JEE Main

27th Jan Shift 1

Questions

Q.1. What is the valence shell electronic configuration of Neodymium?

A) $[Xe]4f^4 6s^2$ B) $[Xe]4f^4 6s^1$

C) $[Xe]4f^4 6s^0$ D) None of the above

Answer: $[Xe]4f^4 6s^2$

Solution: Neodymium(Nd) and its atomic number is 60. It has valence shell electronic configuration as $\left[Xe\right]\ 4f^4\ 6s^2$. It is the fourth member of the lanthanide series and is considered to be one of the rare-earth metals.

Q.2. Which of the following is Polar solvent in nature?

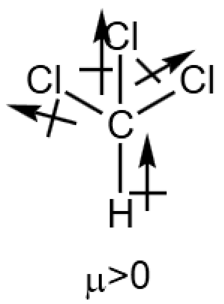
A) CCl_4 B) $CHCl_3$

C) $CH_2=CH_2$ D) CO_2

Answer: $CHCl_3$

Solution: The solvents having an appreciable net dipole moment are known as polar solvents.

$CHCl_3$ is polar due the presence of a net dipole moment. There is a difference in electronegativity of the central atom, carbon, hydrogen, and chlorine. This will create a dipole in molecule. As a dipole has formed in chloroform molecule, due to electronegativity difference, which makes it polar in nature.



Hence,option B is the answer.

Q.3. Which of the following can not show a variable oxidation state?

A) Fluorine B) Chlorine

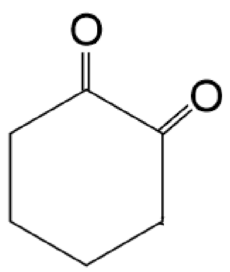
C) Iodine D) Bromine

Answer: Fluorine

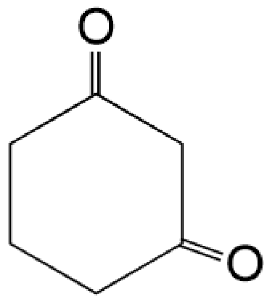
Solution: Because of the absence of d-orbitals, it is not possible for Fluorine to show variable oxidation state. Fluorine does not show variable oxidation states because it is the most electronegative element and has seven valence electrons. Its high electronegativity and strong attraction for electrons prevent it from losing electrons to exhibit different oxidation states. Fluorine always show −1 oxidation state in compounds.

Q.4. Which of the following has the highest enol content?

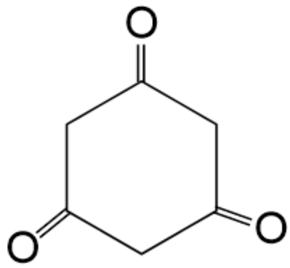
A)



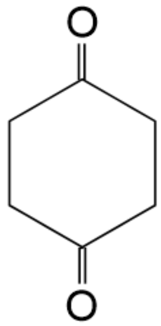
B)



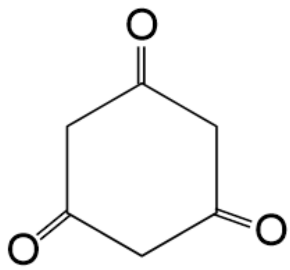
C)



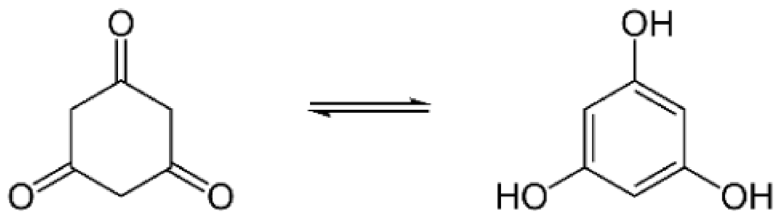
D)

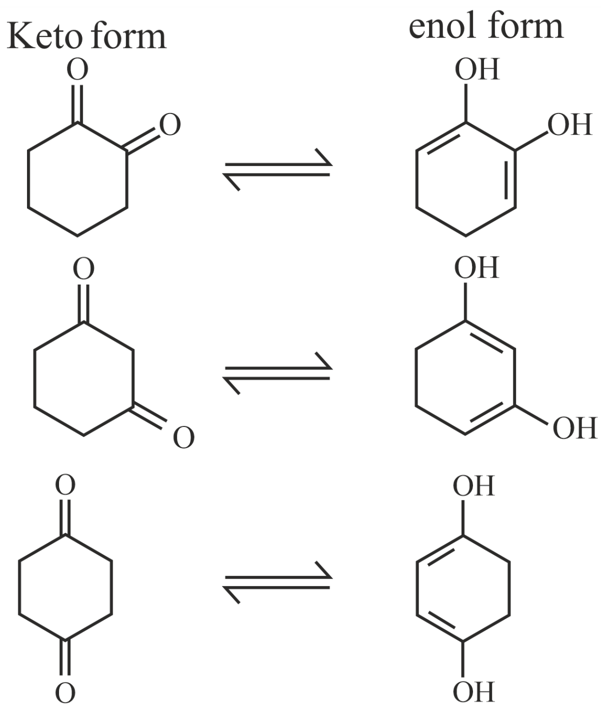


Answer:

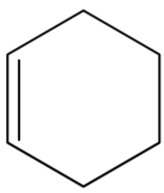


Solution: The compound which forms aromatic compound after enolisation forms more stable enol. Among the given molecules, the following is the more stable enol formation. The enol is phloroglucinol.





Q.5. The compound given below is:



A) Aromatic

B) Alicyclic

C) Acyclic

D) Antiaromatic

Answer: Alicyclic

Solution: This compound does not follow $(4n+2)\pi$ electron rule, Huckel rule. Also $4n\ \pi$ not following here.

Hence, it cannot be aromatic or antiaromatic.

