JEE Main

29th Jan Shift 2

Questions

Q.1. The best reducing agent among the given ions are:

A) Ce^{4+}

B) Lu^{3+}

C) Gd^{2+}

D) Nd^{3+}

Answer: Gd^{2+}

Solution: In Lanthanides, the most stable oxidation state is $+3$.

So, the element in $+2$ oxidation state will be oxidised to $+3$ oxidation state, hence reduces others, which is known as a reducing agent. Hence, Gd^{2+} acts as reducing agent.

Q.2. Why does oxygen shows anomalous behaviour?

A) Large size , high electronegativity

B) Small size , small electronegativity

C) Large size , high electronegativity, presence of vacant d-orbital.

D) Small size , high electronegativity, absence of vacant d-orbital.

Answer: Small size , high electronegativity, absence of vacant d-orbital.

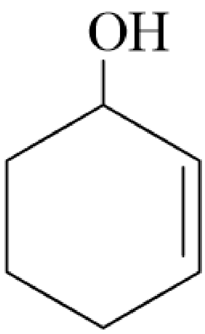
Solution: Due to its small size, Oxygen exhibits high electronegativity, a high charge/ radius ratio, and no d-orbital oxygen shows anomalous behaviour.

Oxygen exists as a diatomic molecule as O_2 comprising a double bond whereas all the other elements of Group 16 exist as polyatomic molecules.

Oxygen is paramagnetic and exists in a gaseous state at room temperature which is completely opposite of the other Group 16 elements.

The small size and high electronegativity of Oxygen enable it in the formation of $p\pi - p\pi$ multiple bonds.

Q.3. IUPAC name of the compound:



A) Cyclohex-2-en-1-ol

B) Hex-2-en-1-ol

C) Cyclohex-1-en-3-ol

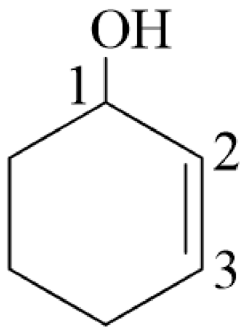
D) 3-Hydroxycyclohexene

Answer: Cyclohex-2-en-1-ol

Solution: The structure of the IUPAC compound is

Secondary prefix+ primary prefix + root word + Primary suffix + Secondary suffix

It is a cyclic compound, with one OH group on position 1 and double bond at position 2.



There is no secondary prefix in the above compound. primary prefix is cyclo. Root word is hex, primary suffix is en and secondary suffix is ol.

Hence, the name is Cyclohex-2-en-1-ol.

Q.4. Which of the following has highest ionisation enthalpy?

A) N B) C

C) Si D) Al

Answer: N

Solution: The first ionization energy is the energy needed to remove the outermost, or highest energy, electron from a neutral atom in the gas phase. It is highest for nitrogen because nitrogen is a more stable half-filled electronic configuration.

Q.5. Nesseler's reagent gives brown colour with

A) CO_2 B) NH_3

C) SO_2 D) CO

Answer: NH_3

Solution: Nesseler's reagent is an alkaline solution of Potassium tetraiodomercurate(II) $(K_2[HgI_4])$. It is prepared by combining potassium iodide (KI) and mercuric chloride

$(HgCl_2)$

It is made slightly alkaline by adding KOH or $NaOH$.

On being reacted with gaseous ammonia it produces brown fumes and on being passed through a solution of ammonia, it gives a dirty brown precipitate. The reaction involved is:

$2K_2[HgI_4] + 3KOH + NH_3 \rightarrow [OHg_2.NH_2]\ I + 7KI + 2H_2O$

Q.6. Which of the following is most acidic?

A) H_3C-CH_3 B) $HC\equiv CH$

C) $H_2C=CH_2$

D) None of the above

Answer: $HC\equiv CH$

Solution: As the s- character increases, electronegativity of the carbon atoms increases. In this way, sp hybridised carbon atom strongly attracts bond pair electrons of $C-H$ bond towards itself and ultimately H^+ ions can be easily donated. Hence, ethyne acts as an acid.

Hence, the answer is option B.

Q.7. Which reagent gives bright red precipitate with Ni^{2+} in basic medium?

A) Nesseler's Reagent B) DMG

C) $KCNS$

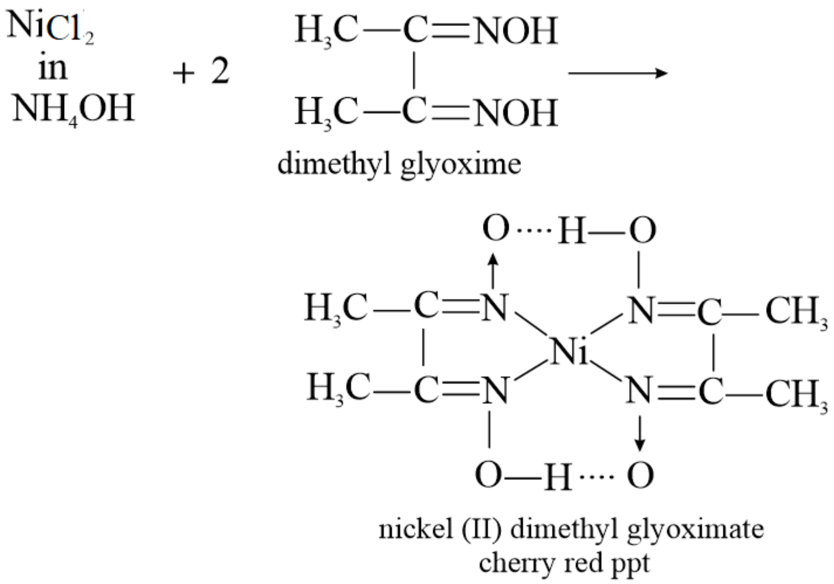
D) $K_4[Fe(CN)_6]$

Answer: DMG

Solution: Estimation of Ni^{2+} is carried out as
Filtrate of group $III + NH_4OH + NH_4Cl$when heated with H_2S, black precipitate of NiS is formed.

This black precipitate of NiS is soluble in $conc.\ HCl$ in presence of oxidising agent like $KClO_3$
$NiS + 2HCl + O \longrightarrow NiCl_2 + H_2O + S$

Now this $NiCl_2$, in basic medium treated with dimethyl glyoxime a cherry red precipitate of bis(dimethylglyoxamate) Nickel(II) complex is formed.



Q.8. IUPAC name of K_2MnO_4 is:

A) Potassium tetraoxomanganate (III)

B) Potassium tetraoxomanganate (VI)

C) Potassium tetraoxomanganese(III)

D) Potassium tetraoxomanganese (VI)

Answer: Potassium tetraoxomanganate (VI)

Solution: The structure of the IUPAC name is

name of cationic part + Name of anionic part.

Metal present in the complex is in anion, so 'ate', must be added at the end of metal name. So options C and D are incorrect.

Oxidation state of Mn

$$\begin{aligned} Mn + 4(-2) &= -2 \\ &= +6 \end{aligned}$$

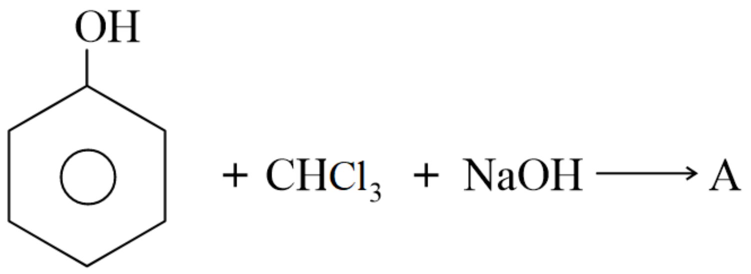
The structure of anionic complex name is

ligand names + metal name(oxidation state)

Hence, the answer is

Potassium tetraoxomanganate (VI)

Q.9.



