# JEE Main

## 30th Jan Shift 2

## Questions

Q.1. Why $KMnO_4$ shows colour?

A) Due to d-d transtion

B) Due to metal to ligand charge transfer

C) Due to ligand to metal charge transfer

D) Due to F-centre

Answer: Due to ligand to metal charge transfer

Solution: The purple colour of $KMnO_4$ ion is due to ligand to metal charge transfer.
In $KMnO_4$, the central manganese atom has $+7$oxidation state. As we know, Manganese has the atomic number of $25$ and since the charge is $+7$, the number of total electrons will be $18$. The electronic configuration can be written as$[Ar]3d^0 4s^0$ . Here, also there are no unpaired electrons and hence d-d transition can't occur.
The oxygen in $KMnO_4$ will donate electrons to the vacant orbitals of manganese and as a result a charge transfer spectra occurs and therefore we can say that the colour of $KMnO_4$ also arises from ligand to metal charge transfer spectra.

Q.2. C is added to a solution of A and B, find the mole fraction of C.

A) $n_C/(n_A + n_B + n_C)$

B) $n_C/(n_A \times n_B + n_C)$

C) $n_C/(n_A \times n_C + n_B)$

D) $n_C/(n_A + n_B)$

Answer: $n_C/(n_A + n_B + n_C)$

Solution: Mole fraction represents the number of moles of a particular component in a mixture divided by the total number of moles in the given mixture.

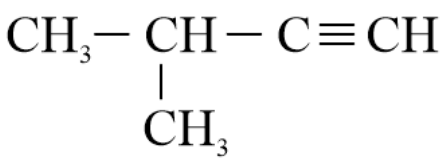
Mole fraction of the component = moles of the component / (moles of mixture)

The molar fraction can be represented by X. If the solution consists of components A , B and C, then the mole fraction of C is

$n_C/(n_A + n_B + n_C)$

Hence option A is the answer.

Q.3. IUPAC name of the compound:



A) 2-Methylbutene

B) 3-Methylbut-1-yne

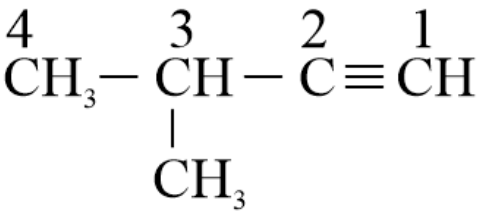
C) 2-Methylbutyne

D) 3-Methylbutane

Answer: 3-Methylbut-1-yne

Solution:

- The longest continuous carbon chain contains four carbon.
- The side chain attached to this chain is methyl group.
- Numbering of the chain starts from the triple bonded carbon.
- The position number of methyl group is 3.



3-Methylbut-1-yne

Hence, the answer is option B.

Q.4. Which reagent on reacting with phenol gives Salicylaldehyde?

A) $CO_2,\ NaOH$

B) $CCl_4,\ NaOH$

C) $CHCl_3, NaOH$

D) $H_2O,\ H^+$

Answer: $CHCl_3, NaOH$

Solution: When phenol gets converted to salicylaldehyde in the presence of $CHCl_3, NaOH$ followed by hydrolysis, the reaction is known as Reimer-Tiemann reaction.

It is an electrophilic substitution reaction, reaction intermediate dichloro carbene is formed and acts as an electrophile.

Hence the correct answer is option C.

Q.5. Find out the correct order of stability for given carbocations.

$(CH_3)_3C^+,\ (CH_3)_2CH^+,\ CH_3CH_2{}^+,\ CH_3^+$

A) $1 > 2 > 3 > 4$

B) $2 > 1 > 3 > 4$

C) $3 > 1 > 2 > 4$

D) $4 > 3 > 2 > 1$

Answer: $1 > 2 > 3 > 4$

Solution: Stability of carbocations depends upon the electron releasing groups. The given carbocations stability can be explained by hyperconjugation effect. More the number of alpha hydrogen, more are number of hyperconjugation structures, hence, more is the stability of carbocation.

Thus, the observed order of stability for carbocations is as follows: tertiary > secondary > primary > methyl

Hence, option A is correct.

Q.6. Which of the following has square pyramidal shape?

A) $PCl_5$

B) $BrF_5$

C) $PF_3$

D) $[Ni(CN)_4]^{2-}$

Answer: $BrF_5$

Solution: In $BrF_5$ molecule:
Number of bond pairs =$5$
Number of lone pairs on central atom =$1$

So, $BrF_5$ molecule is surrounded by six electron pairs. Hybridisation of Br atom in this molecule is $sp^3d^2$. Five positions are occupied by F atoms forming sigma bonds with $sp^3d^2$ hybrid orbitals and one position occupied by lone pair.

So, the molecule has a square pyramidal shape.

Q.7. Arrange the following according to their decreasing oxidising power.

$ClO_4^-,\ IO_4^-, BrO_4^-$

Reduction potential of the above species $E^\circ = 1.19\ V,\ E^\circ = 1.65\ V,\ E^\circ = 1.74\ V$ respectively

A) $ClO_4^- > IO_4^- > BrO_4^-$

B) $IO_4^- > BrO_4^- > ClO_4^-$

C) $BrO_4^- > IO_4^- > ClO_4^-$

D) $BrO_4^- > ClO_4^- > IO_4^-$

Answer: $BrO_4^- > IO_4^- > ClO_4^-$

Solution: The reduction potential of the substance is the ability of the substance to be reduced. So as the reduction potential increases, the reducing ability of the substance increases. It means the oxidising power of the substance (The ability of the substance to make the other substance to lose the electrons increases, which nothing but oxidising power) increases.
So the decreasing order of oxidising power is
$BrO_4^- > IO_4^- > ClO_4^-$.
Hence option $C$ is correct.

Q.8. What is the correct IUPAC name of

$CH_3CH(NH_2)CH_2CH_2CN$

A) $4$-aminopentanenitrile

B) $3$-aminobutanenitrile

C) $3$-aminopentanenitrile

D) $2$-aminobutanenitrile

Answer: $4$-aminopentanenitrile

Solution: The structure of the IUPAC name is

Secondary prefix + Root word + primary suffix + secondary suffix.

In the given compound, the secondary prefix is amino, the root word is pent, the primary suffix is an and the secondary suffix is nitrile.

Hence, the compound is named as

$\overset{5}{C}H_3\overset{4}{C}H\left(NH\right)_2\overset{3}{C}H_2\overset{2}{C}H_2\overset{1}{C}N$

$4$-aminopentanenitrile

Q.9. Shape of $Mn_2(CO)_{10}$

A) Octahedral

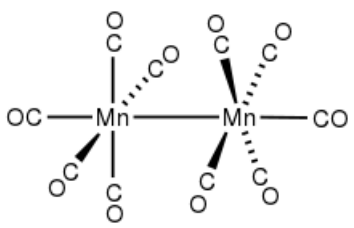
B) Square planar

C) Square Pyramidal

D) None of the above

Answer: Octahedral

Solution: The Steric number is the total number of atoms directly bonded to a central metal atom and the number of lone pairs attached. Steric numbers play an essential role in VSEPR (Valence Shell Electron Pair Repulsion) theory in assessing molecular geometry. It plays a vital role in determining the shape of the molecule.




$Mn - Mn\ \text{bond} = 1\ \text{bond}$
$Mn - CO\ \text{bond} = 5\ \text{bonds}$

Steric number$= 6$

Then the shape will be octahedral.

Q.10. Which of the following solution will have the lowest freezing point?

A) $180$ gm of $CH_3COOH$ in 1L aqueous solution

B) $180$ gm of glucose in 1L aqueous solution

C) $180$ gm of benzoic acid in 1L aqueous solution

D) $180$ gm of sucrose in 1L aqueous solution

Answer: $180$ gm of $CH_3COOH$ in 1L aqueous solution

Solution: Depression in freezing point or elevation in melting point depends upon the number of solute particles rather than the kind of solute particles.

Depression in freezing point ($\Delta T_f$) is directly proportional to number of solute particles.

Also, $\Delta T_f$ is directly proportional to molality.

$\Delta T_f = m \times k_f$

Amongst the given options acetic acid has lowest molecular mass.