It is a cyclic compound, so it can not be acyclic.

Alicyclic compounds are organic compounds that are both aliphatic and cyclic. These are the saturated or unsaturated hydrocarbons containing non-aromatic rings of carbon atoms.

Hence the answer is option B.

Q.6. Which of the following has the highest magnetic moment?

A) $3d^7$ B) $3d^8$

C) $3d^4$ D) $3d^3$

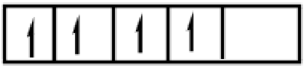
Answer: $3d^4$

Solution: The magnetic moment is given by

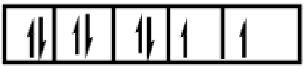
$\sqrt{n(n+2)}$

where, $n=$ number of unpaired electrons

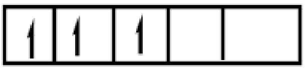
In $3d^4$ the number of unpaired electrons is 4



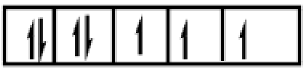
In$3d^8$ the number of unpaired electrons is 2



In $3d^3$ the number of unpaired electrons is 3



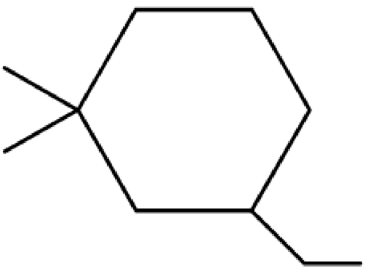
In $3d^7$ the number of unpaired electrons is 3



From the formula of magnetic moment, we can say that higher the number of unpaired electrons (n), more will be the amount of magnetic moment.

Hence option C is the answer.

Q.7. The IUPAC name of the following compound is

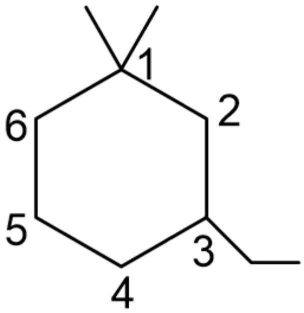


A) 3-ethyl-1,1-dimethylcyclohexane B) 1-ethyl-3,3-dimethylcyclohexane

C) 1-ethyl-3,3-dimethylcyclohexene D) 3-ethyl-1,1-dimethylcyclohexene

Answer: 3-ethyl-1,1-dimethylcyclohexane

Solution:



The parent chain in the above compound is cyclohexane. At position-1, two methyl groups are attached and at position-3 one ethyl group is attached.

Hence, the IUPAC name of the given compound is 3-ethyl-1,1-dimethylcyclohexane.

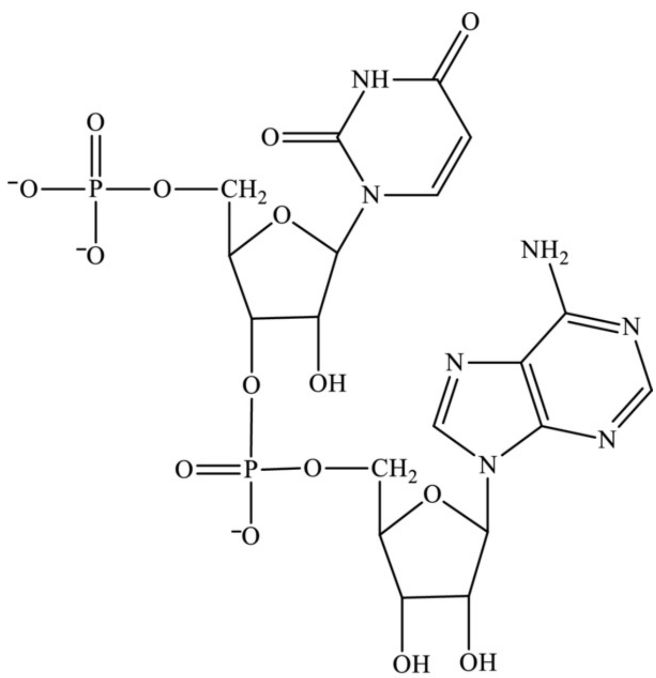
Q.8. Nucleotide pairs are joined by:

A) Peptide linkage B) Glycosidic linkage

C) Water linkage D) Phosphodiester linkage

Answer: Phosphodiester linkage

Solution: The phosphate group of the carbon of one nucleotide and the carbon of another nucleotide typically create a covalent interaction known as a phosphodiester bond.



Dinucleotide

Answer is D.

Q.9. Which of the following has $+4$ oxidation state?

A) $H_2S_2O_7$ B) H_2SO_4

C) H_2SO_3 D) HSO_4^-

Answer: H_2SO_3

Solution: The sum of the oxidation states of all atoms in a species is equal to charge present on that species.

Among the given options, $H_2S_2O_7 (2 + 2x - 14 = 0)$ in this Sulphur exhibits the oxidation state $+6$.

In H_2SO_4 $(2 + x - 8 = 0)$,Sulphur exhibits the oxidation state $+6$

In HSO_4^- $(1 + x - 8 = -1)$Sulphur exhibits the oxidation state $+6$.

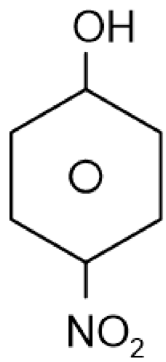
In H_2SO_3 $(2 + x - 6 = 0)$, Sulphur exhibits the oxidation state $+4$.

Hence, option C is the answer.

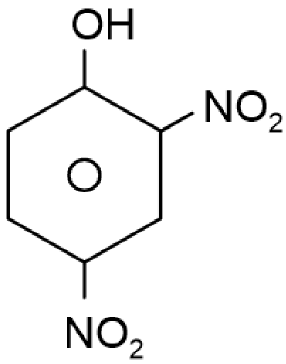
Q.10. Which of the following is most acidic?

