JEE Main

29th Jan Shift 1

Questions

Q.1. Which of the following pair will be formed by the decomposition of $KMnO_4$?

A) MnO_2, H_2O

B) Mn_2O_3, MnO_2

C) K_2MnO_4, MnO_2

D) $KMnO_4$, MnO_2

Answer: K_2MnO_4, MnO_2

Solution: When heated, potassium permanganate decomposes according to the following equation:

$$2\,KMnO_4 \xrightarrow{513\,K} K_2MnO_4 + MnO_2 + O_2$$

Hence, the answer is option C.

Q.2. What is the effect that occurs between lone pair and pi bond?

A) Inductive effect

B) Electromeric effect

C) Hyperconjugation.

D) Resonance Effect.

Answer: Resonance Effect.

Solution: Resonance effect is the polarity produced in a molecule due to interaction between a lone pair of electron and a pi bond or it is produced due to interaction of two pi bonds between two adjacent atoms. Resonance effect can be seen in molecules having conjugated double bond or in molecules having at least one lone pair of electrons and one double bond.

Hence option D is the answer.

Q.3. Energy difference between actual structure and most stable resonating structure having least energy is called

A) Resonance Energy

B) Electromeric Effect

C) Inductive Effect

D) Hyperconjugation

Answer: Resonance Energy

Solution: The resonance energy is defined as a difference in energy between resonance hybrid and most stable resonating structure. The resonance energy of a compound is a measure of the extra stability of the conjugated system compared to the corresponding number of isolated double bonds.

Hence option A is the answer.

Q.4. CO bridge bond is present in:

A) $\left[Cu_2(CO)_6\right]$

B) $\left[Co_2(CO)_8\right]$

C) $\left[Mn_2(CO)_{10}\right]$

D) None of the above

Answer: $\left[Co_2(CO)_8\right]$

Solution: Bridging ligands: A bridging ligand is a ligand that connects two or more metal atoms or ions.

The structure of $\left[Co_2(CO)_8\right]$, there are two CO bridging ligand and one $Co - Co$ bond.

Hence, the answer is option B.
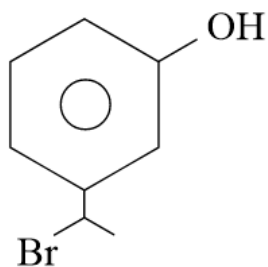
Q.5. The major product formed in the following reaction is

$$\text{OCH}_2\text{CH}_3 \xrightarrow[\text{excess}]{\text{Conc.HBr}}$$

A)

B)

C)

D)

Answer:

Solution: Both ether cleavage and electrophilic addition reaction of alkene takes place with excess HBr. Here ether cleavage takes place by S_N2 mechanism.

Hence option B is the answer.

Q.6. MnO_4^- oxidises I^- in basic medium to:

A) HIO_4

B) HIO_3^-

C) IO_3^-

D) I_2

Answer: IO_3^-

Solution: In neutral or faintly alkaline solution the oxidation of iodide to iodate taken place with potassium permanganate.
$$2\,MnO_4^- + H_2O + I^- \rightarrow 2\,MnO_2 + 2\,OH^- + IO_3^-$$

Q.7. Protein on hydrolysis gives:

A) γ-amino acids

B) α- amino acids

C) β-amino acids

D) δ-amino acids

Answer: α- amino acids

Solution: The smallest unit to which the protein can be broken down is called amino acids. The α-amino acids are the amino acids whose one carbon contains both the amino group (NH_2) and the carboxyl group ($COOH$). This type of carbon is called α-carbon. Thus, the amino acid that contains an α-carbon is called an α-amino acid.

Hence, the answer is option B.

Q.8. Which of the following statements is incorrect?

A) $\Delta G = 0$ for equilibrium reaction.

B) $\Delta G < 0$ for spontaneous reaction.

C) $\Delta G > 0$ for spontaneous reaction.

D) $\Delta G > 0$ for non spontaneous reaction.

Answer: $\Delta G > 0$ for spontaneous reaction.

Solution: Conditions for spontaneous and non spontaneous reactions in terms of free energy change are as follows, $\Delta G > 0$ for non-sponateneous reaction

$\Delta G < 0$ for spontaneous reaction.

$\Delta G = 0$ for equilibrium reaction

Hence option C is the answer.

Q.9. Statement 1: Electronegativity of group 14 elements decreases from Si to Pb

Statement 2: Group 14 has metals, metalloids and non metals.

A) Both Statement 1 and 2 are correct.
B) Both Statement 1 and 2 are incorrect.
C) Statement 1 is correct, Statement 2 is wrong.
D) Statement 2 is correct, Statement 1 is wrong.

Answer: Statement 2 is correct, Statement 1 is wrong.

Solution: The electronegativity decreases as we move down the group but not after silicon. The Electronegativity values for elements from Si to Pb are almost the same.

Group 14 has metals, metalloids and non metals.

Si and Ge are metalloids, Sn and Pb are metals and Carbon is non metal.

Hence option D is the answer.

Q.10. Find product P of the following reaction:

(i) H_2O/H^+
(ii) CrO_3
(iii) NH_2NH_2/KOH

A)

B)

C)
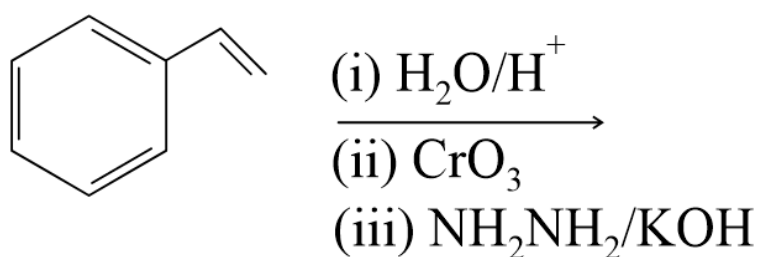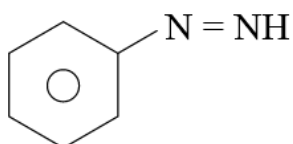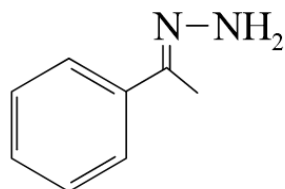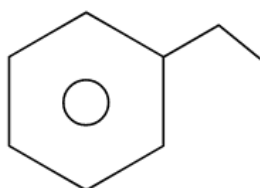
D)
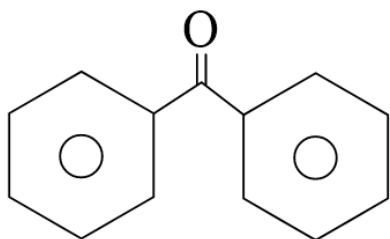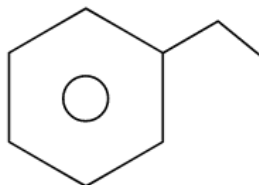
Answer:

Solution: First addition of water takes place according to Markownikoff's rule, alcohol is formed, CrO_3 acts as an oxidising agents which converts alcohol into ketone, then Wolf Kishner reduction occurs.
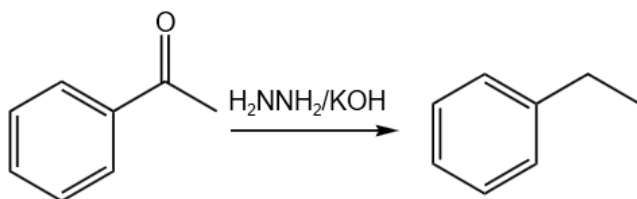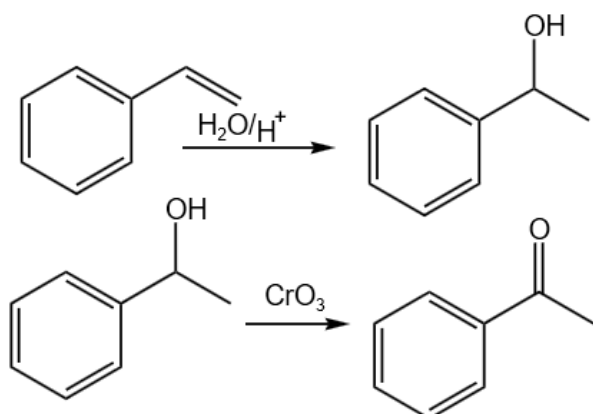
Hence, the answer is C.
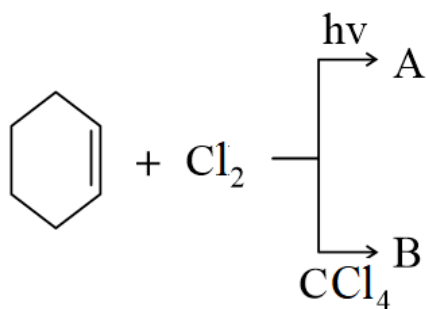
Q.11. Which of the following is an incorrect match?