The molar mass of acetic acid is $60.05$ g $mol^{-1}$.

Hence if the molecular mass is less, molality will be more, so the lowest freezing point will be shown by $180$ gm of acetic acid in 1L aqueous solution.

Q.11. The correct statement(s) for the given hydrides is/are:

$NH_3,\ PH_3,\ AsH_3,\ SbH_3,\ BiH_3$

1. Basicity decreases

2. Thermal stability decreases

3. Reducing character increases

4. Reducing power $NH_3 > BiH_3$

A) 1, 2, 3

B) 1, 2, 4

C) 4, 2, 3

D) None of the above

Answer: 1, 2, 3

Solution: 1. On moving down the group from N to Bi, the atomic size increases. Consequently, the electron density on the central atom decreases and the basic strength decreases.

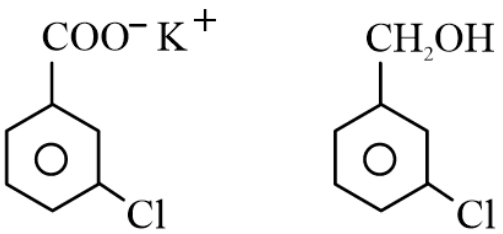
2. With an increase in size, the $M-H$ bond becomes weaker, hence, the thermal stability decreases down the group.

3.The highest bond length will have less bond energy, indicating that it is a strong reducing agent.

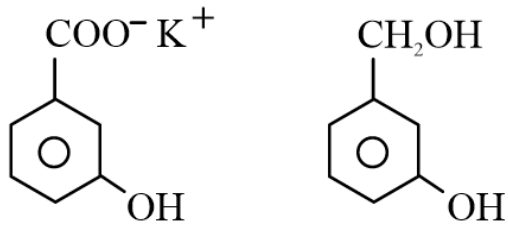
Hence, 1, 2 and 3 are correct.

Q.12. What are the product of the reaction of m-chlorobenzaldehyde with $50\%$ $KOH$

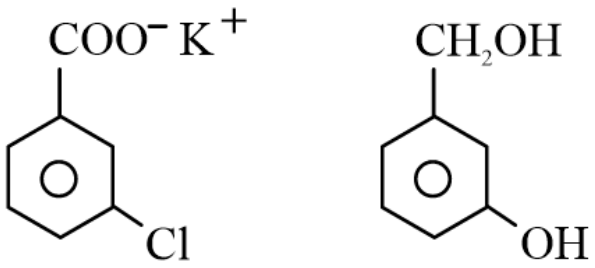
A)



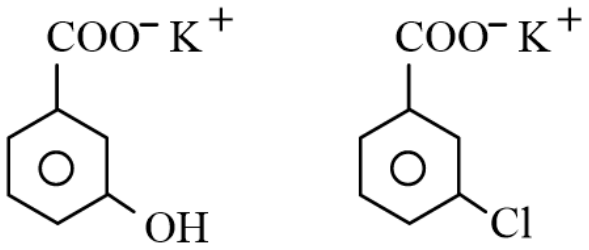

B)



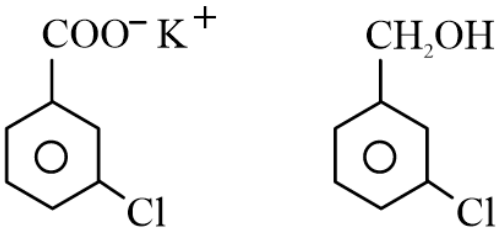
C)



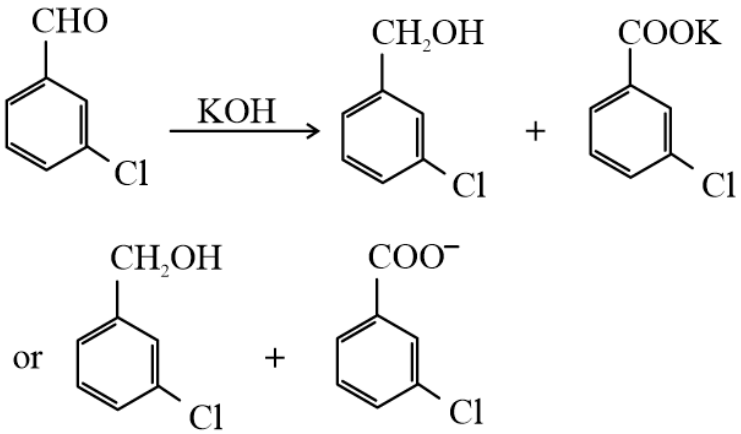

D)




Answer:

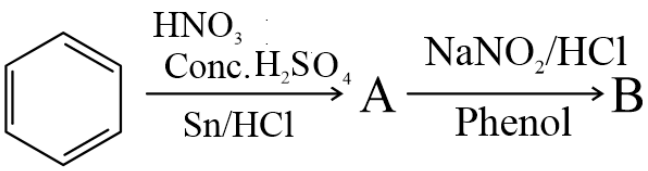



Solution: m-chlorobenzaldehyde undergo disproportionation reaction in the presence of 50% $KOH$. This reaction is known as Cannizzaro's reaction.



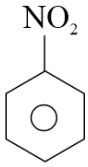

Hence option A is the answer.

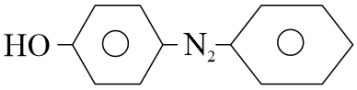
Q.13.




Give A and B.

A)

A = 


B = 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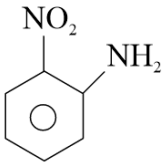
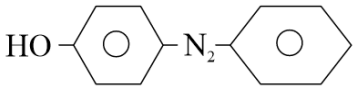
A = 


B = 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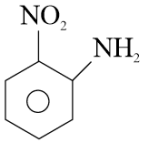
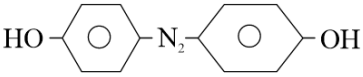
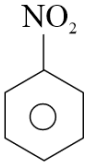
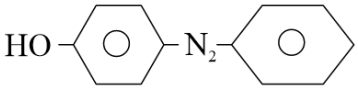
A = 


B = 

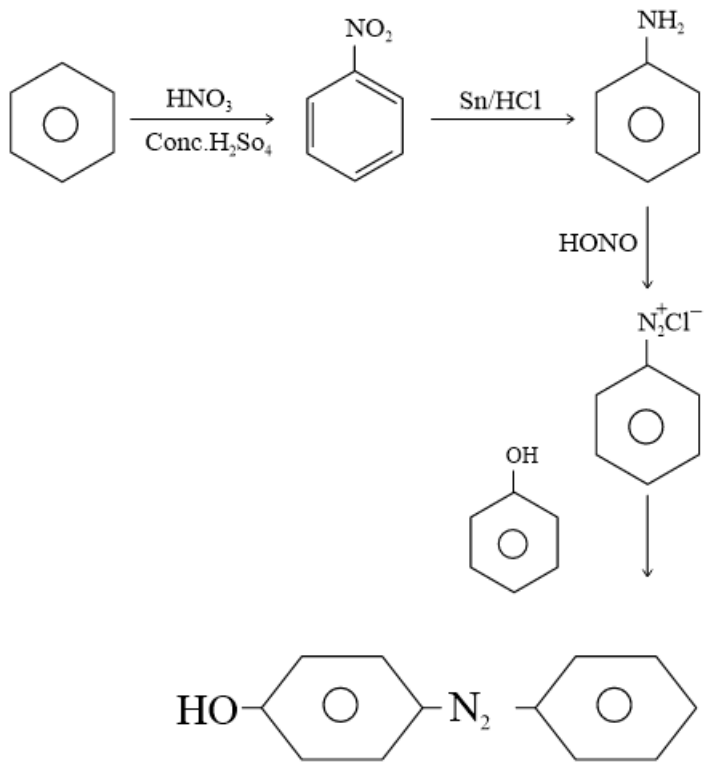

D) None of the above

Answer:

A = 


B = 

Solution: First product A is nitrobenzene formed by nitration of benzene and then reduction takes place and Aniline is formed, then diazotisation takes place, the diazo coupling occurs with phenol.




Q.14. Statement1: Halogen attached to bulkier group undergoes $S_N2$ reaction

Statement 2: Secondary alkyl halide reacts with excess $C_2H_5OH$ undergoes $S_N1$

A) Both statement 1 and 2 are correct.