A) $Bu - OH$

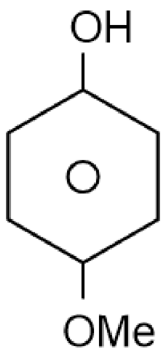
B)



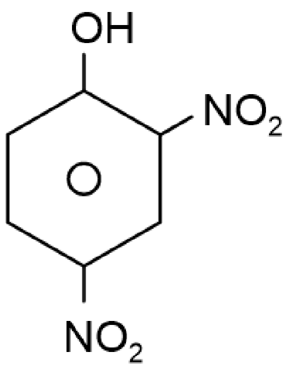
C)



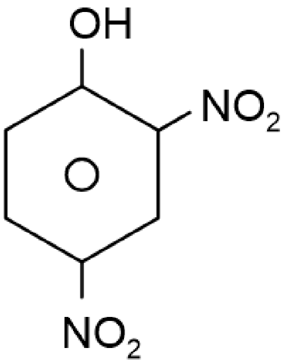
D)



Answer:



Solution:



NO_2 group at ortho and para position withdraws electrons of the O−H bond towards itself by the stronger −R effect.

The molecule with two nitro groups at ortho and para is more acidic than para nitrophenol.

Methoxy group is +R effect group, hence it is less acidic than nitro phenols.

Hence, option C is the answer.

Q.11. Ethanol gives turbidity with Lucas reagent

A) Immidiately

B) After 5 min

C) After 10 min

D) Upon heating

Answer: Upon heating

Solution: Lucas test is used to differentiate and categorise primary, secondary and tertiary alcohols using a solution of anhydrous zinc chloride in concentrated hydrochloric acid. This solution is commonly referred to as the Lucas reagent. Tertiary alcohol responds to the Lucas test by forming turbidity immediately, secondary alcohols form turbidity after 5-10 min and primary alcohols form turbidity after heating only.

Q.12. The radius of the electron in the 3rd orbit of Bohr is R. What is the radius of the 4th orbit in terms of R?

A) $\frac{16}{9}R$

B) $\frac{3}{4}R$

C) $\frac{9}{16}R$

D) None of the above

Answer: $\frac{16}{9}R$

Solution: Radius of n^{th} orbit is given by

$$R_n = a_0\frac{n^2}{Z}$$

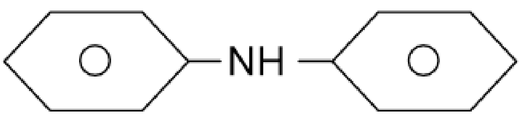
$$\Rightarrow R = a_0(3)^2 = a_0 \times 9$$

$$\Rightarrow R_4 = a_0(4)^2 = a_0 \times 16$$

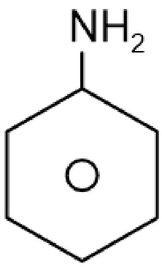
The radius of the 4th orbit in terms of R is $\frac{16}{9}R$

Q.13. Select the strongest Bronsted Base?

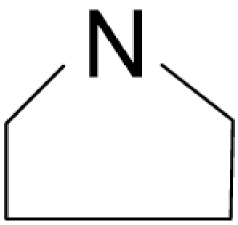
A)



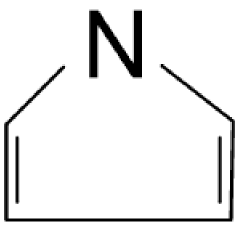
B)



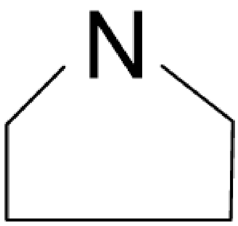
C)



D)



Answer:



Solution: The compound in which lone piar on the nitrogen is readily available for proton is strongest base.

In option A, B and D the lone pair present in the nitrogen is involved in the resonance.

But in option C because of the +I effect the ring system is enhancing the electron density and there is no delocalisation of electrons.

Hence option C is the answer.

Q.14. Assertion: Boron has high melting point (2437K)

Reason : Solid boron has strong crystalline lattice.

A) Assertion is true, Reason is false.

B) Assertion is false, reason is true.

C) Assertion is true and Reason is the correct explanation of Assertion.

D) Assertion is true and Reason is the not the correct explanation of Assertion.

Answer: Assertion is true and Reason is the correct explanation of Assertion.

Solution: Boron has very high melting point because of its small atomic size and very strong crystalline lattice.
It forms strong covalent bonds with the neighbouring atoms. Thus boron atom are closely packed in its solid state, so a large amount of heat is needed to break the bonds between atoms.

Hence option C is the answer.

Q.15. What is the sum of the bond order of CO and NO^{+}?

Answer: 6

Solution: Bond order is the number of chemical bonds between a pair of atoms and indicates the stability of a bond.

Bond Order $= \frac{\text{Number of bonding electrons} - \text{number of antibonding electrons}}{2}$

The given two molecules have the same number of electrons as notrogen molecule.

Hence,

Bond order of $CO = 3$

Bond order of $NO^{+} = 3$

The sum of the bond order of CO and NO^{+}
$= 3 + 3$
$= 6$

Q.16. How many electrons will have $n = 4,\ s = +\frac{1}{2}$?

Answer: 16

Solution: $n = 4,\ s = +\frac{1}{2}$

$\text{Total orbitals} = n^2$
$= 4^2$
$= 16$

Each orbital can accommodate two electrons. Hence, the number of electrons with $s = +\frac{1}{2}$ will be 16 electrons.