A) Cryolite-Na_3AlF_6

B) Fluorspar-BF_3

C) Carnallite-$KCl.MgCl_2.6H_2O$

D) Fluorapatite-$3Ca_3(PO_4)_2.CaF_2$

Answer: Fluorspar-BF_3

Solution: Fluorine is present mainly as insoluble fluorides (fluorspar CaF_2, cryolite Na_3AlF_6 and fluoroapatite $3Ca_3(PO_4)_2.CaF_2$). Sea water contains chlorides, bromides and iodides of sodium, potassium, magnesium and calcium, but is mainly sodium chloride solution (2.5% by mass). The deposits of dried up seas contain these compounds, e.g., sodium chloride and carnallite, $KCl.MgCl_2.6H_2O$.

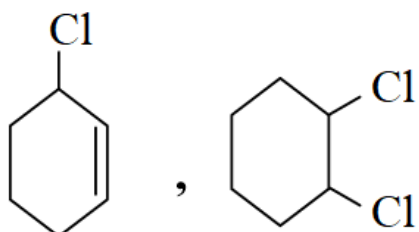Q.12.   In the following reactions, find the products A and B respectively.

A)

B)

C)

D)

Answer:

Solution: In the presence of sunlight chlorine undergo free radical substitution reaction. In the given reaction allylic substitution reaction takes place. In the presence of carbon tetra chloride the reaction between alkene and chlorine is electrophilic addition reaction. Hence, the products are

Q.13. Which of the following elements are confirmed by the appearance of blood red colour with $FeCl_3$ in Lassaigne's test.

A) Presence of S only

B) Presence of N and S.

C) Presence of N only

D) Presence of P only

Answer: Presence of N and S.

Solution: Lassaigne's test detects nitrogen, sulphur, and halogens in organic compounds. In a fusion tube with the organic compound, a small piece of Na metal is heated.

In the case of nitrogen and Sulphur both are present in an organic compound, sodium thiocyanate is formed. It gives blood red colour and no Prussion blue since there are no free cyanide ions.

$$Na + C + N + S \rightarrow NaSCN$$
$$Fe^{3+} + SCN^- \rightarrow [Fe(SCN)]^{2+}$$
$$\text{Blood Red}$$

Hence option B is the answer.

Q.14. The correct set of 4 quantum numbers of valence electrons of $Rb - 37$.

A) $n = 5, \, l = 0, \, m = 0, \, s = \pm\dfrac{1}{2}$

B) $n = 4, \, l = 0, \, m = 0, \, s = \pm\dfrac{1}{2}$

C) $n = 5, \, l = 1, \, m = 1, \, s = \pm\dfrac{1}{2}$

D) $n = 5, \, l = 2, \, m = 0, \, s = \pm\dfrac{1}{2}$

Answer: $n = 5, \, l = 0, \, m = 0, \, s = \pm\dfrac{1}{2}$

Solution: The set of numbers used to describe the position and energy of the electron in an atom are called quantum numbers. There are four quantum numbers, namely, principal, azimuthal, magnetic and spin quantum numbers.

Rb electronic configuration is: $[Kr]5s^1$

Hence,

$n = 5, \, l = 0, \, m = 0, \, s = \pm\dfrac{1}{2}$

Q.15. Given that $K_{net} = (K_1 . K_2)/K_3$. The activation energy values are given as $E_{a1} = 40\,kJ/mol$, $E_{a2} = 50\,kJ/mol$ and $E_{a3} = 60\,kJ/mol$ respectively. Calculate the value of $E_{a(net)}$ in kJ/mol.

A) $30\ kJ/mol$

B) $70\ kJ/mol$

C) $50\ kJ/mol$

D) $100\ kJ/mol$

Answer: $30\ kJ/mol$

Solution: According to Arrhenius concept,

$K = Ae^{-Ea/RT}$

Given that,

$K_{net} = (K_1 . K_2)/K_3$

Using Arrhenius equation we can expand above equation as

$$e^{-E_{a(net)}/RT} = \frac{e^{-E_{a1}/RT} \times e^{-E_{a2}/RT}}{e^{-E_{a3}/RT}}$$

Hence,
$E_{a(net)} = E_{a1} + E_{a2} - E_{a3}$
$\Rightarrow E_{a(net)} = 40 + 50 - 60 = 30\,kJ\,/\,mol$

Q.16.
A container contains $1\,g$ of H_2 gas and $1\,g$ of O_2 gas. What is the ratio of the partial pressure of H_2 and O_2 $\left(\dfrac{p_{H_2}}{p_{O_2}}\right)$

A) $8:1$

B) $4:1$

C) $16:1$

D) $1:1$

Answer: $16:1$

Solution: According to Dalton's law of partial pressures, the total pressure by a mixture of gases is equal to the sum of the partial pressures of each of the constituent gases.

$p_0 = p_T X_0$

$\dfrac{p_{H_2}}{p_{O2}} = \dfrac{p_T X_{H_2}}{p_T X_{O_2}}$

$\dfrac{p_{H_2}}{p_{O_2}} = \dfrac{X_{H_2}}{X_{O2}} = \dfrac{\frac{n_{H_2}}{n_T}}{\frac{n_{O_2}}{n_T}} = \dfrac{\frac{1}{2}}{\frac{1}{32}} = \dfrac{16}{1}$

Q.17. Osmotic pressure at $273K$ is $7 \times 10^5\,atm$ what will be the osmotic pressure at $283K$

A) 7.25

B) 6.40

C) 7.65

D) 8.0

Answer: 7.25

Solution: Osmotic pressure can be calculated as,

$\pi = CRT$

Here given,

$T_1 = 273K$
$T_2 = 283K$

$\pi_1 = CRT_1$
By putting the values in this equation, we get,
$7 \times 10^5 = C \times R \times 273$
$CR = \dfrac{7 \times 10^5}{273}$
$\pi_2 = CRT_2$
$\quad = \dfrac{7 \times 10^5}{273} \times 283$
$\quad = 7.25$

Q.18. Consider the following reaction:

$\xrightarrow[\text{CS}_2]{\text{Br}_2}$ A (intermediate)
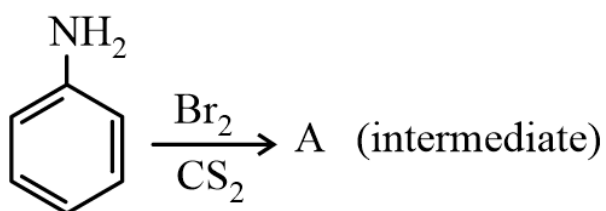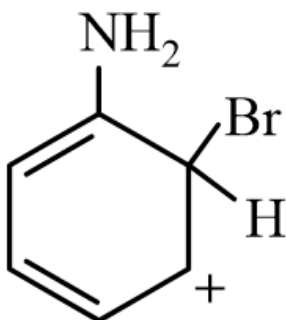
A)
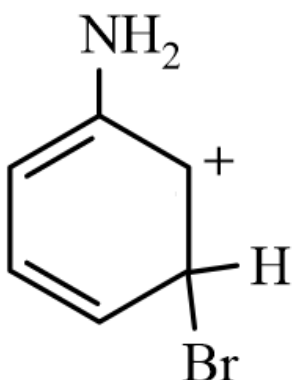
B)
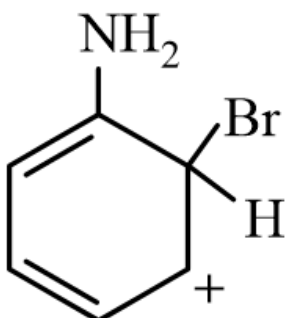
C) Both A and B

D) None of these

Answer:
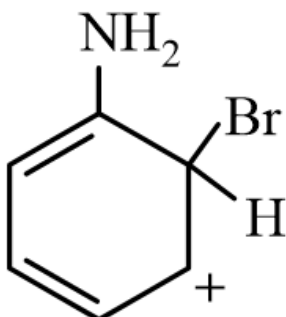
Solution: Arenium ion: A carbocation formed by electrophilic attack on an aromatic ring. A reactive intermediate in electrophilic aromatic substitution.