B) Both statement 1 and 2 are false.

C) Statement 1is true statement 2 is false

D) Statement 1is false statement 2 is true.

Answer: Statement 1is false statement 2 is true.

Solution: According to Statement 2: Secondary alkyl halide reacts with excess $C_2H_5OH$ undergoes $S_N1$ mechanism is correct because, $C_2H_5OH$ is polar protic solvent is favoured by $S_N1$ mechanism and it is moderately strong nucleophile.

According to Statement1: Halogen attached to bulkier group does not undergoes $S_N2$ reaction, rather than smaller groups undergoes $S_N2$ reaction.

Hence option D is the answer.

Q.15. Which of the following is a purification method based on the difference in solubilities of compounds?

A) Sublimation

B) Distillation

C) Column Chromatography

D) Crystallisation

Answer: Crystallisation

Solution: Crystallization is based on the principles of solubility: compounds (solutes) tend to be more soluble in hot liquids (solvents) than they are in cold liquids. If a saturated hot solution is allowed to cool, the solute is no longer soluble in the solvent and forms crystals of pure compound.

Hence the answer is option D.

Q.16. Statement 1: $H_2Te$ is more acidic than $H_2S$.

Statement 2:$H_2Te$ has more BDE than $H_2S$.

A) Both Statement 1 and 2 are correct

B) Both statement 1 and 2 are incorrect

C) Statement 1 is incorrect and 2 is correct

D) Statement 1 is correct and 2 is incorrect

Answer: Statement 1 is correct and 2 is incorrect

Solution: The bond dissociation enthalpy of $H_2Te$ is lower than that of $H_2S$. As a result, less energy is required to break the $H_2Te$ bond, releasing is easier, and the acidic nature of $H_2Te$ is greater. $H_2S$ , on the other hand, has a high bond dissociation energy and so has lower acidic nature.

Hence, statement 1 correct and 2 is incorrect.

Q.17. Statement $1$: There is a regular increase in chemical reactivity from Group $1$ to group $18$

Statement $2$: Oxides of group $1$ elements are basic and oxides of group $17$ are acidic.

A) Both statement $1$ and $2$ are correct.

B) Statement $1$ is true Statement $2$ is false.

C) Statement $1$ is false Statement $2$ is true.

D) Both statement $1$ and $2$ are false.

Answer: Statement $1$ is false Statement $2$ is true.

Solution: According to statement $1$ There is a irregularity in chemical reactivity from Group $1$ to group $18$.

Hence statement $1$ is false.

The basic nature of the oxides generally increases with an increase in the electropositivity of metal forming oxide. Oxides of all elements of group $1$ are basic, and group $17$ are acidic. Down the group the basic nature of oxides increases and acidic nature decreases.

Hence statement $2$ is true.

Hence option C is the answer.

Q.18. In the given reactions A and B are

$$CrO_2Cl_2 + NaOH \rightarrow A + NaCl + H_2O$$
$$H_2SO_4 + A + H_2O_2 \rightarrow B$$

A) $CrO_5$ and $Na_2CrO_4$

B) $Na_2CrO_4$ and $CrO_5$

C) $Na_2CrO_4$ and $CrO_3$

D) $Na_2CrO_7$ and $Na_2CrO_5$

Answer: $Na_2CrO_4$ and $CrO_5$

Solution: Chromyl chloride dissolves in NaOH solution to give a yellow solution. The chromate solution changes to blue colour chromium peroxide with hydrogen peroxide.

The reaction will be as follows,

$$CrO_2Cl_2 + 4NaOH \rightarrow Na_2CrO_4 + 2NaCl + 2H_2O$$
$$CrO_4{}^{2-} + 2H^+ + 2H_2O_2 \rightarrow CrO_5 + 3H_2O$$

Hence, option B is the answer.

Q.19. Number of spectral lines in $He^+$ for transition from $n = 5$ to $n = 1$ are:

Answer: 10

Solution: The formula used here is:

Number of spectral lines

$$= \frac{n(n-1)}{2}$$
$$= \frac{5(5-1)}{2}$$
$$= \frac{20}{2} = 10$$

Number of spectral lines in $He^+$ for transition from $n = 5$ to $n = 1$ are $10$

Q.20. The number of elements which give a flame test for the given elements below:

$Sr,\ Cu,\ Co,\ Ca,\ Ni,\ Be$

Answer: 3

Solution: The flame test is used to visually determine the identity of an unknown metal or metalloid ion based on the characteristic colour the salt turns the flame of a Bunsen burner. The heat of the flame excites the electrons of the metal ions, causing them to emit visible light.

$\mathrm{Sr}-\mathrm{Crimson\ \ red\ \ colour}$
$\mathrm{Cu}-\mathrm{Blue\ \ green\ \ colour}$
$\mathrm{Ca}-\mathrm{Brick\ \ red\ \ colour}$

Hence, the answer is $3$.

Q.21. How many of the following shows disproportionation reaction?

$\mathrm{H_2O_2},\ \mathrm{Ag},\ \mathrm{Cu^+},\ \mathrm{K^+},\ \mathrm{F_2},\ \mathrm{Cl_2}, \mathrm{ClO_4^-}$

Answer: 3

Solution: Disproportionation reaction are the reactions in which the same element/compound get oxidised and reduced simultaneously.

1)$\mathrm{Cu^+} \rightarrow \mathrm{Cu^{2+}} + \mathrm{Cu}$

2) $2\mathrm{H_2\overset{-1}{O}_2} \rightarrow \mathrm{H_2\overset{-2}{O}} + \overset{0}{\mathrm{O}}_2$

3) $3\ \mathrm{Cl_2} + 6\ \mathrm{OH^-} \rightarrow 5\,\mathrm{Cl^-} + \mathrm{ClO_3^-} + 3\ \mathrm{H_2O}$

Here, the chlorine reactant is in oxidation state $0$, and if we look at the products, the chlorine in the $\mathrm{Cl^-}$ ion has an oxidation number of $-1$, where it has undergone reduction. Meanwhile, the oxidation number of the chlorine in the $\mathrm{ClO_3^-}$ ion is $+5$, which means it has been oxidised.

Hence, $\mathrm{H_2O_2},\ \mathrm{Cu^+},\ \mathrm{Cl_2}$ shows disproportionation reaction

Q.22. Bag $A$ contains $7$ white balls and $3$ red balls. Bag $B$ contains $3$ white balls and $2$ red balls. A ball is chosen randomly and found to be red, then find the probability that it is taken from bag $A$

A) $\frac{1}{7}$

B) $\frac{3}{7}$

C) $\frac{2}{7}$

D) $\frac{4}{7}$

Answer: $\frac{3}{7}$

Solution: Given,

Bag $A$ contains $7$ white balls and $3$ red balls. Bag $B$ contains $3$ white balls and $2$ red balls,

A ball is chosen randomly and found to be red,

So, total outcomes$= P(\text{chosing bag } A) \times P(\text{getting red ball}) + P(\text{chosing bag } B) \times P(\text{getting red ball})$

$= \frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{2}{5}$

$= \frac{3}{20} + \frac{2}{10}$

$= \frac{7}{20}$

Now, favourable outcome that it is taken from bag $A = \frac{3}{20}$

So, the required probability is given by,

$$P(E) = \frac{\frac{3}{20}}{\frac{7}{20}} = \frac{3}{7}$$

Q.23. If $f(x) = \log\left(\frac{2x+3}{4x^2-x-3}\right) + \cos^{-1}\left(\frac{2x+1}{x+2}\right)$ and domain of $f(x)$ is $[\alpha, \beta)$, then $5\alpha - 4\beta$ is

A) $2$

B) $1$

C) $-1$

D) $-2$

Answer: $-2$

Solution: Given: $f(x)=\log\left(\frac{2x+3}{4x^2-x-3}\right)+\cos^{-1}\left(\frac{2x+1}{x+2}\right)$

Now defining $\log$ function we get,

$\Rightarrow \frac{2x+3}{4x^2-x-3}>0$

$\Rightarrow \frac{2x+3}{(4x+3)(x-1)}>0$

$\Rightarrow x\in\left(\frac{-3}{2},\frac{-3}{4}\right)\cup(1,\infty)\quad\ldots(i)$

