JEE Main

31st Jan Shift 1

Questions

Q.1. Which of the following options contain amphoteric oxide(s) only?

A) SnO_2 , SiO

B) SnO_2, PbO_2

C) SiO_2

D) CO, SiO

Answer: SnO_2, PbO_2

Solution: All members of group 14 when heated in oxygen form oxides. There are mainly two types of oxides, i.e., monoxide and dioxide of formula MO and MO_2 respectively. SiO only exists at high temperature. Oxides in higher oxidation states of elements are generally more acidic than those in lower oxidation states. The dioxides CO_2, SiO_2 and GeO_2 are acidic, whereas SnO_2 and PbO_2 are amphoteric in nature. Among monoxides, CO is neutral.

Q.2. Find out the final product C

$$CH_3-CH_2-CH_2-Br \xrightarrow[KOH]{alc} A \xrightarrow{HBr} B \xrightarrow[KOH]{aq} C$$

A) Propan-1-ol

B) Propane-2-ol

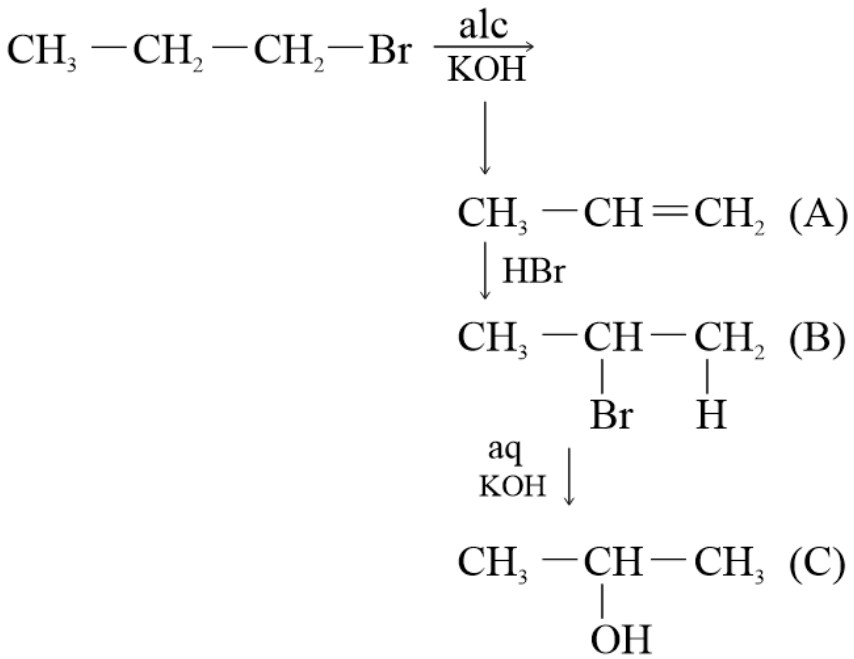
C) Propene

D) Propane

Answer: Propane-2-ol

Solution: 1-bromopropane on reaction with alcoholic KOH gives propene. Propene converts to 2-bromopropane with HBr according to Markonikov's rule. 2-bromopropane converts to propan-2-ol with aqueous KOH.

The reaction will proceed as follows,



Hence option B is the answer.

Q.3. Colour of Lead (II) iodide formed in the confirmation test for Pb^{2+} ion

A) Yellow

B) Red

C) Cryatalline white

D) None of the above

Answer: Yellow

Solution: If the colorless potassium iodide solution (or any other source of iodide ions in solution) to a solution of lead(II) nitrate, a bright yellow precipitate of lead(II) iodide is produced.

Q.4. Which of the following compound is white in colour?

A) $ZnSO_4$ B) $CuSO_4$

C) $FeSO_4$ D) $FeCl_3$

Answer: $ZnSO_4$

Solution: $ZnSO_4$ is white in colour due to absence of unpaired electron.

Copper sulphate crystals are blue in color.

Iron sulphate is blue-green

Ferric chloride is an orange to brown-black solid.

Henc, the answer is option A.

Q.5. On which factor, electrical conductivity of the cell does not depend

A) Concentration of electrolyte B) Amount of electrolyte added

C) Nature of electrode D) Temperature

Answer: Nature of electrode

Solution: Electrical conductivity is nothing but the measure of the capability of the material to pass the flow of electric current.

The electronic conductivity depend on

1. Nature of the added electrolyte

2.The size of the ions produced and their hydration

3.The number of electrons in the valence shell of atoms of metal

4. Concentration and amount of the electrolyte added.

5. Temperature.

It does not depend on nature of the electrode.

Q.6. Decreasing order of electron gain enthalpy of the following elements (magnitude only).

1-Sulphur, 2-Bromine, 3-Fluorine, 4-Argon

A) $1 > 2 > 3 > 4$ B) $3 > 2 > 1 > 4$

C) $4 > 1 > 2 > 3$ D) $3 > 4 > 1 > 2$

Answer: $3 > 2 > 1 > 4$

Solution: Argon will have the least electron gain enthalpy as it has a stable electronic configuration.

Out of Fluorine and Bromine, Fluorine will have more electron gain enthalpy as when we go down a group, electron gain enthalpy decreases.

Hence, the order will be $3 > 2 > 1 > 4$

Q.7. Assertion: Noble gas have very high boiling point

Reason: Noble gas have weak dispersion forces.

A) Assertion is correct, Reason is wrong B) Assertion is wrong, Reason is correct.

C) Both Assertion and Reason are correct. D) Both Assertion and Reason are wrong.

Answer: Assertion is wrong, Reason is correct.

Solution: Due to minimal dispersion forces, noble gases have relatively low boiling points.This indicates that only weak van der Waals interactions or weak London dispersion forces exist between the atoms of noble gases in the liquid or solid state.In general, the boiling and melting points increase from He to Rn as the van der Waals force increases due to an increase in the size of the atom on going down the group.

Hence Assertion is wrong, Reason is correct.

Q.8. Which of the following gives a positive deviation from Raoult's Law?

A) $CHCl_3 + C_6H_5NH_2$ B) $CHCl_3 + CH_3COCH_3$

C) $C_2H_5OH + H_2O$

D) None of the above

Answer: $C_2H_5OH + H_2O$

Solution: If the vapour pressure of a solution is higher than what is expected from Raoult's law, it is called a positive deviation from Raoult's law. A solution of ethyl alcohol and water shows a positive deviation from Raoult's law because there is hydrogen bonding present in the ethyl alcohol solution and water molecules tend to occupy the space between them due to which some hydrogen bonds break.

Options A and B are examples of negative deviation from Raoult's law.

Q.9. Magnetic behaviour of Ni^{2+} (Coordination number 4) with strong field ligand:

A) Diamagnetic

B) Paramagnetic

C) Ferrimagnetic

D) Ferromagnetic

Answer: Diamagnetic

Solution: In diamagnetic complexes, all electrons are paired.

In the presence of a strong field ligand like CN^- ions, all the electrons are paired up in Ni^{2+} ion. The empty $3d$, $3s$ and two $4p$ orbitals undergo dsp^2 hybridization to make bonds with CN^- ligands in square planar geometry. Thus, $[Ni(CN)_4]^{2-}$ is diamagnetic.

Hence, the answer is A.

Q.10. Which of the following does not give colour with conc. sulphuric acid?

A) $NaBr$

B) CaF_2

C) $NaNO_3$

D) I^-

Answer: CaF_2

Solution: On heating $NaBr$ with concentrated sulphuric acid gives brown colour bromine vapours.

On heating $NaNO_3$ with concentrated sulphuric acid will release a brown colour NO_2 gas

On heating I^- with concentrated sulphuric acid gives a dark purple colour.

On heating with concentrated sulphuric acid with CaF_2 does not give any colour because it produces colourless hydrogen fluoride. Hence option B is the answer.

Q.11. The Adsorption method is used for purification in:

A) Distillation

B) Sublimation

C) Extractional method

D) Chromatography

Answer: Chromatography

Solution: Chromatography is used to separate a mixture of chemicals in a liquid or gaseous form by virtue of differences in absorbency. This technique is based on the principle of selective adsorption.
Adsorption refers to the collecting of molecules on the external surface or internal surface (walls of capillaries or crevices) of solids or by the surface of liquids.

Hence, option D is correct.

Q.12. Which of the following has six electrons in Carbon?

A) Carbocation

B) Carbanion

C) Carbon free radical

D) None of the above.