Hence, B is the answer.

Q.19. Statement 1: Ionisation energy decreases along a period.

Statement 2: In a period, effective nuclear charge dominates over the shielding effect.

A)   Both Statement 1 and 2 are correct.

B)   Both Statement 1 and 2 are incorrect.

C)   Statement 1 is correct Statement 2 is incorrect.

D)   Statement 2 is correct Statement 1 is incorrect.

Answer: Statement 2 is correct Statement 1 is incorrect.

Solution: As we move from **left to right across a period, the ionisation energy of elements increases**. This is due to the decrease in the size of atoms across a period. The valence electrons get closer to the nucleus of an atom as we move from left to right due to increased nuclear charge.

More the electron shells, greater is the shielding effect experienced by the outermost electrons. Hence screening or shielding effect increases in a group as shells increases from top to bottom but in a period it decreases from left to right.

Hence option D is the answer.

Q.20.

	Complexes		Metals
A	Vitamin B12	p	Ti
B	Wilkinson catalyst	q	Co
C	Ziegler - Natta catalyst	r	Fe
D	Haemoglobin	s	Rh

A)   A-q, B-s, C-r, D-p

B)   A-q, B-s, C-p, D-r

C)   A-s, B-q, C-r, D-p

D)   A-q, B-r, C-s, D-p

Answer: A-q, B-s, C-p, D-r

Solution: Haemoglobin, the red pigment of blood which acts as oxygen carrier is a coordination compound of iron. Vitamin B12, cyanocobalamine, the anti– pernicious anaemia factor, is a coordination compound of cobalt. The rhodium complex, $[(Ph_3P)_3RhCl]$, a Wilkinson catalyst, is used for the hydrogenation of alkenes. Ziegler-Natta catalysts is $TiCl_4 + Et_3Al$.

Q.21. Which of the following pairs will be paramagnetic and have bond order as 1?

H_2, He_2^+, N_2^{2-}, O_2^{2-}, B_2, F_2, Ne_2
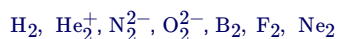
Answer: 1

Solution: Paramagnetic molecules are those molecules in which unpaired electrons are present, while diamagnetic molecules are those in which unpaired electrons are absent.
The molecular orbital electronic configuration of B_2 is.

$B_2: \sigma 1s^2,\ \sigma^* 1s^2,\ \sigma 2s^2,\ \sigma^* 2s^2,\ \pi 2p_x^1,\ \pi 2p_y^1$
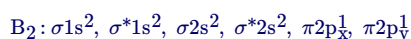
Bond order $= \dfrac{\text{Bonding electrons} - \text{antibonding electrons}}{2}$

$= \dfrac{6-4}{2} = \dfrac{2}{2} = 1$

Hence. the answer is 1

Q.22. Calculate the Molarity of the solution having density 1.5 g/ml. %(w/w) of Solute is 36% and molecular weight of solute is 36 g/mol.

Answer: 15

Solution:

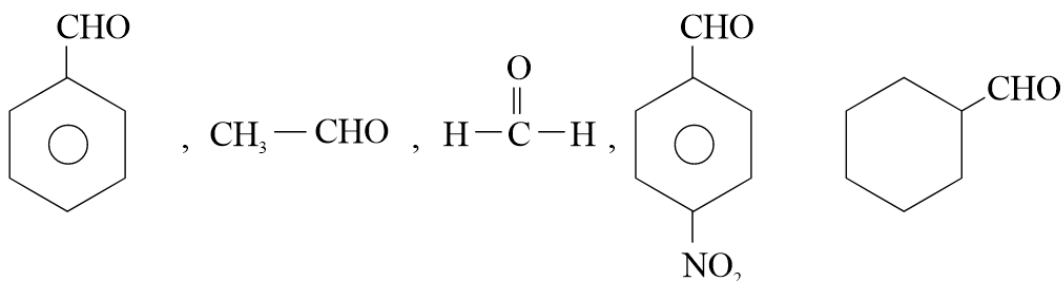Given density of solution= $1.5\ \text{g/ml} = \dfrac{\text{mass of solution}}{\text{Volume of solution}}$

$\text{Molarity} = \dfrac{\text{No. of moles of solute}}{\text{Volume of solution in L}}$

$\text{Percentage w/w} = \dfrac{\text{Mass of solute in solution}}{\text{Total mass of solution}} \times 100$

$\text{Molarity} = \dfrac{\text{w/w\%} \times \text{d} \times 1000}{100 \times \text{molar mass of solute}} = \dfrac{36 \times 1.5 \times 1000}{100 \times 36}$

$\Rightarrow \text{Molarity} = 15\text{M}$

Q.23. How many of the below compounds give Fehling solution test?

Answer: 3

Solution: In Fehling's solution, the reaction between copper(II) ions and aliphatic aldehyde is represented as;
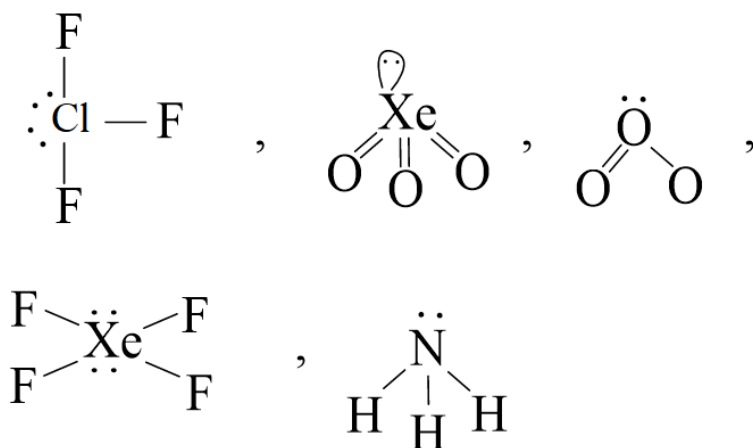
$$\text{RCHO} + 2\,\text{Cu}^{2+} + 5\,\text{OH}^- \rightarrow \text{RCOO}^- + \text{Cu}_2\text{O} + 3\text{H}_2\text{O}$$
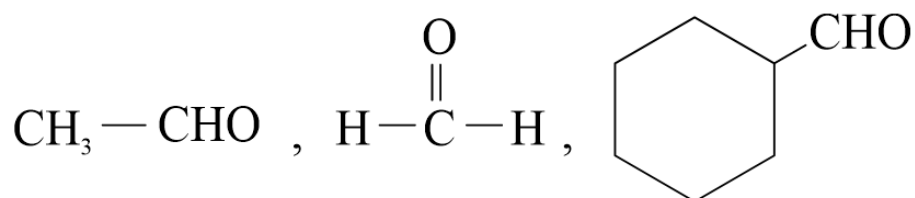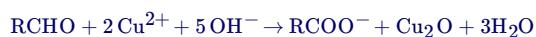
Hence, the answer is 3.

Q.24. How many of the following compounds have only one lone pair in central atom?

$\text{ClF}_3,\ \text{XeO}_3,\ \text{BrF}_5,\ \text{XeF}_4,\ \text{O}_3,\ \text{NH}_3$

Answer: 4

Solution: The structures of the given compounds are as follows,

Hence XeO_3, BrF_5, O_3, NH_3 have only one lone pair in central atom.

Q.25. How many of the following compounds contain Sulphur atoms?