The product A in the above reaction is

A) Benzene-$1,2$-diol

B) Benzene-$1,3$-diol

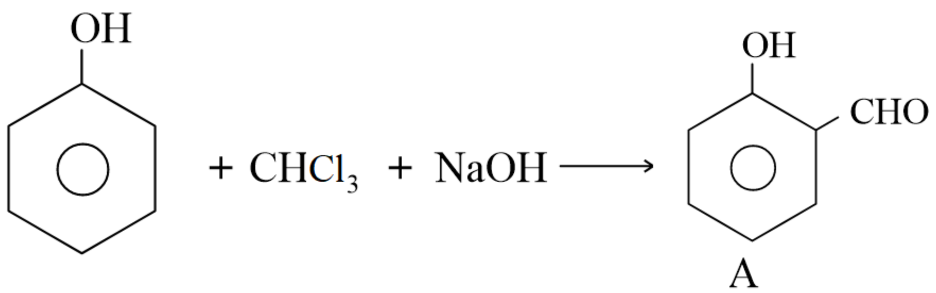
C) Salicylaldehyde

D) Salicylic Acid

Answer: Salicylaldehyde

Solution: When phenol will react with $\mathrm{CHCl_3}$ in the presence of NaOH, salicyaldehyde will be formed.

The reaction will take place as follows,



Q.10. Match the following:

1. Nucleotide	a. $\alpha-\mathrm{D}-\mathrm{glucose}$
2. Starch	b. $\beta-\mathrm{D}-\mathrm{glucose}$
3. Cellulose	c.$\alpha-\mathrm{amino\ \ acids}$
4. Protein	d. Pentose

A) $1-\mathrm{d},\ 2-\mathrm{a},\ 3-\mathrm{b},\ 4-\mathrm{c}$

B) $1-\mathrm{a},\ 2-\mathrm{d},\ 3-\mathrm{b},\ 4-\mathrm{c}$

C) $1-\mathrm{d},\ 2-\mathrm{a},\ 3-\mathrm{c},\ 4-\mathrm{b}$

D) $1-\mathrm{d},\ 2-\mathrm{b},\ 3-\mathrm{a},\ 4-\mathrm{c}$

Answer: $1-\mathrm{d},\ 2-\mathrm{a},\ 3-\mathrm{b},\ 4-\mathrm{c}$

Solution: A nucleotide is made up of three components: a nitrogenous base, a pentose sugar, and one or more phosphate groups.

Starch is formed from $\alpha-$glucose, while cellulose is made of $\beta-$glucose.

Proteins are made up of hundreds or thousands of smaller units called $\alpha-$amino acids.

Q.11. What type of Chromatography depends upon differential adsorption?

A) Paper Chromatography

B) Thin layer Chromatography

C) Column Chromatography

D) B and C both

Answer: B and C both

Solution: Thin Layer Chromatography is a chromatography technique where the mobile phase moves over an adsorbent. The adsorbent is a thin layer which is applied to a solid support for the separation of components.

The separation takes place through differential migration which occurs when the solvent moves along the powder spread on the glass plates.

The basic principle involved in column chromatography is to adsorb solutes of the solution with the help of a stationary phase and further separate the mixture into discrete components.

Q.12. Which reagent is used for the below given reaction:

$$R-X \xrightarrow{\text{Reagent}} R-NC$$

A) KCN

B) $CuCN$

C) $AgCN$

D) None of the above

Answer: $AgCN$

Solution: $AgCN$ is covalent in nature so it does not dissociate in to Ag^{+} and CN^{-} and on nitrogen electron density is high.

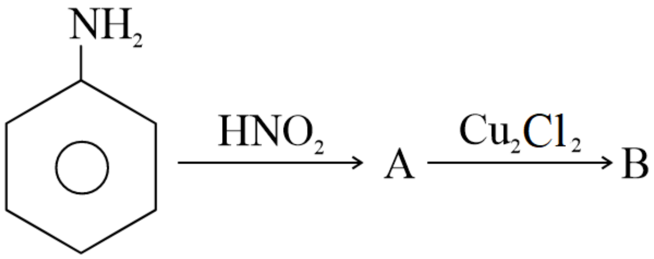
$R-X+AgCN \rightarrow RNC+AgX$

KCN is ionic in nature, so it dissociates completely.

$R-X+KCN \rightarrow RCN+KX$

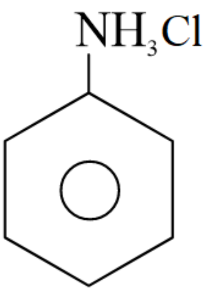
Hence the answer is C.

Q.13.

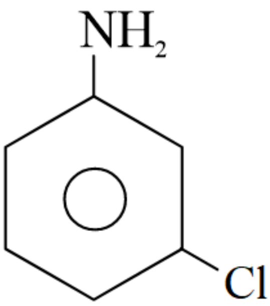


Find the product B in the above reaction is

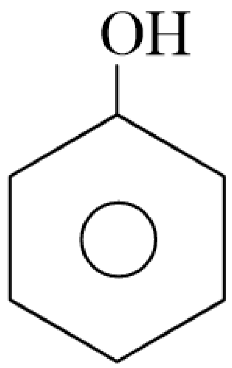
A)



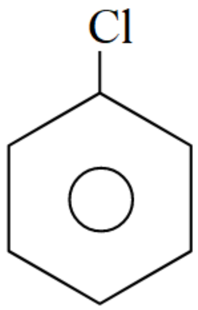
B)



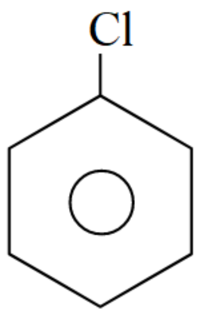
C)



D)

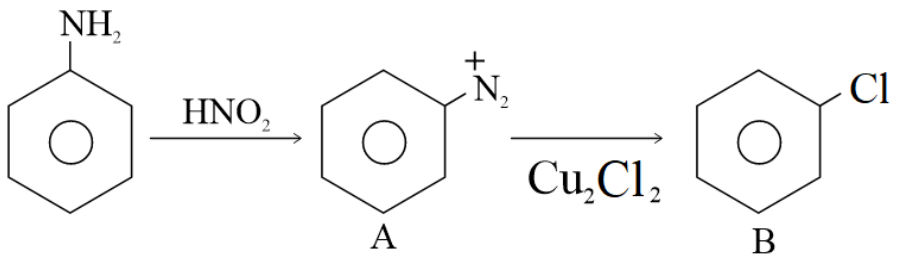


Answer:



Solution: When aniline reacts with HNO_2 , benzene diazonium salt is formed. The diazonium salt converts to chlorobenzene in the second step.

The reaction will takes places as,



The **Sandmeyer reaction** is a type of substitution reaction that is widely used in the production of aryl halides from aryl diazonium salts. Copper salts like chloride, bromide or iodide ions are used as catalysts in this reaction.

Q.14. Match the following:

Compound	pka
A. p-Nitrophenol	1. 10
B. m-Nitrophenol	2. 16
C. Ethanol	3. 7.1
D. Phenol	4. 8.3

A) A-1, B-2, C-3, D-4
B) A-4, B-3, C-2, D-1
C) A-3, B-4, C-2, D-1
D) A-3, B-4, C-1, D-2

Answer: A-3, B-4, C-2, D-1

Solution: The lower the pKa, the stronger the acid and the greater the ability to donate a proton in an aqueous solution.

Electron withdrawing groups increases the acidic nature and Phenols with Electron withdrawing groups at ortho and para position are more acidic than Electron withdrawing group at meta position.

Hence, the acidic nature order is p-nitrophenol>m-nitrophenol>phenol>ethanol.

Hence, the answer is option C.

Q.15. Find the total number of sigma and pi bond in 2-Formyl hex-4-en-oic acid.

A) 22
B) 20
C) 18
D) 24

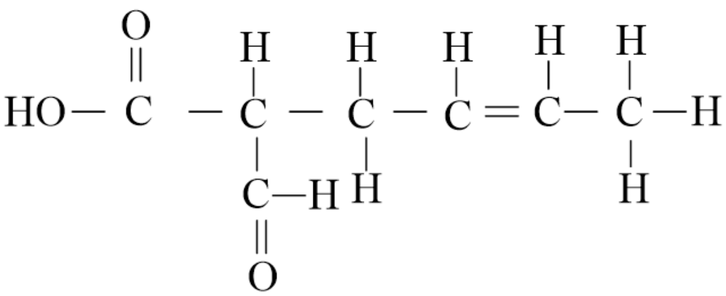
Answer: 22

Solution: A single bond is a result of the axial overlap of bonding orbitals. Hence, it contributes a sigma bond. A multiple bond (double or triple bond) contain one sigam bond and remaining are pi bonds.

