

Shift 1

Memory Based Paper



Chemistry

Q.1. In Kjeldahl's estimation of nitrogen, CuSO_4 acts as:

- A) Reducing agent
- B) Oxidising agent
- C) Boiling point elevator
- D) Catalyst

Answer: Catalyst

Solution: Kjeldahl's method is used for the estimation of nitrogen. The organic compound is heated with conc. H_2SO_4 in presence of K_2SO_4 and a little CuSO_4 to convert all the nitrogen into $(\text{NH}_4)_2\text{SO}_4$. K_2SO_4 raises the boiling point of H_2SO_4 while CuSO_4 catalyses the reaction.

Hence, D is the answer.

Q.2. Which of the following is most likely attacked by an electrophile?

- A) C_6H_6
- B) $\text{C}_6\text{H}_5\text{OH}$
- C) $\text{C}_6\text{H}_5\text{CH}_3$
- D) $\text{C}_6\text{H}_5\text{Cl}$

Answer: $\text{C}_6\text{H}_5\text{OH}$

Solution: Any molecule, ion or atom that is deficient in electrons in some manner can act as an electrophile. In other words, the reagent which attacks the negative of the molecule or loves electrons is called electrophile.

Phenol is most easily attacked by an electrophile because presence of $-\text{OH}$ group increases electron density at o- and p-positions.

Q.3. In case of isoelectronic species, the size of F^- , Ne and Na^+ is affected by:

- A) Nuclear charge (Z)
- B) Principle quantum number (n)
- C) Electron-electron interaction
- D) None of the above factors as their sizes are the same

Answer: Nuclear charge (Z)

Solution: For isoelectronic species, the more nuclear charges, the less will be the size of the species. The valence electrons will experience a greater attractive force due to the increase in nuclear charge.

Hence, the size of isoelectronic species - F^- , Ne and Na^+ is affected by nuclear charge (Z).

Thus, the ionic size of Na^+ is the lowest, and the ionic size of F^- ion is the highest.

Hence, the answer is A.

Q.4. Which of the following is homoleptic complex?

- A) $[\text{Ni}(\text{CN})_4]^{2-}$
- B) $[\text{Pt}(\text{NH}_3)_2\text{Cl}_2]$
- C) $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]$
- D) $[\text{Co}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$

Answer: $[\text{Ni}(\text{CN})_4]^{2-}$

Solution: Homoleptic complexes are compounds in which all the ligands bound to the central metal atom or ion are identical.

Homoleptic complexes have only one type of ligand.

$[\text{Ni}(\text{CN})_4]^{2-}$ has only one type of ligand.

Hence, option A is the answer.

Q.5. Statement 1: PH_3 will have a lower boiling point than NH_3 .

Statement 2: There are strong Van der Waals forces in NH_3 and strong hydrogen bonding in PH_3 .

- A) Both the statements are correct
- B) Both the statements are incorrect



C) Statement 1 is correct and statement 2 is incorrect D) Statement 1 is incorrect and statement 2 is correct

Answer: Statement 1 is correct and statement 2 is incorrect

Solution: In ammonia (NH_3), the molecules are linked by intermolecular hydrogen bonding. Due to this, we need a high amount of energy to break this hydrogen bond for boiling.

But PH_3 molecules are not associated through hydrogen bonding, so it boils easily as compared to NH_3 .

That is why the boiling point of PH_3 is lower than NH_3 .

Hence, the statement 1 is correct.

However, due to the lower electronegativity of larger PH_3 molecules, they are unable to form hydrogen bonds between themselves.

Hence, the statement 2 is incorrect.

Q.6. Statement 1: $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$ is green in colour.

Statement 2: $[\text{Ni}(\text{CN})_4]^{2-}$ is colourless.

- A) Both Statement 1 and 2 are correct B) Both Statement 1 and 2 are wrong.
C) Statement 1 is correct Statement 2 is wrong. D) Statement 1 is wrong Statement 2 is correct.

Answer: Both Statement 1 and 2 are correct

Solution: In both the complexes the Ni^{2+} ion has $3d^8$ valence shell electronic configuration.

In $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$, H_2O is a weak field ligand. Therefore, there are unpaired electrons in Ni^{2+} . In this complex, the d- electrons from the lower energy level can be excited to the higher energy level i.e., the possibility of $d-d$ transition is present. Hence, it is green.

In $[\text{Ni}(\text{CN})_4]^{2-}$, the electrons are all paired as CN^- is a strong field ligand. Therefore, $d-d$ transition is not possible. Hence, it is colourless.

Hence, option A is the answer.

Q.7. Which of the following is correct for adiabatic free expansion against vacuum?

- A) $q \neq 0$, $\Delta U = 0$, $w = 0$ B) $q = 0$, $\Delta U = 0$, $w = 0$
C) $q = 0$, $\Delta U \neq 0$, $w = 0$ D) $q = 0$, $\Delta U \neq 0$, $w \neq 0$

Answer: $q = 0$, $\Delta U = 0$, $w = 0$

Solution: For an adiabatic free expansion of an ideal gas, the gas is kept in an insulated container and then permitted to enlarge in a vacuum. Since there is no external pressure for the gas to enlarge against, the work done by or on the system is zero. Since this process does not involve any exchange of heat transfer or work, the first law of thermodynamics states that the net internal energy change of the system is zero.

For a free expansion, the work done will be zero.
 $w = 0$

For an adiabatic process, the heat will be zero.
 $q = 0$

So, from the first law of thermodynamics, the change in internal energy is the total of heat and the work done.

$$\Delta U = q + w$$

Substituting the values of q and w , we get

$$\Delta U = 0 + 0$$

$$\Delta U = 0$$

Q.8. Which of the following compounds have trigonal bipyramidal shape?

PF_5 , PBr_5 , $[\text{PtCl}_4]^-$, SF_6 , BF_3 , BrF_5 , PCl_5 , $[\text{Fe}(\text{CO})_5]$

- A) PF_5 , PBr_5 , PCl_5 , $[\text{Fe}(\text{CO})_5]$ B) PF_5 , PBr_5 , BrF_5 , PCl_5

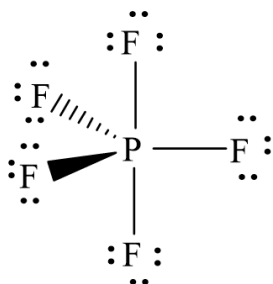


C) PF_5 , PCl_5 , BrF_5

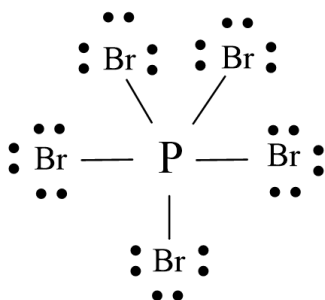
D) PF_5 , PBr_5 , BrF_5 , PCl_5 , $[\text{Fe}(\text{CO})_5]$

Answer: PF_5 , PBr_5 , PCl_5 , $[\text{Fe}(\text{CO})_5]$

Solution: PF_5 involves sp^3d hybridisation and hence has trigonal bipyramidal structure.



