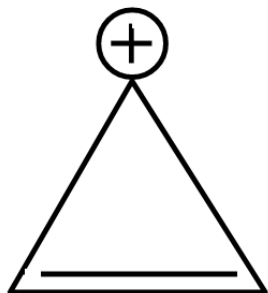


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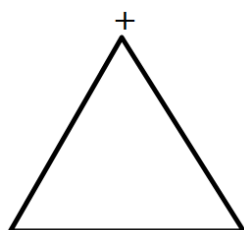
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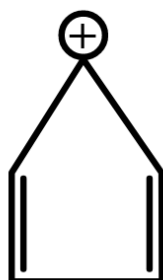
A)



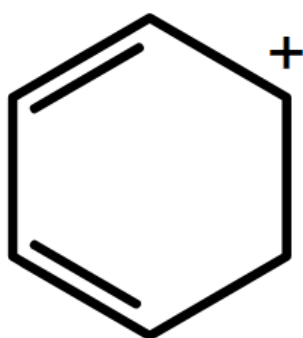
B)



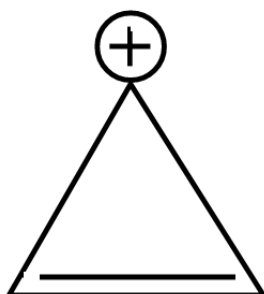
C)



D)



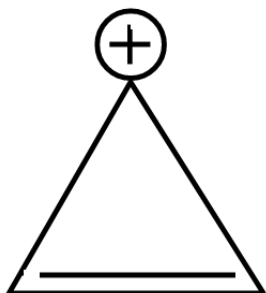
Answer:





Solution: Conjugated carbocations are more stable than non-conjugated carbocation. Among the conjugated carbocations, aromatic systems are more stable.

Option A is aromatic as it follows the Huckel's rule.



In option B there is no conjugation.

Option C is anti-aromatic.

Option D is non-aromatic.

Hence, Option A will be the most stable species.

Q.4. Which of the following compounds will not give Fehling test?

- | | |
|------------|------------|
| A) Lactose | B) Maltose |
| C) Glucose | D) Sucrose |

Answer: Sucrose

Solution: Fehling's test is based on the principle that the presence of aldehydes and alpha hydroxy carbonyl compounds can be detected easily due to their reducing property.

Carbohydrates that contain free anomeric hydroxy groups can reduce Fehling's solution.

Sucrose does not contain a free anomeric hydroxy group. So, it cannot reduce Fehling's solution. It is a non-reducing sugar.

Hence option D is the answer.

Q.5. Which of the following set contains both diamagnetic ions?

- | | |
|--|--|
| A) Ni^{2+} , Cu^{2+} | B) Eu^{3+} , Gd^{3+} |
| C) Ce^{4+} , Pm^{3+} | D) Cu^{+} , Zn^{2+} |

Answer: Cu^{+} , Zn^{2+}

Solution: A diamagnetic material is one in which all the electrons in the substance are coupled. If the substance contains unpaired electrons, it is a paramagnetic substance. If the atoms have a net magnetic moment, the paramagnetism that results outweighs the diamagnetism.

Ni^{2+} , Cu^{2+} ions have d^8 and d^9 configurations respectively.

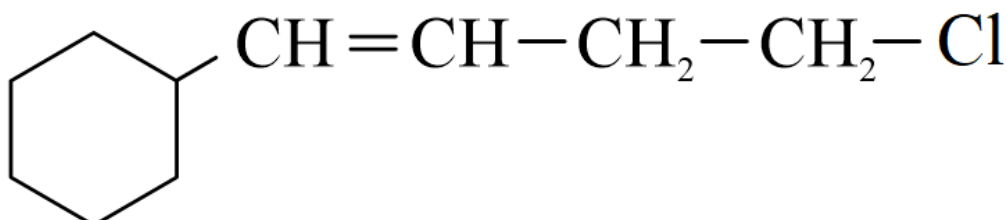
Eu^{3+} , Gd^{3+} ions have f^6 and f^7 configurations respectively.

pm^{3+} ion has f^4 configuration.

Cu^{+} , Zn^{2+} ions have d^{10} configuration, so all electrons are paired, hence, these are diamagnetic species.

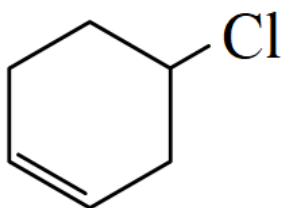
Q.6. Which of the following has allylic halogen?

A)

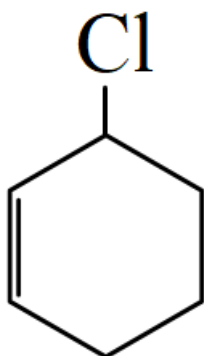




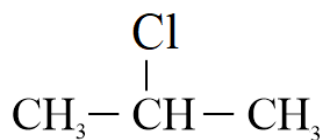
B)



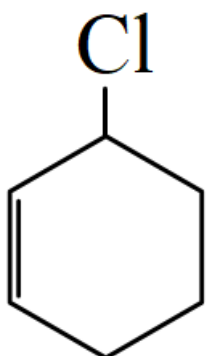
C)



D)

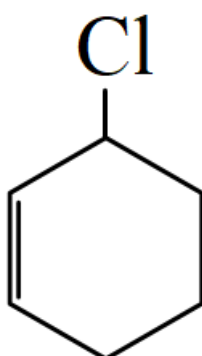


Answer:



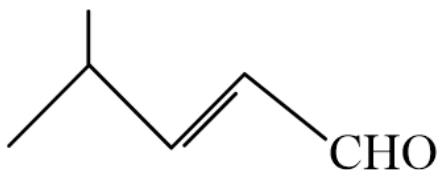
Solution: Allylic halides are the compounds in which the halogen atom is bonded to sp^3 -hybridised carbon atom next to carbon-carbon double bond ($C=C$).

Hence option C is the answer.





Q.7. IUPAC name of the given compound is:



A) 4 – methylpent – 2 – en – 1 – al

B) 3 – methylpent – 2 – en – 1 – al

C) 4 – methylhex – 2 – en – 1 – al

D) 4 – methylpent – 2 – en – 1 – ol

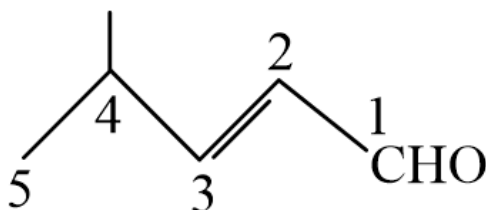
Answer: 4 – methylpent – 2 – en – 1 – al

Solution: The structure of the IUPAC name is

Secondary prefix + root word + primary suffix + secondary suffix.

In the given compound, primary prefix is methyl, root word is pent, primary suffix is en and secondary suffix is al.

Numbering should start from a functional group. the functional group should get the least number.