Also for cosine function, $-1\le\frac{2x+1}{x+2}\le 1$

$\Rightarrow \frac{2x+1}{x+2}+1\ge 0$ and $\frac{2x+1}{x+2}-1\le 0$

$\Rightarrow \frac{3x+3}{x+2}\ge 0$ and $\frac{x-1}{x+2}\le 0$

$\Rightarrow x\in(-\infty,-2)\cup[-1,\infty)\quad\ldots(ii)\quad$ and $\quad x\in(-2,1]\quad\ldots(iii)$

Using $(i)$, $(ii)$ and $(iii)$,

$\Rightarrow x\in\left[-1,\frac{-3}{4}\right)$

So, on comparing we get,

$\Rightarrow 5\alpha-4\beta=5(-1)-4\left(\frac{-3}{4}\right)=-2$

Q.24. $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$, $\left|\vec{b}\right|^2=6$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$. If $\vec{a}.\vec{b}=3$ then $\left(\alpha^2+\beta^2\right)\left|\vec{a}\times\vec{b}\right|^2$ is

A) $36$

B) $18$

C) $32$

D) $8$

Answer: $18$

Solution: We know that, $\vec{a}.\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$

$\Rightarrow 3=\sqrt{1^2+\alpha^2+\beta^2}\left(\sqrt{6}\right)\cos\frac{\pi}{4}$

$\Rightarrow \sqrt{1+\alpha^2+\beta^2}=\sqrt{3}$

$\Rightarrow 1+\alpha^2+\beta^2=3$

$\Rightarrow \alpha^2+\beta^2=2$

Also, $\left|\vec{a}\times\vec{b}\right|=\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|=\sqrt{3}\sqrt{6}\sin\frac{\pi}{4}=\frac{3\sqrt{2}}{\sqrt{2}}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|=3$

$\Rightarrow \left(\alpha^2+\beta^2\right)\left|\vec{a}\times\vec{b}\right|^2=2\times 9=18$

Q.25. If $\left|\vec{b}\right|=2$, $\left|\vec{b}\times\vec{a}\right|=2$ then $\left|\vec{b}\times\vec{a}-\vec{b}\right|^2$ is

A) $8$

B) $0$

C) $10$

D) $16$

Answer: $8$

Solution: Given,

$\left|\vec{b}\right| = 2,\ \left|\vec{b} \times \vec{a}\right| = 2$

Now, solving $\left|\vec{b} \times \vec{a} - \vec{b}\right|^2$

$\left|\vec{b} \times \vec{a} - \vec{b}\right|^2 = \left|\vec{b} \times \vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\left(\vec{b} \times \vec{a}\right) \cdot \vec{b}$

Now, we know that, $\left(\vec{b} \times \vec{a}\right) \cdot \vec{b} = \left(\vec{b} \times \vec{a}\right) \cdot \vec{a} = 0$

So, $\left|\vec{b} \times \vec{a} - \vec{b}\right|^2 = 2^2 + 2^2 - 2 \times 0 = 8$

Q.26. If $f$ is differentiable function and $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)},\ f'(1) = 2024$ then

A) $xf'(x) - 2024f(x) = 0$

B) $2024f'(x) = f(x)$

C) $f'(x) - 2024f(x) = 0$

D) None of these

Answer: $xf'(x) - 2024f(x) = 0$

Solution: Given,

$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$

And we know that, if $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ then $f(x) = x^n$

So, differentiating the function we get,

$f'(x) = nx^{n-1}$ and given $f'(1) = 2024$

Hence, on comparing we get, $n = 2024$

Now, $xf'(x) = x \times 2024x^{2023} = 2024x^{2024}$

$\Rightarrow xf'(x) - 2024x^{2024} = 0$

$\Rightarrow xf'(x) - 2024f(x) = 0$

Q.27. If $x\left(x^2 + 3|x| + 5|x-1| + |x-2|\right) = 0$, then the number of solutions of the given equation is _____.

A) $4$

B) $1$

C) $3$

D) $8$

Answer: $1$

Solution: Given: $x\left(x^2+3|x|+5|x-1|+|x-2|\right)=0$

$\Rightarrow x=0$ is one root of the given equation.

Case-I: $x<0$

$\Rightarrow x^2-3x-5(x-1)-6(x-2)=0$

$\Rightarrow x^2-14x+17=0$

Here $D>0$, so the equation gives two roots, but both are positive, so no solution.

Case-II: $0<x<1$

$\Rightarrow x^2+3x-5(x-1)-6(x-2)=0$

$\Rightarrow x^2-8x+17=0$

Here $D<0$, so no real solution.

Case-III: $1<x<2$

$\Rightarrow x^2+3x+5(x-1)-6(x-2)=0$

$\Rightarrow x^2+2x+7=0$

Here $D<0$, so no real solution.

Case-IV: $x>2$

$\Rightarrow x^2+3x+5(x-1)+6(x-2)=0$

$\Rightarrow x^2+14x-17=0$

All the roots of this equation are less than $2$, so no solution possible.

Hence, only $1$ solution is possible for the given equation.

Q.28. Let $\sum_{r=0}^{n}\frac{{}^nC_r\cdot{}^nC_r}{r+1}=\alpha$, $\sum_{r=0}^{n+1}\frac{{}^nC_r\cdot{}^{n+1}C_r}{r+1}=\beta$ and if $4\beta=7\alpha$, then find the value of $n$

A) $5$

B) $7$

C) $6$

D) $8$

Answer: $6$

Solution: Given,

$\alpha = \sum_{r=0}^{n} \frac{{}^{n}C_r . {}^{n}C_r}{r+1}$

$\Rightarrow \alpha = \frac{1}{n+1} \sum_{r=0}^{n} \frac{n+1}{r+1} {}^{n}C_r \cdot {}^{n}C_r$

$\Rightarrow \alpha = \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1} \cdot {}^{n}C_r$

$\Rightarrow \alpha = \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{n-r} \cdot {}^{n}C_r$

$\Rightarrow \alpha = \frac{1}{n+1} \cdot {}^{2n+1}C_n$

And $\beta = \sum_{r=0}^{n+1} \frac{{}^{n}C_r . {}^{n+1}C_r}{r+1}$

$\Rightarrow \beta = \frac{1}{n+1} \sum_{r=0}^{n+1} \frac{n+1}{r+1} {}^{n}C_r \cdot {}^{n+1}C_r$

$\Rightarrow \beta = \frac{1}{n+1} \sum_{r=0}^{n+1} {}^{n+1}C_{r+1} \cdot {}^{n+1}C_r$

$\Rightarrow \beta = \frac{1}{n+1} \sum_{r=0}^{n+1} {}^{n+1}C_{n-r} \cdot {}^{n+1}C_r$

$\Rightarrow \beta = \frac{{}^{2n+2}C_n}{n+1}$

Also given $4\beta = 7\alpha$

$\Rightarrow 4 \cdot \frac{{}^{2n+2}C_n}{n+1} = 7 \cdot \frac{1}{n+1} \cdot {}^{2n+1}C_n$

$\Rightarrow \frac{2n+2}{n+2} = \frac{7}{4}$

$\Rightarrow n = 6$

Q.29. Consider an $\mathrm{AP}: 3, 7, 11, 15, \ldots$ and let $S_n$ denote the sum of first $n$ terms. If $40 < \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 42$, then determine $n$.

A) $9$

B) $5$

C) $4$

D) $11$

Answer: $9$

Solution: Given, $\mathrm{AP}: 3, 7, 11, 15, \ldots$

$\Rightarrow a_n = 3 + (n-1)(4)$

$\Rightarrow a_n = 4n - 1$

$\Rightarrow S_n = \frac{n}{2}[6 + 4n - 4]$

$\Rightarrow S_n = \frac{n \times 2(2n+1)}{2}$

$\Rightarrow S_n = 2n^2 + n$

Now, $40 < \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 42$

$\Rightarrow \sum_{k=1}^{n} S_k = 2\sum_{n=1}^{n} n^2 + \sum_{n=1}^{n} n$

$\Rightarrow \sum_{k=1}^{n} S_k = \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}$

$\Rightarrow \sum_{k=1}^{n} S_k = \frac{n(n+1)}{2}\left[1 + \frac{4n+2}{3}\right]$

$\Rightarrow 40 < \frac{6}{n(n+1)} \times \frac{n(n+1)}{2}\left[\frac{5+4n}{3}\right] < 42$

$\Rightarrow 40 < 4n + 5 < 42$

$\Rightarrow 35 < 4n < 37$

$\Rightarrow n = 9$

Q.30. A question paper has three sections $A, B, C$ having $6, 8, 4$ questions respectively. If a student has to answer $15$ questions attempting atleast four from each section. Find the number of ways the paper can be answered by a student.