PBr_5 have trigonal bipyramidal shape.



In PCl_5 the central atom is surrounded by five bond pairs and no lone pairs.
Hybridisation of central atom in PCl_5 molecule is sp^3d giving trigonal bipyramidal shape.

In $[\text{Fe}(\text{CO})_5]$, **oxidation state of CO is zero.**

It has hybridisation $=\text{sp}^3\text{d}$.

So it has trigonal bipyramidal geometry.

Hence option A is the answer.

Q.9. What is the complementary strand for the given strand "ATGCTTCA" of DNA?

- A) TACGAAGT B) TACGAAGA
C) TAGCAACA D) TAGCTACT

Answer: TACGAAGT

Solution: As we know that in DNA molecule, adenine (A) always pairs with thymine (T) and cytosine (C) always pairs with guanine (G). Thus, sequence of bases in one strand : ATGCTTCA then the sequence of bases in the complementary strand : TACGAAGT

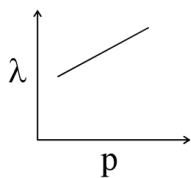
Hence, option A is the answer.

Q.10. Select the correct relationship of the Van't Hoff factors of the three sodium chloride samples given below.

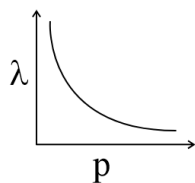
Sample Concentration	Van't Hoff factor
0.1 M	i_1
0.01M	i_2
0.001M	i_3



D)



Answer:



Solution: The relation between de Broglie wave length and momentum is

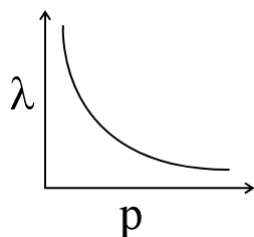
$$\lambda = \frac{h}{p}$$

$$y = \frac{h}{x}$$

$$xy = h$$

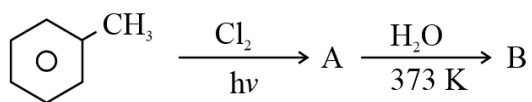
$$xy = c$$

$xy = c$ is represented as hyperbola.



Hence, A is the answer.

Q.13.



Identify the products A and B.

A) $C_6H_5CHCl_2$, C_6H_5CHO

B) $C_6H_5CHCl_2$, C_6H_5OH

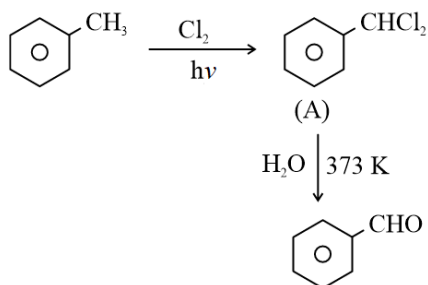
C) C_6H_5Cl , C_6H_5CHO

D) None of the above.

Answer: $C_6H_5CHCl_2$, C_6H_5CHO



Solution: When Toluene reacts with chlorine in the presence of light to give benzal chloride and benzal chloride on reaction with H_2O at 373 K forms Benzaldehyde. The overall reaction is as follows,



Hence option A is the answer.

Q.14. Statement 1: -NH_2 is a strong activating group.

Statement 2: Aniline does not give Friedel craft alkylation or acylation.

- A) Both statements are correct
- B) Both statements are incorrect
- C) Statement 1 is correct and 2 is incorrect
- D) Statement 1 is incorrect and 2 is correct

Answer: Both statements are correct

Solution: NH_2 is a highly activating group in organic chemistry due to its electron-donating nature. It is often used in synthetic chemistry to increase the reactivity of aromatic compounds towards electrophilic substitution reactions.

Aniline forms salt with the Lewis acid catalyst i.e., AlCl_3 , which is used in Friedel-Crafts reaction.

Nitrogen of aniline acquires a positive charge and hence acts as a strong deactivating group for further reaction.

Hence, both statements are correct.

Q.15. For A_2B , the lowest oxidation state of B is -2 . Find the number of valence shell electrons in B.

- A) 2 B) 6
C) 8 D) 4

Answer: 6

Solution: If the lowest oxidation state of an atom is -2 , this typically implies that the atom can gain two electrons to reach a stable configuration. Therefore, the number of valence electrons in that atom would be six, because gaining two electrons would fill its outer shell, synonymous with elements of the oxygen family (group 16) in the periodic table, which includes oxygen, sulphur, selenium, tellurium, and polonium.

Q.16. For ionic reaction in organic compounds, which type of bond cleavage occurs?

- A) Heterolytic Cleavage
B) Homolytic Cleavage
C) Free Radical
D) No cleavage of bond

Answer: Heterolytic Cleavage

Solution: Heterolytic fission, also known as heterolysis, is a type of bond fission in which a covalent bond between two chemical species is broken in an unequal manner, resulting in the bond pair of electrons being retained by one of the chemical species (while the other species does not retain any of the electrons from the bond pair). When a neutrally charged molecule undergoes heterolytic fission, one of the products will have a positive charge whereas the other product will have a negative charge.

Hence option A is the answer.



Q.17. Which of the following is a disproportionation reaction?

1. $\text{Cu}^+ \rightarrow \text{Cu}^{2+} + \text{Cu}$
2. $\text{MnO}_4^{2-} \rightarrow \text{MnO}_4^- + \text{MnO}_2$
3. $\text{H}_2\text{O}_2 \rightarrow \text{O}_2 + \text{H}_2\text{O}$
4. $\text{CrO}_4^{2-} \rightarrow \text{Cr}^{3+} + \text{H}_2\text{O}$

- A) 1, 2, 3 and 4 B) 1 and 3 only
C) 1 and 2 only D) 1, 2 and 3 only

Answer: 1, 2 and 3 only

Solution: Disproportionation reaction is basically a type of redox reaction involving simultaneous reduction and oxidation of atoms of the same element from one oxidation state (OS) to two different oxidation states.