Pyrrole,Furan,Thiophene,Cysteine,Tyrosine,Pyridine

Answer: 2

Solution: The structures of the compounds pyrrole, Furan, thiophene, pyridine, cysteine and tyrosine respectively are follows,
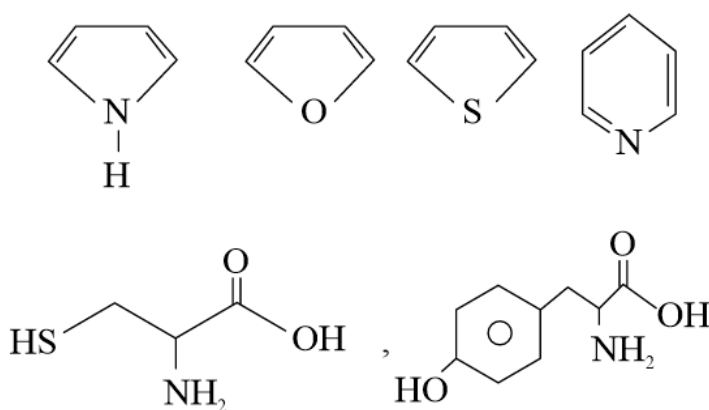
Hence from the given compounds, Thiophene and Cysteine are the compounds containing Sulphur.

Q.26. Let a die rolled till 2 is obtained. The probability that 2 obtained on even number toss is equal to

A) $\frac{5}{6}$

B) $\frac{5}{11}$

C) $\frac{1}{11}$

D) $\frac{6}{11}$

Answer: $\frac{5}{11}$

Solution: Given,

A die rolled till 2 is obtained,

Now, let probability that 2 obtained on even number toss $= k$

Now, probability of getting 2 on one throw is $P(2) = \frac{1}{6}$ and $P(\overline{2}) = \frac{5}{6}$

So, $k = \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \ldots\ldots\ldots\ldots\ldots\infty$

$\Rightarrow k = \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \ldots\ldots\ldots\ldots\ldots\infty$

$\Rightarrow k = \dfrac{\frac{5}{6} \cdot \frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2}$

$\Rightarrow k = \frac{5}{11}$

Q.27.

The value of the limit $\displaystyle\lim_{x \to \frac{\pi}{2}} \dfrac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos t^{\frac{1}{3}} \, dt}{\left(x - \frac{\pi}{2}\right)^2}$ will be

A) $\dfrac{3\pi}{8}$

B) $\dfrac{3\pi^2}{8}$

C) $\dfrac{3\pi}{4}$

D) $\dfrac{3\pi^2}{4}$

Answer: $\dfrac{3\pi^2}{8}$

Solution:

Let, $y = \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos t^{\frac{1}{3}} dt}{\left(x - \frac{\pi}{2}\right)^2}$

$\Rightarrow y = \lim\limits_{h \to 0} \dfrac{\int_{\left(\frac{\pi}{2}-h\right)^3}^{\left(\frac{\pi}{2}\right)^3} \cos t^{\frac{1}{3}} dt}{\left(\frac{\pi}{2}-h-\frac{\pi}{2}\right)^2}$

$\Rightarrow y = \lim\limits_{h \to 0} \left(\dfrac{\int_{\left(\frac{\pi}{2}-h\right)^3}^{\left(\frac{\pi}{2}\right)^3} \cos t^{\frac{1}{3}} dt}{h^2} \right)$

Applying L-Hospital's rule,

$\Rightarrow y = \lim\limits_{h \to 0} \left(\dfrac{0 + \cos\left(\frac{\pi}{2}-h\right) \times 3 \times \left(\frac{\pi}{2}-h\right)^2}{2h} \right)$

$\Rightarrow y = \lim\limits_{h \to 0} \left(\dfrac{\sin h \times 3 \times \left(\frac{\pi}{2}-h\right)^2}{2h} \right)$

We know that, $\lim\limits_{h \to 0}\left(\dfrac{\sin h}{h}\right) = 1$

$\Rightarrow y = \lim\limits_{h \to 0} \left(\dfrac{3 \times \left(\frac{\pi}{2}-h\right)^2}{2} \right)$

$\Rightarrow y = \dfrac{3\pi^2}{8}$

Q.28.

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$, where $\alpha, \beta \in N$, then find the possible value of α

A)   5

B)   12

C)   3

D)   9

Answer: 5

Solution: Given,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix} \text{ and } |2A|^3 = 2^{21}$$

Now, $|2A|^3 = 2^{21}$

$\Rightarrow |2A| = 2^7$

$\Rightarrow 2^3 |A| = 2^7$

$\Rightarrow |A| = 16$

$\Rightarrow \alpha^2 - \beta^2 = 16$

Now, taking value from the option we get,

$\alpha = 5$ as $\alpha, \beta \in N$, so for other value of α, β is not a natural number.

Q.29. If $f(x) = \int \dfrac{\operatorname{cosec} x + \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx$ and $\lim\limits_{x \to \frac{\pi}{2}} f(x) = 1$, then find $f\left(\dfrac{\pi}{4}\right)$.

A) $1 - \dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

B) $1 - \dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

C) $1 - \dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{1}{2}\right)$

D) $\dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{1}{2}\right) - 1$

Answer: $1 - \dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{1}{2}\right)$

Solution: Given:

$$f(x) = \int \dfrac{\operatorname{cosec} x + \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx$$

$$\Rightarrow f(x) = \int \dfrac{\dfrac{1}{\sin x} + \sin x}{\dfrac{1}{\sin x \cos x} + \dfrac{\sin x}{\cos x}\left(\sin^2 x\right)} dx$$

$$\Rightarrow f(x) = \int \dfrac{\dfrac{1 + \sin^2 x}{\sin x}}{\dfrac{1 + \sin^4 x}{\sin x \cos x}} dx$$

$$\Rightarrow f(x) = \int \dfrac{\left(1 + \sin^2 x\right)\cos x}{1 + \sin^4 x} dx$$

Let, $\sin x = t$

$\cos x \, dx = dt$

$$\Rightarrow f(x) = \int \dfrac{1 + t^2}{1 + t^4} dt$$

$$\Rightarrow f(x) = \int \dfrac{1 + \dfrac{1}{t^2}}{t^2 + \dfrac{1}{t^2}} dt$$

$$\Rightarrow f(x) = \int \dfrac{1 + \dfrac{1}{t^2}}{\left(t - \dfrac{1}{t}\right)^2 + 2} dt$$

Let, $t - \dfrac{1}{t} = u$

$$\Rightarrow f(x) = \int \dfrac{1}{u^2 + 2} du$$

$$\Rightarrow f(x) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$\Rightarrow f(x) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + C$$

$$\Rightarrow f(x) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x-\frac{1}{\sin x}}{\sqrt{2}}\right) + C$$

$$\Rightarrow f(x) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x-\operatorname{cosec} x}{\sqrt{2}}\right) + C$$

Now, $\displaystyle\lim_{x \to \frac{\pi}{2}} f(x) = 1$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x-\operatorname{cosec} x}{\sqrt{2}}\right) + C\right] = 1$$

$$\Rightarrow 0 + C = 1$$

$$\Rightarrow C = 1$$

$$\Rightarrow f(x) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x-\operatorname{cosec} x}{\sqrt{2}}\right) + 1$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}-\sqrt{2}}{\sqrt{2}}\right) + 1$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{-1}{2}\right) + 1$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = 1 - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{2}\right)$$

Q.30. In an increasing arithmetic progression $a_1, a_2, \ldots, a_n$ if $a_6 = 2$ and product of a_1, a_5 and a_4 is greatest, then the value of d is equal to _____.

A) 2.0

B) 1.8

C) 0.6

D) 1.6

Answer: 1.6

Solution: Given,

$a_6 = 2$

$\Rightarrow a + 5d = 2$

It is given that $a_1 \times a_5 \times a_4$ is greatest.

Let, $y = a(a + 4d)(a + 3d)$

$\Rightarrow y = (2 - 5d)(2 - 5d + 4d)(2 - 5d + 3d)$

$\Rightarrow y = (2 - 5d)(2 - d)(2 - 2d)$

$\Rightarrow y = \left(4 - 12d + 5d^2\right)(2 - 2d)$

$\Rightarrow y = 8 - 8d - 24d + 24d^2 + 10d^2 - 10d^3$

$\Rightarrow y = 8 - 32d + 34d^2 - 10d^3$

$\Rightarrow \dfrac{dy}{d(d)} = -32 + 68d - 30d^2$

For critical points, $\dfrac{dy}{d(d)} = 0$

$\Rightarrow -32 + 68d - 30d^2 = 0$

$\Rightarrow 15d^2 - 34d + 16 = 0$

$\Rightarrow d = \dfrac{8}{5}, \dfrac{2}{3}$

Product is greatest at $d = \dfrac{8}{5}$.