A) $180$

B) $344$

C) $242$

D) $240$

Answer: $344$

Solution: Given,

A question paper has three sections $A, B, C$ having $6, 8, 4$ questions respectively.

Now, a student has to answer $15$ questions attempting atleast four from each section,

So, the number of methods could be

$(A \to 4,\ B \to 7,\ C \to 4) + (A \to 5,\ B \to 6,\ C \to 4) + (A \to 6,\ B \to 5,\ C \to 4)$

$\Rightarrow {}^6C_4 \times {}^8C_7 \times 1 + {}^6C_5 \times {}^8C_6 \times 1 + {}^6C_6 \times {}^8C_5 \times 1$

$\Rightarrow 120 + 168 + 56 = 344$

Q.31. From the given observation find $\sigma^2$

| $x_i$ | $f_i$ |
|---|---|
| 0 | 3 |
| 1 | 2 |
| 5 | 3 |
| 7 | 2 |
| 10 | 6 |
| 12 | 3 |
| 17 | 3 |

A) $\frac{144}{11}$

B) $\frac{3486}{121}$

C) $\frac{147}{11}$

D) $\frac{3467}{121}$

Answer: $\frac{3486}{121}$

Solution: Finding the mean we get,

$$\frac{\sum f_i x_i}{\sum_{fi}} = \frac{0+2+15+14+60+36+51}{22} = \frac{89}{11}$$

$$\Rightarrow \overline{x} = \frac{89}{11}$$

We know that, $\sigma^2 = \frac{\sum f_i (x_i)^2}{\sum f_i} - (\overline{x})^2$

$$\Rightarrow \sigma^2 = \frac{2+75+98+600+432+867}{22} - \left(\frac{89}{11}\right)^2$$

$$\Rightarrow \sigma^2 = \frac{2074}{22} - \left(\frac{89}{11}\right)^2$$

$$\Rightarrow \sigma^2 = \frac{1037}{11} - \left(\frac{89}{11}\right)^2$$

$$\Rightarrow \sigma^2 = \frac{11407-7921}{11^2} = \frac{3486}{121}$$

Q.32. Consider the system of equations, $x+y+z=5,\ x+2y+\lambda^2 z=9,\ x+3y+\lambda z=\mu$, then

A) System has inconsistent solution for $\lambda=1,\ \mu\neq 13$

B) System has infinite solution for $\lambda=1,\ \mu=12$

C) System is consistent for $\lambda=1,\ \mu\in R$

D) System has infinite solution for $\lambda=1,\ \mu\neq -13$

Answer: System has inconsistent solution for $\lambda=1,\ \mu\neq 13$

Solution: Given: $x+y+z=5,\ x+2y+\lambda^2 z=9,\ x+3y+\lambda z=\mu$

$$\Rightarrow \triangle = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix}$$

$$\Rightarrow \triangle = 2\lambda - 3\lambda^2 - \lambda + \lambda^2 + 1$$

$$\Rightarrow \triangle = -2\lambda^2 + \lambda + 1$$

For infinite solutions, $\triangle = 0$

$$\Rightarrow -2\lambda^2 + \lambda + 1 = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$\Rightarrow (\lambda - 1)(2\lambda + 1) = 0$$

$$\Rightarrow \lambda = 1,\ \frac{-1}{2}$$

Now,

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 1 \\ \mu & 3 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta_1 = \mu - 13 \neq 0$$

Hence, option $(A)$ is the correct answer.

Q.33. If $f(x) = (x-2)^2(x-3)^3$ and $x \in [1,4]$, if $M$ & $m$ are maximum and minimum value respectively, then find the value of $M-m$

Answer: 12

Solution: Given,

$f(x)=(x-2)^2(x-3)^3$

Now differentiating the above function we get,

$f'(x)=2\,(x-2)(x-3)^3+3(x-3)^2(x-2)^2$

$\Rightarrow f'(x)=(x-2)(x-3)^2\,[2\,(x-3)+3\,(x-2)]$

$\Rightarrow f'(x)=(x-2)(x-3)^2\,(5x-12)$

Now, equating $f'(x)=0$ to find critical point we get,

$x=2,\ 3,\ \frac{12}{5}$

Now, $f(2)=f(3)=0$ and $f\left(\frac{12}{5}\right)=\left(\frac{12}{5}-2\right)^2\left(\frac{12}{5}-3\right)^3=\frac{2^3}{5^2}\times\left(-\frac{3^3}{5^3}\right)=-\frac{6^3}{5^5}$

Also, $x\in[1,4]$, so $f(1)=-8\ \&\ f(4)=4$

So, from the above values of the function we get,

Maximum value at $x=4,\ f(4)=4=M$

And minimum value at $x=1,\ f(1)=-8=m$

Hence, $M-m=4+8=12$

Q.34. If $A,B\ \in\left(0,\frac{\pi}{2}\right)$, $3\sin(A+B)=4\sin(A-B)$ and if $\tan A=k\tan B$, then find the value of $k$

Answer: 7

Solution: Given,

$3\sin(A+B)=4\sin(A-B)$

$\Rightarrow \frac{\sin(A+B)}{\sin(A-B)}=\frac{4}{3}$

$\Rightarrow \frac{\sin(A+B)+\sin(A-B)}{\sin(A+B)-\sin(A-B)}=\frac{4+3}{4-3}$

$\Rightarrow \frac{2\sin A\cos B}{2\cos A\sin B}=\frac{7}{1}$

$\Rightarrow \tan A=7\tan B$

So, on comparing we get, $k=7$

Q.35. Find the number of common roots of the equation $z^{1901}+z^{100}+1=0$ and $z^3+2z^2+2z+1=0$

Answer: 2

Solution: Given,

Equation $z^{1901}+z^{100}+1=0$ & $z^3+2z^2+2z+1=0$

Now, solving $z^3+2z^2+2z+1=0$

$z^3+2z^2+2z+1+z^2+z=z^2+z$

$\Rightarrow z^3+3z^2+3z+1=z^2+z$

$\Rightarrow (z+1)^3=z^2+z$

$\Rightarrow (z+1)\left(z^2+z+1\right)=0$

$\Rightarrow z=-1,\ \omega,\ \omega^2$

Now checking the value of $z$ in $z^{1901}+z^{100}+1=0$ we get,

At $z=-1$, $(-1)^{1901}+(-1)^{100}+1\neq 0$

At $z=\omega$, $(\omega)^{1901}+(\omega)^{100}+1=\omega^2+\omega+1=0$

At $z=\omega^2$, $\left(\omega^2\right)^{1901}+\left(\omega^2\right)^{100}+1=\omega+\omega^2+1=0$

Hence, the common roots are $z=\omega$ & $\omega^2$

Q.36. If $f(x)=ae^{2x}+be^x+cx,\ f(0)=-1,\ f'(\log 2)=21,\ \int_0^{\log 4}(f(x)-cx)=\frac{39}{2}$ then find $|a+b+c|$.