The total number of sigma and pi bond in

2-Formyl hex-4-en-oic acid is 22.

The compound can be represented as,



Q.16. The correct statement about $\mathrm{Zn}, \ \mathrm{Cd} \ \text{and} \ \mathrm{Hg}$ is:

A) All are paramagnetic
B) All are solid metals
C) They have high enthalpy of atomization
D) $\mathrm{Zn} \ \text{and} \ \mathrm{Cd}$ cannot show variable oxidation states but Hg can show variable oxidation state

Answer: $\mathrm{Zn} \ \text{and} \ \mathrm{Cd}$ cannot show variable oxidation states but Hg can show variable oxidation state

Solution: Hg is liquid so option B is incorrect.

$\mathrm{Zn}, \ \mathrm{Cd} \ \& \ \mathrm{Hg}$ are diamagnetic so option B is incorrect.

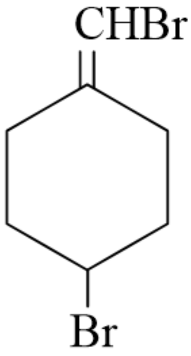
$\mathrm{Zn}, \ \mathrm{Cd} \ \& \ \mathrm{Hg}$ have low enthalpy of atomisation than rest of the series.

$\mathrm{Zn} \ \text{and} \ \mathrm{Cd}$ show $+2$ oxidation state, but Hg shows $0, \ +1 \ \text{and} \ +2$.

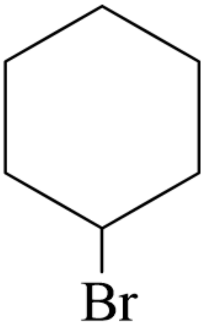
Hence, the answer is option D.

Q.17. Which of the following shows geometrical isomerism?

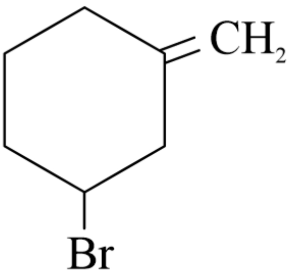
A)



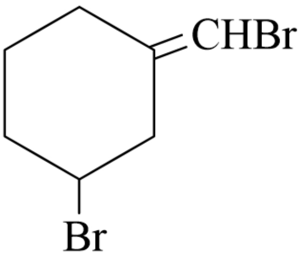
B)



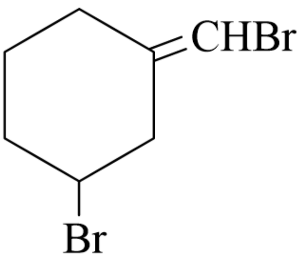
C)



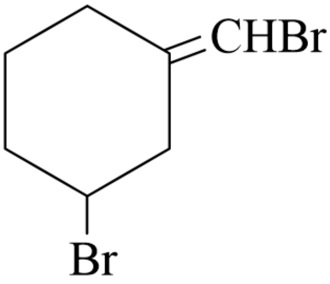
D)



Answer:



Solution: Generally in a geometric isomer, the atoms are bonded by a double bond that does not freely rotate, but it can also happens because of the ring structure.



This compound shows geometrical isomerism.

Hence option D is the answer,

Q.18. Oxidation state of Fe(Iron) in complex formed in Brown ring test.

Answer: 1

Solution: The chemical test called a brown ring test is conducted to find out the presence of Nitrate ions in any solution, in which the solutions form a brown ring in the test tube.

The above test is also called the brown ring test, in which the brown ring complex compound is formulated as $[\mathrm{Fe(H_2O)_5NO}]\,\mathrm{SO_4}$.

Here the oxidation state of Iron in the complex, can be found as:

We can assume that the oxidation state of Iron metal is x.

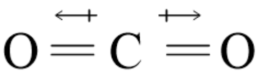
Here the oxidation state of the Sulphate ion is -2, the oxidation state of water is zero and the oxidation state of Nitroso NO is $+1$.Hence, the oxidation state of Iron can be calculated as:
$\Rightarrow \mathrm{x}+(+1)+5(0)=+2$
$\Rightarrow \mathrm{x}=+1$

Q.19. How many of the following have zero dipole moment?

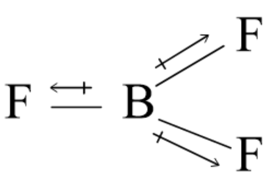
$\mathrm{NH_3},\ \mathrm{H_2O},\ \mathrm{HF},\ \mathrm{CO_2},\ \mathrm{SO_2},\ \mathrm{BF_3},\ \mathrm{CH_4}$

Answer: 3

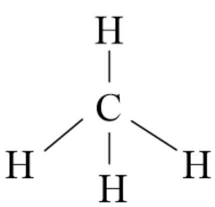
Solution: Dipole moment:- The polarity of a covalent bond can be conveniently measured in terms of a physical quantity called dipole moment. The dipole moment is the vector sum of all the bond moments.



$\mu=0$



$\mu=0$



$\mu=0$

Amongst the given compounds,

$\mathrm{CO_2},\ \ \mathrm{BF_3},\ \mathrm{CH_4}$ have zero dipole moments.

Q.20. 50ml of 0.5 M oxalic acid is completely neutralised by 25ml of $NaOH$ solution. Find out the amount of $NaOH$ in gm present in 25 ml of given $NaOH$ solution.

Answer: 2

Solution: 50ml of 0.5 M oxalic acid is completely neutralised by 25ml of $NaOH$ solution.

hence, NV= constant

Normality of oxalic acid = Molarity$\times$2

$\Rightarrow N_{oxalic\ \ acid} = 0.5 \times 2 = 1N$

$N_1V_1 = N_2V_2$ for neutralisation reaction.

$50 \times 1 = 25 \times N_{NaOH}$

For sodium hydroxide, molarity is the same as normality.

Molarity of sodium hydroxide = 2 M

The number of moles of sodium hydroxide= $25 \times 2 \times 10^{-3}$ mol.

Hence, the mass of sodium hydroxide $= 25 \times 2 \times 10^{-3} \times 40 = 2g$

Q.21. If the standard enthalpy of vaporaisation of CCl_4 is $30.5\ kJ/mol$. Find the heat absorbed for vaporisation of 294 g of CCl_4.

Give an answer to the nearest integer value in kJ/mol.

Answer: 58

Solution: The enthalpy of vapourisation (ΔH) of CCl_4= $30.5\ kJ/mol$

$q_p = \Delta H = 30.5\ kJ/mol$

No. of moles of $CCl_4 = \frac{given\ \ mass}{molar\ \ mass}$

$$= \frac{294}{154} = 1.9\ mol$$

Hence, the heat required for the vapourisation of 1.9 mol of CCl_4 at constant pressure

$= 1.9 \times 30.5$
$\text{Heat required} = 57.95\ kJ/mol \approx 58\ kJ/mol$

Q.22. Calculate the equilibrium constant for the given reaction at 500 K.

$N_{2(g)} + 3H_{2(g)} \leftrightharpoons 2NH_{3(g)}$

Given molarity of $NH_{3(g)}$, $N_{2(g)}$ and $H_{2(g)}$ at equilibrium is 1.5×10^{-2} M, 2×10^{-2} M and 3×10^{-2} M, respectively.

Give answer to the nearest integer value.