- $$\begin{array}{ccccc} +1 & & +2 & & 0 \\ 1. & \text{Cu}^+ & \rightarrow & \text{Cu}^{2+} & + \text{Cu} \end{array}$$
- $$\begin{array}{ccccc} +6 & & +7 & & +2 \\ 2. & \text{MnO}_4^{2-} & \rightarrow & \text{MnO}_4^- & + \text{MnO}_2 \end{array}$$
- $$\begin{array}{ccccc} -1 & & 0 & & -2 \\ 3. & \text{H}_2\text{O}_2 & \rightarrow & \text{O}_2 & + \text{H}_2\text{O} \end{array}$$
- $$\begin{array}{ccccc} +6 & & +3 & & \\ 4. & \text{CrO}_4^{2-} & \rightarrow & \text{Cr}^{3+} & + \text{H}_2\text{O} \end{array}$$

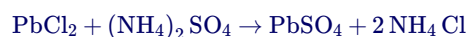
1, 2 and 3 are undergoing both oxidation and reduction, so these are disproportionation reactions.

Q.18. How many moles of precipitate product formed when 72 moles of PbCl_2 react with 50 moles of $(\text{NH}_4)_2\text{SO}_4$?

- A) 72 B) 50
C) 22 D) 122

Answer: 50

Solution: The reaction can be written as follows for the given information,



The precipitate formed in the above reaction is PbSO_4 . According to the above stoichiometric equation, one mole of each lead chloride and ammonium sulphate is required to make one mole of lead sulphate precipitate. Given that 72 mol PbCl_2 and 50 mol $(\text{NH}_4)_2\text{SO}_4$. Here, ammonium sulphate is the limiting reagent. Hence, the moles of precipitated formed is equal to 50 mol.

Q.19. Statement I: Potassium hydrogen phthalate is primary standard for NaOH

Statement II : Phenolphthalein is used to detect completion of this titration.

- A) Both Statement I and II are correct. B) Both Statement I and II are wrong.
C) Statement I is correct and statement II is wrong. D) Statement I is wrong and statement II is correct.

Answer: Both Statement I and II are correct.

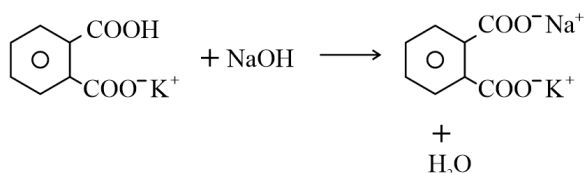


Solution: A primary standard solution is a highly purified compound that is used to standardise a solution of known concentration for use in titrations and other analytical chemistry procedures.

Potassium hydrogen phthalate is primary standard for NaOH. Formula for Potassium hydrogen phthalate is $C_6H_5O_4K$. It behaves as a monobasic acid.

Phenolphthalein is used as an indicator in this titration.

The reaction is,

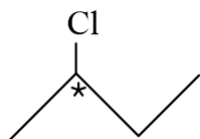


Hence option A is the answer.

Q.20. Find out the total possible optical isomers of 2 – chlorobutane.

Answer: 2

Solution: The number of optical isomers possible for a compound is 2^n where n = number of asymmetric carbon atoms.



As $n=1$ for 2 – chlorobutane, it is an optically active molecule with one chiral carbon atom and it forms dextro and laevo isomers.

$$2^n = 2^1 = 2.$$

Hence, it has two optical isomers.

Q.21. Total number of deactivating group among the following is

– CN, – NHCOCH₃, – COCH₃, – NHCH₃

Answer: 2

Solution: Atoms or groups that make the benzene molecule more reactive by increasing the ring's electron density are called **activating groups**.

An atom or group that makes the benzene molecule less reactive by removing electron density from the ring acts as a **deactivating group**. Deactivating groups direct incoming electrophiles to the meta position. Deactivating groups decreases the rate of reaction relative to H.

Hence, – CN, – COCH₃ are deactivating groups.

Q.22. What is the pH of $\text{CH}_3\text{COO}^-\text{NH}_4^+$? (at 25°C)

Given: K_a of $\text{CH}_3\text{COOH} = 1.8 \times 10^{-5}$, K_b of $\text{NH}_4\text{OH} = 1.8 \times 10^{-5}$

Answer: 7



Solution: Given: K_a of $\text{CH}_3\text{COOH} = 1.8 \times 10^{-5}$, K_b of $\text{NH}_4\text{OH} = 1.8 \times 10^{-5}$

$$\text{p}K_a = -\log K_b = 5 - \log 1.8 = 4.74$$

Both $\text{p}K_a$ and $\text{p}K_b$ are equal.

Both the substances given are weak acid and weak bases. The pH of the salt of weak acid and weak base can be calculated using

$$\begin{aligned}\text{pH} &= \frac{1}{2} (\text{p}K_w + \text{p}K_a - \text{p}K_b) \\ &= \frac{1}{2} (14 + 4.74 - 4.74) \\ &= 7\end{aligned}$$

Q.23. How many oxides are amphoteric in nature?

SnO_2 , PbO_2 , SiO_2 , P_2O_5 , Al_2O_3 , CO_2 , CO , NO , N_2O

Answer: 3

Solution: Amphoteric oxides are the oxides that behave as both acidic and basic oxides. Amphoteric Oxides have features of acidic as well as basic oxides that neutralize both acids and bases.

Amphoteric oxides are: SnO_2 , PbO_2 , Al_2O_3

Acidic oxides are: SiO_2 , P_2O_5 , CO_2

Neutral oxides are: CO , NO , N_2O

Hence, the answer is 3.