Answer: 8

Solution: Given: $f(x)=ae^{2x}+be^x+cx,\ f(0)=-1,\ f'(\log 2)=21,\ \int_0^{\log 4}(f(x)-cx)=\frac{39}{2}$

$\Rightarrow f(0)=a+b=-1 \quad \ldots(i)$

$\Rightarrow f'(x)=2ae^{2x}+be^x+c$

$\Rightarrow f'(\log 2)=2a(4)+b(2)+c=21$

$\Rightarrow 8a+2b+c=21 \quad \ldots(ii)$

Also, $\int_0^{\log 4}\left(ae^{2x}+be^x+cx-cx\right)=\frac{39}{2}$

$\Rightarrow \int_0^{\log 4}\left(ae^{2x}+be^x\right)=\frac{39}{2}$

$\Rightarrow \left[\frac{ae^{2x}}{2}+be^x\right]_0^{\log 4}=\frac{39}{2}$

$\Rightarrow \frac{15a}{2}+3b=\frac{39}{2}$

$\Rightarrow 15a+6b=39$

$\Rightarrow 15a+6(-1-a)=39$

$\Rightarrow 15a-6-6a=39$

$\Rightarrow 9a=45$

$\Rightarrow a=5,\ b=-6$

$\Rightarrow c=-7$

$\Rightarrow |a+b+c|=8$

Q.37. $f(x)=\begin{cases}3x^2+2x+a, & x\leq 1\\ bx+2, & x>1\end{cases}$, $f(x)$ is differentiable $\forall x\in R$. Evaluate $\int_{-2}^{2}f(x)dx$.

Answer: 35

Solution: Given:

$$f(x)=\begin{cases}3x^2+2x+a, & x\le 1\\ bx+2, & x>1\end{cases}$$

Now, for function to be continuous,

$\Rightarrow 3(1)^2+2\times 1+a=b\times 1+2$

$\Rightarrow 5+a=b+2$

$\Rightarrow a-b=-3 \quad \ldots(i)$

Now, differentiating and putting the value of $x$ we get,

$3(2(1))+2=b(1)+0$

$\Rightarrow b=8$

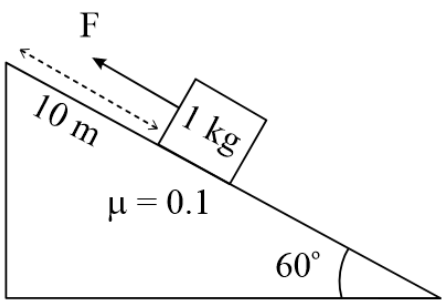
$\Rightarrow a=5$

$\Rightarrow \int_{-2}^{2} f(x)dx=\int_{-2}^{1}\left(3x^2+2x+5\right)dx+\int_{1}^{2}(8x+2)dx$

$\Rightarrow \int_{-2}^{2} f(x)dx=\left[x^3+x^2+5x\right]_{-2}^{1}+\left[4x^2+2x\right]_{1}^{2}$

$\Rightarrow \int_{-2}^{2} f(x)dx=21+14=35$

Q.38. A block of mass $1\ \mathrm{kg}$ is ascended an inclined plane by distance of $10\ \mathrm{m}$ as shown in diagram, with the help of force $F$ along the incline. Find work done against the friction.




A) $5\sqrt{3}\ \mathrm{J}$

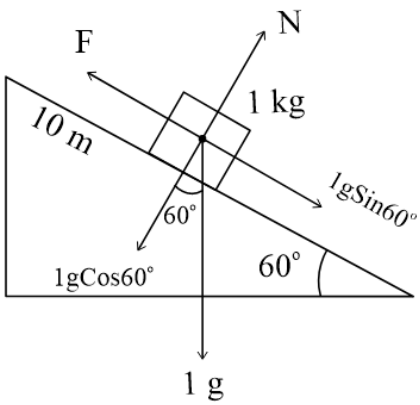
B) $10\ \mathrm{J}$

C) $\left(10-5\sqrt{3}\right)\ \mathrm{J}$

D) $5\ \mathrm{J}$

Answer: $5\ \mathrm{J}$

Solution: Let's consider the following diagram:



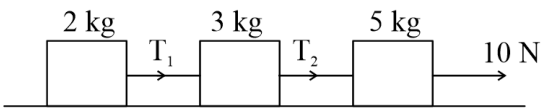

With respect to the above diagram, the frictional force $(f)$ on the block is given by

$$f = \mu mg \cos 60^\circ$$
$$= 0.1 \times 1 \text{ kg} \times 10 \text{ m s}^{-2} \times \frac{1}{2}$$
$$= 0.5 \text{ N}$$

Hence, the work done by the frictional force is given by

$$W = fs$$
$$= 0.5 \text{ N} \times 10 \text{ m}$$
$$= 5 \text{ J}$$

Q.39. A force of $10 \text{ N}$ is applied on a three block system as shown. Find the tensions $T_1$ and $T_2$.




A) $2 \text{ N}, 5 \text{ N}$

B) $4 \text{ N}, 3 \text{ N}$

C) $5 \text{ N}, 2 \text{ N}$

D) $3 \text{ N}, 4 \text{ N}$

Answer: $2 \text{ N}, 5 \text{ N}$

Solution: According to the given diagram, the common acceleration of the system can be calculated as follows:

$$a = \frac{10 \text{ N}}{2 \text{ kg} + 3 \text{ kg} + 5 \text{ kg}}$$
$$= 1 \text{ m s}^{-2}$$

Thus, the tension $(T_1)$ is given by

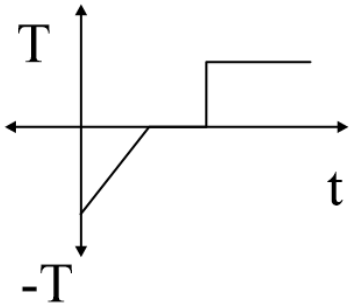
$$T_1 = 2 \text{ kg} \times 1 \text{ m s}^{-2}$$
$$= 2 \text{ N}$$

And, the tension $(T_2)$ can be found out as follows:

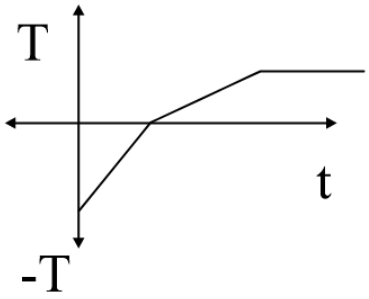
$$T_2 - T_1 = 3 \text{ kg} \times 1 \text{ m s}^{-2}$$
$$\Rightarrow T_2 = 2 \text{ N} + 3 \text{ N}$$
$$= 5 \text{ N}$$

Q.40. Ice at temperature $-10^\circ \text{ C}$ is converted to stream at $100^\circ \text{ C}$, the curve plotted between temperature $(T^\circ \text{ C})$ versus time $(t)$ when it is being heated by constant power source can be depicted by

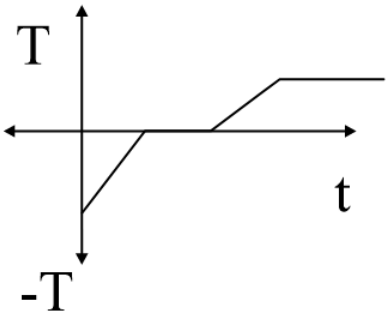
A)



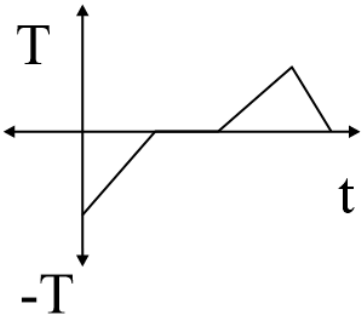

B)



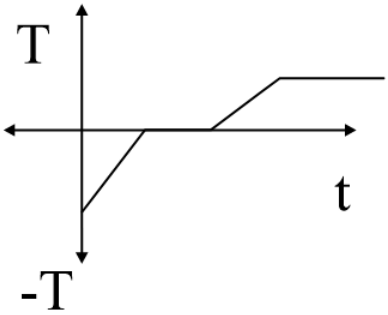

C)



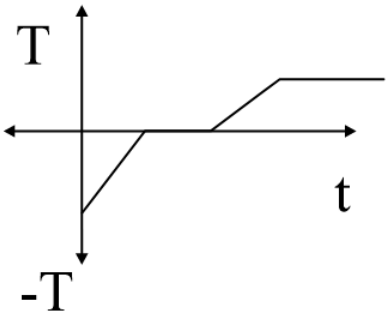

D)




Answer:



Solution:




During the process of changing the temperature of ice from $-10^{\circ}\ \mathrm{C}$ to $0^{\circ}\ \mathrm{C}$, there will be an increase in temperature and the variation is linear.