Answer: 417

Solution:
$$K_c = \frac{[NH_3]^2}{[H_2]^3[N_2]}$$
$$= \frac{\left(\frac{3}{2}\times10^{-2}\right)^2}{\left(3\times10^{-2}\right)^3\times2\times10^{-2}}$$
$$= \frac{3^2\times10^{-4}}{2^2\times3^3\times10^{-6}\times2\times10^{-2}}$$
$$= \frac{1}{4\times2\times3} \times 10^4 = \frac{10000}{24} = 416.6 \approx 417$$

Q.23. Given set $= \{1, 2, 3, 4, 5, \ldots\ldots\ldots, 50\}$ one number is selected randomly from set. Find the probability that number is multiple of 4 or 6 or 7

A) $\frac{18}{50}$

B) $\frac{21}{50}$

C) $\frac{8}{25}$

D) $\frac{21}{25}$

Answer: $\frac{21}{50}$

Solution: Let $A=\{1,2,3,4,5,\ldots\ldots\ldots,50\}$

Now, the probability of choosing a number which is a multiple of 4 will be $P(4)=\frac{12}{50}$

Similarly, the probability of choosing a number which is a multiple of 6 will be $P(6)=\frac{8}{50}$

And the probability of choosing a number which is a multiple of 7 will be $P(7)=\frac{7}{50}$

Also, the probability of choosing a number which is a multiple of 4 & 6 will be $P(4\cap 6)=\frac{4}{50}$

And the probability of choosing a number which is a multiple of 6 & 7 will be $P(6\cap 7)=\frac{1}{50}$

And the probability of choosing a number which is a multiple of 4 & 7 will be $P(4\cap 7)=\frac{1}{50}$

And finally, the probability of choosing a number which is a multiple of $4,\ 6$ & 7 will be $P(4\cap 6\cap 7)=0$

Hence, $P(4\cup 6\cup 7)=\frac{12}{50}+\frac{8}{50}+\frac{7}{50}-\frac{4}{50}-\frac{1}{50}-\frac{1}{50}+0$

$\Rightarrow P(4\cup 6\cup 7)=\frac{21}{50}$

Q.24. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sqrt{1-\sin 2x}dx$ is ______.

A) $2\sqrt{2}+\sqrt{3}-1$

B) $\sqrt{2}-\sqrt{3}+1$

C) $\sqrt{2}+\sqrt{3}-1$

D) $2\sqrt{2}-\sqrt{3}-1$

Answer: $2\sqrt{2}-\sqrt{3}-1$

Solution:

Let, $y=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sqrt{1-\sin 2x}dx$

$\Rightarrow y=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sqrt{(\sin x-\cos x)^2}dx$

$\Rightarrow y=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}|\sin x-\cos x|dx$

$\Rightarrow y=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}(\cos x-\sin x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(\sin x-\cos x)dx$

$\Rightarrow y=[\sin x+\cos x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}+[-\sin x-\cos x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$\Rightarrow y=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}-\frac{1}{2}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$

$\Rightarrow y=2\sqrt{2}-\sqrt{3}-1$

Q.25. If $A=\{1,2,3,4\}$ then the minimum number of elements added to make it equivalence relation on set A containing $(1,3)$ and $(1,2)$ is

A) 9

B) 8

C) 12

D) 16

Answer: 8

Solution: Given,

$R=\{(1,3),(1,2)\}$

To make R as Reflexive, the required elements are $(1,1),\ (2,2),\ (3,3)$ and $(4,4)$

To make R as Symmetric, the required elements are $(3,1)$ and $(2,1)$

To make R as Transitive, the required elements are $(3,2)$ and $(2,3)$

So, 8 elements are required to make R and equivalence relation.

Q.26. If $P(3,2,3),\ Q(4,6,2),\ R(7,3,2)$ are the vertices of $\triangle PQR$, then find $\angle QPR$.

A) $\frac{\pi}{3}$

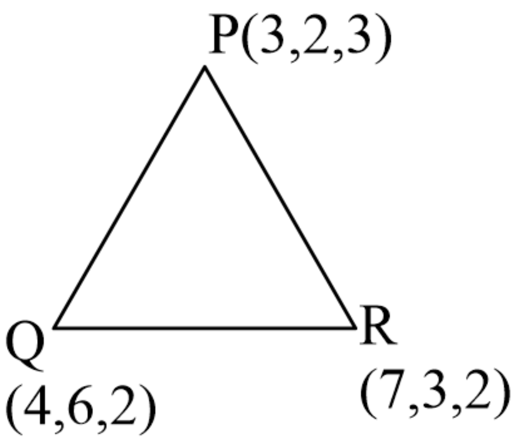
B) $\cos^{-1}\frac{1}{18}$

C) $\frac{\pi}{6}$

D) $\cos^{-1}\frac{7}{18}$

Answer: $\frac{\pi}{3}$

Solution: Given,



Direction Ratios:

$\overrightarrow{QP}=-1,-4,1$

$\overrightarrow{RP}=-4,-1,1$

$\cos\angle QPR=\frac{\overrightarrow{QP}\cdot\overrightarrow{RP}}{\left|\overrightarrow{QP}\right|\left|\overrightarrow{RP}\right|}$

$\Rightarrow\cos\angle QPR=\frac{4+4+1}{\sqrt{1+16+1}\sqrt{16+1+1}}$

$\Rightarrow\cos\angle QPR=\frac{9}{18}=\frac{1}{2}$

$\Rightarrow\angle QPR=\frac{\pi}{3}$

Q.27. If $\ln a,\ \ln b,\ \ln c$ are in $A.P$ and $\ln a-\ln 2b,\ \ln 2b-\ln 3c,\ \ln 3c-\ln a$ are in $A.P$ then $a:b:c$ is

A) $7:7:3$

B) $1:2:3$

C) $4:6:9$

D) $9:6:4$

Answer: $9:6:4$

Solution: Given,

$\ln\ a,\ \ln\ b,\ \ln\ c$ are in $A.P$

$\Rightarrow 2\ln b=\ln a+\ln c$

$\Rightarrow b^2=ac\ \ldots.(1)$

And $\ln a-\ln 2b,\ \ln 2b-\ln 3c\ ,\ \ln 3c-\ln a$ are in $A.P$

$\Rightarrow 2\ln\frac{2b}{3c}=\ln\frac{a}{2b}+\ln\frac{3c}{a}$

$\Rightarrow\left(\frac{2b}{3c}\right)^2=\frac{a}{2b}\times\frac{3c}{a}$

$\Rightarrow\left(\frac{2b}{3c}\right)^2=\frac{3c}{2b}$

$\Rightarrow 2b=3c$

$\Rightarrow 4b^2=9c^2\ \&\ 6b=9c.\ldots.(2)$

Now solving the equation $(1)\ \&\ (2)$ we get,

$4ac=9c^2\Rightarrow 4a=9c\ \ldots.(3)$

Then from equation $(2)\ \&\ (3)$ we get,

$4a=6b=9c=k$

$\Rightarrow a=\frac{k}{4},\ b=\frac{k}{6}\ \&\ c=\frac{k}{9}$

$\Rightarrow a:b:c=\frac{1}{4}:\frac{1}{6}:\frac{1}{9}=9:6:4$

Q.28. If $r=|z|,\ \theta=\arg(z)$ and $z=2-2i\tan\left(\frac{5\pi}{8}\right)$, then find (r,θ)

A) $\left(2\sec\frac{3\pi}{8},\ \frac{3\pi}{8}\right)$

B) $\left(2\sec\frac{5\pi}{8},\ \frac{3\pi}{8}\right)$

C) $\left(2\tan\frac{3\pi}{8},\ \frac{5\pi}{8}\right)$

D) $\left(2\tan\frac{3\pi}{8},\ \frac{3\pi}{8}\right)$

Answer: $\left(2\sec\frac{3\pi}{8},\ \frac{3\pi}{8}\right)$

Solution: Given,

$$\Rightarrow z=2\left[1-i\tan\left(\frac{5\pi}{8}\right)\right]$$

$$\Rightarrow z=2\left[1-\frac{i\sin\left(\frac{5\pi}{8}\right)}{\cos\left(\frac{5\pi}{8}\right)}\right]$$

$$\Rightarrow z=\frac{2}{\cos\left(\frac{5\pi}{8}\right)}\left[\cos\left(\frac{5\pi}{8}\right)-i\sin\left(\frac{5\pi}{8}\right)\right]$$

$$\Rightarrow z=\frac{2}{\cos\left(\pi-\frac{3\pi}{8}\right)}\left[\cos\left(\pi-\frac{3\pi}{8}\right)-i\sin\left(\pi-\frac{3\pi}{8}\right)\right]$$

$$\Rightarrow z=\frac{2}{-\cos\left(\frac{3\pi}{8}\right)}\left[-\cos\left(\frac{3\pi}{8}\right)-i\sin\left(\frac{3\pi}{8}\right)\right]$$

$$\Rightarrow z=\frac{2}{\cos\left(\frac{3\pi}{8}\right)}\left[\cos\left(\frac{3\pi}{8}\right)+i\sin\left(\frac{3\pi}{8}\right)\right]$$

