JEE Main

27th Jan Shift 2

Questions

Q.1. The quantity which changes with temperature.

A) Molarity

B) Mole Fraction

C) Mass percentage

D) Molality

Answer: Molarity

Solution: Molarity is the number of moles in solute per unit volume of solution.

As the temperature rises, the volume of the solution expands and the volume of the solution increases. When the temperature increases the molarity of the solution will decrease as temperature depends on volume.

Hence option A is the answer.

Q.2. Which of the following can not act as an oxidising agent?

A) MnO_4^-

B) SO_4^{2-}

C) N^{3-}

D) BrO_3^-

Answer: N^{3-}

Solution: An oxidising agent is a substance that oxidises other substances involved in the reaction by gaining or accepting electrons from them.

So the lowest oxidation state substance will go to the higher oxidation state, which means it reduces others. Hence, it acts as a reducing agent. -3 is the lowest oxidation state for nitrogen. Hence, it only acts as a reducing agent.

The answer is C.

Q.3. Phenolic group can be identified by a positive

A) Lucas Test

B) Carbylamine test

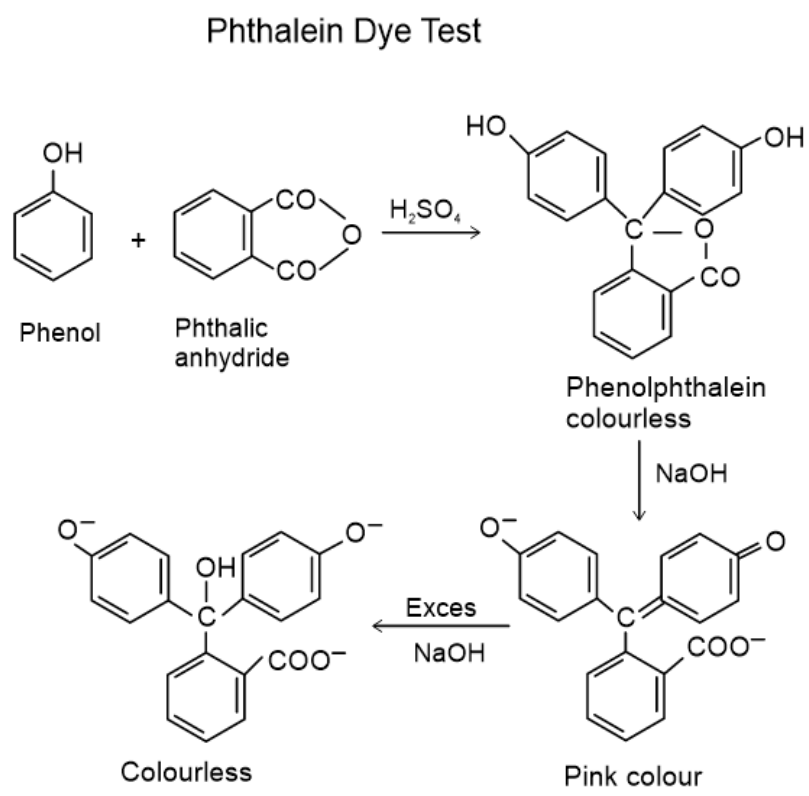
C) Tollen's Test

D) Pthalein Test

Answer: Pthalein Test

Solution: Phenol on heating with phthalic anhydride in the presence of concentrated sulphuric acid forms a colourless condensation compound called phenolphthalein.

On further reaction with dilute sodium hydroxide solution gives a pink colour fluorescent compound called fluorescein. Characteristic colours are produced by different phenolic compounds which can be viewed under white background.



Q.4. Which type of protein can not be denatured when heated?

A) Primary B) Secondary

C) Tertiary D) Quaternary

Answer: Primary

Solution: When a protein in its native form, is subjected to physical change like change in temperature or chemical change like change in pH, the hydrogen bonds are disturbed. Due to this, globules unfold and helix get uncoiled and protein loses its biological activity. This is called denaturation of protein. During denaturation, secondary and tertiary structures are destroyed but primary structure remains intact. Primary structure, such as the sequence of amino acids held together by covalent peptide bonds, is not disrupted by denaturation.

Q.5. The second Homologue of mono carboxylic acid is:

A) $HCOOH$ B) CH_3CH_2COOH

C) CH_3COOH D) $CH_3CH_2CH_2COOH$

Answer: CH_3COOH

Solution: Homologous series refers to a set of organic compounds that have similar structural properties but differ from adjacent members by the $-CH_2$ group. In the mono carboxylic acid homologous series, the first compound is $HCOOH$(Formic acid) and the second compound is CH_3COOH(Acetic acid).

Hence, the answer is C.

Q.6. Identify the following species of which d^2sp^3 hybridisation is shown by the central atom?

A) SF_6 B) BrF_5

C) $[PtCl_4]^{2-}$ D) $[Co(NH_3)_6]^{3+}$

Answer: $[Co(NH_3)_6]^{3+}$

Solution: Co is in $+3$ oxidation state with the configuration $3d^6$. In the presence of NH_3 a strong ligand, the $3d$ electrons pair up, leaving two d-orbitals empty. Hence, the hybridisation is d^2sp^3 forming an inner orbital octahedral complex. In SF_6, sulphur undergo sp^3d^2 hybridisation. In BrF_5, bromine undergo sp^3d^2 hybridisation. In $[PtCl_4]^{2-}$, platinum undergo dsp^2 hybridisation.

Hence, the answer is D.

Q.7. Identify the incorrect pair:

A) Haber process-Iron

B) Polythene-$TiCl_4 / Al(CH_3)_3$

C) Photography- $AgBr$

D) Wacker process-$PtCl_2$

Answer: Wacker process-$PtCl_2$

Solution: $TiCl_4$ with $Al(CH_3)_3$ forms the basis of the Ziegler catalysts used to manufacture polyethylene (polythene).

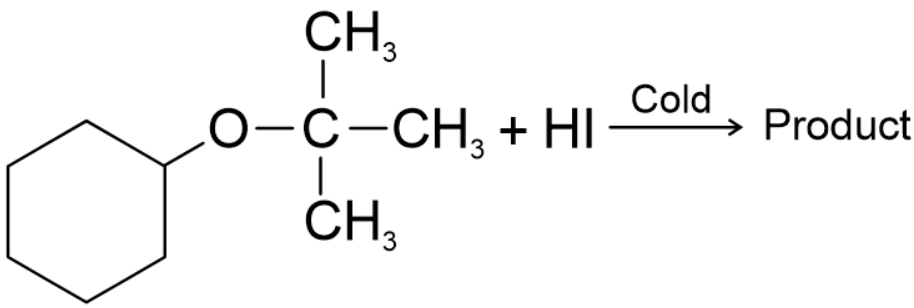
Iron catalysts are used in the Haber process for the production of ammonia from N_2/H_2 mixtures.

The photographic industry relies on the special light-sensitive properties of $AgBr$.