$\Rightarrow d = \dfrac{8}{5} = 1.6$

Q.31. If relation $R : (a, b)\ R\ (c, d)$ if and only if $ad - bc$ is divisible by 5, $(a, b, c, d \in Z)$ then R is

A) Reflexive

B) Equivalence Relation

C) Transitive, Reflexive but not Symmetric

D) Symmetric, Reflexive but not Transitive

Answer: Symmetric, Reflexive but not Transitive

Solution: Given,

Relation $R :\ (a, b)\ R\ (c, d)$

And $ad - bc = 5k$ {where $k \in Z$}

Now, for reflexive $R :\ (a, b)\ R\ (a, b)$

$\Rightarrow ab - ba = 0$ which is divisible by 5

So, the relation is reflexive,

Now, for symmetric $R :\ (c, d)\ R\ (a, b)$

$\Rightarrow bc - ad = -5k$ which is divisible by 5

So, the relation is symmetric,

Now for transitive relation, $R :\ (a, b)\ R\ (c, d)\ R\ (e, f)$

Taking example $(a, b) \equiv (1, 2),\ (c, d) \equiv (10, 10),\ (e, f) \equiv (4, 4)$

Now, $ad - bc = 10 - 20 = -10$

$cf - ed = 40 - 40 = 0$

But $af - be = 8 - 4 = 4$ which is not divisible by 5,

Hence, the relation is reflexive, symmetric but not transitive.

Q.32.

If $f(x) = \begin{cases} 2 + 2x, & x \in (-1, 0) \\ 1 - \dfrac{x}{3}, & x \in \left[0, 3\right) \end{cases}$ & $g(x) = \begin{cases} x, & x \in [0, 1) \\ -x, & x \in (-3, 0) \end{cases}$, then the range of $f(g(x))$ will be,

A) $(0,1]$ B) $[-1,1]$

C) $[0,1]$ D) $(-1,1)$

Answer: $(0,1]$

Solution: Given,

$$f(x) = \begin{cases} 2+2x, & x \in (-1,0) \\ 1-\dfrac{x}{3}, & x \in \left[0,3\right) \end{cases} \quad \& \quad g(x) = \begin{cases} x, & x \in [0,1) \\ -x, & x \in (-3,0) \end{cases}$$

Now, finding

$$f(g(x)) = \begin{cases} 2+2g(x), & g(x) \in (-1,0) \\ 1-\dfrac{g(x)}{3}, & g(x) \in \left[0,3\right) \end{cases}$$

$$\Rightarrow f(g(x)) = \begin{cases} 2+2|x|, & |x| \in (-1,0) \Rightarrow x \in \phi \\ 1-\dfrac{|x|}{3}, & |x| \in \left[0,3\right) \end{cases}$$

$$\Rightarrow f(g(x)) = 1 - \frac{|x|}{3}, \ x \in (-3,1)$$

$$\Rightarrow f(g(x)) \in (0,1]$$

Q.33. Let A be a square matrix such that $AA^T = 1$. Then $\frac{1}{2}A\left[\left(A+A^T\right)^2 + \left(A-A^T\right)^2\right]$ is equal to

A) $A^2 + A^T$ B) $A^2 + I$

C) $A^3 + A^{-1}$ D) $A^3 + I$

Answer: $A^3 + A^{-1}$

Solution: Given,

$$AA^T = 1$$

$$\Rightarrow A^T = A^{-1}$$

Now solving,

$$\frac{1}{2}A\left[\left(A+A^T\right)^2 + \left(A-A^T\right)^2\right] = \frac{1}{2}A \times 2\left[A^2 + \left(A^T\right)^2\right]$$

$$\Rightarrow \frac{1}{2}A\left[\left(A+A^T\right)^2 + \left(A-A^T\right)^2\right] = A^3 + A\left(A^T\right)^2$$

Now using, $AA^T = 1$ we get,

$$\Rightarrow \frac{1}{2}A\left[\left(A+A^T\right)^2 + \left(A-A^T\right)^2\right] = A^3 + A^T$$

Now using, $A^T = A^{-1}$ we get,

$$\Rightarrow \frac{1}{2}A\left[\left(A+A^T\right)^2 + \left(A-A^T\right)^2\right] = A^3 + A^{-1}$$

Q.34. If $f(x) = \dfrac{\left(2^x + 2^{-x}\right)(\tan x)\sqrt{\tan^{-1}\left(2x^2 - 3x + 1\right)}}{\left(7x^2 - 3x + 1\right)^3}$, then $f'(0)$ is equal to:

A) $2\pi^{\frac{3}{2}}$ B) π

         C) $\sqrt{\dfrac{\pi}{4}}$

D) $\sqrt{\pi}$

Answer: $\sqrt{\pi}$

Solution:

Given: $f(x) = \dfrac{\left(2^x + 2^{-x}\right)(\tan x)\sqrt{\tan^{-1}\left(2x^2 - 3x + 1\right)}}{\left(7x^2 - 3x + 1\right)^3}$

$\Rightarrow f(x) = \left[\left(2^x + 2^{-x}\right)\left(7x^2 - 3x + 1\right)^{-3}\sqrt{\tan^{-1}\left(2x^2 - 3x + 1\right)}\right](\tan x)$

$\Rightarrow f'(x) = \left[\left(2^x + 2^{-x}\right)\left(7x^2 - 3x + 1\right)^{-3}\sqrt{\tan^{-1}\left(2x^2 - 3x + 1\right)}\right]\sec^2 x + \tan x\dfrac{d}{dx}\left[\left(2^x + 2^{-x}\right)\left(7x^2 - 3x + 1\right)^{-3}\sqrt{\tan^{-1}\left(2x^2 - 3x + 1\right)}\right]$

$\Rightarrow f'(0) = \left[\left(2^0 + 2^0\right)(7 \times 0 - 3 \times 0 + 1)^{-3}\sqrt{\tan^{-1}(1)}\right]\sec^2 0$

$\Rightarrow f'(0) = 2 \times \sqrt{\dfrac{\pi}{4}}$

$\Rightarrow f'(0) = \sqrt{\pi}$

Q.35. If $z = \dfrac{1}{2} - 2i$ such that $|z + 1| = \alpha z + \beta(1 + i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in N$ then $\alpha + \beta$ is

A) 3

B) 5

C) 7

D) 9

Answer: 3

Solution:

Given: $|z + 1| = \alpha z + \beta(1 + i)$, $z = \dfrac{1}{2} - 2i$

$\Rightarrow \left|\dfrac{1}{2} - 2i + 1\right| = \alpha\left(\dfrac{1}{2} - 2i\right) + \beta(1 + i)$

$\Rightarrow \left|\dfrac{3}{2} - 2i\right| = \dfrac{\alpha}{2} - 2\alpha i + \beta + \beta i$

$\Rightarrow \sqrt{\dfrac{9}{4} + 4} = \left(\dfrac{\alpha}{2} + \beta\right) + i\left(-2\alpha + \beta\right)$

$\Rightarrow \dfrac{5}{2} = \left(\dfrac{\alpha}{2} + \beta\right) + i\left(-2\alpha + \beta\right)$

$\Rightarrow 2\alpha = \beta, \dfrac{5}{2} = \left(\dfrac{\alpha}{2} + \beta\right)$

$\Rightarrow 2\alpha = \beta, \dfrac{5}{2} = \dfrac{5\alpha}{2}$

$\Rightarrow \alpha = 1, \beta = 2$

$\Rightarrow \alpha + \beta = 3$

Q.36. If the value of $\dfrac{^{11}C_1}{2} + \dfrac{^{11}C_2}{3} + \dfrac{^{11}C_3}{4} \ldots \ldots \ldots + \dfrac{^{11}C_9}{10} = \dfrac{m}{n}$ where m & n are coprime, then find the value of $m + n$

Answer: 2041

Solution: Given,

$$\frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \frac{^{11}C_3}{4}\ldots\ldots + \frac{^{11}C_9}{10} = \frac{m}{n}$$

Now, we know that,

$$(1+x)^{11} = {}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + \ldots\ldots {}^{11}C_{11} x^{11}$$