The IUPAC name of the compound is 4 – methylpent – 2 – en – 1 – al

Q.8. What is the geometry of Aluminium Chloride in aqueous solution?

A) Square Planar

B) Octahedral

C) Square pyramidal

D) Tetrahedral

Answer: Octahedral

Solution: Aluminium Chloride in acidified aqueous solution forms octahedral $[Al(H_2O)_6]^{3+}$ ion.

In this complex the 3d orbital of Al are involved and the hybridisation state of Al is sp^3d^2

Hence option B is the answer.

Q.9. Choose the correct option

Molecule	Shape
(a) BrF_5	(i) See-saw
(b) H_2O	(ii) T-shape
(c) ClF_3	(iii) Bent
(d) SF_4	(iv) Square Pyramidal

A) (a)-iv, (b)-iii, (c)-i, (d)-ii

B) (a)-iv, (b)-iii, (c)-ii, (d)-i

C) (a)-iii, (b)-iv, (c)-ii, (d)-i

D) (a)-iii, (b)-iv, (c)-i, (d)-ii

Answer: (a)-iv, (b)-iii, (c)-ii, (d)-i



Solution: SF_4 is sp^3d hybridised and has see-saw shape.

BrF_5 is **square pyramidal** with five bond pairs and a lone pair.

H_2O has a tetrahedral arrangement of molecules or an angular geometry. This is mainly because the repulsion from the lone pair combination is more than bond-pair repulsion. Additionally, the existing pairs do not lie in the same plane. One pair is below the plane and the other one is above. This bond geometry is commonly known as a distorted tetrahedron. Hence it has Bent shape.

Chlorine trifluoride has 10 electrons around the central chlorine atom. This means there are five electron pairs arranged in a trigonal bipyramidal shape with a 175° $\text{F}-\text{Cl}-\text{F}$ bond angle. There are two equatorial lone pairs making the final structure T-shaped.

Hence option B is the answer.

Q.10. Statement 1: For the hydrogen atom, 3p and 3d are degenerate.

Statement 2: Degenerate orbitals have same energy.

- A) Both statement 1 and 2 are correct. B) Both statement 1 and 2 are incorrect.
C) Statement 1 is correct and 2 is incorrect. D) Statement 1 is incorrect and 2 is correct,

Answer: Both statement 1 and 2 are correct.

Solution: 3p and 3d-orbitals all have the same energy for hydrogen.

For hydrogen like species, energy only depends on the principal quantum number n.

Hence, statement 1 is correct.

Electron orbitals having the same energy levels are called degenerate orbitals.

Hence, statement 2 is correct.

Q.11. Statement 1: From N to P there is a considerable increase in covalent radius when compared to As to Bi.

Statement 2: Covalent radii and ionic radii increase down the group for a particular oxidation state.

- A) Statement 1 is correct Statement 2 is wrong B) Statement 2 is correct Statement 1 is wrong
C) Both Statements 1 and 2 are wrong D) Both Statements 1 and 2 are correct

Answer: Both Statements 1 and 2 are correct

Solution: On moving down the group from N to P there is a considerable increase in covalent radii. On the other hand from As to Bi the increment in covalent radii is very small. Due to the presence of d and/or f orbitals in heavier elements, the nucleus has tight hold on valence electrons due to poor shielding of d and/or f orbitals. Therefore, the increase in covalent radii is not significant in heavier elements.

Hence statement 1 is correct.

Covalent radii and ionic radii increases down the group for a particular oxidation state. Statement 2 is correct.

Hence option D is the answer.

Q.12. Statement 1: Structure of allylic halide is $\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{X}$.

Statement 2: In allylic halide, halide is attached to sp^2 hybridized carbon.

- A) Both statement 1 and 2 are correct. B) Both statement 1 and 2 are incorrect.
C) Statement 1 is correct and 2 are incorrect. D) Statement 1 is incorrect and 2 are correct.

Answer: Statement 1 is correct and 2 are incorrect.

Solution: Allylic halides: These are the compounds where the halogen group is attached to a sp^3 hybridised carbon atom next to the carbon which is already in a double bond with another carbon atom.



Hence, statement 1 is correct.

In allylic halide, halide is attached to sp^3 hybridized carbon. Hence, statement 2 is incorrect.



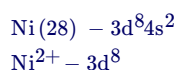
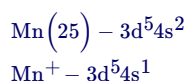
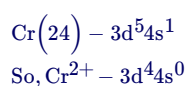
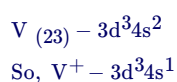
Q.13. Choose the correct option

Element	Electronic Configuration
(a) V^+	(i) $3d^5 4s^1$
(b) Mn^+	(ii) $3d^4 4s^0$
(c) Cr^{2+}	(iii) $3d^3 4s^1$
(d) Ni^{2+}	(iv) $3d^8$

- A) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv) B) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)
 C) (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii) D) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

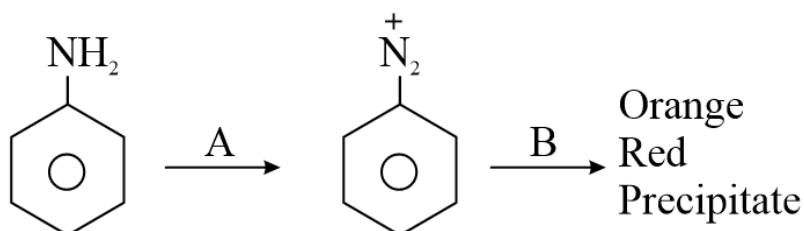
Answer: (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

Solution: The valence shell electronic configuration of the elements is as follows-



Hence, option A is the answer.

Q.14. Consider the following sequence of reaction:

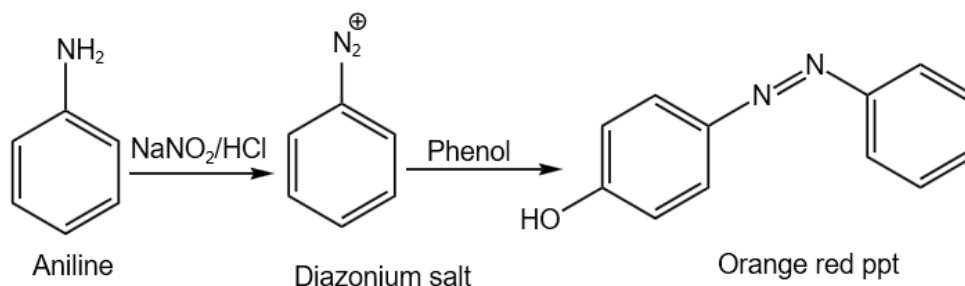


Select the correct option with respect to A and B.

- A) HNO_3 , Phenol B) $NaNO_2/HCl$, Phenol
 C) HNO_3 , aniline D) $NaNO_3/HCl$, aniline

Answer: $NaNO_2/HCl$, Phenol

Solution: Aniline can be converted to Phenol by using Nitrous acid which converts Aniline to Benzenediazonium chloride which upon reaction with phenol orange red dye is formed..