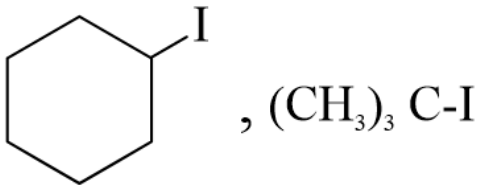
In the Wacker process, $PdCl_2$ is used, it is not the $PtCl_2$.

Hence, it is a incorrect pair.

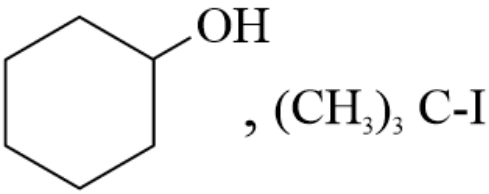
Q.8. Product for the below reaction are



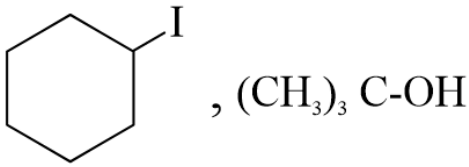
A)



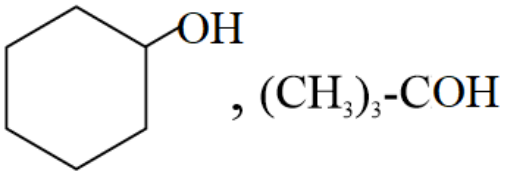
B)



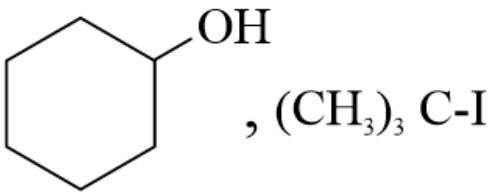
C)



D)



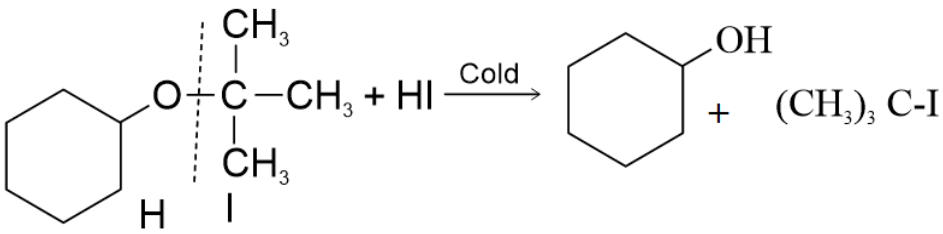
Answer:



Solution: The given compound undergoes S_N1 mechanism, we get the following products.

Hydrogen of the HI will get attached to the cyclohexane ring and iodine group of HI will give tertiary alkyl iodide.

The reaction is as follows-



Q.9. In which of the options all the elements have d^{10} configuration in their ground state.

A) $Cu,\ Zn,\ Cd,\ Ag$

B) $Cd,\ Au, Hg, Ni$

C) $Sc,\ Ti,\ Fe, Zn$

D) $Fe,\ Cr,\ Co, Ni$

Answer: $Cu,\ Zn,\ Cd,\ Ag$

Solution: The electronic configuration can be written as,

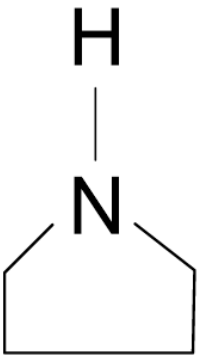
$$Cu_{29} = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^1$$
$$Zn_{30} = \left[Ar\right] 3d^{10} 4s^2$$
$$Cd_{48} = \left[Kr\right]\ 4d^{10} 5s^2$$
$$Ag_{47} = \left[Kr\right]\ 4d^{10} 5s^1$$

Hence, option A is the answer.

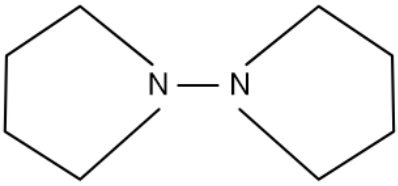
Q.10. $Cl-(CH_2)_4-Cl \xrightarrow{NH_3} \xrightarrow{NaOH} A + H_2O + NaCl$

What is A?

A)



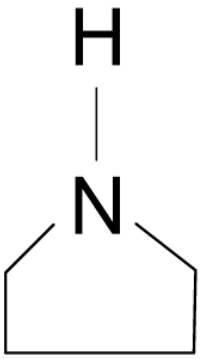
B)



C) $NH_2-(CH_2)_4-NH_2$

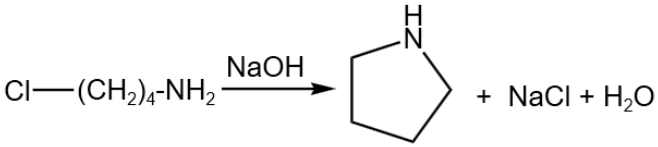
D) $Cl^-NH_3{}^+-(CH_2)_4-NH_3{}^+Cl^-$

Answer:



Solution: $Cl-(CH_2)_4-Cl \xrightarrow{NH_3} Cl-(CH_2)_4-NH_2$

$Cl-(CH_2)_4-NH_2$ will be formed as an intermediate, which cyclises to form pyrrolidine as the product.



Hence, the answer is A.

Q.11. Which of the following will not give S_N1 reaction?

A) $PhCH_2Cl$

B) $CH_2=CH-CH_2-Cl$

C) $(CH_3)CCl$

D) $CH_3-CH=CHCl$

Answer: $CH_3-CH=CHCl$

Solution: In the slow, rate-determining step of the reaction, the bond between the carbon atom and the leaving group breaks to produce a carbocation and a leaving group.

Thermodynamically less stable carbocation will not give S_N1 reaction.

Here,

$CH_3 - CH = CH^+$, positive charge on sp^2 carbon which makes the carbocation unstable.

Q.12. Find the longest wavelength in Paschen series in terms of R

A) $123/2R$

B) $144/7\ R$

C) $170/R$

D) $16/R$

Answer: $144/7\ R$

Solution: Given $R = 1.097 \times 10^7 m^{-1}$

$\text{The Rydberg formula :}$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right)$$

here $n_1 = 3, n_2 = 4$

By putting the values in the equation,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{7}{144}\right)$$

Hence option B is the answer.

Q.13. $Ph - CH = CH_2 \xrightarrow{BH_3, H_2O_2} P$

Give the product P.

A) $PhCH_2CH_2BH_2$

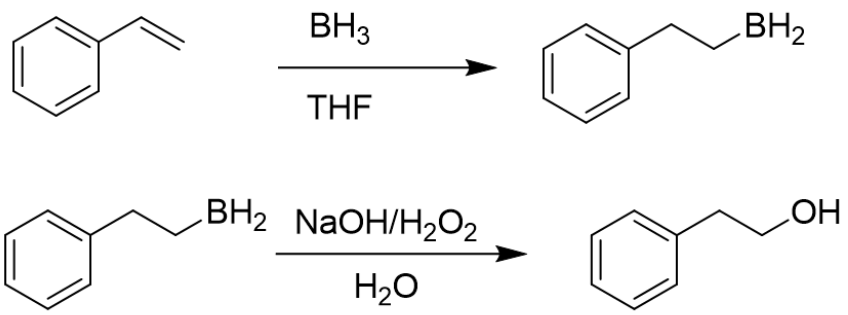
B) $Ph - OH$

C) $PhCH_2CH_2OH$

D) None of the above

Answer: $PhCH_2CH_2OH$

Solution: The hydroboration oxidation reaction is indirect addition of water by antimarkonikov rule. Primary alcohol is produced as shown below.