Now integrating the above expression both side we get,

$$\int_0^1 (1+x)^{11}\, d\,x = \int_0^1 {}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + \ldots\ldots {}^{11}C_{11} x^{11}\, d\,x$$

$$\Rightarrow \left[\frac{(1+x)^{12}}{12}\right]_0^1 = \left[{}^{11}C_0 x + {}^{11}C_1 \frac{x^2}{2} + {}^{11}C_2 \frac{x^3}{3} + \ldots\ldots {}^{11}C_{11}\frac{x^{12}}{12}\right]_0^1$$

$$\Rightarrow \frac{2^{12}-1}{12} - 1 - 1 - \frac{1}{12} = \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \frac{^{11}C_3}{4}\ldots\ldots + \frac{^{11}C_9}{10}$$

$$\Rightarrow \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \frac{^{11}C_3}{4}\ldots\ldots + \frac{^{11}C_9}{10} = \frac{2^{12}-2-24}{12}$$

$$\Rightarrow \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \frac{^{11}C_3}{4}\ldots\ldots + \frac{^{11}C_9}{10} = \frac{4096-26}{12}$$

$$\Rightarrow \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \frac{^{11}C_3}{4}\ldots\ldots + \frac{^{11}C_9}{10} = \frac{4070}{12}$$

$$\Rightarrow \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \frac{^{11}C_3}{4}\ldots\ldots + \frac{^{11}C_9}{10} = \frac{2035}{6}$$

Hence, on comparing we get, $m + n = 2041$

Q.37. Rank of the word '$GTWENTY$' in dictionary is _____.

Answer: 553

Solution: $GTWENTY$

Words starting from $E = \frac{6!}{2!} = \frac{720}{2} = 360$

Words starting with $GE = \frac{5!}{2} = 60$

Words starting with $GN = \frac{5!}{2} = 60$

Words starting with $GTE = 4! = 24$

Words starting with $GTN = 4! = 24$

Words starting with $GTT = 4! = 24$

Then the next word will be $GTWENTY$ and its rank will be $360 + 60 + 60 + 24 + 24 + 24 + 1 = 553$

Q.38. If $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2\cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin 2023x}}\right) dx = \frac{\pi}{4}(\pi + \alpha) - 2$, then find α.

Answer: 3

Solution: Given,

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin 2023x}} \right) dx = \frac{\pi}{4}(\pi + \alpha) - 4$$

Now let, $I_1 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} \right) dx \ \dots\dots (i)$

Now using the property $\int_a^b f(x)\,\mathrm{d}\,x = \int_a^b f(a+b-x)\,\mathrm{d}\,x$ we get,

$$\Rightarrow I_1 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^{-x}} \right) dx \ \dots\dots (ii)$$

Now adding above equations we get,

$$\Rightarrow 2I_1 = 2\int_0^{\frac{\pi}{2}} \left(x^2 \cos x \right) dx \ \left\{ \text{as } \int_{-a}^a f(x)\,\mathrm{d}\,x = 2\int_0^a f(x)\,\mathrm{d}\,x, \text{ if } f(x) \text{ is even} \right\}$$

$$\Rightarrow I_1 = \left[x^2(\sin x) \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x \cdot \sin x\,dx$$

$$\Rightarrow I_1 = \frac{\pi^2}{4} - \left[[-2x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} 2\cos x\,dx \right]$$

$$\Rightarrow I_1 = \frac{\pi^2}{4} - 2$$

Similarly solving, $I_2 = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+e^{\sin 2023x}} \right) dx$

$$\Rightarrow I_2 = \int_0^{\frac{\pi}{2}} \left(1 + \sin^2 x \right) dx$$

$$\Rightarrow I_2 = \int_0^{\frac{\pi}{2}} \left(1 + \frac{1-\cos 2x}{2} \right) dx$$

$$\Rightarrow I_2 = \frac{1}{2} \int_0^{\frac{\pi}{2}} (3 - \cos 2x)\,dx$$

$$\Rightarrow I_2 = \frac{1}{2} \left[3x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow I_2 = \frac{3\pi}{4}$$

$$\Rightarrow I_1 + I_2 = \frac{\pi^2}{4} - 2 + \frac{3\pi}{4}$$

$$\Rightarrow \frac{\pi}{4}(\pi + 3) - 2 = \frac{\pi}{4}(\pi + \alpha) - 2$$

$$\Rightarrow \alpha = 3$$

Q.39.   If $\alpha,\ \beta$ are the roots of $x^2 - x + 2$, such that $\mathrm{Im}\,(\alpha) > \mathrm{Im}\,(\beta)$, then find $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$

Answer: 13

Solution: Given,

Equation $x^2 - x + 2$ has roots α & β,

So, $\alpha + \beta = 1$, $\alpha\beta = 2$

$\Rightarrow \alpha^4 + \beta^4 = \left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2$

$\Rightarrow \alpha^4 + \beta^4 = \left[(\alpha + \beta)^2 - 2\alpha\beta\right]^2 - 2\alpha^2\beta^2$

$\Rightarrow \alpha^4 + \beta^4 = (1 - 4)^2 - 2 \times 4$

$\Rightarrow \alpha^4 + \beta^4 = 1$

It is given that, α is a root of $x^2 - x + 2$.

$\Rightarrow \alpha^2 - \alpha + 2 = 0$

$\Rightarrow \alpha^2 = \alpha - 2$

$\Rightarrow \alpha^4 = \alpha^2 + 4 - 4\alpha$

$\Rightarrow \alpha^4 = \alpha - 2 + 4 - 4\alpha$

$\Rightarrow \alpha^4 = 2 - 3\alpha$

Now, $\alpha^6 - 5\alpha^2 = \alpha^2\left(\alpha^4 - 5\right)$

$\Rightarrow \alpha^6 - 5\alpha^2 = (\alpha - 2)(2 - 3\alpha - 5)$

$\Rightarrow \alpha^6 - 5\alpha^2 = (\alpha - 2)(-3\alpha - 3)$

$\Rightarrow \alpha^6 - 5\alpha^2 = -3\left(\alpha^2 - \alpha - 2\right)$

$\Rightarrow \alpha^6 - 5\alpha^2 = -3(\alpha - 2 - \alpha - 2)$

$\Rightarrow \alpha^6 - 5\alpha^2 = 12$

$\Rightarrow \alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2 = 13$

Q.40. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-collinear vectors. $\vec{a} + 6\vec{b}$ is collinear with $\vec{c}$, $\vec{b} + 5\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0$, then find $\alpha + \beta$.

Answer: 36

Solution: Given,

$\vec{a} + 6\vec{b}$ is collinear with $\vec{c}$.

$\Rightarrow \vec{a} + 6\vec{b} = \lambda\vec{c}$

$\Rightarrow \vec{a} + 6\vec{b} + 30\vec{c} = (\lambda + 30)\vec{c}$ ...(i)

$\vec{b} + 5\vec{c}$ is collinear with $\vec{a}$

$\Rightarrow \vec{b} + 5\vec{c} = \mu\vec{a}$

$\Rightarrow 6\vec{b} + 30\vec{c} = 6\mu\vec{a}$

$\Rightarrow \vec{a} + 6\vec{b} + 30\vec{c} = 6\mu\vec{a} + \vec{a}$

$\Rightarrow \vec{a} + 6\vec{b} + 30\vec{c} = \vec{a}(6\mu + 1)$ ...(ii)

$\Rightarrow \vec{a}(6\mu + 1) = (\lambda + 30)\vec{c}$

But, $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-collinear vectors.

$\Rightarrow \lambda = -30$ and $\mu = -\dfrac{1}{6}$

$\Rightarrow \vec{a} + 6\vec{b} + 30\vec{c} = 0$

So, on comparing with $\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0$ we get,

$\Rightarrow \alpha = 6$ and $\beta = 30$

$\Rightarrow \alpha + \beta = 36$

Q.41. If an object has the same weight at same distance above and below the surface of earth, find its distance from the surface of the earth.

A) $\left(\sqrt{5} - 1\right)R$

B) $\dfrac{\left(\sqrt{5}-1\right)R}{2}$

C) $\dfrac{\left(\sqrt{3}-1\right)R}{2}$

D) $\dfrac{R}{2}$

Answer: $\dfrac{\left(\sqrt{5}-1\right)R}{2}$

Solution: Let $g =$ acceleration due to gravity at the surface.