Hence option B is the answer.



Q.15. The K_{sp} of $Mg(OH)_2$ is 1×10^{-12} , $0.01 \text{ M } Mg^{2+}$ ions will precipitate at the limiting pH equal to.....(at 25°C).

- A) 9
B) 9.5
C) 8
D) None of the above

Answer: 9

Solution: $Mg(OH)_2 \rightleftharpoons Mg^{2+} + 2 OH^-$

$$K_{sp} = [Mg]^{2+} [OH^-]^2$$

$$10^{-12} = 0.01 \times [OH^-]^2$$

$$10^{-10} = [OH^-]^2$$

$$[OH^-] = \sqrt{10^{-10}} = 10^{-5}$$

$$-\log [OH^-] = pOH = 5$$

$$pH = 14 - 5 = 9$$

Hence, the answer is option A.

Q.16. A mixture is heated with dil. H_2SO_4 and the lead acetate paper turns black by the evolved gas. The mixture contains,

- A) Sulphite
B) Sulphide
C) Sulphate
D) Thiosulphate

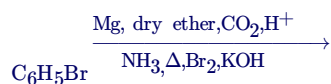
Answer: Sulphide

Solution: It must be sulphide, as hydrogen sulphide gas turns lead acetate paper black.



Hence option B is the answer

Q.17. Find the final product of the reaction given below,

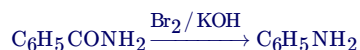
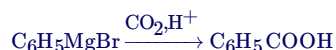


- A) $C_6H_5NH_2$
B) $C_6H_5CH_2NH_2$
C) C_6H_5COOH
D) $C_6H_5CONH_2$

Answer: $C_6H_5NH_2$

Solution: Bromobenzene give Grignard reagent with Magnesium metal. The Grignard reagent gives carboxylic acid with carbon dioxide. The carboxylic acid converts to amide with ammonia. The next step is Hoffmann bromamide reaction.

Hoffmann bromamide reaction mechanism generally includes the use of an alkali as a strong base to attack the amide, leading to the deprotonation and the subsequent generation of an anion. This reaction is used for the conversion of a primary amide to a primary amine with one less carbon atom. This is accomplished by heating the primary amide with a mixture of a halogen (chlorine or bromine), a strong base, and water.



Hence option A is the answer.

Q.18. Ethanal reacts with semi carbazide, find the number of N – atoms in the product.

- A) 3
B) 2
C) 4
D) 1



Answer: 3

Solution: Acetaldehyde or ethanal reacts with semicarbazide, there is removal of water. The reaction is given below.



In the given product, the number of Nitrogen atoms are 3.

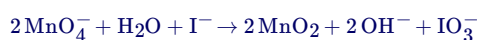
Hence, the answer is 3, option A.

Q.19. Find out the sum of the coefficients of all the species involved in the balanced equation:



Answer: 9

Solution: Balanced chemical reaction will be:



Coefficients on the reactant side are 4 and the coefficients on the product side are 5.

Hence, the sum of the coefficients of all the species is = 4 + 5 = 9

Q.20. Find out the maximum number of hybrid orbitals formed when 2s and 2p orbitals are mixed.

Answer: 4

Solution: The atomic orbitals combine to form a new set of equivalent orbitals known as hybrid orbitals. Unlike pure orbitals, hybrid orbitals are used in bond formation. The phenomenon is known as hybridisation, which can be defined as the process of intermixing of the orbitals of slightly different energies to redistribute their energies, resulting in the formation of a new set of orbitals of equivalent energies and shape.

The number of hybrid orbitals is equal to the number of atomic orbitals that get hybridised.

When 2s and 2p are mixed 4 hybrid orbitals are formed.

Q.21. IUPAC name Ununium element lies in which group in the periodic table?

Answer: 11

Solution: Ununium is the IUPAC name given to the element Roentgenium and the atomic number 111. In the periodic table in the 7th period it is located and placed in the 11th group. Roentgenium is a radioactive element and its isotopes are unstable in nature.

Q.22. 250 mL solution of CH_3COONa of molarity 0.35 M is prepared. What is the mass of CH_3COONa required in grams.

Give an answer to the nearest integer value.

Answer: 7

Solution: 0.35 M solution means 0.35 moles in 1 L of solution.

1000 mL of solution contains 0.35 moles

$$1 \text{ mL of solution} \rightarrow \frac{0.35}{1000} \text{ moles}$$

$$250 \text{ mL of solution} \rightarrow \frac{0.35}{1000} \times 250 = 0.0875$$

$$\text{No. of moles} = \frac{\text{Mass}}{\text{Molar mass}}$$

$$0.0875 = \frac{\text{Mass}}{82}$$

$$\text{Mass of sodium acetate} = 7.1 \text{ g} \approx 7 \text{ g}$$

Q.23. The ratio of magnitude of potential energy and kinetic energy for 5th excited state of hydrogen atom is x : 1, then the value of x is

Answer: 2



Solution: 5th excited state means, 6th energy level.

The energy of nth orbit of hydrogen atom = $\frac{-13.6}{n^2}$ eV

The potential energy of electron in hydrogen atom is $\frac{-27.2}{n^2}$ eV and its kinetic energy is $+\frac{13.6}{n^2}$ eV.

The ratio of the kinetic energy and the potential energy of electron in the hydrogen atom is:

$$\frac{|PE|}{|KE|} = \frac{27.2}{13.6} = 2 : 1$$

Hence, the answer is 2.

Q.24. If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is

- A) $\frac{2}{\sqrt{5}}$ B) $\frac{1}{\sqrt{5}}$
C) $2\sqrt{5}$ D) $\sqrt{5}$

Answer: $\frac{2}{\sqrt{5}}$

Solution: Given: $2b = \frac{1}{2} \times 2ae$

$$\Rightarrow 2b = ae$$

$$\Rightarrow 4b^2 = a^2e^2$$

$$\Rightarrow \frac{4b^2}{a^2} = e^2$$

$$\Rightarrow 4(1 - e^2) = e^2$$

$$\Rightarrow 4 - 4e^2 = e^2$$

$$\Rightarrow e = \frac{2}{\sqrt{5}}$$

Q.25. Let $A(2, 3, 5)$ and $C(-3, 4, -2)$ be opposite vertices of a parallelogram $ABCD$. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the area of the parallelogram is equal to

- A) $\frac{\sqrt{474}}{2}$ B) $\sqrt{474}$
C) $\sqrt{237}$
D) $\frac{\sqrt{237}}{2}$

Answer: $\frac{\sqrt{474}}{2}$

Solution: Area of the parallelogram is given by $\frac{1}{2} |\overrightarrow{AC} \times \overrightarrow{BD}|$.