Hence, the answer is option C.

Q.14. Statement 1 :Ce^{4+} is stable because of noble gas configuration.

Statement 2: Ce^{4+} is good reducing agent as it can go to +3 oxidation state.

A) Statement 1 is incorrect but statement 2 is correct.

B) Statement 1 is correct but statement 2 is incorrect.

C) Both the statements are wrong.

D) Both Statement $1, 2$ are correct.

Answer: Statement 1 is correct but statement 2 is incorrect.

Solution: The electronic configuration of
$Ce(58) = [Xe]\,4f^1 5d^1 6s^2$

$Ce^{4+} = [Xe]\,4f^0 5d^0 6s^0$

Since in $+4$ oxidation state, all orbitals are empty and it gains the nearest inert gas xenon configuration.

Ce^{4+} ion is a strong oxidising agent because $+3$ oxidation state is more stable than $+4$ oxidation state .

Hence Statement 1 is correct and Statement 2 is wrong.

Q.15. Steam volatile and water immiscible compounds can be separated by

A) Distillation

B) Fractional Distillation

C) Steam Distillation.

D) Distillation under reduced Pressure.

Answer: Steam Distillation.

Solution: **Steam Distillation**: This technique is used for separating substances which are immiscible with water, volatile in steam & having high vapour pressure at the boiling temperature of water.

Steam Distillation is a separation process for temperature-sensitive substances. It is a particular type of distillation. Another way around is a method for separating miscible liquid bases depending upon their volatilities. For instance aromatic compounds. It is a physical process.

Hence, option C is the answer.

Q.16. C_2H_6 newman projection find incorrect information.

A) Infinite Conformers

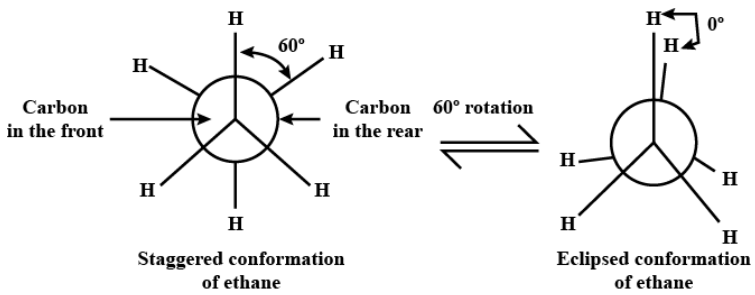
B) Interconvertible

C) Dihedral angle in staggered in 60^0

D) Eclipsed is more stable.

Answer: Eclipsed is more stable.

Solution: Conformations represent conformers which are readily interconvertible and thus non-separable. When an ethane molecule rotates about its carbon – Carbon single bond, two extreme conformations can result in the staggered conformation and the eclipsed conformation.



Newman projections of ethane

Eclipsed conformer is the least stable conformer due to repulsions.

Q.17. Number of Non polar molecules. H_2O, CO_2, CH_4, SO_2, $CHCl_3$, PF_3, NH_3, HF, H_2

Answer: 3

Solution: The molecules which have perfect geometry and no lone pairs are zero dipole moment molecules. Those molecules termed as non-polar molecules. Amongst the given compounds, CH_4, CO_2, H_2 are non poar molecules.

H_2O has two lone pairs and two bond pairs. SO_2 has two sigma bond pairs and one lone pair. $CHCl_3$ has net dipole moment because the bond moments do not cancel with each other. PF_3 and NH_3 have one lone pair and three bond pairs. In HF bond moment is the dipole moment. Hence, all these molecules have some net dipole moment.

Q.18. For a first order reaction $t_{99.9\%} = x\ t_{50\%}$, find the value of x.

Answer: 10

Solution: For first order reaction:

$$kt = \log \frac{[A_0]}{[A_t]}$$

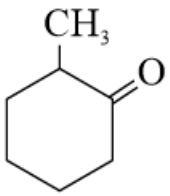
$$\frac{t_{99.9\%}}{t_{50\%}} = x = \frac{\log \frac{[100]}{[100-99.9]}}{\log \frac{[100]}{[50]}}$$

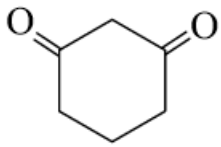
$$= \log \frac{\frac{100}{0.1}}{\log 2}$$

$$= \frac{\log 1000}{\log 2}$$

$$= \frac{3}{0.3} = 10$$

Q.19. How many compounds given below have chiral carbon?

(i) 

(ii) 

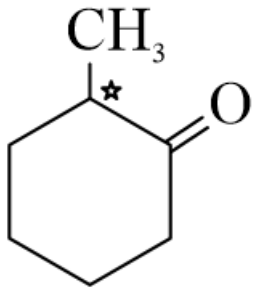
(iii) $CH_3-CH_2-\overset{\overset{I}{|}}{CH}-CH_2-CH_3$

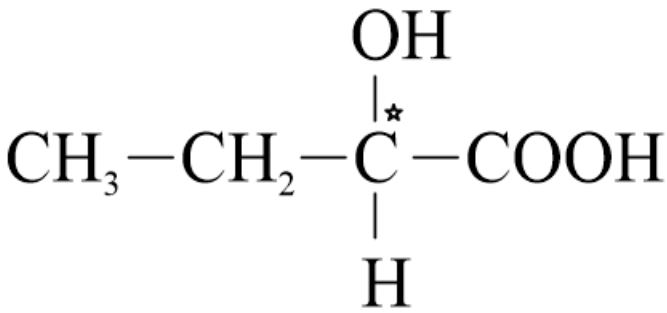
(iv) $CH_3-CH_2-\overset{\overset{OH}{|}}{\underset{\underset{H}{|}}{C}}-COOH$

Answer: 2

Solution: The chiral carbon atoms are carbon atoms that are attached to four different atoms or groups of atoms losses all symmetry, that are placed at the corners of a tetrahedron. The configuration of such a tetrahedral unit is chiral. The structure may exist in either a right-handed configuration or a left-handed configuration. This type of configurational stereoisomers is termed enantiomorphism.

(i)





Amongst the given compounds, (i),(iv) have chiral carbon in it.

Q.20. How many of the given gases below have a noble gas configuration?

1. Fe^{2+}
2. Cs^{+}
3. Sr^{2+}
4. Pb^{2+}

Answer: 2

Solution: The electronic configuration of $Fe^{2+} = [Ar]\,3d^6 4s^0$

The electronic configuration of Pb^{2+}= $[Xe]\,6s^2\,4f^{14}\,5d^{10}$

Cs^{+} and Sr^{2+} have noble gas configuration, which are Xe and Kr, respectively.