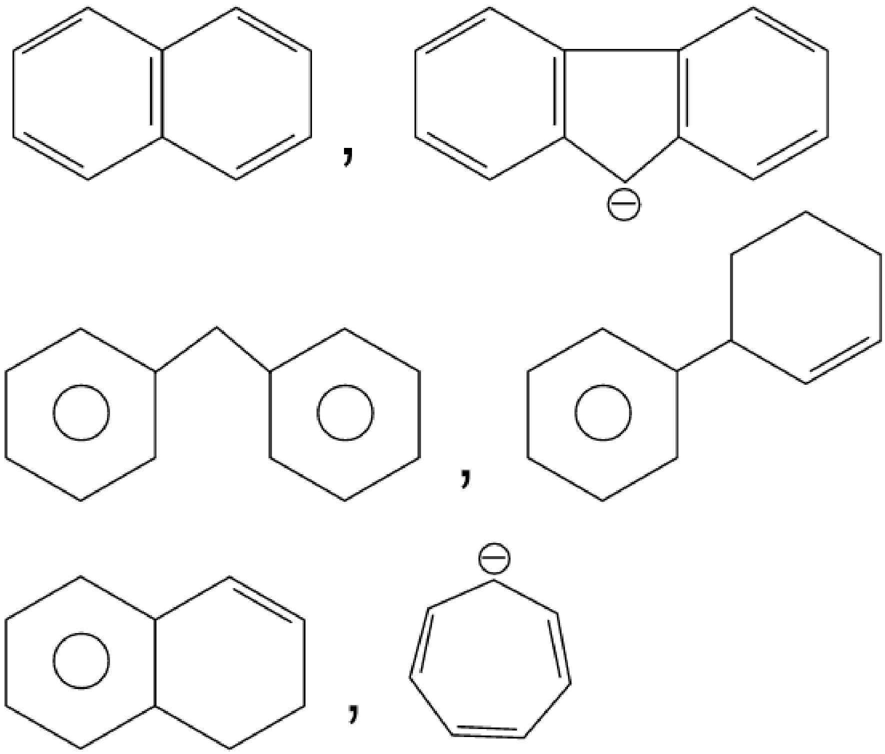
Q.17. How many grams of methane are required to produce 22 g CO_2 by combustion?

Answer: 8

Solution: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Moles of $CO_2 = \frac{22}{44} = 0.5\ \text{mol}$
1 mol of CO_2 is produced from 1 mol of CH_4.
So,0.5 mol of CO_2 will be produced from 0.5 mol of CH_4.
Mass of CH_4 formed $= \text{Moles} \times \text{molar mass}$
Mass of CH_4 formed
$= 0.5\ \text{mol} \times 16\ {}^{g}/_{mol}$
$= 8\ \text{gm}$

Q.18. How many of the following are aromatic compounds?

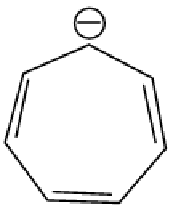


Answer: 5

Solution: Aromatic compounds are chemical compounds that consist of conjugated planar ring systems accompanied by delocalised pi-electron clouds in place of individual alternating double and single bonds.

Aromatic compounds should follow, Huckel's rule. i.e having $4n + 2\ \pi$ electrons.

From the above mentioned compounds, only tropylium anion is not an aromatic in compound due to the presence of 8 electrons which is not following $(4n + 2)\pi$ rule.



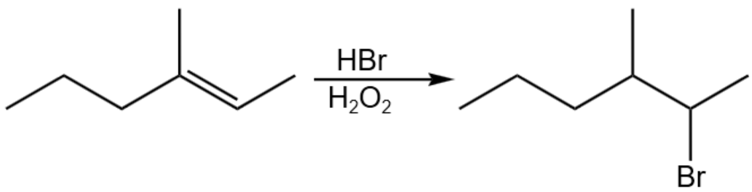
Hence the number of Aromatic compounds is 5.

Q.19. How many stereoisomers are possible for the product P.

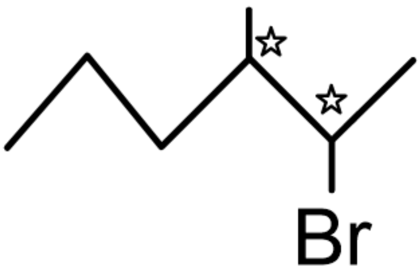
$$3 - \text{methylhex} - 2 - \text{ene} \xrightarrow[H_2O_2]{HBr} P$$

Answer: 4

Solution:



The above product is formed by Kharasch effect. Kharasch effect is the addition of HBr to unsymmetrical alkenes in the presence of Peroxide. It gives a product contrary to what Markovnikov addition would give.



2 chiral carbons, here, so, total number of stereoisomers will be $2^{\mathrm{n}} = 2^2 = 4$

Q.20. If ${}^{n-1}C_r = \left(k^2 - 8\right){}^nC_{r+1}$, find the range of values of k

A) $\left(-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

B) $\left[-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

C) $\left[-3, -2\sqrt{2}\right]$

D) $\left[2\sqrt{2}, 3\right]$

Answer: $\left[-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

Solution: We know that,

$(r+1) \cdot {}^nC_{r+1} = n \cdot {}^{n-1}C_r$

$\Rightarrow \frac{{}^{n-1}C_r}{{}^nC_{r+1}} = \frac{r+1}{n} = k^2 - 8 \quad \ldots(i)$

Also, $n \geq r+1$

$\Rightarrow \frac{r+1}{n} \leq 1 \quad \ldots(ii)$

Using (i) and (ii)