$$\Rightarrow z=2\sec\frac{3\pi}{8}e^{i\cdot\frac{3\pi}{8}}$$

Now on comparing with $z=|z|e^{i\theta}$ we get,

$$\Rightarrow \theta=\frac{3\pi}{8},\ r=2\sec\frac{3\pi}{8}$$

Q.29. In which interval the function $f(x)=\frac{x}{x^2-6x-16}$ is increasing

A) $\left[1,\frac{3}{7}\right)\cup\left(\frac{5}{4},\infty\right)$

B) ϕ

C) $\left(\frac{5}{4},\infty\right)$

D) $\left[\frac{3}{4},\frac{5}{4}\right]$

Answer: ϕ

Solution: Given,

$$f(x)=\frac{x}{x^2-6x-16}$$

Now differentiating the above function we get,

$$\Rightarrow f'(x)=\frac{\left(x^2-6x-16\right)\cdot 1-x\cdot(2x-6)}{\left(x^2-6x-16\right)^2}$$

$$\Rightarrow f'(x)=\frac{-x^2-16}{\left(x^2-6x-16\right)^2}$$

$$\Rightarrow f'(x)<0\ \forall\ x$$

So, the function is always decreasing

Hence, $x\in\phi$

Q.30. If first term of a non constant GP be $\frac{1}{8}$ and every term is AM of next two, then $\sum_{r=1}^{20}T_r-\sum_{r=1}^{18}T_r$ is

A) -2^{15}

B) 2^{-10}

C) 2^{-5}

D) 2^{-20}

Answer: -2^{15}

Solution: Let $\text{GP}:\ a,\ ar,\ ar^2,\ ar^3,\ldots$

Now, given: $2ar=ar^2+ar^3$

$\Rightarrow 2=r+r^2$

$\Rightarrow r^2+r-2=0$

$\Rightarrow (r+2)(r-1)=0$

$\Rightarrow r=-2,\ 1\,[\text{Since GP is not constant so this value is cancelled}]$

$\Rightarrow r=-2$

$\Rightarrow \sum_{r=1}^{20}Tr-\sum_{r=1}^{18}Tr=\frac{a\left(1-r^{20}\right)}{1-r}-\frac{a\left(1-r^{18}\right)}{1-r}$

$\Rightarrow \sum_{r=1}^{20}Tr-\sum_{r=1}^{18}Tr=\frac{a\left[r^{18}-r^{20}\right]}{1-r}$

$\Rightarrow \sum_{r=1}^{20}Tr-\sum_{r=1}^{18}Tr=\frac{\left[2^{18}-2^{20}\right]}{8\times 3}$

$\Rightarrow \sum_{r=1}^{20}Tr-\sum_{r=1}^{18}Tr=\frac{2^{18}(1-4)}{24}$

$\Rightarrow \sum_{r=1}^{20}Tr-\sum_{r=1}^{18}Tr=-2^{15}$

Q.31. The mean of 5 observations is $\frac{24}{5}$ and variance is $\frac{194}{25}$. If the mean of first four observations is $\frac{7}{2}$, then the variance of first four observations is _____.

A) $\frac{3}{2}$

B) $\frac{2}{3}$

C) $\frac{5}{2}$

D) $\frac{5}{4}$

Answer: $\frac{5}{4}$

Solution: Given:

$\overline{x}=\frac{24}{5}$ and variance is $\frac{194}{25}$

$\Rightarrow \sum_{i=1}^{5}x_i=\frac{24}{5}\times 5$

$\Rightarrow \sum_{i=1}^{5}x_i=24$

Also, variance

$\Rightarrow \frac{\sum_{i=1}^{5}(x_i)^2}{5}-\left(\frac{24}{5}\right)^2=\frac{194}{25}$

$\Rightarrow \sum_{i=1}^{5}(x_i)^2=154$

Now, 5^{th} observation $=24-\text{sum of first 4 terms}$

$=24-\frac{7}{2}\times 4=10$

So, new variance $=\frac{\sum_{i=1}^{4}(x_i)^2}{4}-\left(\frac{7}{2}\right)^2$

$=\frac{154-100}{4}-\frac{49}{4}$

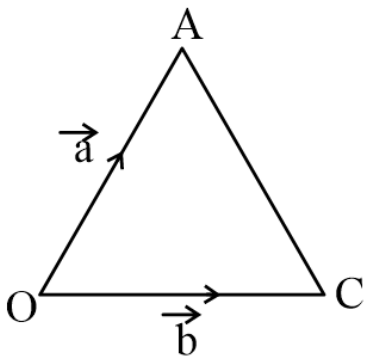
$=\frac{5}{4}$

Q.32. If $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OC}=\vec{b}$ and area of $\triangle OAC$ is S and a parallelogram with sides parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ and diagonal $\overrightarrow{OB}=12\vec{a}+4\vec{b}$ has area equal to B, then $\frac{B}{S}$ is equal to _____.

A) 96
B) 48
C) 24
D) 12

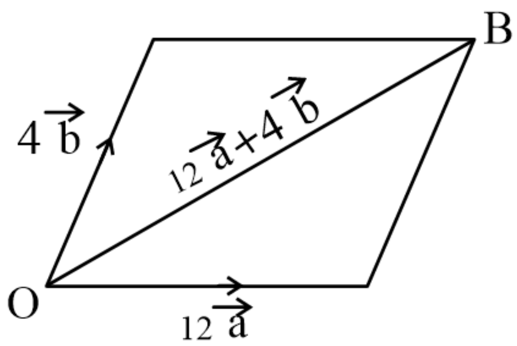
Answer: 96

Solution: Given,



Area of $\triangle OAC$ is given by,

$S=\frac{1}{2}\left|\vec{a}\times\vec{b}\right| \quad \ldots(i)$



Also, area of parallelogram is given by,

$B=\left|12\vec{a}\times 4\vec{b}\right|$

$\Rightarrow B=48\left|\vec{a}\times\vec{b}\right| \quad \ldots(ii)$

$\Rightarrow \frac{B}{S}=\frac{48}{\frac{1}{2}}=96$

Q.33. $\left(x\cos\left(\frac{y}{x}\right)\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$, where $\sin\left(\frac{y}{x}\right)=\log|x|+\frac{\alpha}{2}$ and $f(1)=\frac{\pi}{3}$, then $\alpha^2=$_____.

A) 3
B) 4
C) 2
D) 1

Answer: 3

Solution: Given: $\left(x\cos\left(\frac{y}{x}\right)\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$

$\Rightarrow \cos\left(\frac{y}{x}\right)\frac{dy}{dx}=\frac{y}{x}\cos\left(\frac{y}{x}\right)+1$

Putting, $y=vx$

$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow \cos v\left(v+x\frac{dv}{dx}\right)=v\cos v+1$

$\Rightarrow v+x\frac{dv}{dx}=v+\frac{1}{\cos v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1}{\cos v}$

$\Rightarrow (\cos v)dv=\frac{dx}{x}$

$\Rightarrow \int(\cos v)dv=\int\frac{dx}{x}$

$\Rightarrow \sin v=\log|x|+c$

$\Rightarrow \sin\left(\frac{y}{x}\right)=\log|x|+c$

Now, $f(1)=\frac{\pi}{3}$

$\Rightarrow \sin\left(\frac{\pi}{3}\right)=\log|1|+c$

$\Rightarrow \frac{\sqrt{3}}{2}=c$

$\Rightarrow \sin\left(\frac{y}{x}\right)=\log|x|+\frac{\sqrt{3}}{2}$

So, on comparing we get,

$\Rightarrow \alpha=\sqrt{3}$

$\Rightarrow \alpha^2=3$

Q.34. Find the remainder when $64^{32^{32}}$ is divided by 9

Answer: 1

Solution: Given,

$64^{32^{32}}=(63+1)^{32\times32\times32....32\text{times}}$

$\Rightarrow 64^{32^{32}}=(63+1)^{32\lambda}$

Now we know that by binomial theorem,

$(1+63)^{32\lambda}={}^{32\lambda}C_0\cdot1+{}^{32\lambda}C_1\cdot1\cdot63+{}^{32\lambda}C_2\cdot1\cdot63^2+\ \ldots\ldots$

$\Rightarrow (1+63)^{32\lambda}=1+9\alpha$

So, when $64^{32^{32}}$ is divided by 9 we get 1 as remainder.