Q.24. Given the following cell reaction:



Given that the pressure of hydrogen gas is 2 atm and the concentration of H^+ is 1M. If the reduction potential of the reaction is given by $-x \times 10^{-3}$ V. Find out x.

$$\left(\frac{2.303RT}{F} \right) = 0.06$$

Answer: 9

Solution: The Nernst equation for the given cell reaction $2\text{H}^+ + 2\text{e}^- \rightarrow \text{H}_2$ is

$$E = E^\circ - \frac{0.06}{2} \times \log \frac{P_{\text{H}_2}}{[\text{H}^+]^2}$$

$$E = 0 - \frac{0.06}{2} \log \frac{P_{\text{H}_2}}{[\text{H}^+]^2}$$

$$E = -0.03 \times 0.3$$

$$= -0.009$$

$$= -9 \times 10^{-3}$$

Hence, $x = 9$

Mathematics

Q.25. Number of ways of arranging 5 officers in 4 rooms is

A) 4^5

B) 5^4

C) 1048

D) 625

Answer: 4^5



Q.28. The value of the integral $\int_0^{\frac{\pi}{4}} \frac{x dx}{\cos^4 2x + \sin^4 2x}$ is

- A) $\frac{\pi^2}{16}$ B) $\frac{\pi^2}{32\sqrt{2}}$
C) $\frac{\pi^2}{8\sqrt{2}}$ D) $\frac{\pi^2}{4\sqrt{2}}$

Answer: $\frac{\pi^2}{32\sqrt{2}}$

Solution: Let, $I = \int_0^{\frac{\pi}{4}} \frac{x dx}{\cos^4 2x + \sin^4 2x} \dots (i)$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\left(\frac{\pi}{4} - x\right) dx}{\cos^4 2\left(\frac{\pi}{4} - x\right) + \sin^4 2\left(\frac{\pi}{4} - x\right)}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\left(\frac{\pi}{4} - x\right) dx}{\cos^4 2x + \sin^4 2x} \dots (ii)$$

Adding (i) and (ii),

$$\Rightarrow 2I = \int_0^{\frac{\pi}{4}} \frac{\frac{\pi}{4} dx}{\cos^4 2x + \sin^4 2x}$$

$$\Rightarrow I = \frac{\pi}{8} \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 2x + \sin^4 2x}$$

$$\Rightarrow I = \frac{\pi}{8} \int_0^{\frac{\pi}{4}} \frac{\sec^4 2x dx}{1 + \tan^4 2x}$$

Putting, $\tan 2x = t$

$$\Rightarrow 2 \sec^2 2x dx = dt$$

$$\Rightarrow I = \frac{\pi}{16} \int_0^{\infty} \frac{1+t^2}{1+t^4} dt$$

$$\Rightarrow I = \frac{\pi}{16} \int_0^{\infty} \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$$

Let, $t - \frac{1}{t} = u$

$$\Rightarrow \left(1 + \frac{1}{t^2}\right) dt = du$$

$$\Rightarrow I = \frac{\pi}{16} \int_0^{\infty} \frac{du}{u^2+2}$$

$$\Rightarrow I = \frac{\pi}{16} \times \frac{1}{\sqrt{2}} \left[\tan^{-1} \frac{u}{\sqrt{2}} \right]_0^{\infty}$$

$$\Rightarrow I = \frac{\pi}{16} \times \frac{1}{\sqrt{2}} \times \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi^2}{32\sqrt{2}}$$

Q.29. A bag contains 8 balls (black and white). If four balls are chosen without replacement then $2W$ and $2B$ are found, then the probability that number of white and black balls are same in bag is equal to:

- A) $\frac{1}{7}$ B) $\frac{1}{2}$
C) $\frac{2}{7}$ D) $\frac{3}{5}$

Answer: $\frac{2}{7}$



Solution: Given: $(t+1)dx = (2x + (t+1)^3)dt$

$$\Rightarrow \frac{dx}{dt} = \frac{2x + (t+1)^3}{(t+1)}$$

$$\Rightarrow \frac{dx}{dt} - \frac{2x}{(t+1)} = (t+1)^2$$

$$\Rightarrow \text{IF} = e^{\int \frac{-2}{t+1} dt} = e^{-2 \log|t+1|}$$

$$\Rightarrow \text{IF} = \frac{1}{(t+1)^2}$$

Solution of the differential equations is given by,

$$\Rightarrow x \times \frac{1}{(t+1)^2} = \int (1) dt$$

$$\Rightarrow \frac{x}{(t+1)^2} = t + c$$

$$\text{Now, } x(0) = 2$$

$$\Rightarrow c = 2$$

$$\Rightarrow x = (t+2)(t+1)^2$$

$$\Rightarrow x(1) = (1+2)(1+1)^2 = 3 \times 4$$

$$\Rightarrow x(1) = 12$$

Q.32. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} dx = a\pi + b \cdot \log(3+2\sqrt{2})$ then find the value of $a+b$

A) 2

B) 4

C) 6

D) 8

Answer: 4



Solution: Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} dx \dots (1)$

Using $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ we get, $\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos(\frac{\pi}{2}-\frac{\pi}{2}-x)}{(1+e^{\sin(\frac{\pi}{2}-\frac{\pi}{2}-x)})(1+\sin^4(\frac{\pi}{2}-\frac{\pi}{2}-x))} dx$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+e^{-\sin x})(1+\sin^4 x)} dx \dots (2)$$

Now, adding both above equations we get,

$$\Rightarrow 2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+\sin^4 x)} dx$$

$$\Rightarrow 2I = 2 \int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+\sin^4 x)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+\sin^4 x)} dx$$

Now, let $\sin x = t \Rightarrow \cos x dx = dt$

$$\Rightarrow I = \int_0^1 \frac{8\sqrt{2}}{1+t^4} dt$$

$$\Rightarrow I = 4\sqrt{2} \int_0^1 \frac{1+\frac{1}{t^2} \left(1-\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}} dt$$

$$\Rightarrow I = 4\sqrt{2} \left[\int_0^1 \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt - \int_0^1 \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt \right]$$

$$\Rightarrow I = 4\sqrt{2} \left[\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t^2-1}{\sqrt{2}t} \right) - \frac{1}{2\sqrt{2}} \ln \left(\frac{t^2+1-\sqrt{2}t}{t^2+1+\sqrt{2}t} \right) \right]_0^1$$

$$\Rightarrow I = 4\sqrt{2} \left[\frac{1}{\sqrt{2}} \tan^{-1} (0) - \frac{1}{2\sqrt{2}} \ln \left(\frac{2-\sqrt{2}}{2+\sqrt{2}} \right) - \left(\frac{1}{\sqrt{2}} \tan^{-1} (-\infty) - \frac{1}{2\sqrt{2}} \ln (1) \right) \right]$$

$$\Rightarrow I = 4\sqrt{2} \left[0 - \frac{1}{2\sqrt{2}} \ln \left(\frac{2-\sqrt{2}}{2+\sqrt{2}} \right) - \left(\frac{1}{\sqrt{2}} \cdot \left(-\frac{\pi}{2} \right) - 0 \right) \right]$$

$$\Rightarrow I = 4\sqrt{2} \left[\frac{1}{\sqrt{2}} \cdot \frac{\pi}{2} + \frac{1}{2\sqrt{2}} \ln (3+2\sqrt{2}) \right]$$

$$\Rightarrow I = \left[2\pi + 2 \ln (3+2\sqrt{2}) \right]$$

So, on comparing with given equation we get, $a+b=2+2=4$

Q.33. It is given that $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 4$ and $y = 9f(x) \times x^2$. If y is strictly increasing, then find the interval of x .