$$\Rightarrow \text{Area} = \frac{1}{2} |(-5\hat{i} + \hat{j} - 7\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})|$$

$$\Rightarrow \text{Area} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 1 & -7 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\Rightarrow \text{Area} = \frac{1}{2} |17\hat{i} + 8\hat{j} - 11\hat{k}|$$

$$\Rightarrow \text{Area} = \frac{1}{2} \sqrt{289 + 64 + 121}$$

$$\Rightarrow \text{Area} = \frac{\sqrt{474}}{2} \text{ square units.}$$



Q.26. Find the value of the $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^3}{(n^2+k^2)(n^2+3k^2)}$

- A) $\frac{1}{2} \left[\frac{\pi}{3} - \frac{\pi}{\sqrt{2}} \right]$ B) $\frac{1}{2} \left[\frac{\pi}{\sqrt{3}} - \frac{\pi}{4} \right]$
 C) $\frac{1}{2} \left[\frac{\pi}{3\sqrt{3}} - \frac{\pi}{4} \right]$ D) $\left[\frac{\pi}{\sqrt{3}} - \frac{\pi}{4} \right]$

Answer: $\frac{1}{2} \left[\frac{\pi}{\sqrt{3}} - \frac{\pi}{4} \right]$

Solution: Given,

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n^3}{(n^2+k^2)(n^2+3k^2)} \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{\left(1+\frac{k^2}{n^2}\right)\left(1+3\frac{k^2}{n^2}\right)} \end{aligned}$$

Now, using limit as a sum integral we get,

$$\begin{aligned} &= \int_0^1 \frac{dx}{(1+x^2)(1+3x^2)} \\ &= \frac{1}{2} \left[\int_0^1 \frac{3}{(1+3x^2)} - \frac{1}{(1+x^2)} dx \right] \\ &= \frac{1}{2} \left[\int_0^1 \frac{1}{\left(\left(\frac{1}{\sqrt{3}}\right)^2 + x^2\right)} - \frac{1}{(1+x^2)} dx \right] \\ &= \frac{1}{2} \left[\sqrt{3} \tan^{-1}(\sqrt{3}x) - \tan^{-1}x \right]_0^1 \\ &= \frac{1}{2} \left[\sqrt{3} \tan^{-1}(\sqrt{3}) - \tan^{-1}1 \right] \\ &= \frac{1}{2} \left[\sqrt{3} \cdot \frac{\pi}{3} - \frac{\pi}{4} \right] \\ &= \frac{1}{2} \left[\frac{\pi}{\sqrt{3}} - \frac{\pi}{4} \right] \end{aligned}$$

Q.27. If a straight line passes through A (9,0) and makes a angle 30° with the x -axis, if it is rotated by 15° degree clockwise then find the new equation of line.

- A) $y = \sqrt{3}(x-9)$ B) $y = (2 - \sqrt{3})(x-9)$
 C) $y = 2\sqrt{3}(x-9)$ D) $y = 2(x-9)$

Answer: $y = (2 - \sqrt{3})(x-9)$



Solution: Given,

A straight line passes through $A(9, 0)$ and makes an angle 30° with the x -axis,

And it is rotated by 15° degree clockwise

So, the new angle with the x -axis will be $\theta = 30^\circ - 15^\circ = 15^\circ$

Hence, the equation of the line will be,

$$y - 0 = \tan 15^\circ (x - 9)$$

$$\Rightarrow y = \tan (45^\circ - 30^\circ) (x - 9)$$

$$\Rightarrow y = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} (x - 9)$$

$$\Rightarrow y = (2 - \sqrt{3}) (x - 9)$$

Q.28. The probability of selecting two integers x, y from set $\{0, 1, 2, 3, \dots, 10\}$ and $|x - y| > 5$ is

A) $\frac{15}{121}$

B) $\frac{45}{121}$

C) $\frac{30}{121}$

D) $\frac{30}{119}$

Answer: $\frac{30}{121}$

Solution: Given,

Set $\{0, 1, 2, 3, \dots, 10\}$

So, total number of outcomes are $(11 \times 11) = 121$

Favourable outcomes are $(0, 6), (0, 7), (0, 8), (0, 9), (0, 10), (1, 7), (1, 8), (1, 9), (1, 10), (2, 8), (2, 9), (2, 10), (3, 9), (3, 10), (4, 10)$

And the same number of elements when x and y are interchanged.

So, the number of favourable outcomes are 30.

So, the required probability is given by $P(E) = \frac{30}{121}$.

Q.29. $g(x)$ is a non-constant twice differentiable function. $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$ and $f(x) = \frac{1}{2}[g(x) + g(2-x)]$

A) $f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 1$

B) $f''(x) = 0$ for exactly one value of $x \in (0, 1)$

C) $f''(x) = 0$ for atleast 1 value of $x \in (0, 2)$

D) $f''(x) = 0$ for no value of $x \in (0, 1)$

Answer: $f''(x) = 0$ for atleast 1 value of $x \in (0, 2)$



Solution: Given:

$$f(x) = \frac{1}{2} [g(x) + g(2-x)]$$

$$\Rightarrow f'(x) = \frac{1}{2} [g'(x) - g'(2-x)]$$

$$\Rightarrow f'\left(\frac{1}{2}\right) = \frac{1}{2} \left[g'\left(\frac{1}{2}\right) - g'\left(\frac{3}{2}\right)\right]$$

$$\Rightarrow f'\left(\frac{1}{2}\right) = 0$$

$$\text{Also, } f'\left(\frac{3}{2}\right) = \frac{1}{2} \left[g'\left(\frac{3}{2}\right) - g'\left(\frac{1}{2}\right)\right]$$

$$\Rightarrow f'\left(\frac{3}{2}\right) = 0$$

$$\text{Now, } f'(1) = \frac{1}{2} [g'(1) - g'(1)]$$

$$\Rightarrow f'(1) = 0$$

So, $f'(x)$ has three roots and thus $f''(x)$ will have atleast two roots.

Q.30. The range of r for which circles $(x+1)^2 + (y+2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ coincide at two distinct points.

A) $0 < r < 3$

B) $3 < r < 7$

C) $5 < r < 9$

D) $\frac{1}{2} < r < 4$

Answer: $3 < r < 7$

Solution: Given,

$$S_1 \equiv (x+1)^2 + (y+2)^2 = r^2 \text{ and } S_2 \equiv x^2 + y^2 - 4x - 4y + 4 = 0 \text{ or } S_2 \equiv (x-2)^2 + (y-2)^2 = 2^2$$

Now, if two circles intersect at two distinct points then $|r_1 - r_2| < C_1 C_2 < r_1 + r_2$.

$$\Rightarrow |r - 2| < \sqrt{3^2 + 4^2} < r + 2$$

$$\Rightarrow |r - 2| < 5 < r + 2$$

$$\Rightarrow |r - 2| < 5 \text{ and } r + 2 > 5$$

$$\Rightarrow -5 < r - 2 < 5 \text{ and } r > 3$$

$$\Rightarrow -3 < r < 7 \text{ and } r > 3$$

$$\Rightarrow 3 < r < 7$$

Q.31. In an arithmetic progression if sum of first 20 terms is 790 and sum of the first 10 terms is 145 then find the value of $S_{15} - S_5$ {where S_n denotes sum of n terms}