$\Rightarrow 0 < k^2 - 8 \leq 1$

$\Rightarrow 8 < k^2 \leq 9$

$\Rightarrow k \in \left[-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$

Q.21. If $\cos 2x - a \sin x = 2a - 7$, then find the range of a

A) $a \in [-2, 6]$

B) $a \in [2, 6]$

C) $a \in [-6, 2]$

D) $a \in [6, -2]$

Answer: $a \in [2, 6]$

Solution: Given,

$\cos 2x - a\sin x = 2a - 7$

$\Rightarrow 1 - 2\sin^2 x - a\sin x = 2a - 7$

$\Rightarrow 2\sin^2 x + a\sin x + 2a - 8 = 0$

$\Rightarrow \sin x = \frac{-a \pm \sqrt{a^2 - 8(2a-8)}}{4}$

$\Rightarrow \sin x = \frac{-a \pm (a-8)}{4}$

$\Rightarrow \sin x = \frac{-2a+8}{4}\ \{\text{as } \sin x \neq -2\}$

Now we know that,

$-1 \leq \sin x \leq 1$

$\Rightarrow -1 \leq \frac{-a+4}{2} \leq 1$

$\Rightarrow 2 \leq a \leq 6$

Q.22. If $3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \ldots$ to $\infty = 8$, then p equals -

A) 1

B) 5

C) 7

D) 9

Answer: 9

Solution:

Given that $3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \ldots$ to $\infty = 8$

Let $S = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \ldots$ to ∞

$= \left(3 + \frac{3}{4} + \frac{3}{4^2} + \ldots\ldots\right) + \left(\frac{p}{4} + \frac{2p}{4^2} + \ldots\right)$

Let $S_1 = 3 + \frac{3}{4} + \frac{3}{4^2} + \ldots = 3\left(1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\right)$

$= 3\left(\frac{1}{1 - \frac{1}{4}}\right) = 4$

And $S_2 = \frac{p}{4} + \frac{2p}{4^2} + \frac{3p}{4^3} + \ldots$

$\Rightarrow \frac{S_2}{4} = \frac{p}{4^2} + \frac{2p}{4^3} + \frac{3p}{4^4} + \ldots$

$\Rightarrow S_2 - \frac{S_2}{4} = \frac{p}{4} + \frac{p}{4^2} + \frac{p}{4^3} + \ldots\ldots$

$\Rightarrow \frac{3S_2}{4} = p\left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots\ldots\right) = p\left(\frac{1/4}{1-1/4}\right) = \frac{p}{3}$

$\Rightarrow S_2 = \frac{4p}{9}$

So, $S = S_1 + S_2 = 8$

$\Rightarrow 4 + \frac{4p}{9} = 8$

$\Rightarrow \frac{4p}{9} = 4 \ \Rightarrow p = 9$.

Q.23. The vertices of a triangle $\triangle ABC$ are $A(1,2),\ B(-3,4)$ and $C(5,8)$ then orthocentre of $\triangle ABC$ is

A) $\Rightarrow\left(-\frac{3}{2},1\right)$

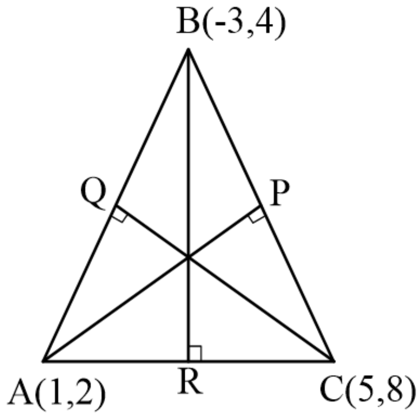
B) $\Rightarrow\left(1,\frac{3}{2}\right)$

C) $\left(\frac{3}{2},1\right)$

D) $\Rightarrow\left(\frac{3}{2},-1\right)$

Answer: $\left(\frac{3}{2},1\right)$

Solution: Given,



$AP:\ (y-2)=\frac{-1}{\frac{8-4}{5+3}}(x-1)$

$\Rightarrow AP:\ (y-2)=-2(x-1)=-2x+2$

$\Rightarrow AP:\ 2x+y=4$

$CQ:\ (y-8)=\frac{-1}{\frac{2-4}{1+3}}(x-5)$

$\Rightarrow CQ:\ (y-8)=2(x-5)=2x-10$

$\Rightarrow CQ:\ 2x-y=2$

Now the orthocentre is given by intersection of the line AP & CQ

$\Rightarrow 4-2x=2x-2$

$\Rightarrow 6=4x$

$\Rightarrow x=\frac{3}{2}$

$\Rightarrow y=2\times\frac{3}{2}-2=1$

$\Rightarrow\left(\frac{3}{2},1\right)$ is orthocentre.

Q.24. The value of k for $(2k,3k),\ (0,0),\ (1,0)$ and $(0,1)$ to be on the circle is

A) $\frac{7}{13}$

B) $\frac{5}{12}$

C) $\frac{5}{13}$

D) $\frac{7}{12}$

Answer: $\frac{5}{13}$

Solution: Equation of circle passing through $(0,0),\ (0,1),\ (1,0)$ is

$(x-0)(x-1)+(y-0)(y-1)=0$

$\Rightarrow x^2+y^2-x-y=0$

Point $(2k,3k)$ lies on a circle. So,

$4k^2+9k^2-2k-3k=0.$

$\Rightarrow k=\frac{5}{13}$

Q.25. Find the length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ bisected at the point $\left(\frac{1}{2},\ \frac{2}{5}\right)$.

A) $\frac{7\sqrt{41}}{5}$

B) $\frac{\sqrt{41}}{5}$

C) $\frac{7}{5}$

D) $7\sqrt{41}$

Answer: $\frac{7\sqrt{41}}{5}$

Solution: Given equation of ellipse is

$\frac{x^2}{25}+\frac{y^2}{16}=1 \ldots .(1)$

We know that the equation of chord of an ellipse where mid-point is (x_1, y_1) is given by

$T=S_1$

$\Rightarrow \frac{x x_1}{25}+\frac{y y_1}{16}-1=\frac{x_1^2}{25}+\frac{y_1^2}{16}-1$

Since $(x_1, y_1)=\left(\frac{1}{2}, \frac{2}{5}\right)$

$\therefore$ We have $=\frac{x\left(\frac{1}{2}\right)}{25}+\frac{y\left(\frac{2}{5}\right)}{16}=\frac{\left(\frac{1}{2}\right)^2}{25}+\frac{\left(\frac{2}{5}\right)^2}{16}$