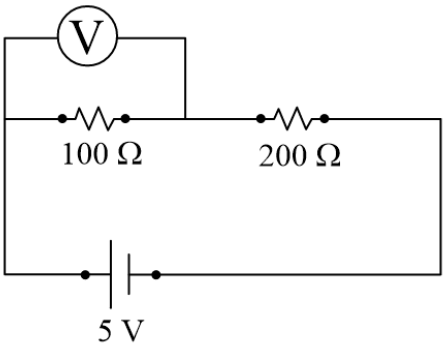
During the process of changing the $0^{\circ}\ \mathrm{C}$ ice to $0^{\circ}\ \mathrm{C}$ water, there will be no change of temperature as only latent heat is required for the process.

Again, to increase the temperature of water from $0^{\circ}\ \mathrm{C}$ to $100^{\circ}\ \mathrm{C}$, the variation of temperature will be linear with time.

Finally, to change the $100^{\circ}\ \mathrm{C}$ water to $100^{\circ}\ \mathrm{C}$ steam, there will be no change in temperature as it only involves the change of phase.

Thus, this is the correct variation of the temperature with time for the given process.

Q.41. In the given circuit, reading of voltmeter is $1\ \mathrm{V}$. The resistance of the voltmeter is




A) $200\ \Omega$

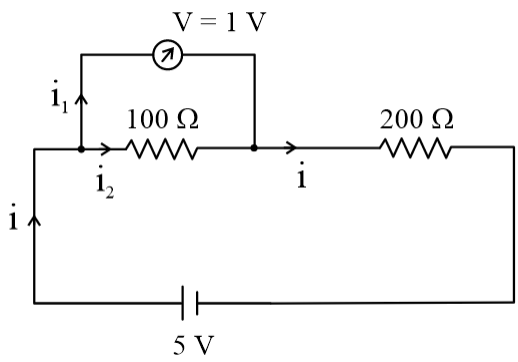
B) $100\ \Omega$

C) $50\ \Omega$

D) $200\sqrt{5}\ \Omega$

Answer: $100\ \Omega$

Solution: Let's consider the following diagram:




In accordance with the above diagram, the current flowing through the entire circuit is given by

$$i=\frac{5\text{ V}-1\text{ V}}{200\ \Omega}$$
$$=0.02\text{ A}$$

Also, the current through $100\ \Omega$ resistor is given by

$$i_2=\frac{1\text{ V}}{100\ \Omega}$$
$$=0.01\text{ A}$$

Thus, the current through the voltmeter is

$$i_1=i-i_2$$
$$=0.01\text{ A}$$

If $r$ be the resistance of the voltmeter, it follows that

$$r=\frac{1\text{ V}}{i_1}$$
$$=\frac{1\text{ V}}{0.01\text{ A}}$$
$$=100\ \Omega$$

Q.42. The slope of graph between the stopping potential $V_0$ and frequency of incident photon $\nu$ in photoelectric effect is

($h=$Planck's constant, $e=$charge on electron)

A) $\frac{e}{h}$

B) $\frac{h}{2e}$

C) $\frac{h}{e}$

D) $\frac{2h}{e}$

Answer: $\frac{h}{e}$

Solution: As we know, $E=\phi+KE_{max}$

$\Rightarrow h\nu=\phi+eV_0$

$\Rightarrow eV_0=h\nu-\phi$

$\Rightarrow V_0=\left(\frac{h}{e}\right)\nu-\frac{\phi}{e}$

Clearly, the relation is linear with slope $=\frac{h}{e}$.

Q.43. $1000$ drops of surface energy $E_1$ coalesce to form a bigger drop of surface energy $E_2$. Find the value of $\frac{E_2}{E_1}\times 10^3$.

A) $100$

B) $200$

C) $300$

D) $400$

Answer: $100$

Solution: As we know, surface energy $E = TA$, where $T$ =surface tension and $A$ =surface area.

Now, total volume of the drops will remain same. Therefore,

$$1000\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$
$$\Rightarrow R = 10r$$

Now, $\frac{E_2}{E_1} \times 10^3 = \frac{T\left(4\pi R^2\right)}{T\left(1000 \times 4\pi r^2\right)} \times 10^3 = \left(\frac{R}{r}\right)^2 = 100$

Q.44. If dimension of $\left[c^p G^{\frac{-1}{2}} h^{\frac{1}{2}}\right] = [M]$, where $c$ is speed of light, $G$ is the universal gravitational constant and $h$ is Planck's constant. Find the value of $p$.

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{5}$

Answer: $\frac{1}{2}$

Solution: Given: $\left[c^p G^{\frac{-1}{2}} h^{\frac{1}{2}}\right] = [M]$

Writing dimensions, we get

$$\left[\mathrm{LT}^{-1}\right]^p \left[M^{-1} L^3 \mathrm{T}^{-2}\right]^{\frac{-1}{2}} \left[\mathrm{ML}^2 \mathrm{T}^{-1}\right]^{\frac{1}{2}} = [M]$$
$$\Rightarrow M^{\frac{1}{2}+\frac{1}{2}} L^{p-\frac{3}{2}+1} T^{-p+1-\frac{1}{2}} = M$$

Comparing both sides, we get

$$p - \frac{1}{2} = 0$$
$$\Rightarrow p = \frac{1}{2}$$

Q.45. Two particles are projected from a tower of height $400\ \mathrm{m}$ at angles $45^\circ$ and $60^\circ$ with the horizontal. If they have same time of flight, then find the ratio of their velocities.

A) $\sqrt{\frac{3}{4}}$

B) $\sqrt{\frac{3}{2}}$

C) $\sqrt{\frac{5}{2}}$

D) $1$

Answer: $\sqrt{\frac{3}{2}}$

Solution: Along vertical direction, we can write

$$s = ut + \frac{1}{2}at^2.$$

For first particle:

$$-400 = v_1 \sin 45^\circ t - \frac{1}{2}gt^2 \quad \ldots Eq(1)$$

For second particle:

$$-400 = v_2 \sin 60^\circ t - \frac{1}{2}gt^2 \quad \ldots Eq(2)$$

Now, from equation (1) & (2), considering time of flight is same, we get

$$v_1 \sin 45^\circ = v_2 \sin 60^\circ$$
$$\Rightarrow \frac{v_1}{v_2} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \sqrt{\frac{3}{2}}$$

Q.46. An electron in the $5^{\text{th}}$ excited state of $\text{He}^+$ atom moves to the $1^{\text{st}}$ excited state. Find the number of possible spectral lines formed.

A) $3$

B) $4$

C) $2$

D) $6$

Answer: $6$

Solution: The energy of an electron in the $n^{\text{th}}$ excited state is given by

$$E_n = -\frac{13.6Z^2}{n^2} \quad \ldots(1)$$

where, $Z$ is the atomic number.

When the given electron jumps from the $5^{\text{th}}$ excited state $(n=6)$ to the $1^{\text{st}}$ excited state$(n=2)$, it has four possible energy states in between.

Hence, the required number of possible spectral line is given by

$$\begin{aligned} n &= {}^4C_2 \\ &= \frac{4!}{2!2!} \\ &= 6 \end{aligned}$$

Q.47. A point source is placed at origin. If intensity at a distance of $2\ \text{cm}$ from the source is $I$, then the intensity at a distance $4\ \text{cm}$ from the source will be:

A) $I$

B) $\frac{I}{2}$

C) $\frac{I}{4}$

D) $\frac{I}{16}$

Answer: $\frac{I}{4}$

Solution: For point source, intensity is inversely proportional to square of distance between source and observation point.

$$I \propto \frac{1}{r^2}$$

where $I$ is intensity and $r$ is distance between source and observation point.

So, we can write

$$\frac{I_1}{I_2} = \frac{{r_2}^2}{{r_1}^2}$$

Given, Intensity at distance $(r_1 = 2\ \text{cm})$ is $I_1 = I$.

If distance is doubled$(r_2 = 4\ \text{cm})$, then intensity is:

$$\frac{I}{I_2} = \frac{4^2}{2^2}$$

$$\Rightarrow I_2 = \frac{I}{4}$$

Q.48. For a given planet, $R_P = \frac{1}{3}R_E$ and $M_P = \frac{1}{6}M_E$. Find the escape velocity(in $\text{km s}^{-1}$) to the nearest integer for the planet, if the escape velocity for Earth is $11.2\ \text{km s}^{-1}$.