- A) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ B) $\left(-\infty, \frac{-1}{\sqrt{5}}\right] \cup \left(\frac{1}{\sqrt{5}}, 0\right)$
 C) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$ D) $\left(-\sqrt{\frac{2}{5}}, 0\right) \cup \left(\sqrt{\frac{2}{5}}, \infty\right)$

Answer: $\left(-\sqrt{\frac{2}{5}}, 0\right) \cup \left(\sqrt{\frac{2}{5}}, \infty\right)$



Solution: Given: $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 4 \dots (i)$

Replacing x by $\frac{1}{x}$.

$$5f\left(\frac{1}{x}\right) + 4f(x) = \left(\frac{1}{x}\right)^2 - 4 \dots (ii)$$

Using, $(i) \times 5 - (ii) \times 4$

$$\Rightarrow 25f(x) + 20f\left(\frac{1}{x}\right) - 16f(x) - 20f\left(\frac{1}{x}\right) = 5x^2 - 20 - \frac{4}{x^2} + 16$$

$$\Rightarrow 9f(x) = 5x^2 - 4 - \frac{4}{x^2}$$

Now, $y = 9f(x) \times x^2$

$$\Rightarrow y = \left(5x^2 - 4 - \frac{4}{x^2}\right) \times x^2$$

$$\Rightarrow y = 5x^4 - 4x^2 - 4$$

$$\Rightarrow y' = 20x^3 - 8x$$

$$\Rightarrow y' = 4x(5x^2 - 2)$$

For function to be increasing $y' > 0$

$$\Rightarrow x \in \left(-\sqrt{\frac{2}{5}}, 0\right) \cup \left(\sqrt{\frac{2}{5}}, \infty\right)$$

Q.34. If the hyperbola $x^2 - y^2 \operatorname{cosec}^2 \theta = 5$ and ellipse $x^2 \operatorname{cosec}^2 \theta + y^2 = 5$ has eccentricity e_H & e_E respectively and $e_H = \sqrt{7}e_E$, then θ will be

A) $\frac{\pi}{3}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{2}$

D) $\frac{\pi}{6}$

Answer: $\frac{\pi}{3}$

Solution: Given,

The hyperbola $x^2 - y^2 \operatorname{cosec}^2 \theta = 5$

$$\Rightarrow \frac{x^2}{5} - \frac{y^2}{5 \sin^2 \theta} = 1$$

And ellipse $x^2 \operatorname{cosec}^2 \theta + y^2 = 5$

$$\Rightarrow \frac{x^2}{5 \sin^2 \theta} + \frac{y^2}{5} = 1 \text{ \{ here } a < b \text{ as } \sin^2 \theta \leq 1 \}}$$

$$\text{So, } e_H = \sqrt{1 + \sin^2 \theta} \text{ \& } e_E = \sqrt{1 - \sin^2 \theta}$$

Also given, $e_H = \sqrt{7}e_E$

$$\Rightarrow \sqrt{1 + \sin^2 \theta} = \sqrt{7} \sqrt{1 - \sin^2 \theta}$$

$$\Rightarrow 1 + \sin^2 \theta = 7 - 7 \sin^2 \theta$$

$$\Rightarrow 8 \sin^2 \theta = 6$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

Q.35. Let $S = \{1, 2, 3, \dots, 20\}$, $R_1 = \{(a, b) : a \text{ divide } b\}$, $R_2 = \{(a, b) : a \text{ is integral multiple of } b\}$ and $a, b \in S$ then $n(R_1 - R_2) = ?$



- A) 42 B) 46
C) 44 D) 40

Answer: 46

Solution: Given: $S = \{1, 2, 3, \dots, 20\}$, $R_1 = \{(a, b) : a \text{ divide } b\}$, $R_2 = \{(a, b) : a \text{ is integral multiple of } b\}$

$$\Rightarrow R_1 = \{(1, 1), (1, 2), \dots, (1, 20), (2, 2), (2, 4), \dots, (2, 20), (3, 3), (3, 6), \dots, (3, 18), (4, 4), (4, 8), \dots, (4, 20), (5, 5), (5, 10), \dots, (5, 20)\}$$

$$\Rightarrow n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 + 2 + 10 = 66$$

Now, finding

$$R_1 \cap R_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), \dots, (20, 20)\}$$

$$\Rightarrow n(R_1 \cap R_2) = 20$$

$$\Rightarrow n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$$

$$\Rightarrow n(R_1 - R_2) = 66 - 20 = 46$$

Q.36. If the shortest distance between the two lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ & $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1 then find the sum of possible values of λ

- A) $4\sqrt{3}$ B) $8\sqrt{3}$
C) $\sqrt{3}$ D) $2\sqrt{3}$

Answer: $2\sqrt{3}$

Solution: Given,

Equations of line,

$$\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1} \text{ \& } \frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$$

Now, we know that shortest distance between two lines is given by,

$$S.D = \frac{|(a_2 - a_1) \cdot \vec{b_1} \times \vec{b_2}|}{|\vec{b_1} \times \vec{b_2}|}$$

$$\Rightarrow 1 = \frac{\begin{vmatrix} \lambda - \sqrt{3} & 1 & -1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$\Rightarrow 1 = \frac{(\lambda - \sqrt{3})3 + 3 - 3}{|3\hat{i} + 3\hat{j} + 3\hat{k}|}$$

$$\Rightarrow |(\lambda - \sqrt{3})| = \sqrt{3}$$

$$\Rightarrow \lambda = 0, 2\sqrt{3}$$

Hence, the sum is $2\sqrt{3}$

Q.37. If the system of equations $2x + 3y - z = 5$, $x + \alpha y + 3z = -4$ and $3x - y + \beta z = 7$ have many solutions, then the value of $|13\alpha\beta|$ will be