Hence, the answer is 2.

Q.21. 20th term from the end of $20,\ 19\frac{1}{4},\ 18\frac{1}{2},\ 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is

A) -112

B) -120.5

C) -115

D) -100

Answer: -115

Solution: Given: $20,\ 19\frac{1}{4},\ 18\frac{1}{2},\ 17\frac{3}{4}, \ldots, -129\frac{1}{4}$

$\Rightarrow a = 20,\ d = 18\frac{1}{2} - 19\frac{1}{4} = \frac{-3}{4}$

$\Rightarrow -\frac{517}{4} = 20 + (n-1)\left(\frac{-3}{4}\right)$

$\Rightarrow -517 = 80 + 3 - 3n$

$\Rightarrow 3n = 600$

$\Rightarrow n = 200$

We know that,

20th term from the end is $(200 - 20 + 1) = 181^{\text{th}}$ term from the beginning.

$\Rightarrow a_{181} = 20 + 180 \times \left(-\frac{3}{4}\right)$

$\Rightarrow a_{181} = 20 - 135 = -115$

Q.22. The integral $\int \frac{x^8 - x^2}{\left(x^{12}+3x^6+1\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)} dx$ is equal to

A) $\frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right| + c$

B) $\frac{1}{6}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right| + c$

C) $\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right| + c$

D) $\frac{1}{9}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right| + c$

Answer: $\frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right| + c$

Solution: Let,

$$I = \int \frac{x^8 - x^2}{\left(x^{12}+3x^6+1\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)} dx$$

$$\Rightarrow I = \int \frac{x^2 - \frac{1}{x^4}}{\left(x^6+3+\frac{1}{x^6}\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)} dx$$

$$\Rightarrow I = \int \frac{x^2 - \frac{1}{x^4}}{\left(\left(x^3+\frac{1}{x^3}\right)^2+1\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)} dx$$

Now let $x^3 + \frac{1}{x^3} = t \Rightarrow x^2 - \frac{1}{x^4}dx = \frac{dt}{3}$

$$\Rightarrow I = \frac{1}{3}\int \frac{1}{\left(t^2+1\right)\tan^{-1} t} dt$$

Now, let $\tan^{-1} t = z \Rightarrow \frac{1}{1+t^2}dt = dz$

$$\Rightarrow I = \frac{1}{3}\int \frac{1}{z} dz$$

$$\Rightarrow I = \frac{1}{3}\ln|z| + c$$

$$\Rightarrow I = \frac{1}{3}\ln\left|\tan^{-1} t\right| + c$$

$$\Rightarrow I = \frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right| + c$$

Q.23. If $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ and $y(0)=2$, then find $y(2)$.

A) e^2

B) e

C) 0

D) 2

Answer: 0

Solution: $\frac{dy}{dx}=\frac{x+y-2}{x-y}$

$\Rightarrow \frac{dy}{dx}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

Let, $X=x-1,\ Y=y-1$

$\Rightarrow \frac{dy}{dx}=\frac{X+Y}{X-Y}$

Putting, $Y=vX$

$\Rightarrow v+X\frac{dv}{dX}=\frac{1+v}{1-v}$

$\Rightarrow X\frac{dv}{dX}=\frac{1+v}{1-v}-v$

$\Rightarrow X\frac{dv}{dX}=\frac{1+v-v+v^2}{1-v}$

$\Rightarrow X\frac{dv}{dX}=\frac{1+v^2}{1-v}$

$\Rightarrow \left(\frac{1-v}{1+v^2}\right)dv=\frac{dX}{X}$

$\Rightarrow \int\left(\frac{1-v}{1+v^2}\right)dv=\int\frac{dX}{X}$

$\Rightarrow \int\frac{1}{1+v^2}dv-\int\frac{v}{1+v^2}dv=\int\frac{dX}{X}$

$\Rightarrow \tan^{-1}v-\log\sqrt{1+v^2}=\log|X|+C$

$\Rightarrow \tan^{-1}\frac{Y}{X}-\log\sqrt{1+\frac{Y^2}{X^2}}=\log|X|+C$

$\Rightarrow \tan^{-1}\frac{y-1}{x-1}-\log\sqrt{1+\frac{(y-1)^2}{(x-1)^2}}=\log|x-1|+C$

It is given that, $y(0)=2$.

$\Rightarrow \tan^{-1}(-1)-\log\sqrt{2}=\log|-1|+C$

$\Rightarrow \frac{-\pi}{4}-\log\sqrt{2}=C$

$\Rightarrow \tan^{-1}\frac{y-1}{x-1}-\log\sqrt{1+\frac{(y-1)^2}{(x-1)^2}}=\log|x-1|-\frac{\pi}{4}-\log\sqrt{2}$

Now taking $x=2$, we get,

$\Rightarrow \tan^{-1}(y-1)-\log\sqrt{1+(y-1)^2}=0-\frac{\pi}{4}-\log\sqrt{2}$

$\Rightarrow \tan^{-1}(y-1)-\log\sqrt{1+(y-1)^2}=-\frac{\pi}{4}-\log\sqrt{2}$

Now on comparing both side we get,

$\Rightarrow y(2)=0$

Q.24. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in $\left[0, \frac{n\pi}{2}\right]$ for least value of $n \in N$, then the value of $\sum_{k=1}^{n} \frac{k}{2^n}$ will be

A) $\frac{9}{2^9}$

B) $\frac{11}{2^{12}}$

C) $\frac{7}{2^7}$

D) $\frac{91}{2^{13}}$

Answer: $\frac{91}{2^{13}}$

Solution: Given,

$2\tan^2\theta - 5\sec\theta = 1$

$\Rightarrow 2\sec^2\theta - 2 - 5\sec\theta = 1$

$\Rightarrow 2\sec^2\theta - 5\sec\theta - 3 = 0$

$\Rightarrow (2\sec\theta + 1)(\sec\theta - 3) = 0$

$\Rightarrow \sec\theta = 3 \left\{\text{as } \sec\theta \neq \frac{-1}{2}\right\}$

$\Rightarrow \cos\theta = \frac{1}{3}$

Now for $\cos\theta = \frac{1}{3}$ will have 7 solution in $\left[0, \frac{13\pi}{2}\right]$ as for $[0, 2\pi],\ [2\pi, 4\pi],\ \&\ [4\pi, 6\pi]$ will have 2 solution in each domain and 1 solution in $\left[6\pi, \frac{13\pi}{2}\right]$

So, $n = 13$

Now solving,

$\sum_{k=1}^{n} \frac{k}{2^n}$

$= \frac{1}{2^{13}}(1 + 2 + 3 \ldots\ldots\ldots + 13)$

$= \frac{1}{2^{13}}\left(\frac{13 \times 14}{2}\right)$

$= \frac{91}{2^{13}}$

Q.25. Coefficient of x^{2012} in $(1-x)^{2008}\left(1 + x + x^2\right)^{2007}$ is

A) 0

B) 2012

C) $(2012)^7$

D) 1

Answer: 0

Solution: Let, $y = (1-x)^{2008}\left(1 + x + x^2\right)^{2007}$

$\Rightarrow y = (1-x)\left(1 - x^3\right)^{2007}$

$\Rightarrow y = (1-x)\left[1 - {}^{2007}C_1 x^3 + {}^{2007}C_2 x^6 + \ldots\right]$

Since all the terms will have power of x as multiples of 3 and 2012 is not a multiple of 3, also for $x \cdot x^{2011}$ not be present as 2011 not a multiple of 3

Therefore, the coefficient of $x^{2012}$is 0.