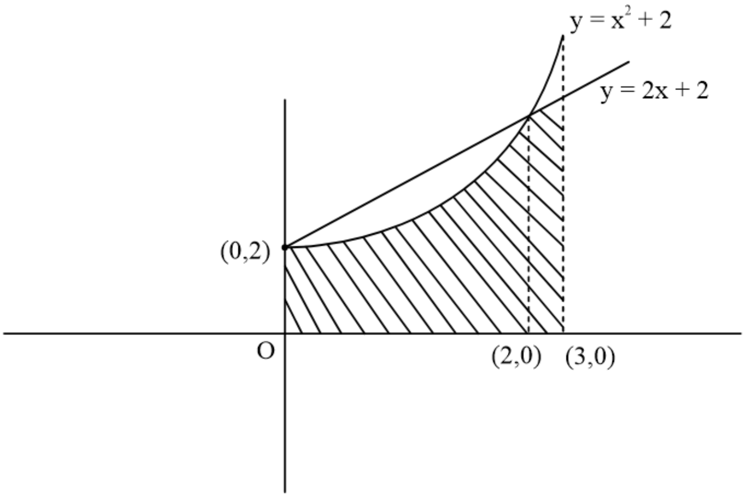
Q.35. If area bounded by $0\le y\le\min\left\{x^2+2,\ 2x+2\right\},\ x\in[0,3]$ is A, then the value of $12A$ will be

Answer: 164

Solution: Given,

$0 \leq y \leq \min \left\{\mathrm{x}^2+2,\ 2\mathrm{x}+2\right\},\ \mathrm{x} \in [0,3]$

Now, plotting the diagram of the above expression we get,



Now, from above diagram we get,

$A=\int_0^2 x^2+2\ \mathrm{d}x+\int_2^3 2x+2\ \mathrm{d}x$

$\Rightarrow A=\left[\frac{x^3}{3}+2x\right]_0^2+\left[x^2+2x\right]_2^3$

$\Rightarrow A=\left[\frac{8}{3}+4\right]+[9+6-4-4]$

$\Rightarrow A=\frac{20}{3}+7=\frac{41}{3}$

$\Rightarrow 12A=164$

Q.36. The number of ways to distribute 8 identical books into 4 distinct bookshelf is (where any bookshelf can be empty)

Answer: 165

Solution: Given,

$n=8,\ r=4$

$\Rightarrow x_1+x_2+x_3+x_4=8$

The number of ways of distribution is given by,

$N={}^{n+r-1}C_{r-1}$

$\Rightarrow N={}^{11}C_3=\frac{11\times10\times9}{3\times2\times1}$

$\Rightarrow N=165$

Q.37. If $f(x)=\log\left(\frac{1-x^2}{1+x^2}\right)$, then value of $225\left[f'(x)-f''(x)\right]$ at $x=\frac{1}{2}$ is ______.

Answer: 736

Solution:

Given: $f(x)=\log\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow f(x)=\log\left(1-x^2\right)-\log\left(1+x^2\right)$

$\Rightarrow f'(x)=\frac{-2x}{\left(1-x^2\right)}-\frac{2x}{\left(1+x^2\right)}$

$\Rightarrow f'(x)=(-2x)\left[\frac{1+x^2+1-x^2}{1-x^4}\right]$

$\Rightarrow f'(x)=\left(\frac{4x}{x^4-1}\right)$

$\Rightarrow f''(x)=\frac{\left(x^4-1\right)(4)-(4x)\left(4x^3\right)}{\left(x^4-1\right)^2}$

$\Rightarrow f''(x)=\frac{4\left(-3x^4-1\right)}{\left(x^4-1\right)^2}$

$\Rightarrow 225\left[f'(x)-f''(x)\right]=225\left[\left(\frac{4x}{x^4-1}\right)-\frac{4\left(-3x^4-1\right)}{\left(x^4-1\right)^2}\right]$

Putting, $x=\frac{1}{2}$

$\Rightarrow 225\left[f'(x)-f''(x)\right]=225\left[\left(\frac{2}{-\frac{15}{16}}\right)-\frac{4\left(-\frac{3}{16}-1\right)}{\left(\frac{1}{16}-1\right)^2}\right]$

$\Rightarrow 225\left[f'(x)-f''(x)\right]=225\left[\left(\frac{-32}{15}\right)-\frac{4\left(\frac{-19}{16}\right)}{\left(\frac{-15}{16}\right)^2}\right]$

$\Rightarrow 225\left[f'(x)-f''(x)\right]=225\left[\left(\frac{-32}{15}\right)+\frac{4\times19\times16}{225}\right]$

$\Rightarrow 225\left[f'(x)-f''(x)\right]=[-480+1216]=736$

$\Rightarrow 225\left[f'(x)-f''(x)\right]=736$

Q.38. If $\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$, then find the sum of the roots.

Answer: 1

Solution: Given: $\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$

$\Rightarrow \frac{\cos 2x\left(3+\cos^2 2x\right)}{\left(\cos^2 x\right)^3-\left(\sin^2 x\right)^3}=x^3-x^2+6$

$\Rightarrow \frac{\cos 2x\left(3+\cos^2 2x\right)}{\left(\cos^2 x-\sin^2 x\right)\left(\cos^4 x+\sin^4 x+\sin^2 x\cos^2 x\right)}=x^3-x^2+6$

$\Rightarrow \frac{\left(3+\cos^2 2x\right)}{\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x+\sin^2 x\cos^2 x}=x^3-x^2+6$

$\Rightarrow \frac{\left(3+\cos^2 2x\right)}{1-\sin^2 x\cos^2 x}=x^3-x^2+6$

$\Rightarrow \frac{\left(3+\cos^2 2x\right)}{1-\frac{(2\sin x\cos x)^2}{4}}=x^3-x^2+6$

$\Rightarrow 4\left(\frac{3+\cos^2 2x}{4-\sin^2 2x}\right)=x^3-x^2+6$

$\Rightarrow 4\left(\frac{3+1-\sin^2 2x}{4-\sin^2 2x}\right)=x^3-x^2+6$

$\Rightarrow 4=x^3-x^2+6$

$\Rightarrow x^3-x^2+2=0$

So, the sum of roots is 1.

Q.39. An electromagnetic wave has electric field given by $\vec{E}=\left(9.6\ \hat{j}\right)\sin\left[2\pi\left\{30\times10^6 t-\frac{1}{10}x\right\}\right]\ \mathrm{N\ C^{-1}}$ where, x and t are in S.I. units. The maximum magnetic field is:

A) $10^{-7}\ \mathrm{T}$ B) $9.6\times10^{-8}\ \mathrm{T}$

C) $3.2\times10^{-8}\ \mathrm{T}$ D) $1.7\times10^{-8}\ \mathrm{T}$

Answer: $3.2\times10^{-8}\ \mathrm{T}$

Solution: Comparing the given equation with the standard equation of electric field of electromagnetic wave, we get

$E_0=9.6\ \mathrm{N\ C^{-1}}$

We know that the electric field and magnetic field in an EM-wave are related by the expression

$E_0=\mathrm{c}B_0$

Hence,

$B_0=\frac{E_0}{c}$

$=\frac{9.6}{3\times10^8}=3.2\times10^{-8}\ \mathrm{T}$

Q.40. A planet at distance r from the Sun takes 200 days to complete one revolution around the Sun. What will be the time period for a planet at a distance $\frac{r}{4}$ from the Sun?

A) 12.5 days B) 25 days

C) 50 days D) 100 days

Answer: 25 days

Solution: Kepler's Third Law: the squares of the orbital periods of the planets are directly proportional to the cubes of the semi-major axes of their orbits.

Therefore,

$$\left(\frac{T_1}{T_2}\right)^2=\left(\frac{R_1}{R_2}\right)^3$$

$$\Rightarrow\left(\frac{200}{T_2}\right)^2=\left(\frac{r}{\frac{r}{4}}\right)^3$$

$$\Rightarrow T_2=\frac{200}{8}=25\text{ days}$$

Q.41. In a simple pendulum of length 10 m, string is initially kept horizontal and the bob is released. If 10% of energy is lost till the bob reaches the lowermost position, then find the speed of bob at the lowermost position.