Answer: Carbocation

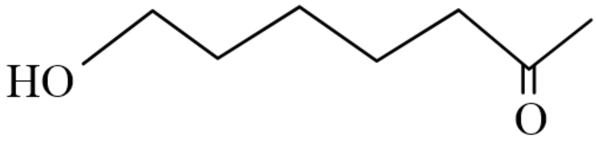
Solution: Carbanion has 8 electrons, example is CH_3^-

Carbon-free radical has 7 electrons, example is $\overset{\odot}{C}H_3$.

Carbocation has 6 electrons in it, example is CH_3^+.

The positively charged carbon atom in a carbocation is a "**sextet**", i.e. it has only six electrons in its outer valence shell instead of the eight valence electrons, which ensures maximum stability (octet rule).

Q.13. IUPAC name of the given compound is:



A) 1-Hydroxyheptan-2-one

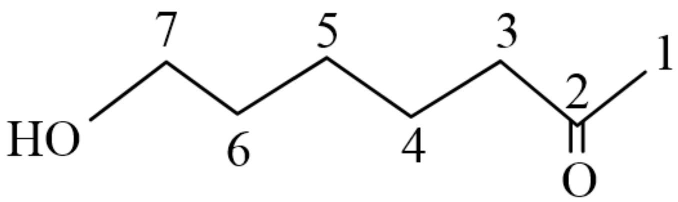
B) 7-Hydroxyheptan-2-one

C) 2-Oxoheptan-7-ol

D) None of the above

Answer: 7-Hydroxyheptan-2-one

Solution:



- The longest continuous carbon chain contain 7 carbon hence, root word is hept.
- Hydroxy group is attached at 7th carbon.
- The major functional group is keto group, hence, the secondary suffix is one.

Hence, the IUPAC name of the given compound is 7-Hydroxyheptan-2-one

Q.14. Statement 1: pka of phenol is 10 and pka of ethyl alcohol is 15.6

Statement 2: Ethyl alcohol is more acidic than phenol

A) Both statements 1 and 2 are correct

B) Both statements 1 and 2 are incorrect

C) Statement 1 is correct and 2 is incorrect

D) Statement 1 is incorrect and 2 is correct

Answer: Statement 1 is correct and 2 is incorrect

Solution:

$$\underset{\text{Phenol}}{\mathrm{Ph-OH}} \rightarrow \underset{\text{Phenoxide ion}}{\mathrm{Ph-O^{-}}} + \mathrm{H^{+}}$$
$$\underset{\text{Ethanol}}{\mathrm{Et-OH}} \rightarrow \underset{\text{Ethoxide ion}}{\mathrm{Et-O^{-}}} + \mathrm{H^{+}}$$

When a molecule of phenol loses a proton, it forms a phenoxide ion which is stabilized by resonance as the negative charge is delocalized over the aromatic nucleus. No such resonance is present when an ethanol loses a proton to form an ethoxide ion. Hence, phenol is more acidic than ethyl alcohol or ethanol.

Hence, the answer is option C.

Q.15. Number of stereoisomers of $[\mathrm{Pt(en)_2Cl_2}]$

A) 3

B) 2

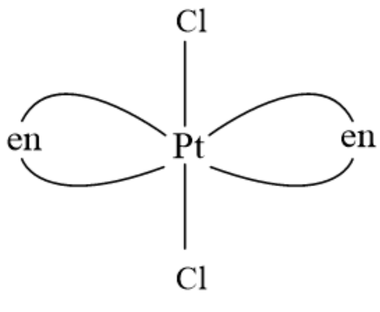
C) 5

D) 4

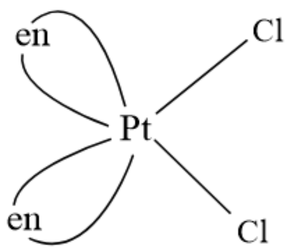
Answer: 3

Solution: The name of this complex is Dichloridobis(ethane-$1, 2$-diammine)platinum(II).

Geometrical isomers are



trans isomer



cis isomer

Hence, total geometrical isomers=2

Out of cis and trans isomers, cis isomer exhibit optical isomerism. Hence, cis isomer has optical isomers.

Number of stereoisomers of $\left[Pt(en)_2 Cl_2\right]$=$2+1=3$

Q.16. How many of the following can be used as electrodes in batteries?

Zinc, Zinc-mercury amalgam, Lead, Graphite.

A) 4
B) 3
C) 1
D) 2

Answer: 4

Solution: Natural graphite has found uses in zinc-carbon batteries, in electric motor brushes, and various specialised applications.

Zinc-mercury amalgam is used as an electrode in button batteries.

Zinc and lead are also used as electrodes in batteries.

Q.17. The spin only magnetic momentum of complex ion $\left[Ni\left(NH_3\right)_6\right]^{2+}$ is (in BM)

A) 2.8 BM
B) 1.73 BM
C) 3.9 BM
D) 0 BM

Answer: 2.8 BM

Solution: The oxidation state of Nickel in the complex is +2. The electronic configuration of Ni^{2+} is $3d^8$. Here, pairing of electrons does not take place, as it cannot provide two inner d-orbitals after pairing. Hence, the number of unpaired electrons are 2.

The spin only magnetic momentum can be calculated as follows,

$\mu=\sqrt{n(n+2)} BM=\sqrt{2(2+2)}=2.83$ BM

Q.18. $\left[Fe(SCN)\right]^{2+}(aq) \rightleftharpoons Fe^{3+}(aq)+SCN^{-}(aq)$

What is the equilibrium constant of the above reaction?

A) $K_c = \frac{[Fe^{3+}]^3[SCN^-]}{[[Fe(SCN)]^{2+}]}$

B) $K_c = \frac{[Fe^{3+}]^3[SCN^-]}{[[Fe(SCN)]^{2+}]^2}$

C) $K_c = \frac{[Fe^{3+}][SCN^-]}{[[Fe(SCN)]^{2+}]}$

D) $K_c = \frac{[Fe^{3+}][SCN^-]^2}{[[Fe(SCN)]^{2+}]}$

Answer: $K_c = \frac{[Fe^{3+}][SCN^-]}{[[Fe(SCN)]^{2+}]}$

Solution: The equilibrium constant of a chemical reaction is the value of its reaction quotient at chemical equilibrium. It is the ratio of products to reactants at equilibrium.

$[Fe\,(SCN)]^{2+}(aq) \rightleftharpoons Fe^{3+}(aq) + SCN^-(aq)$

The equilibrium constant expression for the equilibria:

$$K_c = \frac{[Fe^{3+}][SCN^-]}{[[Fe(SCN)]^{2+}]}$$

Q.19. How many of the following compounds have sp^3 hybridised central atom?

$H_2O,\ NH_3,\ SiO_2,\ SO_2,\ CO$ and BF_3

Answer: 3

Solution: H_2O has sp^3 hybridisation and angular shape due to 2 lone pair of electrons.

NH_3 has sp^3 hybridised nitrogen atom and pyramidal shape due to 1 lone pair of electrons.

SiO_2 has sp^3 hybridised atoms, it is a network solid and the shape is tetrahedral around each silicon atom.

SO_2 has sp^2 hybridised sulphur atom and the shape is V-shaped due to 1 lone pair of electrons.

CO has sp hybridised carbon atom and shape is linear.

BF_3 has sp^2 hybridised boron atom and the shape is trigonal planar.

Hence, the answer is 3.

Q.20. If one Faraday of electricity is used in the discharging of Cu^{2+}, then find the mass in gm of Cu deposited.

Answer: 32

Solution: One Faraday of electricity means it is equivalent to one mole of electrons charge.

$Cu^{2+} + 2e^- \rightarrow Cu$

mass of Cu= $63.54\ g/mol$

Hence, for $2F$ of electricity, $63.54\ g$ of copper can be deposited.

For one Faraday of electricity, the copper deposited $= \frac{63.5}{2} = 31.75 \cong 32\ g$

Q.21. Mole of CH_4 required for the formation of $22\ g$ of CO_2 is $m \times 10^{-2}$, The value of m is:

Answer: 50

Solution: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Gram molecular mass of $CO_2 = 12 + 2(16) = 44\ g/mol$

From the reaction, it is clear that 1 mole of methane on complete combustion produces $44\ g$ (1 mole) of carbon dioxide.