A) 395

B) 400

C) 405

D) 385

Answer: 395



Solution: Given,

In an arithmetic progression sum of first 20 terms is 790

So, let a be the first term and d be common difference,

$$\text{So, } S_{20} = \frac{20}{2} [2a + 19d] = 790 \dots (1)$$

And sum of the first 10 terms is 145

$$\text{So, } S_{10} = \frac{10}{2} [2a + 9d] = 145 \dots (2)$$

By solving equation (1) & (2) we get,

$$a = -8 \text{ \& } d = 5$$

$$\text{Then, } S_{15} - S_5 = \frac{15}{2} [2 \times (-8) + 14 \times 5] - \frac{5}{2} [2 \times (-8) + 4 \times 5]$$

$$\Rightarrow S_{15} - S_5 = 405 - 10 = 395$$

Q.32. If $2 \sin x (x - 1) + \cos x (x^2 - 2x) = \frac{dy}{dx}$ and satisfies $f(2) = 2$, then $y = ?$

- A) $-(x^2 - 2x) \cos x + 2$ B) $(x^2 - 2x) \sin x + 2$
 C) $-(x^2 - 2x) \sin x + 2$ D) $(x^2 - 2x) \cos x + 2$

Answer: $(x^2 - 2x) \sin x + 2$

Solution: Given:

$$2(x - 1) \cdot \sin x + (x^2 - 2x) \cdot \cos x = \frac{dy}{dx}$$

Now, integrating both side we get,

$$\int 2(x - 1) \cdot \sin x + (x^2 - 2x) \cdot \cos x dx = \int dy$$

$$\Rightarrow \int 2 \sin x \cdot (x - 1) + \int \cos x \cdot (x^2 - 2x) dx = y$$

$$\Rightarrow \int 2 \sin x \cdot (x - 1) + (x^2 - 2x) \cdot \sin x - \int (2x - 2) \cdot (\sin x) dx + C = y$$

$$\Rightarrow (x^2 - 2x) \sin x + C = y$$

Now, given $f(2) = 2$

$$\Rightarrow 2 = (2^2 - 2 \times 2) (\sin(2)) + C$$

$$\Rightarrow C = 2$$

$$\Rightarrow y = (x^2 - 2x) \sin x + 2$$

Q.33. Find the value of $9I$ if $I = \int_0^9 \left[\sqrt{\frac{10x}{x+1}} \right] dx$ where $[.]$ denotes the greatest integer function.

- A) 155 B) 152
 C) 146 D) 148

Answer: 155



Solution: Given:

$$I = \int_0^9 \left[\sqrt{\frac{10x}{x+1}} dx \right]$$

The square of integers are 0, 1, 4, 9, ...

$$\Rightarrow 0 = \frac{10x}{x+1} \Rightarrow x = 0 \quad \dots (i)$$

$$\Rightarrow 1 = \frac{10x}{x+1} \Rightarrow x + 1 = 10x$$

$$\Rightarrow x = \frac{1}{9} \quad \dots (ii)$$

$$\Rightarrow 4 = \frac{10x}{x+1} \Rightarrow 4x + 4 = 10x$$

$$\Rightarrow x = \frac{2}{3} \quad \dots (iii)$$

$$\Rightarrow 9 = \frac{10x}{x+1} \Rightarrow 9x + 9 = 10x$$

$$\Rightarrow x = 9 \quad \dots (iv)$$

Now, from the above equations using the law $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^d f(x) dx + \int_d^b f(x) dx$

$$\Rightarrow I = \int_0^9 (0) dx + \int_{\frac{1}{9}}^{\frac{2}{3}} (1) dx + \int_{\frac{2}{3}}^9 (2) dx$$

$$\Rightarrow I = \frac{2}{3} - \frac{1}{9} + 2 \left(9 - \frac{2}{3} \right)$$

$$\Rightarrow I = \frac{155}{9}$$

$$\Rightarrow 9I = 155$$

Q.34. The domain of $y = \cos^{-1} \left| \frac{2-|x|}{4} \right| + \log(3-x)^{-1}$ is $(-\alpha, \beta] - \{\gamma\}$ then the value of $\alpha + \beta + \gamma$ is

A) 11

B) 9

C) 2

D) 6

Answer: 11

Solution: Given:

$$y = \cos^{-1} \left| \frac{2-|x|}{4} \right| + \log(3-x)^{-1}$$

$$\Rightarrow -1 \leq \left| \frac{2-|x|}{4} \right| \leq 1$$

$$\Rightarrow -4 \leq 2 - |x| \leq 4$$

$$\Rightarrow -6 \leq -|x| \leq 2$$

$$\Rightarrow -2 \leq |x| \leq 6$$

$$\Rightarrow |x| \leq 6$$

$$\Rightarrow -6 \leq x \leq 6 \quad \dots (i)$$

Also, $3 - x > 0$ and $3 - x \neq 1$

$$\Rightarrow x \neq 2 \quad \dots (ii) \text{ and } x < 3 \quad \dots (iii)$$

Using (i), (ii) and (iii) we get,

$$\Rightarrow x \in [-6, 3) - \{2\}$$

So, on comparing with $x \in (-\alpha, \beta] - \{\gamma\}$ we get,

$$\Rightarrow \alpha + \beta + \gamma = 6 + 3 + 2 = 11$$



Q.35. If (α, β, γ) be the foot of the perpendicular from the point $(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ then $19(\alpha + \beta + \gamma)$ will be

Answer: 101

Solution: Given,

$P(\alpha, \beta, \gamma)$ be the foot of the perpendicular from the point $Q(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$

Now, let $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$

So, any point on the line is given by $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$, so let the point be $P(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

Now, direction ratio of $PQ = (5\lambda - 4, 2\lambda - 1, 3\lambda - 7)$

Now, the line and PQ will be perpendicular,

So, $5(5\lambda - 4) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$$\Rightarrow 38\lambda - 43 = 0$$

$$\Rightarrow \lambda = \frac{43}{38}$$

So, the point $P(\alpha, \beta, \gamma) = P\left(5 \times \frac{43}{38} - 3, 2 \times \frac{43}{38} + 1, 3 \times \frac{43}{38} - 4\right)$

Then $19(\alpha + \beta + \gamma) = 19\left[5 \times \frac{43}{38} - 3 + 2 \times \frac{43}{38} + 1 + 3 \times \frac{43}{38} - 4\right]$

$$\Rightarrow 19(\alpha + \beta + \gamma) = 19\left[\frac{101+124-23}{38}\right]$$

$$\Rightarrow 19(\alpha + \beta + \gamma) = 101$$

Q.36. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to

Answer: 0

Solution:

$$\text{Given: } f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^4 x & \sin^2 2x \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$$\Rightarrow f(x) = 9 \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$\Rightarrow f(x) = 9(2\cos^4 x + 2\sin^4 x + 3 + \sin^2 2x)$$