A) $6\sqrt{5}\text{ m s}^{-1}$

B) 6 m s^{-1}

C) $4\sqrt{2}\text{ m s}^{-1}$

D) $7\sqrt{5}\text{ m s}^{-1}$

Answer: $6\sqrt{5}\text{ m s}^{-1}$

Solution: At the point of release, the potential energy of the pendulum bob is given by(considering the bottom point as reference)

$$U=mgl\quad\ldots(1)$$

At the bottommost point the kinetic energy of the pendulum bob is given by

$$K=\frac{1}{2}mv^2\quad\ldots(2)$$

According to the given problem, it can be written that

$$0.9U=K\quad\ldots(3)$$

From equations (1), (2) and (3), it follows that

$$0.9mgl=\frac{1}{2}mv^2$$
$$\Rightarrow v=\sqrt{1.8gl}$$
$$=\sqrt{1.8\times10\times10}\text{ m s}^{-1}$$
$$=6\sqrt{5}\text{ m s}^{-1}$$

Q.42. The intensity at each slit are equal for a YDSE and it is maximum I_{max} at central maxima. If I is intensity for phase difference $\frac{7\pi}{2}$ between two waves at screen. Then $\frac{I}{Imax}$is

A) $\frac{1}{4}$

B) $\frac{1}{2}$

C) $\frac{1}{\sqrt{2}}$

D) $\frac{3}{8}$

Answer: $\frac{1}{2}$

Solution: The formula to calculate the intensity of the fringe pattern observed on the screen is given by

$$I=4I_0\cos^2\frac{\varphi}{2}\quad\ldots(1)$$

where, $4I_0=I_{max}$ is the maximum intensity and φ is the phase difference.

From equation (1), it follows that

$$\frac{I}{Imax}=\cos^2\frac{7\pi}{4}$$
$$=\frac{1}{2}$$

Q.43. Two equal charges of masses m_1 & m_2 are sent in a transverse magnetic field by accelerating through the same potential difference. Find the ratio of their radii inside.

A) $\frac{m_1}{m_2}$

B) $\frac{m_2}{m_1}$

C) $\sqrt{\frac{m_1}{m_2}}$

D) $\sqrt{\frac{m_2}{m_1}}$

Answer: $\sqrt{\frac{m_1}{m_2}}$

Solution: Kinetic energy gained by accelerating potential,

$\frac{1}{2}mv^2 = qV \quad ....Eq(1).$

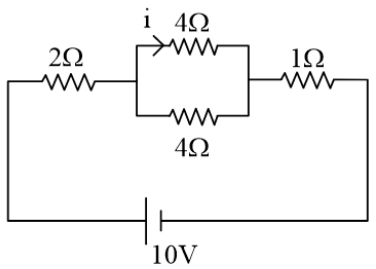
Now, radius of circular path in magnetic field is given by,

$$\frac{mv^2}{r} = qvB$$
$$\Rightarrow \frac{1}{2}mv^2 = \frac{qvBr}{2}$$
$$\Rightarrow qV = \frac{qvBr}{2}$$

As, qV is same for both particles, therefore

$$\frac{qv_1Br_1}{2} = \frac{qv_2Br_2}{2}$$
$$\Rightarrow \frac{r_1}{r_2} = \frac{v_2}{v_1}$$
$$\Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{\frac{2qV}{m_2}}}{\sqrt{\frac{2qV}{m_1}}} \text{ From equation } (1)$$
$$= \sqrt{\frac{m_1}{m_2}}$$

Q.44. In the following circuit, find the value of the current i.



A) 2 A

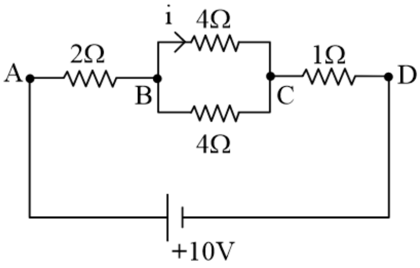
B) 1 A

C) 4 A

D) 3 A

Answer: 1 A

Solution: Let's consider the following diagram:



The equivalent resistance (R_{eq}) for the given circuit can be calculated as follows:

$$R_{eq}=2\ \Omega+\frac{1}{\frac{1}{4\ \Omega}+\frac{1}{4\ \Omega}}+1\ \Omega$$
$$=5\ \Omega$$

Thus, the current (i) through the entire circuit is given by

$$i=\frac{10\ \mathrm{V}}{5\ \Omega}$$
$$=2\ \mathrm{A}$$

Since across points B and C, two equal resistors are connected in parallel, the current through each resistor will be the same.

Hence, the required current is given by

$$i=1\ \mathrm{A}\ .$$

Q.45. Two rods of the same length and material is applied with the forces F and $\frac{F}{2}$ respectively. If the cross-sectional radii are R and $\frac{R}{2}$, find the ratio of the extensions of the rod.

A) $1:2$

B) $2:1$

C) $4:1$

D) $1:4$

Answer: $1:2$

Solution: The ratio of the cross-sectional areas of the rods can be calculated as follows:

$$\frac{A_1}{A_2}=\frac{\pi R^2}{\pi\left(\frac{R}{2}\right)^2}$$
$$=\frac{4}{1}$$

The formula to calculate the Young's modulus for the two rods can be written as

$$Y=\frac{F_1 L}{A_1 l_1}\quad\ldots(2)\quad[\text{for 1st rod}]$$
$$Y=\frac{F_2 L}{A_2 l_2}\quad\ldots(3)\quad[\text{for 2nd rod}]$$

Equating equations (2) and (3), we have

$$\frac{F_1 L}{A_1 l_1}=\frac{F_2 L}{A_2 l_2}$$
$$\Rightarrow\frac{l_1}{l_2}=\frac{F_1 A_2}{F_2 A_1}$$
$$=\frac{F}{\frac{F}{2}}\times\frac{1}{4}$$
$$=\frac{1}{2}$$

Q.46. Alternating voltage and current in an AC circuit is given as:

$V=100\sin\omega t$ V

$I=100\sin\left(\omega t+\frac{\pi}{3}\right)$ mA

Find the average power dissipated in the circuit.

A) 2.5 W

B) 5 W

C) 10 W

D) 20 W

Answer: 2.5 W

Solution: Given,
Instantaneous voltage is, $\mathrm{V}=100\sin\omega\mathrm{t}$ V
Instantaneous current is, $I=100\sin\left(\omega t+\frac{\pi}{3}\right)$ mA
It is clear from the given quantities that the current is leading voltage by $\phi=\frac{\pi}{3}$ rad.

Power factor is given by, $\cos(\phi)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$

Now average power is given as, $P=\frac{V_O I_O}{2}\cos(\phi)$, where $V_O=100$ V & $I_O=100$ mA represents peak voltage and peak current respectively.

$\Rightarrow P=\frac{100\times100\times10^{-3}}{2}\times\frac{1}{2}=2.5$ W

Q.47. Two blocks of equal volume have the same elongation for the deforming forces F_1 and F_2. Find the ratio of the forces $\frac{F_1}{F_2}$. Given the ratio of their cross-sectional area as $A_1:A_2=4:1$.

A) $1:4$

B) $4:1$

C) $1:16$

D) $16:1$

Answer: $16:1$

Solution: As the volume is same for the given blocks, it can be written that

$A_1L_1=A_2L_2$

$\Rightarrow\frac{L_1}{L_2}=\frac{A_2}{A_1}\quad\ldots(1)$

where, $L_1,\ L_2$ are the respective lengths of the blocks.

The formula to calculate the Young's modulus for both the blocks can be written as

$Y_1=\frac{F_1L_1}{A_1l}\quad\ldots(2)$

$Y_2=\frac{F_2L_2}{A_2l}\quad\ldots(3)$

Since both the blocks are made of the same material, equating equation (2) and (3), we have

$\frac{F_1L_1}{A_1l}=\frac{F_2L_2}{A_2l}$

$\Rightarrow\frac{F_1}{F_2}=\frac{A_1L_2}{A_2L_1}$

$=\frac{A_1^2}{A_2^2}$

$=\left(\frac{4}{1}\right)^2$

$=\frac{16}{1}$

Q.48. The time period of a particle performing SHM is 6π s. Find the time taken by the particle to move from $x=A$ to $x=\frac{A}{2}$, where A is the amplitude of oscillation.