Q.26. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability that the first draw gives all white balls and second draw gives all black balls is:

A) $\frac{1}{495}$

B) $\frac{5}{812}$

C) $\frac{3}{715}$

D) $\frac{2}{335}$

Answer: $\frac{3}{715}$

Solution: Total number of balls in the bag $= 6 + 9 = 15$.

Number of white balls $= 6$.

Number of black balls $= 9$.

Four balls are drawn at random at a time.

The probability that first draw give four white balls

$$\frac{{}^{6}C_4}{{}^{15}C_4} = \frac{1}{91}$$

The balls are not replaced.

Now, the probability that second draw give four black balls

$$\frac{{}^{9}C_4}{{}^{11}C_4} = \frac{21}{55}$$

The probability that the first draw give four white balls and second draw give four black balls

$$= \frac{1}{91} \times \frac{21}{55} = \frac{3}{715}$$

Hence, the required probability is $\frac{3}{715}$.

Q.27. Let $f : R - \left\{\frac{-1}{2}\right\} \to R$ and $g : R - \left\{\frac{-5}{2}\right\} \to R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$, then the domain of the function $f(g(x))$ is:

A) $R - \left\{\frac{-1}{2}\right\}$

B) $\left[\frac{-1}{2}, \frac{-5}{2}\right]$

C) $R - \left\{\frac{-5}{2}\right\}$

D) R

Answer: $R - \left\{\frac{-5}{2}\right\}$

Solution: Given,

$f: R-\left\{\frac{-1}{2}\right\} \rightarrow R$

Now for $f(g(x)), g(x) \neq -\frac{1}{2}$

$\Rightarrow \frac{|x|+1}{2x+5} \neq -\frac{1}{2}$

Case $\mathrm{I}: x \geq 0$

$\Rightarrow \frac{x+1}{2x+5} = -\frac{1}{2}$

$\Rightarrow 2x+2=-2x-5$

$\Rightarrow 4x=-7$

$\Rightarrow x=\frac{-7}{4}$ [Not possible as $x \geq 0$]

Case $\mathrm{II}: x<0$

$\Rightarrow \frac{-x+1}{2x+5} = -\frac{1}{2}$

$\Rightarrow -2x+2=-2x-5$

$\Rightarrow 2=-5$ [Not possible]

So, domain of $f(g(x))=$domain of $g(x)=R-\left\{\frac{-5}{2}\right\}$

Q.28. Taking principal values of inverse trigonometric functions, the positive real values of x satisfying $\tan^{-1}x+\tan^{-1}(2x)=\frac{\pi}{4}$ is

A) $\frac{\sqrt{17}+3}{4}$

B) $\frac{\sqrt{5}-1}{2}$

C) $\frac{\sqrt{17}-3}{4}$

D) $\frac{\sqrt{5}+1}{2}$

Answer: $\frac{\sqrt{17}-3}{4}$

Solution: Given,

$\tan^{-1}x+\tan^{-1}(2x)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{3x}{1-2x^2}\right)=\frac{\pi}{4}$

$\Rightarrow \frac{3x}{1-2x^2}=1$

$\Rightarrow 2x^2+3x-1=0$

$\Rightarrow x=\frac{-3\pm\sqrt{17}}{4}$

$\Rightarrow x=\frac{\sqrt{17}-3}{4}\left\{\text{as } x=\frac{\sqrt{17}-3}{4} \text{ is positive}\right\}$

Q.29. If α, β are roots of $x^2-x-1=0$ and $S_n=2024\cdot\alpha^n+2024\cdot\beta^n$ then S_3 is equal to

A) 8096

B) 4048

C) 1012

D) 2024

Answer: 8096

Solution: Given: α, β are the roots of $x^2 - x - 1 = 0$.

$\Rightarrow \alpha^2 = \alpha + 1,\ \beta^2 = \beta + 1 \quad \ldots(i)$

Also, $\alpha + \beta = 1,\ \alpha\beta = -1 \quad \ldots(ii)$

Now, $S_n = 2024\ \alpha^n + 2024\ \beta^n$

$\Rightarrow S_3 = 2024\ \alpha^3 + 2024\ \beta^3$

$\Rightarrow S_3 = 2024\ \alpha(\alpha + 1) + 2024\ \beta(\beta + 1) \qquad [\text{Using } (i)]$

$\Rightarrow S_3 = 2024\ \left(\alpha^2 + \alpha\right) + 2024\ \left(\beta^2 + \beta\right)$

$\Rightarrow S_3 = 2024\alpha + 2024\alpha^2 + 2024\beta + 2024\beta^2$

$\Rightarrow S_3 = 2024(\alpha + \beta) + 2024\left[(\alpha + \beta)^2 - 2\alpha\beta\right]$

$\Rightarrow S_3 = 2024 + 2024(1 + 2)$

$\Rightarrow S_3 = 8096$

Q.30. The mean of 15 observations is 12 and standard deviation is 3. If 12 is replaced by 10 then the new mean is μ and variance is σ^2, then what is the value of $15\left(\mu + \mu^2 + \sigma^2\right)$?

Answer: 2429

Solution: Given,

$\overline{x} = 12,\ \text{standard deviation} = 3,\ n = 15$

Now using the mean formula we get,

$\Rightarrow \frac{\sum x_i}{15} = 12$

$\Rightarrow \sum x_i = 180$

So, new mean will be

$\Rightarrow \mu = \frac{180 - 12 + 10}{15}$

$\Rightarrow \mu = \frac{178}{15}$

Also, variance of the original data is 9.

$\Rightarrow 9 = \frac{\sum (x_i)^2}{15} - (\overline{x})^2$

$\Rightarrow 9 = \frac{\sum (x_i)^2}{15} - 144$

$\Rightarrow 153 \times 15 = \sum (x_i)^2$

$\Rightarrow \sum (x_i)^2 = 2295$

Now, finding the new sum we get,

$\Rightarrow \sum (x_i)^2 - 12^2 + 10^2 = 2295 - 144 + 100$

$\Rightarrow \sum (x_i)^2 - 12^2 + 10^2 = 2251$

So, new variance is given by,

$\sigma^2 = \frac{2251}{15} - \left(\frac{178}{15}\right)^2$

$\Rightarrow 15\left(\mu + \mu^2 + \sigma^2\right) = 15\left[\frac{2251}{15} + \frac{178}{15}\right] = 2429$

Q.31. A is a 2×2 matrix, I is 2×2 identity matrix and $|A - \lambda I| = 0$ gives values of λ as -1 and 3. Then the trace of A^2 is equal to _____.