Now, differentiating above function we get,

$$\Rightarrow f'(x) = 9[8\cos^3 x(-\sin x) + 8\sin^3 x(\cos x) + 2\sin 2x(\cos 2x)(2)]$$

$$\Rightarrow f'(0) = 9[0 + 0 + 0]$$

$$\Rightarrow f'(0) = 0$$

Q.37. The value of maximum area possible of a $\triangle ABC$ where $A(0, 0)$, $B(x, y)$ and $C(-x, y)$ such that $y = -2x^2 + 54x$ (in square units) is:

Answer: 5832



Solution: Let, area of triangle ABC be denoted by P .

$$\Rightarrow P = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$\Rightarrow P = \frac{1}{2} |0 - 0 + 2xy|$$

$$\Rightarrow P = |xy|$$

$$\Rightarrow P = |x(-2x^2 + 54x)|$$

$$\Rightarrow P = |-2x^3 + 54x^2|$$

$$\Rightarrow \frac{dP}{dx} = |-6x^2 + 108x|$$

For critical points, $\frac{dP}{dx} = 0$

$$\Rightarrow |-6x^2 + 108x| = 0$$

$$\Rightarrow x = 0, 18$$

Area will be minimum at $x = 0$.

So, the maximum area will be,

$$P = |-2(18)^3 + 54(18)^2|$$

$$\Rightarrow P = |18^2 \times (54 - 36)| = 5832.$$

Q.38. In the expansion of $\left(2^{\frac{1}{2}} + 3^{\frac{1}{6}}\right)^{824}$ the number of rational terms is

Answer: 138

Solution: General term of $\left(2^{\frac{1}{2}} + 3^{\frac{1}{6}}\right)^{824}$ will be given by,

$${}^{824}C_r 2^{\frac{824-r}{2}} 3^{\frac{r}{6}}.$$

For rational terms, r should be a multiple of 6.

$$\Rightarrow r = 0, 6, 12, \dots, 822$$

$$\Rightarrow 822 = 0 + (n-1)6$$

$$\Rightarrow n = 138$$

Q.39. If $z = x + iy$, $xy \neq 0$ satisfies the equation $z + i\bar{z} = 0$ then $|z^2|$ is equal to

Answer: 1



Solution: Given: $z^2 + i\bar{z} = 0$

$$\Rightarrow z^2 = -i\bar{z}$$

Now taking $z = x + iy$ we get,

$$\Rightarrow x^2 - y^2 + 2ixy = -ix - y$$

Now on comparing both side we get,

$$\Rightarrow x^2 - y^2 = -y \text{ \& } 2xy = -x$$

$$\Rightarrow x^2 - y^2 = -y \text{ \& } x(2y + 1) = 0$$

$$\Rightarrow x^2 - \left(\frac{-1}{2}\right)^2 = -\left(\frac{-1}{2}\right) \text{ \& } y = \frac{-1}{2}$$

$$\Rightarrow x^2 = \left(\frac{3}{4}\right) \text{ \& } y = \frac{-1}{2}$$

$$\Rightarrow x = \pm \frac{\sqrt{3}}{2} \text{ \& } y = \frac{-1}{2}$$

$$\text{Hence, } z = \pm \frac{\sqrt{3}}{2} + i\left(\frac{-1}{2}\right)$$

$$\Rightarrow z^2 = \frac{3}{4} - \frac{1}{4} \pm 2i\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow z^2 = \frac{1}{2} \pm i\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow |z^2| = \left(\sqrt{\frac{1}{4} + \frac{3}{4}}\right)^2 = 1$$

Q.40. The electrostatic potential due to a short electric dipole at a distance r varies as:

A) r^2

B) r

C) $\frac{1}{r}$

D) $\frac{1}{r^2}$

Answer: $\frac{1}{r^2}$

Solution: The potential at a distance r due to a short dipole is $V_{dip}(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$, where \vec{p} = dipole moment.

Thus, $V_{dip}(r)$ is proportional to $\frac{1}{r^2}$.

Q.41. At what temperature the RMS velocity of hydrogen gas is equal to that of oxygen gas at 47° C ?

A) 20 K

B) 40 K

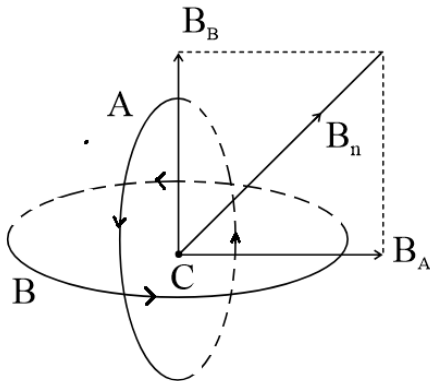
C) 35 K

D) 100 K

Answer: 20 K



Solution: Let's consider the following diagram:



The magnetic field at C due to ring A is given by

$$B_A = \frac{\mu_0 I}{2R} \quad \dots (1)$$

The magnetic field at C due to ring B is given by

$$B_B = \frac{\mu_0 I}{2R} \quad \dots (2)$$

Hence, the net magnetic field at C is given by

$$\begin{aligned} B_n &= \sqrt{B_A^2 + B_B^2} \\ &= \sqrt{\left(\frac{\mu_0 I}{2R}\right)^2 + \left(\frac{\mu_0 I}{2R}\right)^2} \\ &= \frac{\mu_0 I}{\sqrt{2}R} \end{aligned}$$

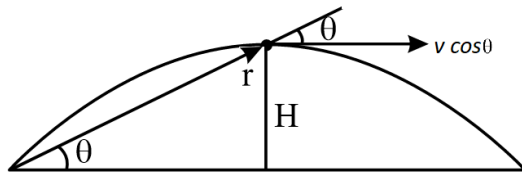
Q.44. A particle of mass m is projected from ground with speed v at an angle of 30° with the horizontal. Find its angular momentum about the point of projection when it reaches its maximum height.

- | | |
|-----------------------|-------------------------------|
| A) $\frac{mv^3}{16g}$ | B) $\frac{\sqrt{3}mv^3}{16g}$ |
| C) $\frac{mv^3}{3g}$ | D) $\frac{\sqrt{3}mv^3}{8g}$ |

Answer: $\frac{\sqrt{3}mv^3}{16g}$



Solution:



Let range be R .