$\Rightarrow \frac{x}{50}+\frac{y}{40}=\frac{1}{100}+\frac{1}{100}$

$\Rightarrow \frac{x}{50}+\frac{y}{40}=\frac{2}{100}$

$\Rightarrow \frac{x}{5}+\frac{y}{4}=\frac{1}{5}$ or $4x+5y=4 \ldots .(2)$

From equation $(1) \frac{x^2}{25}+\frac{y^2}{16}=1$

$\Rightarrow 16x^2+25y^2=400$

$\Rightarrow 16x^2+(4-4x)^2=400$ $[\text{From } (2), 5y=4-4x]$

$\Rightarrow 16x^2+16+16x^2-32x=400$

$\Rightarrow 32x^2-32x-384=0$

$\Rightarrow x^2-x-12=0$

$\Rightarrow (x-4)(x+3)=0$

$\Rightarrow x=4,-3$

If $x=4,\ 5y=4-4 \cdot 4$

$\Rightarrow y=-\frac{12}{5}$

And if $x=-3,\ 5y=4-4(-3)$

$\Rightarrow y=\frac{16}{5}$

$\therefore$ The points of intersection of chord and ellipse are

$\left(4, \frac{-12}{5}\right)$ & $\left(-3, \frac{16}{5}\right)$

Length of chord $=\sqrt{(-3-4)^2+\left(\frac{16}{5}+\frac{12}{5}\right)^2}$

$=\sqrt{(-7)^2+\left(\frac{28}{5}\right)^2}$

$=\sqrt{49+\frac{784}{25}}=\sqrt{\frac{1225+784}{25}}=\sqrt{\frac{2009}{25}}=\frac{7\sqrt{41}}{5}$

Q.26. If $f(x)-f(y)=\log\left(\frac{x}{y}\right)+x-y$, then find $\sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)$.

A) 2390

B) 1245

C) 2890

D) None of these

Answer: 2890

Solution: $f(x)-f(y)=\log\left(\frac{x}{y}\right)+x-y$

$\Rightarrow f(x)-f(y)=\log x-\log y+x-y$

$\Rightarrow f(x)-\log x-x=f(y)-\log y-y=C(\text{let})$, where C is any arbitrary constant

$\Rightarrow f(x)=C+\log x+x$

$\Rightarrow f'(x)=\frac{1}{x}+1$

$\Rightarrow f'\left(\frac{1}{k^2}\right)=\frac{1}{\frac{1}{k^2}}+1=k^2+1$

$\Rightarrow \sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)=\left(1^2+2^2+3^2+\ldots+20^2\right)+[1+1++\ldots+1\,(20\text{ times})]$

$\Rightarrow \sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)=\frac{20\times21\times41}{6}+20=10\times7\times41+20$

$\Rightarrow \sum_{k=1}^{20} f'\left(\frac{1}{k^2}\right)=2890$

Q.27. Shortest distance between the parabola $y^2=4x$ and $x^2+y^2-4x-16y+64=0$ is

A) $2\sqrt{3}-2$

B) $3\sqrt{2}-3$

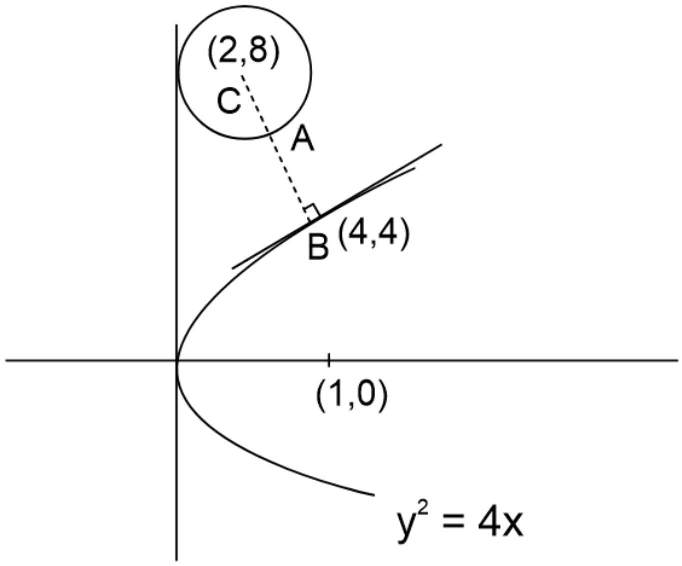
C) $2\sqrt{5}-2$

D) $4\sqrt{5}-3$

Answer: $2\sqrt{5}-2$

Solution: We know that,

Shortest distance between $y^2=4x$ & $x^2+y^2-4x-16y+64=0$ is given by common normal,



So, the equation of normal of parabola $y^2=4ax$ is given by

$y=mx-2am-am^3$

$\Rightarrow y=mx-2m-m^3\ \left\{\text{for } y^2=4x,\ a=1\right\}$

Now the normal will pass through the centre of the circle $x^2+y^2-4x-16y+64=0$ which is $(2,8)$

So, $8=2m-2m-m^3$

So, on solving we get $m=-2$

Now the equation of normal will be $y+2x=12$

And the point of contact of normal and parabola is given by intersection $y+2x=12$ & $y^2=4x$ which is $B(4,4)$

Now the shortest distance will be $=AB=BC-r$

$=\sqrt{(2-4)^2+(8-4)^2}-2=2\sqrt{5}-2$

Q.28. If the sum of the coefficients in the expansion of $\left(1-3x+10x^2\right)^n$ is a and sum of coefficients in the expansion of $\left(1+x^2\right)^n$ is b, then

A) $a=3b$

B) $a=b^3$

C) $b=a^3$

D) none of these

Answer: $a=b^3$

Solution: We have $a=$sum of the coefficients in the expansion of

$\left(1-3x+10x^2\right)^n=\left(1-3+10\right)^n=\left(8\right)^n=\left(2\right)^{3n}$

(putting $x=1$)

Now, $b=$sum of the coefficients in the expansion of

$\left(1+x^2\right)^n=\left(1+1\right)^n=2^n$

Clearly, $a=b^3$

Q.29. If $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$, then find $f'(10)$.