For same weight, the value of g at both places must be the same. Therefore,

$g_{height} = g_{depth}$

$\Rightarrow \dfrac{gR^2}{(R+h)^2} = g\left(1 - \dfrac{h}{R}\right)$

$\Rightarrow \left(1 - \dfrac{h}{R}\right)\left(1 + \dfrac{h^2}{R^2} + \dfrac{2h}{R}\right) = 1$

$\Rightarrow \dfrac{h^3}{R^3} + \dfrac{h^2}{R^2} - \dfrac{h}{R} = 0$

$\Rightarrow \dfrac{h}{R}\left(\dfrac{h^2}{R^2} + \dfrac{h}{R} - 1\right) = 0$

$\Rightarrow \dfrac{h}{R} = \dfrac{\sqrt{5}-1}{2}$

$\Rightarrow h = \dfrac{\left(\sqrt{5}-1\right)R}{2}$

Q.42. A body of mass 100 kg travelled 10 m before coming to rest. If $\mu = 0.4$, work done against the friction is (motion is happening in horizontal plane and take $g = 10$ m s^{-2})

A) 4500 J

B) 4200 J

C) 4000 J   D) 50000 J

Answer: 4000 J

Solution: The formula to calculate the frictional force (f) on the object is given by

$f = \mu mg$ ...(1)

The work done (W) against the friction is given by

$W = fS$
$\quad = \mu mgS$ ...(2)

From equation (2), it follows that

$W = 0.4 \times 100 \text{ kg} \times 10 \text{ m s}^{-2} \times 10 \text{ m}$
$\quad = 4000 \text{ J}$

Q.43. Statement 1: A capillary tube is first dipped in cold water and then in hot water. The rise is higher in hot water.

Statement 2: As temperature increases, surface tension decreases

(Assume density of water to be constant)

A) Statement 1 is true and Statement 2 is false   B) Statement 1 is false and Statement 2 is true

C) Both statements are true   D) Both statements are false

Answer: Statement 1 is false and Statement 2 is true

Solution: We know that $h = \frac{2S\cos\theta}{r\rho g}$.

The surface tension of hot water is less than the surface tension of cold water. Moreover, due to thermal expansion, the radius of the capillary tube will increase in hot water. Due to both these factors, the height of capillary rise will be less in hot water as compared to that in cold water.

Q.44. If a particle starting from rest having constant acceleration covers distance S_1 in first $(P-1)$ seconds & S_2 in first P seconds, then determine time for which displacement is $S_1 + S_2$.

A) $\sqrt{P - (P-1)^2}$ B) $2P$

C) $\sqrt{2P^2 + 1 - 2P}$

D) $\sqrt{2P^2 + 1 + 2P}$

Answer: $\sqrt{2P^2 + 1 - 2P}$

Solution: Using equation of motion for constant acceleration, we can write
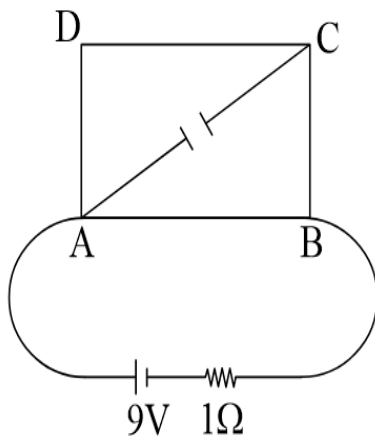
$S_1 = 0 + \frac{1}{2}a(P-1)^2$ ...$Eq(1)$

and

$S_2 = 0 + \frac{1}{2}a(P)^2$ ...$Eq(2)$

From both equations, we get

$S_1 + S_2 = \frac{1}{2}a\left[(P-1)^2 + (P)^2\right]$
$\Rightarrow \frac{1}{2}at^2 = \frac{1}{2}a\left[P^2 - 2P + 1 + P^2\right]$
$\Rightarrow t = \sqrt{2P^2 + 1 - 2P}$

Q.45. In the following circuit, the resistance of the square loop $ABCD$ is 16 Ω. Find the voltage across the capacitor in the steady state.
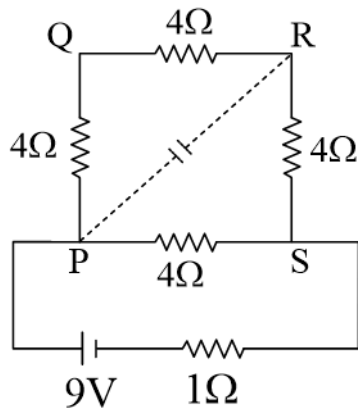

A) 4. 5 V

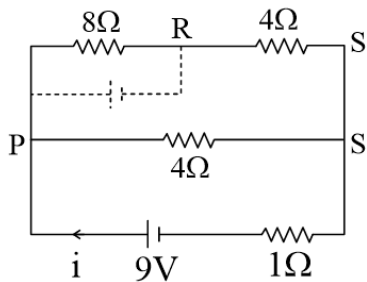B) 3 V

C) 1 V

D) 4 V

Answer: 4. 5 V

Solution: In the steady state, the capacitor becomes an open circuit.

The given circuit can be redrawn as follows:

The equivalent circuit of the above figure can be drawn as below:

The equivalent resistance (R) of the square loop becomes,

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{8+4}$$
$$= \frac{1}{3}$$
$$\Rightarrow R = 3\ \Omega$$

Thus, the equivalent resistance (R_{eq}) for the entire circuit is given by

$$R_{eq} = (3+1)\ \Omega$$
$$= 4\ \Omega$$

The current flowing through the battery is given by

$$i = \frac{9\ \text{V}}{4\ \Omega}$$
$$= 2.25\ \text{A}$$

Using the voltage divider theorem, the current (i') along QRS path can be found as follows:

$$i' = \frac{4\ \Omega}{4\ \Omega + 12\ \Omega} \times i$$
$$= \frac{4}{16} \times \frac{9}{4}\ \text{A}$$
$$= \frac{9}{16}\ \text{A}$$

The potential across the capacitor can be calculated as follows:

$$V_C = i' \times 8\ \Omega$$
$$= \frac{9}{16}\ \text{A} \times 8\ \Omega$$
$$= 4.5\ \text{V}$$

Q.46.   De-Broglie wavelength of a proton and an electron is same. The ratio of kinetic energy of electron to that of proton is

A) $\dfrac{1}{1837}$

B) 1

C) 933.5

D) 1837

Answer: 1837

Solution: De-Broglie wavelength is given by, $\lambda = \dfrac{h}{p}$. Therefore, momentum of proton and electron would be the same.

The relation between kinetic energy and momentum:

$$p = \sqrt{2mK_E}$$

Hence, the ratio of two kinetic energies:

$$\dfrac{(K_E)_e}{(K_E)_p} = \dfrac{m_p}{m_e} (\text{As } p_e : p_p = 1 : 1)$$

$$\Rightarrow \dfrac{(K_E)_e}{(K_E)_p} = \dfrac{m_p}{m_e} = 1837$$

The required ratio is $1837 : 1$

Q.47. If the ratio of centripetal acceleration of two particles moving on the same circular path is $3 : 4$. Find the ratio of their speed.

A) $2 : \sqrt{3}$

B) $\sqrt{2} : 1$

C) $\sqrt{3} : 2$

D) $\sqrt{3} : 1$

Answer: $\sqrt{3} : 2$

Solution: The formula to calculate the centripetal acceleration (a_c) of an object is given by

$$a_c = \dfrac{v^2}{r} \quad \dots (1)$$

For the two objects with velocities v_1 and v_2 and moving on the same circular path, following equation (1), it can be written that

$$\dfrac{(a_c)_1}{(a_c)_2} = \dfrac{v_1{}^2}{v_2{}^2} \quad \dots (2)$$

From equation (2), it follows that

$$\dfrac{v_1{}^2}{v_2{}^2} = \dfrac{3}{4}$$

$$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{\sqrt{3}}{2}$$

Q.48. Given the current at any instant t second is, $i = (20 + 3t)$ A. Find the charge flown in 20 s.