The angular momentum is given by,

$$\vec{L} = \vec{r} \times \vec{p}$$

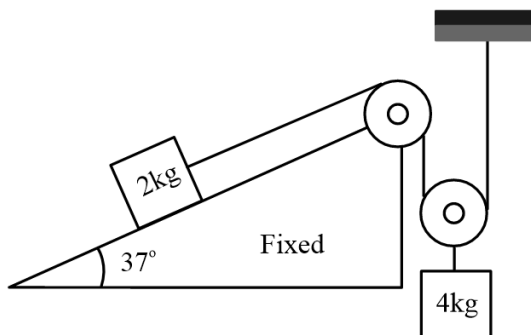
$$\Rightarrow \vec{L} = \left(\frac{H}{\sin \theta} \right) (mv \cos \theta) \sin \theta (-\hat{k}), \text{ where } (-\hat{k}) \text{ is directed inside the plane of the paper.}$$

$$\begin{aligned} \Rightarrow \vec{L} &= \left(\frac{\frac{v^2 \sin^2 \theta}{2g}}{\sin \theta} \right) (mv \cos \theta) \sin \theta (-\hat{k}) \\ &= \frac{mv^3 \cos \theta \sin^2 \theta}{2g} (-\hat{k}) \end{aligned}$$

But, $\theta = 30^\circ$

$$\therefore |\vec{L}| = \frac{\sqrt{3}mv^3}{16g}$$

Q.45. Find the acceleration of 2 kg block shown in the diagram (neglect friction).



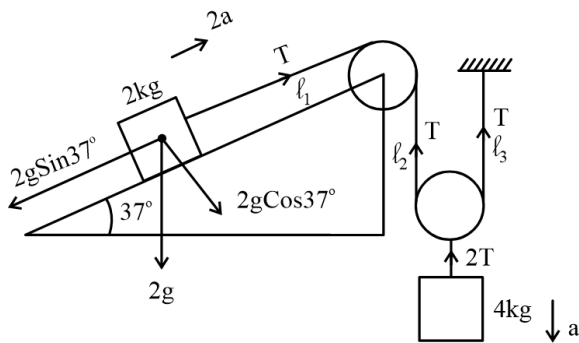
- A) $\frac{2g}{15}$
C) $\frac{2g}{3}$

- B) $\frac{4g}{15}$
D) $\frac{g}{15}$

Answer: $\frac{4g}{15}$



Solution: The FBD for the system can be drawn as follows:



If l_1 , l_2 , l_3 are the lengths of the strings connected to the 2 kg and the 4 kg blocks, it can be written that

$$\begin{aligned} \ddot{l}_1 + \ddot{l}_2 + \ddot{l}_3 &= 0 \\ \Rightarrow -a_1 + a_2 + a_2 &= 0 \\ \Rightarrow a_1 = 2a_2 = 2a \quad (\text{let}) \end{aligned}$$

With reference to the above diagram, the equation of motion of the 4 kg block can be written as

$$4g - 2T = 4a \quad \dots (1)$$

And, the equation of motion for the 2 kg block can be written as

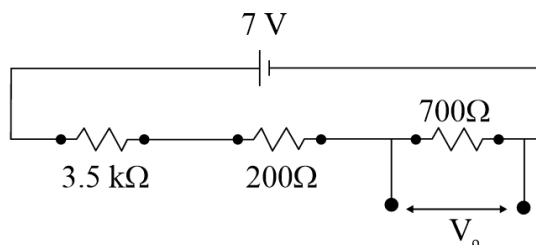
$$\begin{aligned} T - 2g \sin 37^\circ &= 2 \times 2a \\ \Rightarrow T - \frac{6g}{5} &= 4a \quad \dots (2) \end{aligned}$$

By doing the process, $(1) + 2 \times (2)$, we have

$$\begin{aligned} 4g - 2T + \left[2T - \frac{12g}{5} \right] &= 4a + 8a \\ \Rightarrow 4g - \frac{12g}{5} &= 12a \\ \Rightarrow a &= \frac{2g}{15} \end{aligned}$$

Hence, the magnitude of the required acceleration of the 2 kg block is $2a = \frac{4g}{15}$.

Q.46. Find the potential difference across the 700Ω resistance, i.e., V_0 for the following circuit.



- | | |
|----------|----------|
| A) 0.5 V | B) 2 V |
| C) Zero | D) 1.1 V |

Answer: 1.1 V



Solution: The current (I) through the entire circuit can be calculated as follows:

$$I = \frac{7 \text{ V}}{(3500 + 200 + 700) \Omega}$$

$$= \frac{7}{4400} \text{ A}$$

Hence, the required potential difference across the given resistor can be found as follows:

$$V_0 = 700I$$

$$= 700 \times \frac{7}{4400} \text{ V}$$

$$\approx 1.1 \text{ V}$$

Q.47. Which of the following option is correct for the given table?

Column A	Column B
(a) Coefficient of viscosity	(p) $[ML^2T^{-2}]$
(b) Surface Tension	(q) $[ML^2T^{-1}]$
(c) Angular momentum	(r) $[ML^{-1}T^{-1}]$
(d) Rotational kinetic energy	(s) $[ML^0T^{-2}]$

A) $(a) \rightarrow (s); (b) \rightarrow (q); (c) \rightarrow (p); (d) \rightarrow (r)$

B) $(a) \rightarrow (r); (b) \rightarrow (s); (c) \rightarrow (q); (d) \rightarrow (p)$

C) $(a) \rightarrow (q); (b) \rightarrow (s); (c) \rightarrow (r); (d) \rightarrow (p)$

D) $(a) \rightarrow (q); (b) \rightarrow (p); (c) \rightarrow (r); (d) \rightarrow (s)$

Answer: $(a) \rightarrow (r); (b) \rightarrow (s); (c) \rightarrow (q); (d) \rightarrow (p)$



Solution: The formula to calculate the coefficient of viscosity is given by

$$F = 6\pi\eta rv$$

$$\Rightarrow \eta = \frac{F}{6\pi rv} \dots (1)$$

Thus, the dimension of η is

$$[\eta] = \frac{[F]}{[r][v]}$$

$$= \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$

$$= [ML^{-1}T^{-1}]$$

Following the same technique, the dimension of surface tension can be calculated as follows:

$$[T] = \frac{[F]}{[L]}$$

$$= \frac{[MLT^{-2}]}{[L]}$$

$$= [ML^0T^{-2}]$$

The dimension of angular momentum is given by

$$[L] = [mvr]$$

$$= [M][LT^{-1}][L]$$

$$= [ML^2T^{-1}]$$

The dimension of rotational kinetic energy is given by

$$[K] = \left[\frac{1}{2} I \omega^2 \right]$$

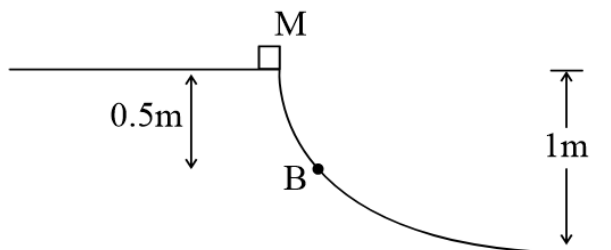
$$= [ML^2][T^{-2}]$$

$$= [ML^2T^{-2}]$$

Hence, the correct option is (a) \rightarrow (r); (b) \rightarrow (s); (c) \rightarrow (q); (d) \rightarrow (p).

Q.48. If a block of mass M is released from the top of a frictionless slide, find the velocity when the block reaches to point B that is 0.5 m below the starting point.