A) 204

B) 200

C) 202

D) 198

Answer: 202

Solution: Let, $f(x) = x^3 + ax^2 + bx + c$

$\Rightarrow f'(x) = 3x^2 + 2ax + b$

$\Rightarrow f'(1) = 3 + 2a + b$

$\Rightarrow 3 + 2a + b = a$

$\Rightarrow a + b = -3 \quad \ldots(i)$

$\Rightarrow f''(x) = 6x + 2a$

$\Rightarrow f''(2) = 12 + 2a$

$\Rightarrow 12 + 2a = b$

$\Rightarrow -3 - a = 12 + 2a$

$\Rightarrow 3a = -15$

$\Rightarrow a = -5,\ b = 2$

$\Rightarrow f'''(x) = 6 = c$

$\Rightarrow f'(10) = 3(10)^2 + 2(-5)(10) + 2 = 300 - 100 + 2$

$\Rightarrow f'(10) = 202$

Q.30. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k},\ \vec{b} = 3\left(\hat{i} - \hat{j} + \hat{k}\right),\ \vec{a} \cdot \vec{c} = 3$ and $\vec{a} \times \vec{c} = \vec{b}$, then $\vec{a} \cdot \left(\left(\vec{c} \times \vec{b}\right) - \vec{b} - \vec{c}\right) =$

Answer: 24

Solution: Given,

$\vec{a} = \hat{i} + 2\hat{j} + \hat{k},\ \vec{b} = 3\left(\hat{i} - \hat{j} + \hat{k}\right),\ \vec{a} \cdot \vec{c} = 3$ and $\vec{a} \times \vec{c} = \vec{b}$

Now solving,

$\vec{a} \times \vec{c} = \vec{b}$

$\Rightarrow \left(\vec{a} \times \vec{c}\right) \cdot \vec{b} = \left|\vec{b}\right|^2$

$\Rightarrow \left(\vec{a} \times \vec{c}\right) \cdot \vec{b} = \left(3\sqrt{1^2 + 1^2 + 1^2}\right)^2$

$\Rightarrow \left(\vec{a} \times \vec{c}\right) \cdot \vec{b} = 27$

$\Rightarrow \vec{a} \cdot \left(\vec{c} \times \vec{b}\right) = 27 \left\{\text{as } \vec{a} \cdot \left(\vec{c} \times \vec{b}\right) = \vec{b} \cdot \left(\vec{a} \times \vec{c}\right)\right\}$

And $\vec{a} \cdot \vec{b} = 3 - 6 + 3 = 0$

So, $\vec{a} \cdot \left(\left(\vec{c} \times \vec{b}\right) - \vec{b} - \vec{c}\right)$

$= \vec{a} \cdot \left(\vec{c} \times \vec{b}\right) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$

$= 27 - 0 - 3 = 24$

Q.31. If $S_1 = 3, 9, 15, \ldots\ldots 25 \text{ terms}$ and $S_2 = 3, 8, 13, \ldots\ldots 37 \text{ terms}$, then the number of common terms in S_1, S_2 is equal to _____

Answer: 5

Solution: Given,

$S_1 = 3, 9, 15, \ldots\ldots 25 \text{ terms}$ and $S_2 = 3, 8, 13, \ldots\ldots 37 \text{ terms}$

So, $a_1 = 3,\ d_1 = 6,\ a_2 = 3,\ d_2 = 5$

Now last term is given by,

$(a_n)_1 = 3 + 24 \times 6 = 147,\ (a_n)_2 = 3 + 36 \times 5 = 183$

$\text{LCM}\,(6, 5) = 30$

So, common terms will be $3, 33, 63, \ldots, a_n$

$\Rightarrow a_n \leq 147$

$\Rightarrow 3 + (n-1)30 \leq 147$

$\Rightarrow 30n \leq 147 + 27$

$\Rightarrow n \leq 5.8$

$\Rightarrow n = 5$

Q.32. $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx = a + b\sqrt{2} + c\sqrt{3}$, then $2a - 3b - 4c$ is equal to ______

Answer: 12

Solution: $I = \int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx$

$\Rightarrow I = \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{2} dx$

$\Rightarrow I = \frac{1}{3}\left[(3+x)^{\frac{3}{2}} - (1+x)^{\frac{3}{2}}\right]_0^1$

$\Rightarrow I = \frac{1}{3}\left\{\left[(3+1)^{\frac{3}{2}} - (1+1)^{\frac{3}{2}}\right]_0^1 - \left[(3)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]_0^1\right\}$

$\Rightarrow I = \frac{1}{3}\left(8 - 2^{\frac{3}{2}} - (3)^{\frac{3}{2}} + 1\right)$

$\Rightarrow I = 3 - \frac{2\sqrt{2}}{3} - \sqrt{3}$

$\Rightarrow 2a - 3b - 4c = 6 + 2 + 4 = 12$

Q.33. If $1 + x + x^2 = 0$ and $(1+x)^7 = a + bx + cx^2$, then find the value of $5\,(3a - 2b - c)$.