Answer: 10

Solution: We know that, if λ is eigen value of A then λ^2 is eigen value of A^2.

As for $A_{2\times2}$: $A^2 - Tr(A) + |A|I = 0$ has two values λ_1 & λ_2

So, $\lambda_1 + \lambda_2 = Tr(A)$ & $\lambda_1\lambda_2 = |A|$

It is given that, $|A - \lambda I| = 0$ and $\lambda = -1,\ 3$.

Now, for A^2, $\lambda_1 = (-1)^2 = 1$ and $\lambda_2 = 3^2 = 9$

So, trace of $A^2 = 1 + 9 = 10$.

Q.32. If two sets A & B, $n(A) = m$ and $n(B) = n$. Also, the difference of number of subsets of A and number of subsets of B is 56, then the value of $2m + n$ is

Answer: 15

Solution: We know that,

The number of subsets of $A = 2^m$ as $n(A) = m$

And the number of subsets of $B = 2^n$ as $n(B) = n$

Now, given $2^m - 2^n = 56$

$\Rightarrow 2^m - 2^n = 64 - 8$

$\Rightarrow 2^m - 2^n = 2^6 - 2^3$

Now on comparing both side we get, $m = 6$ & $n = 3$

Hence, $2m + n = 12 + 3 = 15$

Q.33. If the line $x + y = 0$ is tangent to the circle $(x - \alpha)^2 + (y - \beta)^2 = 50$, then $(\alpha + \beta)^2 =$_____.

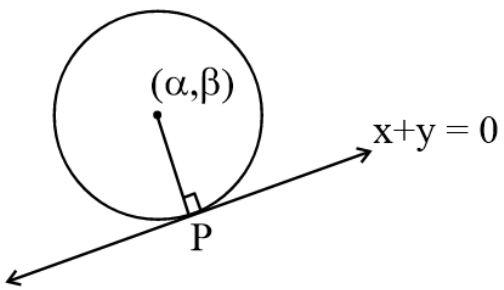
Answer: 100

Solution: Given,

Equation of circle $(x - \alpha)^2 + (y - \beta)^2 = 50$

And tangent line $x + y = 0$

Now plotting the diagram we get,



We know that the perpendicular distance from centre of a circle to its tangent is radius.

$\Rightarrow \left|\frac{\alpha+\beta}{\sqrt{1^2+1^2}}\right| = \sqrt{50}$

$\Rightarrow |\alpha + \beta| = \sqrt{100}$

$\Rightarrow (\alpha + \beta)^2 = 100$

Q.34. If $\lim_{x\to0} \frac{3+\alpha\sin x+\beta\cos x+\ln(1-x)}{3\tan^2 x} = \frac{1}{3}$, then find the value of $2\alpha - \beta$

Answer: 5

Solution: Given,

$$\lim_{x\to 0}\frac{3+\alpha\sin x+\beta\cos x+\ln(1-x)}{3\tan^2 x}=\frac{1}{3}$$

Now, using the expansion formula of trigonometric and logarithmic function we get,

$$\Rightarrow \lim_{x\to 0}\frac{3+\alpha\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-....\infty\right)+\beta\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-....\infty\right)+\left(-x-\frac{x^2}{2!}-\frac{x^3}{3!}-....\infty\right)}{3\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+....\infty\right)^2}=\frac{1}{3}$$

Now taking x & x^2 common from numerator in above series we get,

$$\Rightarrow \lim_{x\to 0}\frac{3+\beta+x(\alpha-1)+x^2\left(\frac{-\beta}{2}-\frac{1}{2}\right)+.....\infty}{3x^2\left(1+\frac{x^2}{3}+\frac{2x^4}{15}+....\infty\right)^2}=\frac{1}{3}$$

Now limit to exist $3+\beta=0\Rightarrow\beta=-3$ and $(\alpha-1)=0\Rightarrow\alpha=1$

Hence, the value of $2\alpha-\beta=2+3=5$

Q.35. Statement 1: Positive zero error is added in measured value

Statement 2: Defect may occur during manufacturing of measuring instruments

A) Statement 1 is true while statement 2 is false

B) Statement 1 is false while statement 2 is true

C) Both statements are true

D) Both statements are false

Answer: Statement 1 is false while statement 2 is true

Solution: Zero error is subtracted from the measurement to get the corrected measurement. Zero error itself could be positive or negative.

Defects are any deviations from the expected quality or performance of a product or service. Manufacturing defects result from errors in the creation of a particular unit of a product that cause it to not meet the quality standards.

Therefore, Statement 1 is false while statement 2 is true.

Q.36. If the work function of a metal is $6.63\ \mathrm{eV}$, then find the threshold frequency for the photoelectric effect.

A) $1.6\times10^{15}\ \mathrm{Hz}$

B) $1.9\times10^{15}\ \mathrm{Hz}$

C) $1.2\times10^{15}\ \mathrm{Hz}$

D) $2.0\times10^{16}\ \mathrm{Hz}$

Answer: $1.6\times10^{15}\ \mathrm{Hz}$

Solution: The formula to calculate the work function of the metal is given by

$\phi=h\nu_0\quad\ldots(1)$

From equation (1), it follows that

$$6.63\ \mathrm{eV}\times\frac{1.6\times10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}}=6.626\times10^{-34}\ \mathrm{J\cdot s}\times\nu_0$$

$$\Rightarrow\nu_0=\frac{6.63\ \mathrm{eV}\times\frac{1.6\times10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}}}{6.626\times10^{-34}\ \mathrm{J\cdot s}}$$

$$\approx1.6\times10^{15}\ \mathrm{Hz}$$

Q.37. If $\left(p-\frac{a}{V^2}\right)(V-b)=nRT$, where P,V,R, and T are pressure, volume, universal gas constant and the temperature, then $\frac{a}{b^2}$ has the same dimensional formula as that of

A) PV

B) R

C) P

D) RT

Answer: P

Solution: According to the given equation, the quantity $\frac{a}{V^2}$ must have the dimension of pressure. So,

$$\left[\frac{a}{V^2}\right]=[P]$$
$$\Rightarrow \frac{[a]}{\left[\mathrm{L}^3\right]^2}=\left[\mathrm{ML}^{-1}\mathrm{T}^{-2}\right]$$
$$\Rightarrow [a]=\left[\mathrm{ML}^5\mathrm{T}^{-2}\right]$$

Also, the dimension of b can be written as

$$[b]=[V]$$
$$=\left[\mathrm{L}^3\right]$$

Thus, the dimension of the quantity $\frac{a}{b^2}$ is given by

$$\left[\frac{a}{b^2}\right]=\frac{\left[\mathrm{ML}^5\mathrm{T}^{-2}\right]}{\left[\mathrm{L}^3\right]^2}$$
$$=\left[\mathrm{ML}^{-1}\mathrm{T}^{-2}\right]$$

From the above calculation, it can be concluded that the required quantity has the same dimension as pressure.

Q.38. Kinetic friction and static friction depends on

A) Surface area

B) Material of the surface

C) Both surface area & material of the surface

D) None of these

Answer: Material of the surface

Solution: Both static and kinetic friction are independent of the area of contact but depend upon the nature and material of the surfaces in contact.