Therefore moles of CH_4 required to produce $22\ g$ of CO_2 are:

$$= \frac{1}{44} \times 22 = 0.5\ mol\ = 50 \times 10^{-2}\ mol$$

Hence, $m = 50$

Q.22. The total number of different alkanes are formed when the following mixture is subjected to electrolysis:

$CH_3COONa_{(aq)}$ and $C_2H_5COONa_{(aq)}$ (do not consider disproportionation reaction)

Answer: 3

Solution: An aqueous solution of sodium or potassium salt of carboxylic acid is electrolysed in this reaction, resulting in the dissociation of the salt into a carboxylate ion and sodium or potassium ions.

The total number of different alkanes are formed when the following mixture is subjected to electrolysis:

$CH_3COONa_{(aq)}$ and $C_2H_5COONa_{(aq)}$

In this ethane, butane and propane are formed.

So, the total number of alkanes formed here will be 3.

Q.23. How many of the following statements are true?

$1)$ Chromate ion is square planar.

$2)$ Green manganate ion is diamagnetic.

$3)$ Dicromate can be prepared using Chromate.

$4)$ Dark green $KMnO_4$ disproportionates in acidic medium and neutral medium.

$5)$ For d- block elements ionic character decreases for increasing oxidation number for metal in oxides.

Answer: 2

Solution: Dichromates are generally prepared from chromate, which in turn are obtained by the fusion of chromite ore $(FeCr_2O_4)$ with sodium or potassium carbonate in free access of air. The reaction with sodium carbonate occurs as follows:

$$4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8\ Na_2CrO_4 + 2\ Fe_2O_3 + 8\ CO_2$$

The yellow solution of sodium chromate is filtered and acidified with sulphuric acid to give a solution from which orange sodium dichromate, $Na_2Cr_2O_7 . 2H_2O$ can be crystallised.

$$2Na_2CrO_4 + 2\ H^+ \rightarrow Na_2Cr_2O_7 + 2\ Na^+ + H_2O$$

The chromate ion is tetrahedral. The manganate and permanganate ions are tetrahedral; the green manganate is paramagnetic with one unpaired electron but the permanganate is diamagnetic.

For d- block elements, the tendency to form ionic compounds decreases with an increase in the oxidation number of the metal. Higher oxidation states give rise to covalent compounds.

Q.24. If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in R$, then $2f(0)+f'(0)$ is equal to

A) 21 B) 42

C) 24 D) 12

Answer: 42

Solution: Given: $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$

$\Rightarrow f(0)=\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}$

$\Rightarrow f(0)=0+4+8$

$\Rightarrow f(0)=12$

$\Rightarrow f'(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ 3x^2-1 & 0 & 2x \end{vmatrix}+\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 6x & 2 & 3x^2 \\ x^3-x & 4 & x^2-2 \end{vmatrix}+\begin{vmatrix} 3x^2 & 4x^2 & 3 \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$

$\Rightarrow f'(0)=\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}+\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}$

$\Rightarrow f'(0)=0-(0+6)+0+0+0+0+3(8)$

$\Rightarrow f'(0)=-6+24=18$

$\Rightarrow 2f(0)+f'(0)=42$

Q.25. If $f(x)=\frac{4x+3}{6x-4}$, $x\neq\frac{2}{3}$ and $fof(x)=g(x)$, where $g:R-\left\{\frac{2}{3}\right\}\rightarrow R$ then $gogogo(4)$ is equal to

A) 1 B) 4

C) 2 D) 3

Answer: 4

Solution: Given: $f(x)=\frac{4x+3}{6x-4}$

$\Rightarrow fof(x)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$

$\Rightarrow fof(x)=\frac{16x+12+18x-12}{24x+18-24x+16}$

$\Rightarrow fof(x)=\frac{34x}{34}$

$\Rightarrow fof(x)=x$

$\Rightarrow g(x)=x$

$\Rightarrow gog(x)=x$

$\Rightarrow gogog(x)=x$

$\Rightarrow gogog(4)=4$

Q.26. If the system of linear equation $x-2y+z=-4,\ 2x+\alpha y+3z=5\ \&\ 3x-y+\beta z=3$ has infinity many solutions then the value of $12\alpha+13\beta$ will be

A) 68 B) 58

C) 90 D) 45

Answer: 58

Solution: Given,

The system of linear equations

$x-2y+z=-4$

$2x+\alpha y+3z=5$

$3x-y+\beta z=3$ has infinity many solutions,

So, $\triangle = \triangle_1 = \triangle_2 = \triangle_3$

Now, finding,

$\triangle_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$

$\Rightarrow 5\beta-9+8\beta-36-9=0$

$\Rightarrow 13\beta=54$

And $\triangle_3 = \begin{vmatrix} 1 & -2 & -4 \\ 2 & \alpha & 5 \\ 3 & -1 & 3 \end{vmatrix} = 0$

$\Rightarrow 3\alpha+5+12-30+8+12\alpha=0$

$\Rightarrow 15\alpha=5$

$\Rightarrow 12\alpha=4$

Hence, the value of $12\alpha+13\beta=4+54=58$

Q.27. Sum of the series $\frac{1}{1-3\times1^2+1^4}+\frac{2}{1-3\times2^2+2^4}+\frac{3}{1-3\times3^2+3^4}+\ldots$ upto 10 terms is

A) $\frac{55}{109}$

B) $\frac{45}{109}$

C) $\frac{-45}{109}$

D) $\frac{-55}{109}$

Answer: $\frac{-55}{109}$

Solution: General term of the given series is given by,

$T_n=\frac{n}{1-3n^2+n^4}$

$\Rightarrow T_n=\frac{n}{\left(n^4-2n^2+1\right)-n^2}$

$\Rightarrow T_n=\frac{n}{\left(n^2-1\right)^2-n^2}$

$\Rightarrow T_n=\frac{n}{\left(n^2-1-n\right)\left(n^2-1+n\right)}$

$\Rightarrow T_n=\frac{1}{2}\frac{\left(n^2-1+n\right)-\left(n^2-1-n\right)}{\left(n^2-1-n\right)\left(n^2-1+n\right)}$

$\Rightarrow T_n=\frac{1}{2}\left[\frac{1}{\left(n^2-1-n\right)}-\frac{1}{\left(n^2-1+n\right)}\right]$

$\Rightarrow \sum_{n=1}^{10}T_n=\frac{1}{2}\left[\left(\frac{1}{-1}-\frac{1}{1}\right)+\left(\frac{1}{1}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{11}\right)+\ldots+\left(\frac{1}{89}-\frac{1}{109}\right)\right]$

$\Rightarrow \sum_{n=1}^{10}T_n=\frac{1}{2}\left[-1-\frac{1}{109}\right]$

$\Rightarrow \sum_{n=1}^{10}T_n=\frac{-55}{109}$

Q.28. An urn contains 15 red, 10 white, 60 orange and 15 green balls. 2 balls are taken with replacement. Find the probability that 1 ball is red and other is white.

A) 0.009

B) 0.03

C) 0.012

D) 0.005

Answer: 0.03

Solution: Given,

An urn contains 15 red, 10 white, 60 orange and 15 green balls,

So, let $R \rightarrow 15,\ W \rightarrow 10,\ O \rightarrow 60,\ G \rightarrow 15$

Now total sample space will be $15 + 10 + 60 + 15 = 100$

The required probability of taking 2 in which 1 is red and other is white is given by,

$$P(E) = \frac{15}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{15}{100}$$

$$\Rightarrow P(E) = \frac{30}{1000}$$

$$\Rightarrow P(E) = 0.03$$

Q.29. Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k},\ \vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be 3 vectors. If a vector $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot \left(\hat{i} - \hat{j} - \hat{k}\right)$ is equal to ______.