Answer: 8

Solution: The escape velocity for the Earth is given by

$$v_E = \sqrt{\frac{2GM_E}{R_E}} \quad \ldots(1)$$

The escape velocity for the planet is given by

$$v_P = \sqrt{\frac{2GM_P}{R_P}} \quad \ldots(2)$$

From equations (2) and (1), it follows that

$$\frac{v_P}{v_E} = \frac{\sqrt{\frac{2GM_P}{R_P}}}{\sqrt{\frac{2GM_E}{R_E}}}$$

$$= \sqrt{\frac{M_P R_E}{M_E R_P}}$$

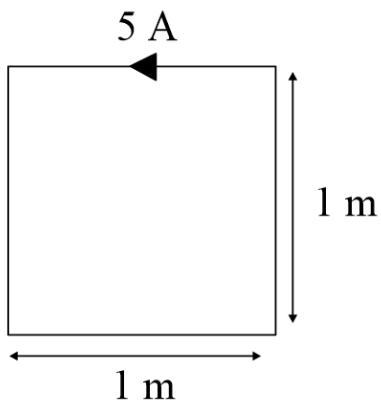
$$= \sqrt{\frac{3}{6}}$$

$$\Rightarrow v_P = \sqrt{\frac{1}{2}} v_E$$

$$= \sqrt{\frac{1}{2}} \times 11.2 \text{ km s}^{-1}$$

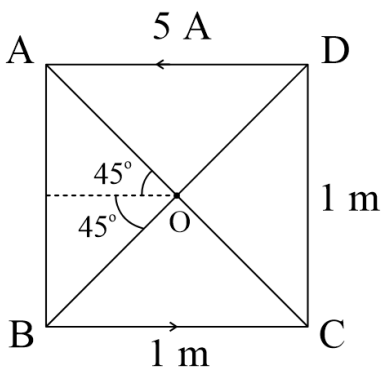
$$\approx 8 \text{ km s}^{-1}$$

Q.49. A square loop of side $1 \text{ m}$ is carrying a current of $5 \text{ A}$ as shown. If the magnetic field at the centre is $x\sqrt{2} \times 10^{-7} \text{ T}$, find $x$.




Answer: 40

Solution: Let's consider the following diagram:




The formula to calculate the magnetic field $(B)$ at a distance $r$ from a finite current conducting wire is given by

$$B=\frac{\mu_0 I}{4\pi r}\left[\sin\theta_1+\sin\theta_2\right] \quad \ldots(1)$$

In accordance with the above figure, the magnetic field at the centre of of the system due to side $AB$ can be written as

$$\begin{aligned} B_{AB} &= \frac{\mu_0 I}{4\pi\times 0.5\text{ m}}\left[\sin 45^\circ+\sin 45^\circ\right] \\ &= \frac{4\pi\times 10^{-7}\times 5}{4\pi\times 0.5\text{ m}}\sqrt{2}\text{ T} \\ &= 10\sqrt{2}\times 10^{-7}\text{ T} \end{aligned}$$

Hence, due to the entire square loop, the magnetic field at the centre is given by

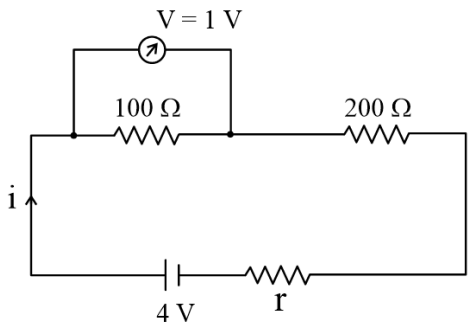
$$\begin{aligned} B_n &= 4B_{AB} \\ &= 4\times 10\sqrt{2}\times 10^{-7}\text{ T} \\ &= 40\sqrt{2}\times 10^{-7}\text{ T} \end{aligned}$$

Thus, $x=40$.

Q.50. A $100\ \Omega$ resistance and a $200\ \Omega$ resistance is connected in series with a $4\text{ V}$ battery. Voltmeter across $100\ \Omega$ reads $1\text{ V}$. Find the internal resistance(in ohm unit) of the battery.

Answer: 100

Solution: Let's consider the following diagram:




Let, the internal resistance of the battery is $r\ \Omega$.

The current $(i)$ flowing through the circuit is given by

$$i=\frac{1\ \mathrm{V}}{100\ \Omega}$$
$$=0.01\ \mathrm{A} \quad ,,,(1)$$

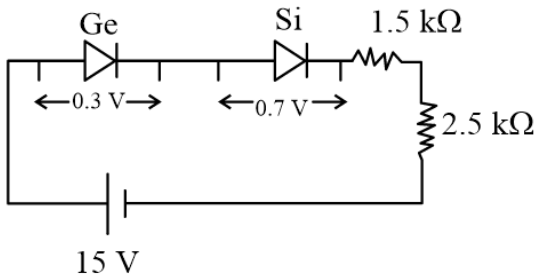
Also, the current through the circuit can be written as

$$i=\frac{4\ \mathrm{V}}{(100+200+r)\ \Omega}$$
$$=\frac{4}{300+r}\ \mathrm{A} \quad \ldots(2)$$

From equations (1) and (2), it follows that

$$0.01=\frac{4}{300+r}$$
$$\Rightarrow 300+r=\frac{4}{0.01}=400$$
$$\Rightarrow r=100\ \Omega$$

Q.51. In the circuit shown the potential drop in forward bias across $Si$ and $Ge$ diode are $0.7\ \mathrm{V}$ and $0.3\ \mathrm{V}$. The voltage drop across the $1.5\ \mathrm{k}\Omega$ resistance for the given circuit is $x \times 10^{-2}\ \mathrm{V}$. Find $x$.




Answer: 525

Solution: The net potential difference across the resistors, for the given circuit will be

$$V=15\ \mathrm{V}-(0.3+0.7)\ \mathrm{V}$$
$$=14\ \mathrm{V}$$

Thus, the current $(i)$ flowing through the circuit is given by

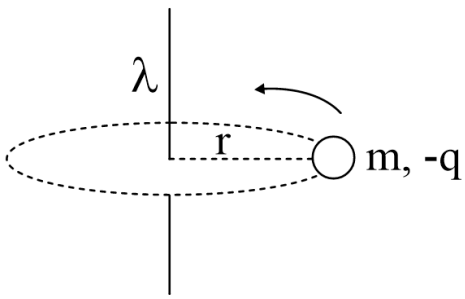
$$i=\frac{14\ \mathrm{V}}{(1.5+2.5)\ \mathrm{k}\Omega}$$
$$=\frac{14}{4}\ \mathrm{mA}$$

Thus, the potential difference across the given resistor can be calculated as follows:

$$V=1.5\ \mathrm{k}\Omega \times \frac{14}{4}\ \mathrm{mA}$$
$$=5.25\ \mathrm{V}=525 \times 10^{-2}\ \mathrm{V}$$

Hence, $x=525$.

Q.52. A negatively charged particle $(m, -q)$ rotates around a positively charged infinite line charge of linear charge density $\lambda$ as shown. The time period of the particle is $\sqrt{\frac{x\pi^3\varepsilon_0 m r^2}{\lambda q}}$. Find the value of $x$.




Answer: 8

Solution: The electric field is given by

$$E = \left(\frac{2k\lambda}{r}\right)$$

The required centripetal force is

$$F = \left(\frac{mv^2}{r}\right)$$

The force due to the field is equal to the required centripetal force,

$$\Rightarrow \left(\frac{2k\lambda}{r}\right)q = \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{2k\lambda q}{m}}$$

Now, time period is given by,

$$T = \frac{2\pi r}{v}$$
$$= \frac{2\pi r}{\sqrt{\frac{2k\lambda q}{m}}}$$
$$= \frac{2\pi r}{\sqrt{\frac{\lambda q}{2\pi\varepsilon_0 m}}}$$
$$= \sqrt{\frac{8\pi^3\varepsilon_0 m r^2}{\lambda q}}$$

Therefore, $x = 8$.