A) 1200 C

B) 1600 C

C) 800 C

D) 1000 C

Answer: 1000 C

Solution: Given, the current is

$$i = 20 + 3t \quad \dots (1)$$

Integrate both sides of equation (1) with respect to time to obtain the charge.

$$\dfrac{dq}{dt} = 20 + 3t$$

$$\Rightarrow \int dq = \int_0^{20} (20 + 3t)dt$$

$$\Rightarrow q = \left[20t + \dfrac{3}{2}t^2 \right]_0^{20}$$

$$= 1000 \text{ C}$$

Q.49. A capacitor of capacitance $100 \ \mu F$ is charged to a potential of 12 V and is connected to an inductor of inductance $6.4 \ mH$. What is the maximum current through the inductor at resonance?

A) 2.0 A

B) 1.5 A

C) 3.2 A

D) 1.2 A

Answer: 1.5 A

Solution: At resonance, the total energy stored within the capacitor and the inductor will be conserved.

After the complete charging of the capacitor, the energy stored within it can be written as

$E_C = \frac{1}{2}CV^2$ $\dots(1)$

During the discharge the energy stored in inductor can be written as,

$E_L = \frac{1}{2}Li^2$ $\dots(2)$

Equate equations (1) and (2) and simplify to obtain the required current.

$\frac{1}{2}CV^2 = \frac{1}{2}Li^2$

$\Rightarrow i = \sqrt{\dfrac{CV^2}{L}}$ $\dots(3)$

From equation (3), it follows that

$i = \sqrt{\dfrac{100 \times 10^{-6} \times (12)^2}{6.4 \times 10^{-3}}}$ A

$= 1.5$ A

Q.50. An electron in a stationary hydrogen atom de-excites from the first excited state to the ground state. Find the recoil speed(in $\mathrm{m\,s^{-1}}$) of the hydrogen atom up to the nearest integer value. (mass of hydrogen atom$= 1.8 \times 10^{-27}$ kg)

A)   2   B)   1

C)   3   D)   5

Answer: 3

Solution: The energy of the emitted photon in the given process can be written as

$E = -13.6 \left(\dfrac{1}{n_i^2} - \dfrac{1}{n_f^2} \right)$ eV $\dots(1)$

From equation (1), it follows that

$E = -13.6 \times \left(\dfrac{1}{2^2} - \dfrac{1}{1^2} \right)$ eV

$= 10.2$ eV

Now, momentum of the photon is given by, $p = \dfrac{E}{c}$. From conservation of momentum, momentum of hydrogen atom will be equal to p.

$p = \dfrac{E}{c}$

$\Rightarrow mv = \dfrac{E}{c}$

$\Rightarrow v = \dfrac{E}{mc}$ $\dots(2)$
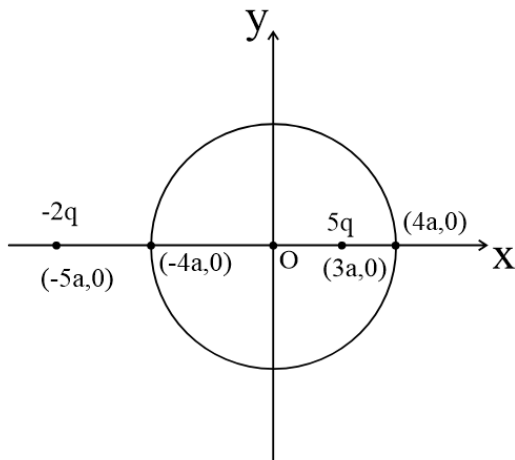
From equation (2), it can be concluded that

$v = \dfrac{10.2\,\mathrm{eV} \times \dfrac{1.6 \times 10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}}}{1.8 \times 10^{-27}\,\mathrm{kg} \times 3 \times 10^8\,\mathrm{m\,s^{-1}}}$

$= 3\ \mathrm{m\,s^{-1}}$

Q.51. A solid sphere of radius $4a$ units is placed with its centre at origin. Two charges $-2q$ at $(-5a,\ 0)$ and $5q$ at $(3a,\ 0)$ is placed. If the flux through the sphere is $\dfrac{xq}{\varepsilon_0}$ in SI units. Find the value of x.

Answer: 5

Solution: Let's consider the following figure:

From the above figure, it is clear that the charge $-2q$ resides outside the Gaussian surface of radius $4a$. Thus, only $5q$ charge is enclosed by the Gaussian surface.

From Gauss's law of electrostatics, it can be written that the flux (φ) through the surface is given by

$$\phi = \frac{q_{enc}}{\varepsilon_0}$$
$$= \frac{5q}{\varepsilon_0}$$

Hence, $x = 5$.

Q.52. The work done for the process depicted by the following $P-V$ diagram is $400x$ J. Find the value of x.
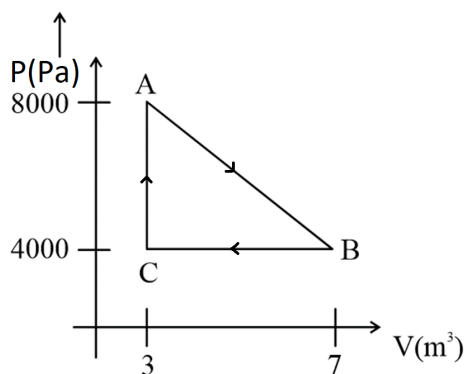
Answer: 20

Solution: The work done for the given process can be measured by calculating the area inside the triangle.

Hence, the work done (W) for the given process can be written as

$$W = \frac{1}{2} \times (V_2 - V_1) \times (P_2 - P_1)$$
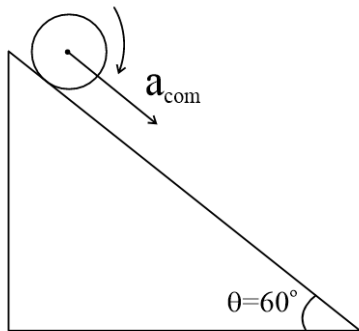$$= \frac{1}{2} \times 4 \times 4000 \text{ J}$$
$$= 8000 \text{ J}$$

Thus, $x = 20$

Q.53. A solid cylinder is released from rest on an inclined plane with inclination angle of $\theta = 60°$. The acceleration of the centre of mass of the cylinder is given by $\frac{g}{\sqrt{\beta}}$ m s^{-2}. Find the value of β.

(Consider pure rolling)

Answer: 3

Solution: Let's consider the following diagram:

The formula to calculate the acceleration of the centre of mass for pure rolling motion in an inclined plane is given by

$$a_{com} = \frac{g\sin\theta}{1 + \frac{I}{MR^2}} \quad \ldots(1)$$

where, g is the acceleration due to gravity, θ is the inclination angle, I is the moment of inertia, M is the mass and R is the radius of the object.

For the solid cylinder, the moment of inertia passing through its central perpendicular axis is given by

$$I = \frac{1}{2}MR^2 \quad \ldots(2)$$

From equations (1) and (2), it follows that

$$a_{com} = \frac{g\sin 60°}{1 + \frac{\frac{1}{2}MR^2}{MR^2}}$$
$$= \frac{2g \times \frac{\sqrt{3}}{2}}{3}$$
$$= \frac{g}{\sqrt{3}}$$

Hence, $\beta = 3$.

Q.54. The ratio of total energy to kinetic energy in SHM when $x = \frac{A}{3}$, where A is amplitude is $\frac{\beta}{8}$. Write the value of β.

Answer: 9

Solution: Required ratio:

$$\frac{T.E.}{K.E.} = \frac{\frac{1}{2}m\omega^2 A^2}{\frac{1}{2}m\omega^2\left(A^2 - x^2\right)}$$
$$= \frac{A^2}{\left(A^2 - x^2\right)} = \frac{9}{8}$$

Therefore, $\beta = 9$.

Q.55. Current through a resistance measured in a simple circuit is $i = (20 \pm 0.2)$ A and potential difference across the resistor is $V = (200 \pm 5)$ V. If the percentage errors in measurement of resistance is $\beta\%$, write the value of 2β.

Answer: 7

Solution: For a simple circuit, applying Ohm's law, we get

$V = iR$

$\Rightarrow R = \dfrac{V}{i}$

Now, for very small changes, we can write

$\dfrac{\Delta R}{R} \times 100 = \dfrac{\Delta V}{V} \times 100 + \dfrac{\Delta i}{i} \times 100$

$= \dfrac{5}{200} \times 100 + \dfrac{0.2}{20} \times 100$

$= 2.5 + 1 = 3.5\%$

Therefore, $2\beta = 7$.