A) 6

B) 16

C) 23

D) 32

Answer: 32

Solution: Given: $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$

$$\Rightarrow \left(\vec{p} - \vec{c}\right) \times \vec{b} = 0$$

$$\Rightarrow \left(\vec{p} - \vec{c}\right) || \vec{b}$$

$$\Rightarrow \left(\vec{p} - \vec{c}\right) = k\vec{b}$$

$$\Rightarrow \vec{p} = k\vec{b} + \vec{c}$$

$$\Rightarrow \vec{p} = (4k + 1)\hat{i} + (k - 3)\hat{j} + (7k + 4)\hat{k}$$

Now, $\vec{p} \cdot \vec{a} = 0$

$$\Rightarrow 3(4k + 1) + (k - 3) - 2(7k + 4) = 0$$

$$\Rightarrow k = -8$$

$$\Rightarrow \vec{p} = -31\hat{i} - 11\hat{j} - 52\hat{k}$$

$$\Rightarrow \vec{p} \cdot \left(\hat{i} - \hat{j} - \hat{k}\right) = -31 + 11 + 52$$

$$\Rightarrow \vec{p} \cdot \left(\hat{i} - \hat{j} - \hat{k}\right) = 32$$

Q.30. If one of the diameter of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is chord of another circle and whose centre is the point of intersection of the line $2x + 3y = 12$ & $3x - 2y = 5$, then the radius of the circle is

A) 20

B) 6

C) 15

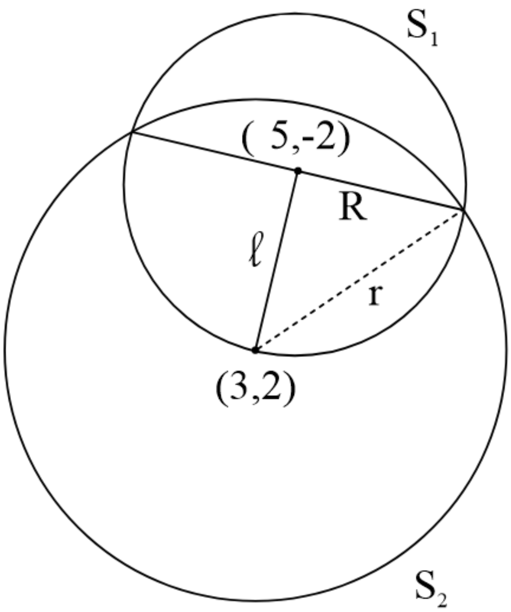
D) $\sqrt{2}$

Answer: 6

Solution: Given,

$S_1 \equiv x^2 + y^2 - 10x + 4y + 13 = 0$ with radius $R = 4$ and its one diameter is chord of other circle S_2 whose centre is given by intersection of $2x + 3y = 12$ & $3x - 2y = 5$ which is $(3, 2)$

Now, plotting the diagram we get,



Now, from above circle the distance between the centres is given by $l = \sqrt{(5-3)^2 + (-2-2)^2} = \sqrt{20}$

Hence, radius of S_2 will be, $r^2 = \left(\sqrt{20}\right)^2 + 4^2 = 20 + 16 = 36$

$\Rightarrow r = 6$

Q.31. Let, $y = y(x)$ be the solution of $\frac{dy}{dx} = \frac{\tan x + y}{\sin x(\sec x - \sin x \tan x)}$, $x \in \left(0, \frac{\pi}{2}\right)$ satisfy the condition $y\left(\frac{\pi}{4}\right) = 2$ then $y\left(\frac{\pi}{3}\right)$ is:

A) $\frac{\sqrt{3}}{2}\log 2 + 2\sqrt{3}$

B) $\frac{\sqrt{3}}{2}\log 3 + 2\sqrt{3}$

C) $\frac{1}{2}\log 3 + 2\sqrt{3}$

D) $\frac{\sqrt{3}}{2}\log 3 + 2$

Answer: $\frac{\sqrt{3}}{2}\log 3 + 2\sqrt{3}$

Solution: Given: $\frac{dy}{dx}=\frac{\tan x+y}{\sin x(\sec x-\sin x\tan x)}$

$\Rightarrow \frac{dy}{dx}=\frac{\tan x+y}{\sin x\left(\frac{1}{\cos x}-\frac{\sin^2 x}{\cos x}\right)}$

$\Rightarrow \frac{dy}{dx}=\frac{\tan x+y}{\sin x\left(\frac{\cos^2 x}{\cos x}\right)}$

$\Rightarrow \frac{dy}{dx}=\frac{\tan x+y}{\sin x\cos x}$

$\Rightarrow \frac{dy}{dx}-\frac{2y}{\sin 2x}=\sec^2 x$

$\Rightarrow \mathrm{IF}=e^{-2\int \operatorname{cosec} 2x dx}$

$\Rightarrow \mathrm{IF}=e^{-\log|\operatorname{cosec} 2x-\cot 2x|}$

$\Rightarrow \mathrm{IF}=\frac{1}{\operatorname{cosec} 2x-\cot 2x}=\operatorname{cosec} 2x+\cot 2x$

$\Rightarrow \mathrm{IF}=\frac{1}{\sin 2x}+\frac{\cos 2x}{\sin 2x}$

$\Rightarrow \mathrm{IF}=\frac{2\cos^2 x}{2\sin x\cos x}$

$\Rightarrow \mathrm{IF}=\cot x$

$\Rightarrow y\times\cot x=\int \cot x\times\sec^2 x dx$

$\Rightarrow y\times\cot x=2\int \operatorname{cosec} 2x dx$

$\Rightarrow y\cot x=\log|\operatorname{cosec} 2x-\cot 2x|+c$

$\Rightarrow 2\cot\frac{\pi}{4}=\log\left|\operatorname{cosec}\frac{\pi}{2}-\cot\frac{\pi}{2}\right|+c$

$\Rightarrow c=2$

$\Rightarrow y\cot x=\log|\operatorname{cosec} 2x-\cot 2x|+2$

$\Rightarrow y\cot\frac{\pi}{3}=\log\left|\operatorname{cosec}\frac{2\pi}{3}-\cot\frac{2\pi}{3}\right|+2$

$\Rightarrow y\times\frac{1}{\sqrt{3}}=\log\left|\frac{2}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right|+2$

$\Rightarrow y\times\frac{1}{\sqrt{3}}=\frac{1}{2}\log 3+2$

$\Rightarrow y=\frac{\sqrt{3}}{2}\log 3+2\sqrt{3}$

Q.32. The value of the limit $\lim_{x\to 0}\frac{e^{|2\sin x|}-|2\sin x|-1}{x^2}$ is

A) 1

B) -1

C) 2

D) Does not exist

Answer: 2

Solution: Given,

$$\lim_{x\to0}\frac{e^{|2\sin x|}-|2\sin x|-1}{x^2}$$

Now using the expansion of $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots\ldots\infty$ we get,

$$=\lim_{x\to0}\frac{\left(1+\frac{|2\sin x|}{1!}+\frac{|2\sin x|^2}{2!}+\ldots..\infty\right)-|2\sin x|-1}{x^2}$$

$$=\lim_{x\to0}\frac{\frac{|2\sin x|^2}{2!}+\frac{|2\sin x|^3}{3!}\ldots..\infty}{x^2}$$

$$=\lim_{x\to0}\frac{2\sin^2 x+\frac{|2\sin x|^3}{3!}\ldots..\infty}{x^2}$$

$$=2\left\{\text{as }\lim_{x\to0}\frac{\sin^2 x}{x^2}=1\text{ and other terms will become zero}\right\}$$

Q.33. The solution of differential equation $y\frac{dx}{dy}=x\left(\log_e x-\log_e y+1\right),\ x>0,\ y>0$ and passing through $(e,1)$ is

A) $2\left|\log_e\frac{x}{y}\right|=y$

B) $\left|\log_e\frac{y}{x}\right|=y^2$

C) $\left|\log_e\frac{x}{y}\right|=y$

D) $\left|\log_e\frac{y}{x}\right|=x$

Answer: $\left|\log_e\frac{x}{y}\right|=y$

Solution: Given: $y\frac{dx}{dy}=x\left(\log_e x-\log_e y+1\right),\ x>0,\ y>0$

$\Rightarrow\frac{dx}{dy}=\left(\frac{x}{y}\right)\left[\log_e\left(\frac{x}{y}\right)+1\right]$

Putting, $x=vy$

$\Rightarrow\frac{dx}{dy}=v+y\frac{dv}{dx}$

$\Rightarrow v+y\frac{dv}{dy}=\left(\frac{vy}{y}\right)\left[\log_e\left(\frac{vy}{y}\right)+1\right]$

$\Rightarrow v+y\frac{dv}{dy}=v\left[\log_e(v)+1\right]$

$\Rightarrow v+y\frac{dv}{dy}=v\log_e(v)+v$

$\Rightarrow y\frac{dv}{dy}=v\log_e(v)$

$\Rightarrow\frac{dv}{v\log_e v}=\frac{dy}{y}$

$\Rightarrow\int\frac{dv}{v\log_e v}=\int\frac{dy}{y}$

Putting, $\log_e v=t$

$\Rightarrow\frac{dv}{v}=dt$

$\Rightarrow\int\frac{dt}{t}=\int\frac{dy}{y}$

$\Rightarrow\log t=\log y+c$

$\Rightarrow\log\left(\log_e\frac{x}{y}\right)=\log y+c$

Using point $(e,1)$

$\Rightarrow\log\left(\log_e e\right)=\log(1)+c$

$\Rightarrow c=0$

$\Rightarrow\log\left(\log_e\frac{x}{y}\right)=\log y$

$\Rightarrow\left|\log_e\frac{x}{y}\right|=y$

Q.34.