Answer: 1120



Solution: Given,

The system of equations,

$$2x + 3y - z = 5$$

$$x + \alpha y + 3z = -4$$

$$3x - y + \beta z = 7 \text{ have many solutions,}$$

$$\text{So, } \Delta = \Delta_1 = \Delta_2 = \Delta_3$$

$$\text{Now, finding } \Delta_2 = \begin{vmatrix} 2 & 5 & -1 \\ 1 & -4 & 3 \\ 3 & 7 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 2(-4\beta - 21) - 5(\beta - 9) - 1(7 + 12) = 0$$

$$\Rightarrow 13\beta = -16$$

$$\text{Now, finding } \Delta_3 = \begin{vmatrix} 2 & 3 & 5 \\ 1 & \alpha & -4 \\ 3 & -1 & 7 \end{vmatrix} = 0$$

$$\Rightarrow 14\alpha - 8 - 57 - 5 - 15\alpha = 0$$

$$\Rightarrow \alpha = -70$$

$$\text{Hence, the value of } |13\alpha\beta| = 70 \times 16 = 1120$$

Q.38. If $S = \left\{ x \in R : \left(\sqrt{3} + \sqrt{2} \right)^x + \left(\sqrt{3} - \sqrt{2} \right)^x = 10 \right\}$ then number of elements in S is

Answer: 2

Solution: Given,

$$\left(\sqrt{3} + \sqrt{2} \right)^x + \left(\sqrt{3} - \sqrt{2} \right)^x = 10$$

$$\text{Now, let } \left(\sqrt{3} + \sqrt{2} \right)^x = t \Rightarrow \left(\sqrt{3} - \sqrt{2} \right)^x = \frac{1}{t}$$

$$\text{So, } \left(\sqrt{3} + \sqrt{2} \right)^x + \left(\sqrt{3} - \sqrt{2} \right)^x = 10$$

$$\Rightarrow t + \frac{1}{t} = 10$$

$$\Rightarrow t^2 - 10t + 1 = 0$$

$$\Rightarrow t = 5 \pm 2\sqrt{6}$$

$$\Rightarrow t = \left(\sqrt{3} - \sqrt{2} \right)^2, \left(\sqrt{3} + \sqrt{2} \right)^2$$

$$\text{Now, taking } \left(\sqrt{3} + \sqrt{2} \right)^x = \left(\sqrt{3} - \sqrt{2} \right)^2 = \left(\sqrt{3} + \sqrt{2} \right)^{-2} \text{ we get, } x = -2$$

$$\text{And equating } \left(\sqrt{3} + \sqrt{2} \right)^x = \left(\sqrt{3} + \sqrt{2} \right)^2 \text{ we get, } x = 2$$

Hence, the number of elements in S is 2

Q.39. $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} \frac{a-b\cos 2x}{x^2}, & x < 0 \\ x^2 + cx + 2, & 0 \leq x \leq 1 \\ 2x + 1, & x > 1 \end{cases}$. If f is continuous and m is the number of points where it is differentiable then $m + a + b + c$.

Answer: 4



Solution: The capacitance of a capacitor is given by

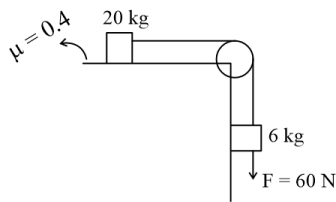
$$C = \frac{Q}{V}.$$

As the potential difference is doubled, the charge on capacitors also increases, maintaining a constant ratio following the above formula, and consequently, the capacitance remains unchanged.

This constancy in capacitance is attributed to its dependence on factors such as plate area, the dielectric medium between plates, and the separation distance between plates. Therefore, when the potential difference is elevated to $2V$, the capacitance remains at its original value.

Hence, the percentage change in capacitance is 0%.

Q.41. Find the acceleration (in m s^{-2}) of the system if an external force of 60 N is applied on 6 kg block as shown. Take $g = 10 \text{ m s}^{-2}$.



A) $\frac{5}{13}$

B) $\frac{20}{13}$

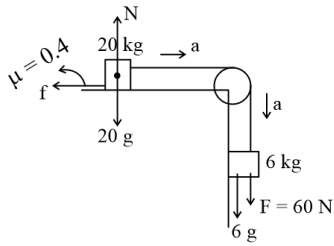
C) $\frac{1}{2}$

D) $\frac{2}{5}$

Answer: $\frac{20}{13}$



Solution: Let's consider the following FBD:



Let's consider that a be the required acceleration of the system.

The net force on the 6 kg block is given by

$$\begin{aligned} F_n &= 60 \text{ N} + 6g \\ &= (60 + 6 \times 10) \text{ N} \\ &= 120 \text{ N} \end{aligned}$$

The amount of frictional force on the 20 kg block is given by

$$\begin{aligned} f &= \mu (20g) \text{ N} \\ &= 0.4 \times 20 \times 10 \text{ N} \\ &= 80 \text{ N} \end{aligned}$$

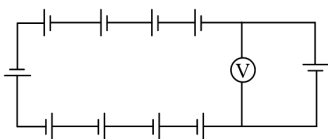
Thus, the equations of motion for the entire connected system can be written as

$$F_n - f = (20 + 6)a \quad \dots (1)$$

From equation (1), it follows that

$$\begin{aligned} 26a &= (120 - 80) \\ \Rightarrow a &= \frac{40}{26} \text{ m s}^{-2} \\ &= \frac{20}{13} \text{ m s}^{-2} \end{aligned}$$

Q.42. In the following figure, all the batteries are identical (5 V, 0.2 Ω) and are connected as shown. Find the reading of the voltmeter(in volt).

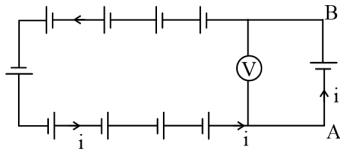


- A) 10 B) 5
C) 0 D) 2

Answer: 0



Solution: Let's consider the following diagram:



The required reading in the voltmeter will be the same as the potential difference across the cell connected between the points A and B, as shown in the above figure.

Let, ε be the emf and r be the resistance of each cell and i be the current through the circuit.

Thus, for the entire loop, KVL implies that

$$10(-ir + \varepsilon) = 0$$

$$\Rightarrow i = \frac{\varepsilon}{r} \dots (1)$$

Between the points A and B, it can be written that

$$V_A - ir + \varepsilon = V_B$$

$$\Rightarrow V_A - V_B = ir - \varepsilon$$

$$= 0$$

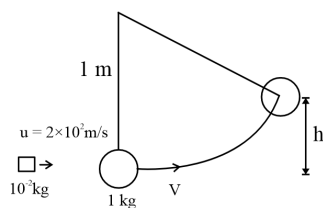
Hence, the reading of the voltmeter will be 0.