A) $\frac{\pi}{2}$ s

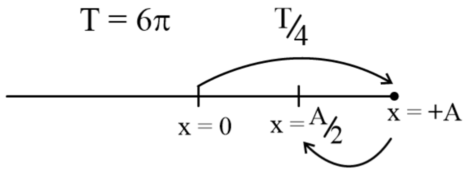
B) π s

C) 3π s

D) $\frac{3\pi}{2}$ s

Answer: π s

Solution: Let's consider the following figure:



The particle will take the time of $\frac{T}{4}$ to move from its mean position to $x=+A$.

Let, the equation of motion of the particle starting from its mean position is given by

$x=A\sin\omega t \quad \ldots(1)$

If t be the time taken by the particle to move from its mean position to $x=\frac{A}{2}$ for the second time (after crossing $x=+A$), we have from equation (1),

$$\frac{A}{2}=A\sin\omega t$$
$$\Rightarrow \sin\omega t=\frac{1}{2}$$
$$\Rightarrow \sin\omega t=\sin\frac{5\pi}{6}$$
$$\Rightarrow \omega t=\frac{5\pi}{6}$$
$$\Rightarrow \frac{2\pi}{T}t=\frac{5\pi}{6}$$
$$\Rightarrow t=\frac{5T}{12}$$

Hence, the time required by the particle to move from $x=A$ to $x=\frac{A}{2}$ is given by

$$t'=t-\frac{T}{4}$$
$$=\frac{5T}{12}-\frac{T}{4}$$
$$=\frac{T}{6}$$
$$=\frac{6\pi\ \mathrm{s}}{6}$$
$$=\pi\ \mathrm{s}$$

Q.49. For an ideal gas, pressure is $1.38\ \mathrm{atm}$ and the number of molecules are $2\times10^{25}\ \mathrm{m}^{-3}$. Find the temperature of the gas.

A) 1000 K

B) 1500 K

C) 250 K

D) 500 K

Answer: 500 K

Solution: The ideal gas equation can be written as

$PV = Nk_BT \quad \ldots(1)$

where, P, V, N, k_B, T are the pressure, volume, number of molecules, Boltzmann constant and the absolute temperature of the gas.

Substitute the values of the known parameters into equation (1) and solve to calculate the required temperature of the gas.

$$P = \frac{N}{V}k_BT$$
$$\Rightarrow T = \frac{P}{\frac{N}{V}k_B}$$
$$= \frac{1.38 \text{ atm} \times \frac{1.01 \times 10^5 \text{ Pa}}{1 \text{ atm}}}{2 \times 10^{25} \text{ m}^{-3} \times 1.36 \times 10^{-23} \text{ J K}^{-1}}$$
$$\approx 500 \text{ K}$$

Q.50. A rod of length 2 m moving with velocity 2 mm s^{-1} along the positive $x-$ axis and magnetic field $B = 2 \text{ T}$ along the negative $z-$ axis. Find the magnitude of emf induced (in mV) in the rod. Length of the rod is along y-axis.

Answer: 8

Solution: In the given problem, the direction of length, the velocity and the magnetic field are mutually perpendicular to each other.

The formula to calculate the magnitude of the motional emf induced in the rod is given by

$\varepsilon = Blv \quad \ldots(1)$

From equation (1), it follows that

$$\varepsilon = 2 \text{ T} \times 2 \text{ m} \times 2 \times 10^{-3} \text{ m s}^{-1}$$
$$= 8 \times 10^{-3} \text{ V}$$
$$= 8 \text{ mV}$$

Q.51. Consider a physical quantity $Q = \frac{a^3b^4}{r^5}$. If the maximum percentage errors in measuring the quantities $a,\ b,\ r$ are $3\%,\ 4\%,\ 2\%$ respectively, what is the magnitude of the maximum percentage error in measuring the quantity Q?

Answer: 35

Solution: Given the quantity is

$Q = \frac{a^3b^4}{r^5} \quad \ldots(1)$

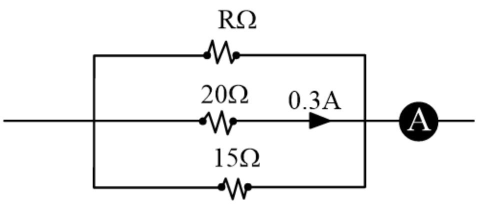
The relative error in measuring Q is given by

$\frac{\Delta Q}{Q} = 3\frac{\Delta a}{a} + 4\frac{\Delta b}{b} + 5\frac{\Delta r}{r} \quad \ldots(2)$

Hence, the required percentage error can be written as:

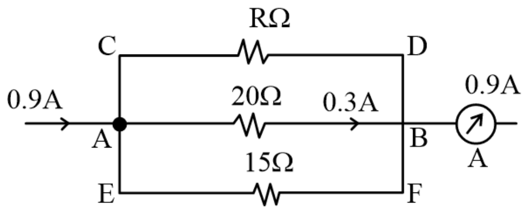
$$\left(\frac{\Delta Q}{Q}\right) \times 100 = 3\left(\frac{\Delta a}{a}\right) \times 100 + 4\left(\frac{\Delta b}{b}\right) \times 100 + 5\left(\frac{\Delta r}{r}\right) \times 100$$
$$= 3 \times 3\% + 4 \times 4\% + 5 \times 2\%$$
$$= 35\%$$

Q.52. In the given circuit, the ammeter reading is 0.9 A. Find the value of the resistance R (in Ω).



Answer: 30

Solution: Let's consider the following diagram:



The potential difference across each resistor will be equal to the potential difference across the branch AB, as shown in the figure.

Thus, the required potential difference is given by

$$V = 20\ \Omega \times 0.3\ \text{A}$$
$$= 6\ V$$

Let the currents through $CD,\ AB$ and EF branches are $i_1,\ i_2,\ i_3$ respectively.

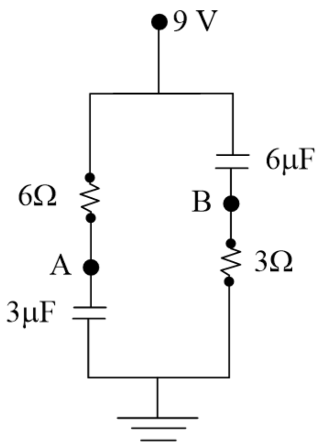
According to the question,

$$i_1 + i_2 + i_3 = 0.9 \quad \ldots(1)$$

From equation (1), it follows that

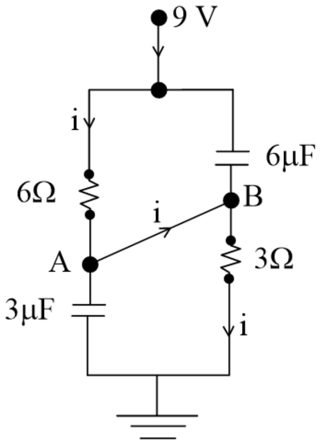
$$\frac{V}{R} + 0.3 + \frac{V}{15} = 0.9$$
$$\Rightarrow \frac{6}{R} = 0.9 - 0.3 - \frac{6}{15} = 0.2$$
$$\Rightarrow R = \frac{6}{0.2}$$
$$= 30\ \Omega$$

Q.53. In the circuit below, find the charge(in μC) on $6\ \mu$F when A and B are sorted.



Answer: 36

Solution: Let's consider the following diagram, when both A and B are sorted:



The direction of current when both the points are sorted is shown in the above diagram.

The current (i) through the sorted circuit is given by

$$
\begin{aligned}
i &= \frac{9\ \mathrm{V}}{6\ \Omega + 3\ \Omega} \\
&= 1\ \mathrm{A}
\end{aligned}
$$

Thus, the potential difference (V) across the resistor $6\ \Omega$, which is also the potential difference across the capacitor $6\ \mu\mathrm{F}$, can be calculated as follows:

$$
\begin{aligned}
V &= 1\ \mathrm{A} \times 6\ \Omega \\
&= 6\ \mathrm{V}
\end{aligned}
$$

Hence, the charge (Q) on the given capacitor can be written as

$$
\begin{aligned}
Q &= CV \\
&= 6\ \mu\mathrm{F} \times 6\ \mathrm{V} \\
&= 36\ \mu\mathrm{C}
\end{aligned}
$$