Let a be the sum of all coefficients in the expansion of $\left(1-2x+2x^2\right)^{2023}\left(3-4x+2x^3\right)^{2024}$ and $b=\lim_{x\to0}\frac{\int_0^x\frac{\log(1+t)}{t^{2024}+1}dt}{x^2}$. If the equation $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ has a common root, where $c,d,e\in R$. Find $d:c:e$.

A) $1:4:1$

B) $1:1:4$

C) $4:1:1$

D) $1:1:1$

Answer: $1:1:4$

Solution: Given: a is the sum of all coefficients in $\left(1-2x+2x^2\right)^{2023}\left(3-4x+2x^3\right)^{2024}$

$\Rightarrow a=(1-2\times 1+2\times 1)^{2023}(3-4\times 1+2\times 1)^{2024}$

$\Rightarrow a=1 \quad \ldots(i)$

Now, $b=\lim_{x\to 0}\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}$

Using L-Hospital's rule and Newton Leibnitz Theorem, we get

$\Rightarrow b=\lim_{x\to 0}\frac{\log(1+x)}{\left(x^{2024}+1\right)2x}$

$\Rightarrow b=\lim_{x\to 0}\frac{1}{2\left(x^{2024}+1\right)}$

$\Rightarrow b=\frac{1}{2} \quad \ldots(ii)$

Also, $2bx^2+ax+4=0$

$\Rightarrow x^2+x+4=0$, which gives complex conjugates as roots.

Let α and $\overline{\alpha}$ be those roots.

Then, $cx^2+dx+e=0$ will also have α and $\overline{\alpha}$ as roots.

$\Rightarrow d:c:e=1:1:4$

Q.35. The distance of the point $Q(0,2,-2)$ from the line passing through the point $P(5,-4,3)$ and perpendicular to line $\left(-3\hat{i}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+5\hat{k}\right)$, $\lambda\in R$ and $\left(\hat{i}-2\hat{j}+\hat{k}\right)+\mu\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ is

A) $\sqrt{74}$

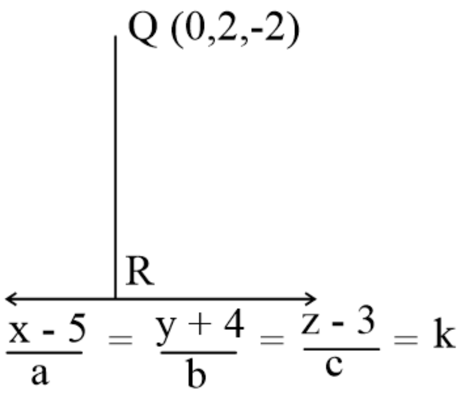
B) $\sqrt{47}$

C) 74

D) 47

Answer: $\sqrt{74}$

Solution: Plotting the diagram of the given data we get,



Now, let a, b, c be the DR's of the line perpendicular to $\left(-3\hat{i} + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$ and $\left(\hat{i} - 2\hat{j} + \hat{k}\right) + \mu\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$.

$\Rightarrow 2a + 3b + 5c = 0, \ -a + 3b + 2c = 0$

Now, solving $2a + 3b + 5c = 0$ & $-a + 3b + 2c = 0$ we get,

$\Rightarrow \frac{a}{-9} = \frac{b}{-9} = \frac{c}{9}$

$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{-1}$

So, the equation will be,

$\Rightarrow \frac{x-5}{1} = \frac{y+4}{1} = \frac{z-3}{-1} \quad \ldots(i)$

Let R be the foot of perpendicular from $Q(0, 2, -2)$ to (i).

$\Rightarrow \frac{x-5}{1} = \frac{y+4}{1} = \frac{z-3}{-1} = k$

$\Rightarrow x = k + 5, \ y = k - 4, \ z = -k + 3$

$\Rightarrow R \equiv (k + 5, \ k - 4, \ -k + 3)$

So, DR's of QR are $(k + 5, \ k - 6, \ 5 - k)$.

Now, using perpendicular condition we get,

$\Rightarrow 1(k + 5) + 1(k - 6) + (-1)(5 - k) = 0$

$\Rightarrow k = 2$

$\Rightarrow R \equiv (7, -2, 1)$

$\Rightarrow QR = \sqrt{49 + 16 + 9}$

$\Rightarrow QR = \sqrt{74}$

Q.36. If S be the set of positive integral values of a for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0 \ \forall \ x \in R$, then the number of elements in S is

Answer: 0

Solution: Given,

$$\frac{ax^2+2\left(a+1\right)x+9a+4}{x^2-8x+32}<0\ \ \forall x\in R$$

For quadratic $x^2-8x+32=0,\ \ D_1=(-8)^2-4\left(32\right)=-64$

Since the discriminant is less than zero and the leading coefficient is positive, this quadratic will always be positive.

Now, solving $ax^2+2\left(a+1\right)x+9a+4<0$

We know that, for a quadratic to be always negative, the coefficient of $x^2<0,\ D<0$.

$\Rightarrow a<0$

But we want positive values.

So, no positive integral value exist.

Q.37. In the expansion of $(1+x)\left(1-x^2\right)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$, the sum of coefficient of $x^3\ \&\ x^{-13}$ is

Answer: 118

Solution: Given,

$$(1+x)\left(1-x^2\right)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$$

$$=(1+x)^2(1-x)\left(\frac{x^3+3x^2+3x+1}{x^3}\right)^5$$

$$=(1+x)^2(1-x)\left(\frac{(1+x)^3}{x^3}\right)^5$$

$$=\frac{(1+x)^{17}(1-x)}{x^{15}}$$

$$=\frac{(1+x)^{17}}{x^{15}}-\frac{(1+x)^{17}}{x^{14}}$$

Now for coefficient of x^3, we will find coefficient of x^{18} in expansion of $(1+x)^{17}$ which is not possible and x^{17} in expansion of $-(1+x)^{17}$ which will be $-{}^{17}C_{17}=-1$

And for coefficient of x^{-13} we will coefficient of x^2 in expansion of $(1+x)^{17}$ which will be ${}^{17}C_2$ and coefficient of x in expansion of $-(1+x)^{17}$ which will be $-{}^{17}C_1$, so the coefficient of x^{-13} will be ${}^{17}C_2-{}^{17}C_1=136-17=119$

Hence, the sum will be $119-1=118$

Q.38. $A=\{1,2,3,4\},\ \ R=\{(1,2),(2,3),(2,4)\},\ \ R\subseteq S$ and S is an equivalence relation and the minimum number of elements to be added to R is n, then the value of n is

Answer: 13

Solution: Given,

$R=\{(1,2),(2,3),(2,4)\}$

Elements required to make R as reflexive are:

$(1,1),(2,2),(3,3),(4,4)$.

Elements required to make R as symmetric are:

$(2,1),(3,2),(4,2)$.

Elements required to make R as transitive are:

$(1,3),(3,1),(1,4),(4,1),(3,4),(4,3)$.

So, the total number of elements required to make R as equivalence is 13.

Q.39. If $ABCD$ is a parallelogram where $A(\alpha, \beta),\ B(1,0),\ C(\gamma, \delta)$ and $D(3,2)$ and $AB = \sqrt{10}$, then the value of $2(\alpha+\beta+\gamma+\delta)$ will be

Answer: 12

Solution: We know that diagonals of a parallelogram bisect each other.

$\Rightarrow$ Mid-point of $AC =$ Mid-point of BD

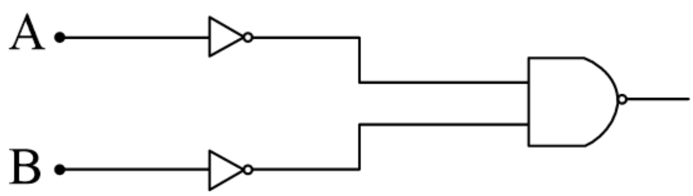
$$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = \left(\frac{1+3}{2}, \frac{0+2}{2}\right)$$

$$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = (2,1)$$

$$\Rightarrow \alpha+\gamma = 4,\ \beta+\delta = 2$$

$$\Rightarrow 2(\alpha+\gamma+\ \beta+\delta) = 12$$

Q.40. Output of the given circuit represents:



A) AND

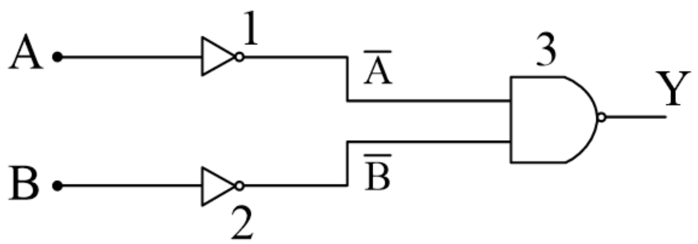
B) OR

C) NOT

D) NOR

Answer: OR

Solution: Let's consider the following diagram:



As can be seen from the above figure, gates 1 and 2 represents NOT gates and gate 3 represents NAND gate.