Q.43. A bullet of mass 10^{-2} kg moving with speed 2×10^2 m s⁻¹ hits a ballistic pendulum of length 1 m and mass 1 kg horizontally and gets embedded in it. Find the maximum height till which it rises.

- A) 0.3 m B) 0.5 m
- C) 0.7 m D) 0.2 m

Answer: 0.2 m

Solution: Let's consider the following diagram:



Let, V be the velocity of the combined system after the bullet is embedded in the pendulum.

From the conservation of momentum, it follows that

$$10^{-2} \text{ kg} \times 2 \times 10^2 \text{ m s}^{-1} + 0 \times 1 \text{ kg} = (10^{-2} \text{ kg} + 1 \text{ kg})V$$

$$\Rightarrow V = \frac{2}{1.01} \text{ m s}^{-1}$$

If h be the maximum height attained by the pendulum after the collision, then it follows that

$$0 + \frac{1}{2} (1 + 10^{-2}) V^2 = (1 + 10^{-2}) gh + 0$$

$$\Rightarrow h = \frac{V^2}{2g}$$

$$= \frac{\left(\frac{2}{1.01}\right)^2}{2 \times 10} \text{ m}$$

$$\approx 0.2 \text{ m}$$

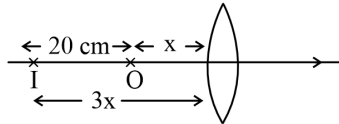
Q.44. Find the focal length of a convex lens if the image is 3 times virtually magnified. Distance between the object and the image is 20 cm.



- A) 12 cm B) 15 cm
C) 18 cm D) 20 cm

Answer: 15 cm

Solution: Let's consider the following diagram:



Let's consider that the object distance is x , as shown in the above figure.

The magnification for the given scenario can be written as

$$m = \frac{v}{u} \dots (1)$$

From equation (1), it follows that

$$\begin{aligned} +3 &= \frac{v}{u} \\ \Rightarrow v &= 3u \\ &= 3(-x) = -3x \end{aligned}$$

According to the problem,

$$\begin{aligned} 3x - x &= 20 \\ \Rightarrow x &= 10 \text{ cm} \end{aligned}$$

The formula to calculate the focal length of the lens is given by

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \dots (2)$$

Equation (2) implies that

$$\begin{aligned} \frac{1}{-3x} - \frac{1}{-x} &= \frac{1}{f} \\ \Rightarrow \frac{1}{f} &= \frac{2}{3x} \\ \Rightarrow f &= \frac{3x}{2} \\ &= \frac{3 \times 10}{2} \text{ cm} \\ &= 15 \text{ cm} \end{aligned}$$

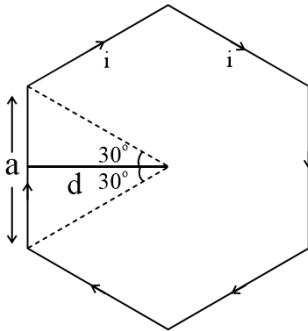
Q.45. Find the magnetic field at the centre of a current carrying regular hexagon wire of side length a and current i .

- A) $\frac{\mu_0 i}{3\pi a}$ B) $\frac{3\mu_0 i}{\pi a}$
C) $\frac{\sqrt{3}\mu_0 i}{\pi a}$
D) $\frac{\mu_0 i}{\sqrt{3}\pi a}$

Answer: $\frac{\sqrt{3}\mu_0 i}{\pi a}$



Solution: Let's consider the following diagram:



With reference to the above diagram, it can be written that

$$\begin{aligned}\tan 30^\circ &= \frac{\left(\frac{a}{2}\right)}{d} \\ \Rightarrow d &= \frac{a}{2 \tan 30^\circ} \\ &= \frac{\sqrt{3}a}{2}\end{aligned}$$

Thus, the magnetic field at the centre due to one side of the hexagon can be written as

$$\begin{aligned}B &= \frac{\mu_0 i}{4\pi d} [\sin 30^\circ + \sin 30^\circ] \\ &= \frac{\mu_0 i}{4\pi \times \frac{\sqrt{3}a}{2}} \times 2 \times \frac{1}{2} \\ &= \frac{\mu_0 i}{2\sqrt{3}\pi a}\end{aligned}$$

Hence, the net magnetic field at the centre due to the entire hexagon is given by

$$\begin{aligned}B_n &= 6B \\ &= 6 \times \frac{\mu_0 i}{2\sqrt{3}\pi a} \\ &= \frac{\sqrt{3}\mu_0 i}{\pi a}\end{aligned}$$

Q.46. Position of a particle moving along x -axis is given by $x = (6t^3 - t^2 - t)$ m. Find the magnitude of the speed(in m s^{-1}) of the particle when its acceleration becomes zero.

- | | |
|--------------------|--------------------|
| A) $\frac{14}{15}$ | B) $\frac{17}{15}$ |
| C) $\frac{18}{19}$ | D) $\frac{19}{18}$ |

Answer: $\frac{19}{18}$



Solution: We know that $R = R_0 A^{1/3}$

Given :

$$A_X = 64$$

$$R_X = 4.8 \text{ fermi}$$

$$R_Y = 4 \text{ fermi}$$

Using the above equation, we can write

$$\frac{R_Y}{R_X} = \left(\frac{A_2}{64} \right)^{1/3}$$

$$\Rightarrow \left(\frac{4}{4.8} \right)^3 = \frac{A_2}{64}$$

$$\Rightarrow A_2 = \left(\frac{5}{6} \right)^3 \times 64 \approx 37$$

Q.49. Dimension of angular momentum is

A) $[ML^2T]$ B) $[ML^2T^{-2}]$

C) $[ML^2T^{-1}]$ D) $[ML^{-2}T^1]$

Answer: $[ML^2T^{-1}]$

Solution: Angular momentum(L) of any particle of mass(m) which moves with linear velocity(v) in circular path of radial distance(r) is product of mass, velocity and radial distance.

$$\therefore L = mvr$$

$$\text{Dimension of angular momentum} = [M] [LT^{-1}] [L] = [ML^2T^{-1}]$$

Q.50. On increasing temperature, the elasticity of a material :

- A) Increases B) Decreases
C) Remains constant D) May increase or decrease

Answer: Decreases

Solution: When the temperature increases, the thermal motion of atoms increases violently. This leads to the weakening of the intermolecular forces.

Hence, it'll be easier for the material to strain and as the modulus of elasticity is inversely proportional to strain, it will decrease as strain increases.

Hence, the modulus of elasticity decreases as temperature increases and vice versa.