(Take $g = 9.8 \text{ m s}^{-2}$)

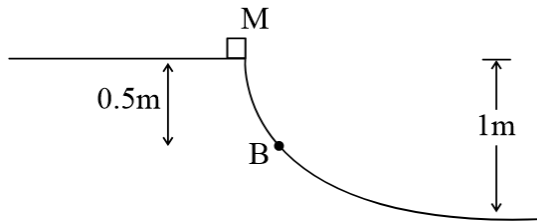


- | | |
|----------------------------|----------------------------|
| A) 3.16 m s^{-1} | B) 3.4 m s^{-1} |
| C) 4.2 m s^{-1} | D) 6.28 m s^{-1} |

Answer: 3.16 m s^{-1}



Solution:



Applying conservation of mechanical energy, we get

$$Mgh = \frac{1}{2}Mv^2$$

$$\Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.5} = 3.16 \text{ m s}^{-1}$$

Q.49. The gravitational potential at a certain height from the surface of the Earth is $-5.12 \times 10^7 \text{ J kg}^{-1}$ and the acceleration due to gravity at that point is 6.4 m s^{-2} . The height from the surface of the Earth is

(Take $R_{\text{earth}} = 6.4 \times 10^6 \text{ m}$)

- A) 8000 m B) 80 km
C) 1600 km D) 800 km

Answer: 1600 km

Solution: Let mass of the earth be M .

The formula to calculate the gravitational potential of an object of mass m , situated at a distance r from the centre of the Earth is given by

$$V = -\frac{GM}{r} \quad \dots (1)$$

Also, the acceleration due to gravity at the same distance can be written as

$$g = \frac{GM}{r^2} \quad \dots (2)$$

From equation (1) and (2), it follows that

$$\frac{V}{g} = \frac{-\frac{GM}{r}}{\frac{GM}{r^2}}$$

$$\Rightarrow r = -\frac{V}{g} \quad \dots (3)$$

Equation (3) implies that

$$r = -\frac{-5.12 \times 10^7 \text{ J kg}^{-1}}{6.4 \text{ m s}^{-2}}$$

$$= 8 \times 10^6 \text{ m}$$

Hence, the required height from the surface of the Earth is given by

$$h = r - R_e$$

$$= 8 \times 10^6 \text{ m} - 6.4 \times 10^6 \text{ m}$$

$$= 1600 \text{ km}$$

Q.50. The distance between an object and its twice magnified real image for a convex lens is 45 cm. Find the focal length of the lens.

- A) 10 cm B) 15 cm
C) 60 cm D) 30 cm

Answer: 10 cm



Solution: The formula to calculate the magnification of the lens is given by

$$m = \frac{v}{u} \dots (1)$$

From equation (1), it follows that

$$\begin{aligned} -2 &= \frac{v}{u} \\ \Rightarrow v &= -2u \dots (2) \end{aligned}$$

Given that,

$$(-u) + (v) = 45 \dots (3)$$

Equation (3) implies that

$$\begin{aligned} -u - 2u &= 45 \\ \Rightarrow u &= -15 \text{ cm} \\ \text{and, } v &= 30 \text{ cm} \end{aligned}$$

The formula to calculate the focal length of the lens is given by

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \dots (4)$$

From equation (4), it follows that

$$\begin{aligned} \frac{1}{30} - \frac{1}{-15} &= \frac{1}{f} \\ \Rightarrow \frac{1}{f} &= \frac{3}{30} \\ \Rightarrow f &= 10 \text{ cm} \end{aligned}$$

Q.51. The ratio of the kinetic energy to the potential energy of an electron in the fifth orbit of a hydrogen like atom is $-\alpha \times 10^{-1}$. Find the value of α .

Answer: 5

Solution: The potential energy of an electron of an atom with atomic number Z in the n^{th} orbit is given by

$$U = -\frac{Ze^2}{4\pi\epsilon_0 r_n} \dots (1)$$

The kinetic energy of the same electron in the same orbit is given by

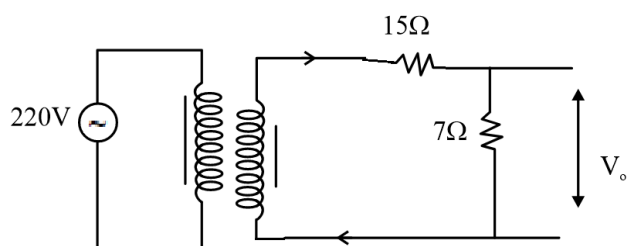
$$K = \frac{1}{2} \left(\frac{Ze^2}{4\pi\epsilon_0 r_n} \right) \dots (2)$$

Thus, the ratio of the kinetic energy to the potential energy of the electron is given by

$$\begin{aligned} \frac{K}{U} &= -\frac{1}{2} \\ &= -0.5 \\ &= -5 \times 10^{-1} \end{aligned}$$

Hence, $\alpha = 5$.

Q.52. In the following circuit, the primary coil has 100 turns and the secondary coil has 10 turns. Find the value of V_0 (in V).



Answer: 7



Solution: The voltages and the number of turns in both primary and the secondary coils are related by the formula:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \dots (1)$$

From equation (1), it follows that

$$V_s = \frac{10}{100} \times 220 \text{ V} \\ = 22 \text{ V}$$

The equivalent resistance in the secondary circuit is given by

$$R = (15 + 7) \Omega \\ = 22 \Omega$$

Thus, the current through the secondary circuit is given by

$$i = \frac{22 \text{ V}}{22 \Omega} \\ = 1 \text{ A}$$

Hence, the required potential difference is given by

$$V_0 = 7 \Omega \times 1 \text{ A} \\ = 7 \text{ V}$$

Q.53. Work function of a material is 3 eV, then find the maximum wavelength(in nm) to the nearest integer for photoemission.

(Take, $hc = 1240 \text{ eV nm}$)

Answer: 413

Solution: The threshold wavelength for a photosensitive material is the maximum possible wavelength of the incident electromagnetic wave for which the emission of the photoelectrons is possible from the surface of the material. The relation of the threshold wavelength and the work function of the material is given as:

$$\phi = h\nu = \frac{hc}{\lambda} = \frac{1240}{\lambda} \\ \Rightarrow \lambda = \frac{1240}{3} = 413 \text{ nm}$$

The threshold wavelength of the given photosensitive material is 413 nm.

Q.54. If in a Hydrogen atom, an electron is excited to an orbit with $E = -0.85 \text{ eV}$, then find the maximum number of transitions to the lower energy level.

Answer: 6

Solution: The energy in the n^{th} orbit is given by

$$E_n = \frac{-13.6 \text{ eV}}{n^2} \dots (1)$$

From equation (1), it follows that

$$-0.85 \text{ eV} = -\frac{13.6}{n^2} \text{ eV} \\ \Rightarrow n^2 = \frac{13.6}{0.85} \\ \approx 16 \\ \Rightarrow n = 4$$

Hence, the maximum number of transition to the lower energy state is given by

$$n = {}^4C_2 \\ = \frac{4!}{2!(4-2)!} \\ = 6$$