Using Boolean identity, the final output Y can be found out as follows:

$$Y = \overline{\overline{A} \cdot \overline{B}}$$
$$= \overline{\overline{A}} + \overline{\overline{B}}$$
$$= A + B$$

Hence, the combination of the gates works together as an OR gate.

Q.41. Two charges Q and $3Q$ are kept in a line separated by a distance R. Electric field is zero at a distance x from Q. Find the value of x.

A) $\left(\frac{1-\sqrt{3}}{2}\right)R$

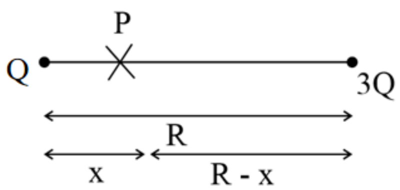
B) $\left(\frac{\sqrt{3}-1}{2}\right)R$

C) $\left(\frac{\sqrt{3}-1}{3}\right)R$

D) $\left(\frac{1-\sqrt{3}}{3}\right)R$

Answer: $\left(\frac{\sqrt{3}-1}{2}\right)R$

Solution: Let's consider the following diagram:



With reference to the above figure, the electric field at P due to Q is given by

$E=\frac{kQ}{x^2}$ $\ldots(1)$

And, the electric field at P due to $3Q$ is given by

$E=\frac{k(3Q)}{(R-x)^2}$ $\ldots(2)$

As the direction of both the fields are opposite, therefore to have zero electric field at P, it follows that

$\frac{kQ}{x^2}=\frac{3kQ}{(R-x)^2}$

$\Rightarrow (R-x)^2=3x^2$

$\Rightarrow 2x^2+2Rx-R^2=0$

$\Rightarrow x=\frac{-2R\pm\sqrt{4R^2-4\times2\times\left(-R^2\right)}}{2\times2}$

$=\frac{-2R\pm\sqrt{12R^2}}{4}$

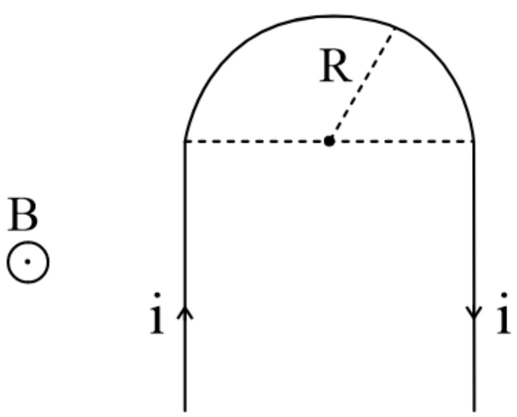
$=\frac{-2R\pm2\sqrt{3}R}{4}$

Since only positive values are allowed for x, it can be written that

$x=\frac{-2R+2\sqrt{3}R}{4}$

$=\frac{\sqrt{3}-1}{2}R$

Q.42. A current carrying wire is placed in an external magnetic field as shown. Find the magnetic force on the given wire.



A) $2iBR\hat{\imath}$

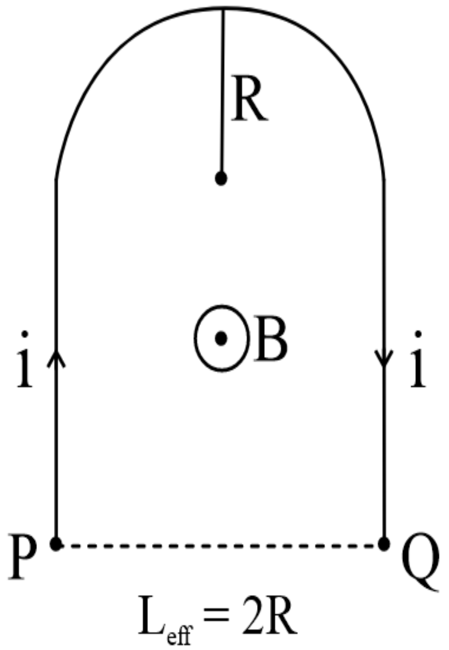
B) $-2iBR\hat{\imath}$

C) $2iBR\hat{\jmath}$

D) $-2iBR\hat{\jmath}$

Answer: $-2iBR\hat{\jmath}$

Solution: Let's consider the following diagram:



If the magnetic force on the wire is to be calculated let's connect the end points of the wire, as shown in the above figure.

Thus, the length between the end points of the wire is

$PQ = 2R$

The formula to calculate the magnetic force on the wire is given by

$\vec{F} = i\vec{L}_{eff} \times \vec{B} \quad \ldots(1)$

From the figure, it is clear that

$\vec{L}_{eff} = 2R\hat{i}$

$\vec{B} = B\hat{k}$

Hence, from equation (1), we have

$\vec{F} = i(2R)\hat{i} \times B\hat{k}$

$= -2iRB\hat{j}$

Q.43. If mass defect in a nuclear reaction is $0.4\ \mathrm{g}$, then find the Q-value of the reaction.

A) $3.16 \times 10^{13}\ \mathrm{J}$

B) $4.16 \times 10^{13}\ \mathrm{J}$

C) $5.16 \times 10^{13}\ \mathrm{J}$

D) $6.16 \times 10^{13}\ \mathrm{J}$

Answer: $3.16 \times 10^{13}\ \mathrm{J}$

Solution: The amount of energy absorbed or released during the nuclear reaction is called Q-value of nuclear reaction.

As $Q = \Delta mc^2$, where c is speed of light.

Therefore,

$Q = \left(0.4 \times 10^{-3}\ \mathrm{kg}\right) \times \left(3 \times 10^{8}\ \mathrm{m\ s^{-1}}\right)^2$

$= 3.6 \times 10^{13}\ \mathrm{J}$

Q.44. A prism has a refractive index $\cot\left(\frac{A}{2}\right)$, where A is the refractive angle of the prism. The minimum deviation due to this prism is

A) $\pi - 2A$

B) $\frac{A}{2}$

C) $\pi - 3A$

D) A

Answer: $\pi - 2A$

Solution: The formula to calculate the refractive index of the material of the prism can be written as

$$\mu = \frac{\sin \frac{A+\delta m}{2}}{\sin \frac{A}{2}} \quad \ldots (1)$$

Simplify equation (1) to obtain the expression for the minimum angle of deviation for the prism.

$$\mu \sin \frac{A}{2} = \sin \left(\frac{A+\delta m}{2}\right)$$
$$\Rightarrow \frac{A+\delta m}{2} = \sin^{-1}\left(\mu \sin \frac{A}{2}\right)$$
$$\Rightarrow \delta_m = 2\sin^{-1}\left(\mu \sin \frac{A}{2}\right) - A \quad \ldots (2)$$

Substitute the given expression for the refractive index into equation (2) and simplify to obtain the required minimum angle of deviation for the prism.

$$\begin{aligned}\delta_m &= 2\sin^{-1}\left(\cot \frac{A}{2} \sin \frac{A}{2}\right) - A \\ &= 2\sin^{-1}\left(\frac{\cos \frac{A}{2}}{\sin \frac{A}{2}} \sin \frac{A}{2}\right) - A \\ &= 2\sin^{-1}\left(\cos \frac{A}{2}\right) - A \\ &= 2\sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{A}{2}\right)\right) - A \\ &= 2\left(\frac{\pi}{2} - \frac{A}{2}\right) - A \\ &= \pi - 2A\end{aligned}$$

Q.45. If the percentage error in measuring length and diameter of a wire is 0.1% each, then the percentage error of the resistance of the wire is:

A) 0.1%

B) 0.2%

C) 0.3%

D) 0.4%

Answer: 0.3%

Solution: The resistance of a wire is given by, $R = \frac{\rho L}{A} = \frac{4\rho L}{\pi d^2}$.