Q.39. Find the total kinetic energy of 1 mole of oxygen gas at 27° C. (Take $R=\frac{25}{3}$ J $\mathrm{mol}^{-1}\mathrm{K}^{-1}$)

A) 12500 J

B) 3125 J

C) 6250 J

D) 625 J

Answer: 6250 J

Solution: The formula to calculate the average kinetic energy of the gas is given by

$$K=\frac{f}{2}nRT \quad \ldots(1)$$

where, f is the degree of freedom of the gas.

From equation (1), it follows that

$$K=\frac{5}{2}\times 1\times \frac{25}{3}\ \mathrm{J\ mol}^{-1}\mathrm{K}^{-1}\times(273+27)\ \mathrm{K}$$
$$=6250\ \mathrm{J}$$

Q.40. An LCR series AC circuit with $L=\frac{100}{\pi}$ mH, $C=\frac{10^{-3}}{\pi}$ F and $R=10\ \Omega$ has $f=50$ Hz. The RMS voltage is 230 V. Find the power factor of the circuit.

A) 1

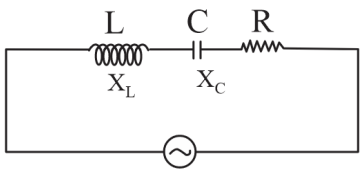
B) 0

C) $\frac{\sqrt{3}}{2}$

D) $\frac{1}{2}$

Answer: 1

Solution: Let's consider the following figure:



The capacitive reactance (X_C) of the circuit is given by

$$X_C = \frac{1}{2\pi f C}$$
$$= \frac{1}{2\pi \times 50 \times \frac{10^{-3}}{\pi}} \Omega$$
$$= 10\ \Omega$$

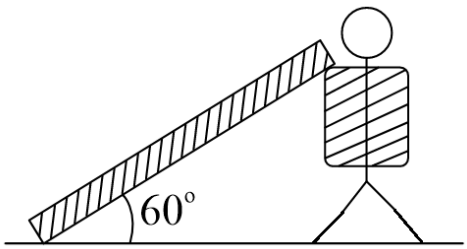
The inductive reactance (X_L) of the circuit is given by

$$X_L = 2\pi f L$$
$$= 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3}\ \Omega$$
$$= 10\ \Omega$$

From the above calculation, it can be concluded that the circuit is in resonance condition. So the phase angle is given by $\phi = 0^\circ$.

Hence, the required power factor is given by $\cos\phi = 1$.

Q.41. A man holds a rod of mass $12\ \mathrm{kg}$ on his shoulder as shown. What is the normal reaction force applied by the shoulder of the man?



A) $30\ \mathrm{N}$

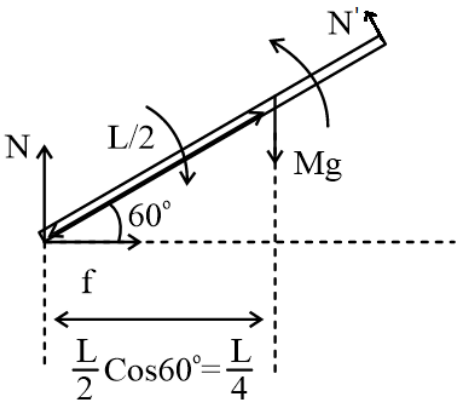
B) $15\ \mathrm{N}$

C) $120\ \mathrm{N}$

D) $60\ \mathrm{N}$

Answer: $30\ \mathrm{N}$

Solution: Let's consider the following diagram:



Here, f is the frictional force, N is the normal reaction force on the side of the rod which touches the ground.

If N' be the normal force on the side of the rod which touches the shoulder of the man, then from the condition of rotational equilibrium(about the point of contact with ground) of the rod, with reference from the above figure, it can be written that

$$\frac{L}{4} \times mg = LN'$$

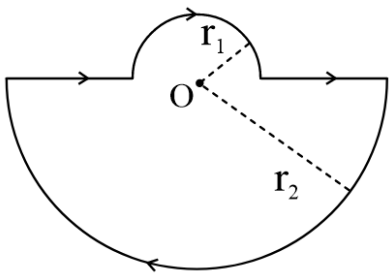
$$\Rightarrow N' = \frac{mg}{4} \quad \ldots(1)$$

From equation (1), it follows that

$$N' = \frac{12\,\mathrm{kg} \times 10\,\mathrm{m\,s^{-2}}}{4}$$

$$= 30\ \mathrm{N}$$

Q.42. Find the magnetic field at point O of the following figure. Given the current through the wire is $4\ \mathrm{A}$, $r_1 = 2\pi\ \mathrm{m}$ and $r_2 = 4\pi\ \mathrm{m}$.



A) $2 \times 10^{-7}\ \mathrm{T}$

B) $10^{-7}\ \mathrm{T}$

C) $3 \times 10^{-7}\ \mathrm{T}$

D) $4 \times 10^{-7}\ \mathrm{T}$

Answer: $3 \times 10^{-7}\ \mathrm{T}$

Solution: The formula to calculate the magnetic field at the centre of a circular current carrying conductor is given by

$$B=\frac{\mu_0 i}{2r} \quad \ldots(1)$$

For the semi-circular loop of radius r_1, the magnetic field (B_1) is given by

$$B_1=\frac{1}{2}\frac{\mu_0 i}{2r_1} \quad \ldots(2)$$

For the semi-circular loop of radius r_2, the magnetic field (B_2) is given by

$$B_2=\frac{1}{2}\frac{\mu_0 i}{2r_2} \quad \ldots(3)$$

Since, the direction of magnetic field due to both circular parts of the wires are the same(inside the plane of the paper), the net magnetic field (B_o) at the centre O can be written as

$$\begin{aligned} B_o &= B_1+B_2 \\ &= \frac{\mu_0 i}{4r_1}+\frac{\mu_0 i}{4r_1} \\ &= \frac{\mu_0 i}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \quad \ldots(4) \end{aligned}$$

From equation (4), it follows that

$$\begin{aligned} B_o &= \frac{4\pi\times10^{-7}\times4}{4}\left(\frac{1}{2\pi}+\frac{1}{4\pi}\right)\ \mathrm{T} \\ &= 3\times10^{-7}\ \mathrm{T} \end{aligned}$$

Q.43. If the primary side of a transformer is connected with $230\ \mathrm{V}$, $50\ \mathrm{Hz}$ AC supply and the ratio of number of turns of primary to the secondary winding is $10:1$. If the load resistance at secondary coil is $46\ \Omega$, then output power of secondary winding is

A) $11.5\ \mathrm{W}$

B) $13\ \mathrm{W}$

C) $16\ \mathrm{W}$

D) $17\ \mathrm{W}$

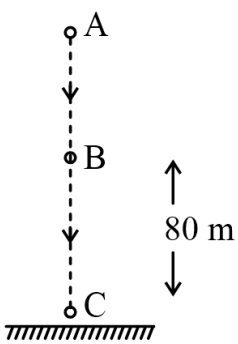
Answer: $11.5\ \mathrm{W}$

Solution: In a transformer, relation of output voltage is given by

$$\begin{aligned} &\frac{V_1}{V_2}=\frac{n_1}{n_2} \\ &\Rightarrow \frac{230}{V_2}=\frac{10}{1} \\ &\Rightarrow V_2=23\ \mathrm{V} \end{aligned}$$

Required output power, $P_2=\frac{(V_2)^2}{R}=\frac{(23)^2}{46}=11.5\ \mathrm{W}$

Q.44. An object is released from point A, as shown in the figure below. Time taken by the object to move from B to C is $2\ \mathrm{s}$. Find the height AC. Given that $BC=80\ \mathrm{m}$.