Q.51. A vernier calliper has 10 main scale divisions coinciding with 11 vernier scale division and equals 5 mm. The least count of the device is :

A) $\frac{5}{110}$ mm B) $\frac{4}{110}$ mm

C) $\frac{3}{110}$ mm D) $\frac{2}{110}$ mm

Answer: $\frac{5}{110}$ mm



Solution: Given here,

$$1 \text{ M. S. D.} = \frac{5}{10} \text{ mm} = 0.5 \text{ mm.}$$

Now, 11 divisions of vernier scale = 10 divisions of main scale.

$$\text{Then, } 1 \text{ V. S. D.} = \left(\frac{10}{11}\right) \text{ M. S. D.}$$

As we know, the difference between one main scale division and one vernier scale division is known as the least count.

$$\text{So, L. C.} = 1 \text{ M. S. D.} - 1 \text{ V. S. D.}$$

$$\text{L. C.} = 1 \text{ M. S. D.} - \left(\frac{10}{11}\right) \text{ M. S. D.}$$

$$= \left(\frac{1}{11}\right) \text{ M. S. D.}$$

$$= \frac{1}{11} \times 0.5 \text{ mm} = \frac{5}{110} \text{ mm}$$

Hence, the least count of callipers is $\frac{5}{110} \text{ mm}$.

Q.52. A gas undergoes a thermodynamic process from state (P_1, V_1, T_1) to state (P_2, V_2, T_2) . For the given process $PV^{\frac{3}{2}} = \text{constant}$, find the work done by the gas.

- A) $\frac{P_2V_2 - P_1V_1}{2}$ B) $\frac{P_1V_1 - P_2V_2}{2}$
 C) $\frac{3(P_1V_1 - P_2V_2)}{2}$ D) $2(P_1V_1 - P_2V_2)$

Answer: $2(P_1V_1 - P_2V_2)$

Solution: The process equation for a polytropic process is,
 $PV^x = \text{constant}$.

The given equation is $PV^{\frac{3}{2}} = C$, which is an equation of polytropic process.

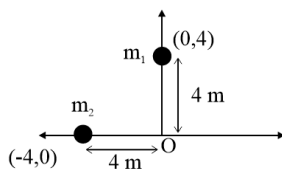
Work done in polytropic process is

$$W = \frac{P_fV_f - P_iV_i}{1-x}$$

$$\Rightarrow W = \frac{P_2V_2 - P_1V_1}{1-\frac{3}{2}}$$

$$= 2(P_1V_1 - P_2V_2)$$

Q.53. Two particles each of mass 2 kg are placed as shown in $x - y$ plane. If the distance of centre of mass from origin is $\frac{4\sqrt{2}}{x} \text{ m}$, find x .



Answer: 2



Solution: With reference to the given diagram, the x – coordinate of the centre of mass is given by

$$X = \frac{2 \times 0 + 2 \times (-4)}{2+2} \text{ m} \\ = -2 \text{ m}$$

Similarly, the y – coordinate of the centre of mass is given by

$$Y = \frac{2 \times 4 + 2 \times 0}{2+2} \text{ m} \\ = 2 \text{ m}$$

Thus, the required distance (r) of the centre of mass from the origin is given by

$$r = \sqrt{X^2 + Y^2} \\ = \sqrt{(-2)^2 + 2^2} \\ = 2\sqrt{2} \\ = \frac{4\sqrt{2}}{2} \text{ m}$$

Hence, $x = 2$.

Q.54. The length of a seconds pendulum, if it is placed at height $2R$ from the surface of the earth (R : radius of earth) is $\frac{10}{x\pi^2}$ m .
Find x .

Answer: 9



Solution: If g and g' are the accelerations due to gravity on the surface and at a distance $2R$ from the surface of the Earth, it can be written that

$$g = \frac{GM}{R^2} \dots (1)$$

And,

$$\begin{aligned} g' &= \frac{GM}{(R+2R)^2} \\ &= \frac{GM}{9R^2} \dots (2) \end{aligned}$$

From equations (1) and (2), it follows that

$$g' = \frac{g}{9}$$

The formula to calculate the time period of a simple pendulum on the surface of the Earth is

$$T = 2\pi\sqrt{\frac{l}{g}} \dots (3)$$

From equation (3),

$$\begin{aligned} l &= g \frac{T^2}{4\pi^2} \\ &= 10 \times \frac{2^2}{4\pi^2} \text{ m} \\ &= \frac{10}{\pi^2} \text{ m} \end{aligned}$$

And, that at a distance $2R$ from the surface of the Earth is

$$T = 2\pi\sqrt{\frac{l'}{g'}} \dots (4)$$

Equations (3) and (4) imply that

$$\begin{aligned} 2\pi\sqrt{\frac{l}{g}} &= 2\pi\sqrt{\frac{l'}{g'}} \\ \Rightarrow l' &= \frac{g'}{g} l \\ &= \frac{1}{9} \times \frac{10}{\pi^2} \text{ m} \\ &= \frac{10}{9\pi^2} \text{ m} \end{aligned}$$

Hence, $x = 9$.

Q.55. If the de-Broglie wavelength of proton and alpha particle is the same, then find the ratio of their speeds.

Answer: 4

Solution: De-broglie wavelength of the particle is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \text{ where, } v \text{ is the speed of the particle.}$$

Given that, the De-broglie wavelength is same for both proton and α -particle.

Mass of α -particle = $4 m_p$ & Mass of proton = m_p

$$\begin{aligned} \lambda_\alpha &= \lambda_p \\ \Rightarrow \frac{h}{(4m_p)v_\alpha} &= \frac{h}{m_p v_p} \\ \Rightarrow \frac{v_p}{v_\alpha} &= 4 \end{aligned}$$

Q.56. Current in a wire is given by $i = (3t^2 + 4t^3)$ A. Find the amount of charge(in C) flown between $t = 1$ s to $t = 2$ s through wire.

Answer: 22



Solution: Let dq be the charge that passes in a small interval of time dt , then $dq = i dt = (3t^2 + 4t^3) dt$.

Hence, the total charge passed between $t = 1$ s and $t = 2$ s is

$$q = \int_1^2 (3t^2 + 4t^3) dt = \left[\frac{3t^3}{3} + \frac{4t^4}{4} \right]_1^2$$

$$\Rightarrow q = (2^3 + 2^4) - (1^3 + 1^4)$$

$$\Rightarrow q = 8 + 16 - 2 = 22 \text{ C, will be the amount of charge flown through the wire.}$$