Here, ρ is resistivity, L is length and A is the cross-section area of wire.

Therefore, for very small changes, we can write

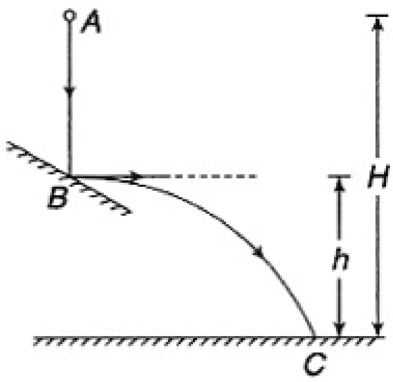
Change in resistance, $\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\frac{\Delta d}{d}$.

Given here, $\frac{\Delta L}{L} \times 100 = \frac{\Delta d}{d} \times 100 = 0.1$.

Hence, the percentage change in resistance is,

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta L}{L} \times 100 + 2\frac{\Delta d}{d} \times 100 = 0.1 + (2 \times 0.1) = 0.3\%.$$

Q.46. A particle P is released from a fixed height H from the ground. It falls on an incline at point B and after striking the plane its velocity becomes horizontal. Find the ratio $\frac{H}{h}$ so that total time of flight becomes maximum.



A) 2

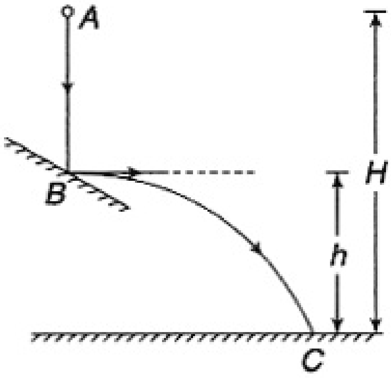
B) 4

C) 6

D) 8

Answer: 2

Solution:



Body is released from A, it hits an incline at B and thereafter moves like a horizontal projectile to fall on the ground at C.

For motion AB

$$-\left(H-h\right)=ut+\frac{1}{2}at^2$$
$$\Rightarrow -\left(H-h\right)=0-\frac{1}{2}gt^2_{AB}$$

$$\Rightarrow t_{AB}=\sqrt{\frac{2(H-h)}{g}}$$

For horizontal projectile, time of flight for BC

$$t_{BC}=\sqrt{\frac{2h}{g}}$$

$$T=t_{AB}+t_{BC}=\sqrt{\frac{2}{g}}\left[\sqrt{H-h}+\sqrt{h}\right]$$

For T to be minimum, $\frac{dT}{dh}=0$

$$\Rightarrow \frac{d}{dh}\left[\sqrt{H-h}+\sqrt{h}\right]=0$$

$$\Rightarrow -\frac{1}{2}\left(H-h\right)^{-\frac{1}{2}}+\frac{1}{2}h^{-\frac{1}{2}}=0$$

$$\Rightarrow \quad h=\frac{H}{2}$$
$$\Rightarrow \frac{H}{h}=2$$

Q.47. In a region of space, the peak electric field due to electromagnetic wave is $50\ \mathrm{N\ C^{-1}}$. Find average energy density in this region.

A) $5.5\times10^{-9}\ \mathrm{J\ m^{-3}}$

B) $2.2\times10^{-8}\ \mathrm{J\ m^{-3}}$

C) $2.1\times10^{-9}\ \mathrm{J\ m^{-3}}$

D) $1.1\times10^{-8}\ \mathrm{J\ m^{-3}}$

Answer: $1.1\times10^{-8}\ \mathrm{J\ m^{-3}}$

Solution: The formula of average energy density is, $U=\frac{1}{2}\varepsilon_0(E_0)^2$.

Given,

$E_0=50\ \mathrm{N\ C^{-1}}\ \&\ \varepsilon_0=8.85\times10^{-12}\ \mathrm{SI\ unit}$

Therefore,

$$U=\frac{1}{2}\times8.85\times10^{-12}\times(50)^2=1.1\times10^{-8}\ \mathrm{J\ m^{-3}}$$

Q.48. For the following equation, force is given by $F=ax^2+bt^{\frac{1}{2}}$. Find the dimension of $\frac{b^2}{a}$.

A) $\left[\mathrm{ML^2T}\right]$

B) $\left[\mathrm{MLT^{-3}}\right]$

C) $\left[ML^3T^{-3}\right]$

D) $\left[MLT^{-1}\right]$

Answer: $\left[ML^3T^{-3}\right]$

Solution: From the principle of dimensional homogeneity, if two quantities are added or subtracted the dimension of both the quantities should be same.

$F = ax^2 + bt^{\frac{1}{2}}$, the dimension of F, ax^2 and $bt^{\frac{1}{2}}$ should be same.

$\Rightarrow \left[F\right] = \left[ax^2\right]$, dimension of F is MLT^{-2}, where F is force.

$$\therefore [a] = \left[\frac{F}{x^2}\right] = \left[\frac{MLT^{-2}}{L^2}\right] = \left[ML^{-1}T^{-2}\right]$$

Similarly, $\left[F\right] = \left[bt^{\frac{1}{2}}\right]$

$$[b] = \left[\frac{F}{t^{\frac{1}{2}}}\right] = \left[\frac{MLT^{-2}}{T^{\frac{1}{2}}}\right] = \left[MLT^{-\frac{5}{2}}\right]$$

Thus,

$$\left[\frac{b^2}{a}\right] = \frac{\left[MLT^{-\frac{5}{2}}\right]^2}{\left[ML^{-1}T^{-2}\right]}$$
$$= \left[ML^3T^{-3}\right]$$

Q.49. An artillery of mass M_1 carries a shell of mass M_2. Initially both are at rest. The artillery fires the shell horizontally on smooth ground. Find the ratio of kinetic energy of artillery and shell.

A) $\frac{M_1}{M_2}$

B) $\frac{M_2}{M_1}$

C) M_1M_2

D) $\frac{1}{M_1M_2}$

Answer: $\frac{M_2}{M_1}$

Solution: If p_1, p_2 are the momentum of the artillery and the shell after firing, then from the conservation of momentum, it follows that

$p_1 - p_2 = 0$
$\Rightarrow p_1 = p_2$

As initially both are at rest, they will move in the opposite directions after firing.

Hence, the ratio of the kinetic energies can be calculated as follows:

$$\frac{K_1}{K_2} = \frac{\frac{(p_1)^2}{2M_1}}{\frac{(p_2)^2}{2M_2}}$$
$$= \frac{M_2}{M_1} \quad [\text{as } p_1 = p_2]$$

Q.50. A block is performing SHM of amplitude A. When it is at $\frac{2A}{3}$ from mean position, its velocity is tripled. Find the new amplitude of motion.

A) $\frac{6A}{5}$

B) $\frac{2A}{5}$

C) $\frac{7A}{3}$

D) $\frac{3A}{5}$

Answer: $\frac{7A}{3}$

Solution: Let the new amplitude of motion is A'. As the angular frequency of oscillation remains the same, it can be written,

$$V_f = 3V_i$$
$$\Rightarrow \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2} = 3\omega\sqrt{(A)^2 - \left(\frac{2A}{3}\right)^2}$$
$$\Rightarrow (A')^2 - \left(\frac{2A}{3}\right)^2 = 9\left[(A)^2 - \left(\frac{2A}{3}\right)^2\right]$$
$$\Rightarrow (A')^2 = \frac{4A^2}{9} + 9 \times \frac{5A^2}{9}$$
$$= \frac{49A^2}{9}$$
$$\Rightarrow A' = \frac{7A}{3}$$

Q.51. Light from two sources of intensities in ratio $1:9$ and phase difference 60° meet on the screen. Find the ratio of net intensity when they are coherent versus when they are not.

A) $\frac{5}{8}$

B) $\frac{8}{5}$

C) $\frac{10}{13}$

D) $\frac{13}{10}$

Answer: $\frac{13}{10}$

Solution: When the light sources are coherent, the net intensity is given by

$$Ic = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$$
$$= I + 9I + 2\sqrt{I \times 9I}\cos 60^\circ$$
$$= 13I$$

When the light sources are non-coherent, the net intensity is given by

$$Inc = I_1 + I_2$$
$$= 10I$$

Hence, the required ratio is

$$\frac{Ic}{Inc} = \frac{13I}{10I}$$
$$= \frac{13}{10}$$

Q.52. Stopping potential is $8\ \mathrm{V}$ if wavelength of incident light is λ and it is $2\ \mathrm{V}$ for 3λ. The threshold wavelength is given by $x \times 10^2\ \mathring{A}$. Find the value of x. Take $hc = 12400\ \mathrm{eV} \cdot \mathring{A}$.