A) $100\ \mathrm{m}$

B) $125\ \mathrm{m}$

C) $40\ \mathrm{m}$

D) $160\ \mathrm{m}$

Answer: $125\ \mathrm{m}$

Solution: For the path BC of the falling object, the formula to calculate the distance travelled is given by

$$S = v_B t + \frac{1}{2} g t^2 \quad \ldots (1)$$

Substitute the values of the known parameters into equation (1) and solve to calculate the velocity of the object at B.

$$80 = v_B \times 2 + \frac{1}{2} \times 10 \times 2^2$$
$$\Rightarrow 2v_B = 80 - 20$$
$$\Rightarrow v_B = 30 \text{ m s}^{-1}$$

For the path AB of the falling object, the final velocity at B can be written as

$$v_B{}^2 = v_A{}^2 + 2gh \quad \ldots (2)$$

where, h is equal to the distance AB.

From equation (2) it follows that

$$30^2 = 0 + 2 \times 10 \times h$$
$$\Rightarrow h = \frac{900}{20}$$
$$= 45 \text{ m}$$

Hence, the total path length is given by

$$AC = 45 \text{ m} + 80 \text{ m}$$
$$= 125 \text{ m}$$

Q.45. There exists a uniform electric field of $20\ \hat{\imath}\ \text{N C}^{-1}$. A dipole of dipole moment $15\ \text{C m}$ is placed at an angle 30° with the electric field. Torque acting on the dipole is

A) 100 N m

B) 150 N m

C) 200 N m

D) 250 N m

Answer: 150 N m

Solution: Given,

The dipole moment $p = 15\ \text{C m}$

Magnitude of the field strength $E = 20 \text{ N C}^{-1}$

Angle between the electric field and the dipole moment $\theta = 30^\circ$

Torque on the dipole,

$$\tau = pE \sin\theta = 15 \times 20 \times \sin 30^\circ$$

$$= 15 \times 20 \times \frac{1}{2}$$

$\therefore$Torque on the dipole$= 150 \text{ N m}$

Q.46. During an adiabatic process, the pressure of gas is proportional to cube of its absolute temperature. The ratio of $\frac{C_p}{C_v}$ is $x : 2$. Write the value of x.

Answer: 3

Solution: In an adiabatic process, $T^\gamma = (\text{constant}) P^{\gamma - 1}$

or $T^{\frac{\gamma}{\gamma-1}} = (\text{constant})\ P$

Given $T^3 = (\text{constant})\ P$

$$\therefore \frac{\gamma}{\gamma - 1} = 3 \Rightarrow \gamma = 3\gamma - 3$$

or $2\gamma = 3 \Rightarrow \gamma = \frac{C_p}{C_v} = \frac{3}{2}$

Therefore, $x = 3$.

Q.47. A uniform ring and a uniform solid sphere of equal mass and equal radius undergoes pure rolling on the same inclined plane and covers equal distances. If the ratio of their translational kinetic energy is $\frac{7}{x}$, then x is

Answer: 10

Solution: Let along the incline plane, acceleration of given object be a. Then,

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(u^2 + 2as\right) = as$$

Now, for pure rolling

$$a = \frac{g\sin\theta}{1+\frac{k^2}{r^2}}, \text{ where } k = \text{radius of gyration.}$$

Therefore, required ratio

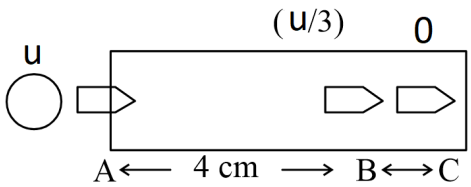
$$= \frac{\frac{g\sin\theta}{1+\frac{\left(k_{ring}\right)^2}{r^2}}}{\frac{g\sin\theta}{1+\frac{\left(k_{sphere}\right)^2}{r^2}}} = \frac{r^2+\left(k_{sphere}\right)^2}{r^2+\left(k_{ring}\right)^2} = \frac{r^2+\frac{2}{5}r^2}{r^2+r^2} = \frac{7}{10}$$

Hence, $x = 10$.

Q.48. A bullet is fired into a wooden plank. Its velocity becomes $\left(\frac{1}{3}\right)^{rd}$ when it penetrates $4\ \text{cm}$. It should penetrate a further distance of $x \times 10^{-1}\ \text{cm}$ for its velocity to become zero. Find the value of x.

Answer: 5

Solution: Let's consider the following diagram:



The formula to calculate the deceleration of the bullet inside the wooden plank can be written as

$$v^2 = u^2 - 2al \quad \ldots(1)$$

From the given information, equation (1) follows to

$$\left(\frac{u}{3}\right)^2 = u^2 - 2a \times 4$$

$$\Rightarrow 8a = \frac{8u^2}{9}$$

$$\Rightarrow a = \frac{u^2}{9}$$

If S is the required distance within the plank, it can be calculated as follows:

$$0 = \left(\frac{u}{3}\right)^2 - 2aS$$

$$\Rightarrow S = \frac{\frac{u^2}{9}}{2\times\frac{u^2}{9}}$$

$$= 0.5\ \text{cm}$$

$$= 5 \times 10^{-1}\ \text{cm}$$

Hence, $x = 5$.

Q.49. Current of $200\ \mathrm{A}$ deflects the coil of a moving coil galvanometer by 60°. Find the current(in A) that causes a deflection of $\frac{\pi}{10}\ \mathrm{rad}$.

Answer: 60

Solution: The deflection (θ) of the moving coil galvanometer is related to the current (i) as given by

$\theta \propto i$

If $i_1,\ i_2$ are the currents corresponding to deflections $\theta_1,\ \theta_2$ respectively, then, from equation (1), it follows that

$$\frac{\theta_1}{\theta_2} = \frac{i_1}{i_2} \quad \ldots(2)$$

From equation (2), it follows that

$$\frac{60^{\circ} \times \frac{\pi\ \mathrm{rad}}{180^{\circ}}}{\frac{\pi}{10}\ \mathrm{rad}} = \frac{200\ \mathrm{A}}{i_2}$$

$$\Rightarrow i_2 = \frac{200\ \mathrm{A}}{\frac{60^{\circ} \times \frac{\pi\ \mathrm{rad}}{180^{\circ}}}{\frac{\pi}{10}\ \mathrm{rad}}}$$

$$= 60\ \mathrm{A}$$