, $\vec{F}_{1}=(5 \hat{\mathrm{i}}+8 \hat{\mathrm{j}}+7 \widehat{\mathrm{k}}) \mathrm{N}$ and $\vec{F}_{2}=(3 \hat{\mathrm{i}}-4 \hat{\mathrm{j}}-3 \widehat{\mathrm{k}}) \mathrm{N}$. Find the magnitude of acceleration of the body.
A) $\sqrt{3} \mathrm{~m} \mathrm{~s}^{-2}$
B) $\sqrt{11} \mathrm{~m} \mathrm{~s}^{-2}$
C) $\sqrt{7} \mathrm{~m} \mathrm{~s}^{-2}$
D) $\sqrt{6} \mathrm{~m} \mathrm{~s}^{-2}$

Answer: $\sqrt{6} \mathrm{~m} \mathrm{~s}^{-2}$
Solution: In this problem: $m=4 \mathrm{~kg}$

$$
\vec{F}_{1}=(5 \hat{\mathrm{i}}+8 \hat{\mathrm{j}}+7 \widehat{\mathrm{k}}) \mathrm{N} \& \vec{F}_{2}=(3 \hat{\mathrm{i}}-4 \hat{\mathrm{j}}-3 \widehat{\mathrm{k}}) \mathrm{N}
$$

Resultant force $=\vec{F}=\vec{F}_{1}+\vec{F}_{2}$

$$
\begin{aligned}
& =(5 \hat{\mathrm{i}}+8 \hat{\mathrm{j}}+7 \widehat{\mathrm{k}})+(3 \hat{\mathrm{i}}-4 \hat{\mathrm{j}}-3 \widehat{\mathrm{k}}) \\
& =(8 \hat{\mathrm{i}}+4 \hat{\mathrm{j}}+4 \widehat{\mathrm{k}})
\end{aligned}
$$

$$
|\vec{F}|=\sqrt{8^{2}+4^{2}+4^{2}}=\sqrt{64+16+16}=4 \sqrt{6} \mathrm{~N}
$$

$$
\Rightarrow F=m a \Rightarrow a=\frac{F}{m}
$$

$$
\therefore a=\frac{4 \sqrt{6} \mathrm{~N}}{4 \mathrm{~kg}}=\sqrt{6} \mathrm{~m} \mathrm{~s}^{-2}
$$

Q.49. A particle is moving in circular path of radius $r$ with speed $v$ such that speed is proportional to radius as $v \propto r^{-\frac{3}{2}}$. Then the time period of revolution depends on $r$ as $T \propto r^{n}$. The value of $n$ is
A) $\frac{5}{2}$
B) $-\frac{1}{2}$
C) $\frac{1}{2}$
D) $-\frac{5}{2}$

Answer: $\frac{5}{2}$

Solution: The relation between the angular velocity and the linear velocity for the given particle is

$$
v=\omega r \ldots(1)
$$

Also, the time period of revolution of the particle is
$T=\frac{2 \pi}{\omega} \quad \ldots(2)$
Equations (1) and (2) imply that
$T=\frac{2 \pi r}{v}$
Since, $v \propto r^{-\frac{3}{2}}$, from equation (3), it can be written that

$$
\begin{aligned}
& T=\frac{2 \pi r}{c r-\frac{3}{2}} \cdot \text { where } c \text { is constant } \\
& =k r^{\frac{5}{2}} \\
& \Rightarrow T \propto r^{\frac{5}{2}}
\end{aligned}
$$

Here, $c, k$ are proportionality constants.
Hence, $n=\frac{5}{2}$.
Q.50. Initially the balance point of the meter bridge is at 40 cm . Now if unknown resistance $(X)$ is shunt by $2 \Omega$ resistance then the new balance point is found to be at $\beta \times 10^{-1} \mathrm{~cm}$. Find the value of $\beta$.


Answer: 625

Solution: Let's consider the following diagram:


For the first case, from the balanced condition of the meter bridge, it can be written that
$\frac{2}{40}=\frac{X}{100-40}$
$\Rightarrow X=60 \times \frac{2}{40} \Omega$
$=3 \Omega$
When the shunt is used, the equivalent resistance for the EB part of the above diagram becomes,

$$
\begin{aligned}
R_{e q} & =\frac{1}{\frac{1}{3 \Omega}+\frac{1}{2 \Omega}} \\
& =\frac{6}{5} \Omega
\end{aligned}
$$

Thus, for the second case, if $l$ be the required balanced length in the meter bridge, it follows that
$\frac{2}{l}=\frac{\frac{6}{5}}{100-l}$
$\Rightarrow 2 \times \frac{5}{6}(100-l)=l$
$\Rightarrow 500-5 l=3 l$
$\Rightarrow 8 l=500$
$\Rightarrow l=\frac{500}{8}$
$=62.5$
$=625 \times 10^{-1} \mathrm{~cm}$
Hence, $\beta=625$.
Q.51. Find the number of significant digits in the value 10.05 .

Answer: 4
Solution: Rule for significant digits:

1. All non-zero numbers are significant.
2. Zeros between two non-zero digits are significant.
3. Leading zeros are not significant.
4. Trailing zeros to the right of the decimal are significant.

Therefore, number of significant digits are 4.
Q.52. A ball of mass 120 g moving with initial velocity $25 \mathrm{~m} \mathrm{~s}^{-1}$ is stopped by an external force $F$ in 0.15 s . Find the magnitude of $F$ in newton :

Answer: 20

Solution: Given that,
Mass of the ball, $\mathrm{m}=120 \mathrm{~g}=0.12 \mathrm{~kg}$
Initial velocity of the ball, $u=25 \mathrm{~m} \mathrm{~s}^{-1}$
Final velocity of the ball, $\mathrm{v}=0 \mathrm{~m} \mathrm{~s}^{-1}$
Time taken for the velocity change, $\mathrm{t}=0.15 \mathrm{~s}$
We know that,
Force $=$ mass $\times$ acceleration
$\Rightarrow \mathrm{F}=\mathrm{m}\left(\frac{\mathrm{v}-\mathrm{u}}{\mathrm{t}}\right)=\frac{0.12(0-25)}{0.15}=-20 \mathrm{~N}$ ( - ve sign represents opposite direction)
Therefore, required magnitude is 20 .