Answer: 124

Solution: The formula to calculate the stopping potential for the first case is given by

$h\nu_1 = \phi + e(V_s)_1$
$\Rightarrow \frac{hc}{\lambda} = \phi + e \times 8$
$= \phi + 8\ \text{eV} \quad \ldots(1)$

For the second case, the equation becomes,

$h\nu_2 = \phi + e(V_s)_2$
$\Rightarrow \frac{hc}{3\lambda} = \phi + e \times 2$
$= \phi + 2\ \text{eV} \quad \ldots(2)$

Dividing equation (1) by equation (2), we have

$\frac{\frac{hc}{\lambda}}{\frac{hc}{3\lambda}} = \frac{\phi + 8\ \text{eV}}{\phi + 2\ \text{eV}}$
$\Rightarrow 3(\phi + 2\ \text{eV}) = \phi + 8\ \text{eV}$
$\Rightarrow 2\phi = 2\ \text{eV}$
$\Rightarrow \phi = 1\ \text{eV} \quad \ldots(3)$

Also, the work function can be written as

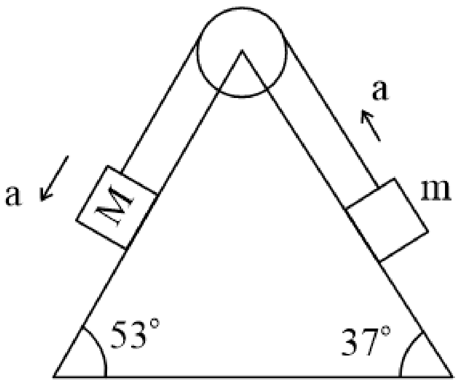
$\phi = \frac{hc}{\lambda_0} \quad \ldots(4)$

Equations (3) and (4) implies that

$\frac{hc}{\lambda_0} = 1\ \text{eV}$
$\Rightarrow \lambda_0 = \frac{hc}{1\ \text{eV}}$
$= \frac{12400\ \text{eV}\cdot\text{Å}}{1\ \text{eV}}$
$= 12400\ \text{Å}$

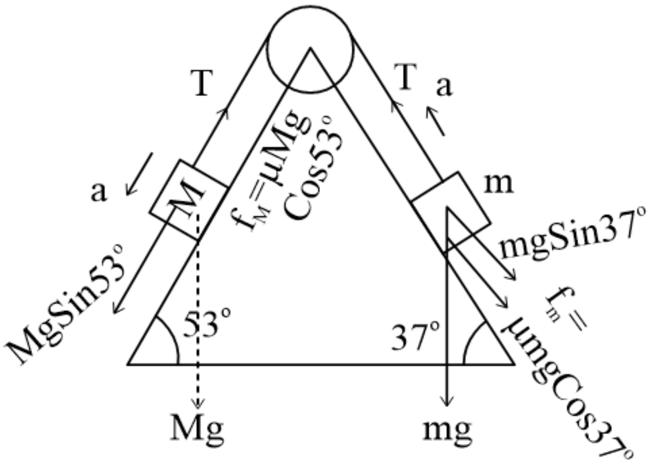
Hence, $x = 124$.

Q.53. For the following diagram, the value of m is given by $\frac{\alpha}{10}\ \text{kg}$, if $M = 10\ \text{kg}$ and the acceleration of the system is $2\ \text{m s}^{-2}$. Find α. Given that $\mu = 0.25$ and $g = 10\ \text{m s}^{-2}$.



Answer: 45

Solution: Let's consider the following diagram:



From the above figure, the equation of motion for mass M can be written as

$Mg\sin 53^\circ - T - \mu Mg\cos 53^\circ = Ma \quad \ldots(1)$

And, the equation of motion for mass m can be written as

$T - mg\sin 37^\circ - \mu mg\cos 37^\circ = ma \quad \ldots(2)$

Adding both the equations, we have

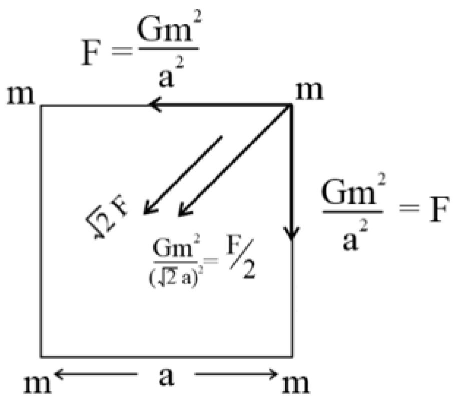
$$(Mg\sin 53^\circ - T - \mu Mg\cos 53^\circ) + (T - mg\sin 37^\circ - \mu mg\cos 37^\circ) = Ma + ma$$
$$\Rightarrow m(a + g\sin 37^\circ + \mu g\cos 37^\circ) = Mg\sin 53^\circ - \mu Mg\cos 53^\circ - Ma$$
$$\Rightarrow m = \frac{M(g\sin 53^\circ - \mu g\cos 53^\circ - a)}{(a + g\sin 37^\circ + \mu g\cos 37^\circ)}$$
$$= \frac{10\text{ kg}\times\left(10\text{ m s}^{-2}\times\sin 53^\circ - 0.25\times 10\text{ m s}^{-2}\times\cos 53^\circ - 2\text{ m s}^{-2}\right)}{2\text{ m s}^{-2} + 10\text{ m s}^{-2}\times\sin 37^\circ + 0.25\times 10\text{ m s}^{-2}\times\cos 37^\circ}$$
$$= \frac{10\times\left(10\times\frac{4}{5} - 0.25\times 10\times\frac{3}{5} - 2\right)}{2 + 10\times\frac{3}{5} + 0.25\times 10\times\frac{4}{5}}\text{ kg}$$
$$= 4.5\text{ kg}$$
$$= \frac{45}{10}\text{ kg}$$

Hence, $\alpha = 45$.

Q.54. Four equal masses m are kept at corners of a square of side a. If net gravitational force on a mass is given by $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$. The value of a in terms of L is given by $a = pL$. Find the value of p.

Answer: 4

Solution: Let's consider the following diagram:



In accordance with the above diagram, the magnitude of the gravitational force on mass m at any corner of the square due to any of the masses situated at the adjacent corners of the square is given by

$$F = \frac{Gm^2}{a^2} \quad \ldots(1)$$

The force on the same mass m due to the mass m situated at the diagonally opposite corner of the square is given by

$$F' = \frac{Gm^2}{\left(\sqrt{2}a\right)^2}$$
$$= \frac{Gm^2}{2a^2} \quad \ldots(2)$$

From equations (1) and (2), it follows that

$$F' = \frac{F}{2} \quad \ldots(3)$$

The net force on m due to all the other three masses can be calculated as follows:

$$Fn = \sqrt{F^2 + F^2} + F'$$
$$= \left(\sqrt{2} + \frac{1}{2}\right)F$$
$$= \left(\frac{2\sqrt{2}+1}{2}\right)\frac{Gm^2}{a^2}$$
$$= \left(\frac{2\sqrt{2}+1}{32}\right)\frac{16Gm^2}{a^2} \quad \ldots(4)$$

Comparing equation (4) with the given expression, it can be concluded that

$$\frac{1}{L^2} = \frac{16}{a^2}$$
$$\Rightarrow a^2 = 16L^2$$
$$\Rightarrow a = 4L$$

Hence, $p = 4$.

Q.55. A ball dropped from height H rebounds up to height h after colliding with horizontal surface. If the coefficient of restitution for collision is $e = \frac{1}{2}$, then the ratio $\frac{H}{h}$ shall be equal to an integer n. Find n.

Answer: 4

Solution: When the ball is dropped from the height H, its velocity just before touching the ground will be

$v=\sqrt{2gH} \quad \ldots(1)$

When the ball bounces back, its velocity just after the collision with the ground will be

$u=\sqrt{2gh} \quad \ldots(2)$

The coefficient of restitution for the given scenario can be written as

$e=\frac{u}{v} \quad \ldots(3)$

From equations (1), (2) and (3), it follows that

$$\frac{1}{2}=\frac{\sqrt{2gh}}{\sqrt{2gH}}$$

$$=\sqrt{\frac{h}{H}}$$

$$\Rightarrow \frac{H}{h}=4$$

Hence, $n=4$.