Answer: 5

Solution: Given,

$1 + x + x^2 = 0$

$\Rightarrow x = \omega, \omega^2$

Now solving,

$(1+x)^7 = a + bx + cx^2$

Now taking, $x = \omega$ we get,

$\Rightarrow (1+\omega)^7 = a + b\omega + c\omega^2$

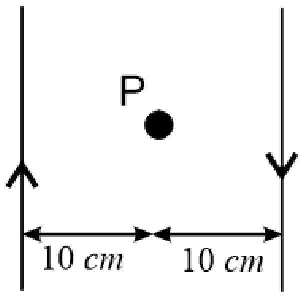
$\Rightarrow -\omega^2 = a + b\omega + c\omega^2$

Now, on comparing both side we get,

$\Rightarrow a = 0,\ b = 0,\ c = -1$

Hence, the value of $5\,(3a - 2b - c) = 5\,(0 - 0 + 1) = 5$

Q.34. Two infinite current carrying wires having current I in opposite directions are shown below. Find the magnitude of magnetic field in SI unit at point P.



A) $\frac{7\mu_0 I}{\pi}$

B) $\frac{\mu_0 I}{\pi}$

C) $\frac{10\mu_0 I}{\pi}$

D) $\frac{5\mu_0 I}{\pi}$

Answer: $\frac{10\mu_0 I}{\pi}$

Solution: The magnetic field at P due to the left wire is given by

$$B_l = \frac{\mu_0 I}{2\pi \times \left(10\ \mathrm{cm} \times \frac{1\ \mathrm{m}}{100\ \mathrm{cm}}\right)} \otimes$$
$$= \frac{5\mu_0 I}{\pi} \otimes$$

Similarly, the magnetic field at P due to the right wire is given by

$$B_r = \frac{\mu_0 I}{2\pi \times \left(10\ \mathrm{cm} \times \frac{1\ \mathrm{m}}{100\ \mathrm{cm}}\right)} \otimes$$
$$= \frac{5\mu_0 I}{\pi} \otimes$$

Hence, the net electric field at P can be written as

$$B = B_l + B_r$$
$$= \frac{5\mu_0 I}{\pi} \otimes + \frac{5\mu_0 I}{\pi} \otimes$$
$$= \frac{10\mu_0 I}{\pi} \otimes$$

Hence, the magnitude of the net magnetic field is $\frac{10\mu_0 I}{\pi}$.

Q.35. If the diameter of earth becomes half(mass remains unchanged), then the acceleration due to gravity at surface of earth becomes

A) twice

B) four times

C) three times

D) half

Answer: four times

Solution: Acceleration due to gravity is given by, $g = \frac{GM}{R^2}$.

As mass is constant, therefore

$$\frac{g'}{g} = \frac{\frac{GM}{(R')^2}}{\frac{GM}{R^2}} = \left(\frac{R}{R'}\right)^2 = \left(\frac{R}{\frac{R}{2}}\right)^2 = 4$$

Hence, $g' = 4g$.

Q.36. Two masses $m_1 = 4\ \mathrm{g}$ and $m_2 = 25\ \mathrm{g}$ have the same kinetic energy. The ratio of their linear momentum is

A) $1:1$

B) $1:5$

C) $1:6$

D) $2:5$

Answer: $2:5$

Solution: The formula to calculate the kinetic energy (K_1) of the first particle is given by

$$K_1 = \frac{1}{2} m_1 {v_1}^2$$
$$= \frac{{p_1}^2}{2m_1} \quad \ldots(1)$$

The formula to calculate the kinetic energy (K_2) of the second particle is given by

$$K_2 = \frac{1}{2} m_2 {v_2}^2$$
$$= \frac{{p_2}^2}{2m_2} \quad \ldots(2)$$

Equate equations (1) and (2) and simplify to obtain the required ratio.

$$\frac{{p_1}^2}{2m_1} = \frac{{p_2}^2}{2m_2}$$
$$\Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$$
$$= \sqrt{\frac{4}{25}}$$
$$= \frac{2}{5}$$

Q.37. A charge $Q = 10^{-6}\ \mathrm{C}$ is placed at the origin. Find the potential difference between two points A and B whose position vectors are $\left(\sqrt{3}\hat{\imath} + \sqrt{3}\hat{\jmath}\right)\ \mathrm{m}$ and $\sqrt{6}\hat{\jmath}\ \mathrm{m}$ repectively.

A) $1000\ \mathrm{V}$

B) $2000\ \mathrm{V}$

C) Zero

D) $500\ \mathrm{V}$

Answer: Zero

Solution: The electric potential at A is given by

$$V_A = \frac{kQ}{r}$$
$$= \frac{k \times 10^{-6}}{\sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2}}$$
$$= \frac{10^{-6}k}{\sqrt{6}}$$

The electric potential at B is given by

$$V_B = \frac{kQ}{r}$$
$$= \frac{k \times 10^{-6}}{\sqrt{6}}$$
$$= \frac{10^{-6}k}{\sqrt{6}}$$

Hence, the potential difference between the points is given by

$$V_{AB} = V_A - V_B$$
$$= 0$$

Q.38. A body of mass $1000\ \mathrm{kg}$ is moving horizontally with velocity $6\ \mathrm{m\ s^{-1}}$. Another body of mass $200\ \mathrm{kg}$ is added gently. Find the new velocity.

A) $2\ \mathrm{m\ s^{-1}}$

B) $3\ \mathrm{m\ s^{-1}}$

C) $4\ \mathrm{m\ s^{-1}}$

D) $5\ \mathrm{m\ s^{-1}}$

Answer: 5 m s^{-1}

Solution: Initial momentum of the system $= m_1 v_1 = 1000 \times 6 = 6000 \text{ kg m s}^{-1}$

Final momentum of the system

$= (m_1 + m_2)v = 1200v$

Now, from conservation of linear momentum, we can write

$6000 \text{ kg m s}^{-1} = 1200v$
$\Rightarrow v = 5 \text{ m s}^{-1}$

Q.39. A rod of length l having resistance R is cut into two equal parts. These parts are connected in parallel, then new resistance will be

A) $\frac{R}{4}$

B) $\frac{R}{2}$

C) R

D) $2R$

Answer: $\frac{R}{4}$

Solution: Resistance of a uniform wire is given by, $R = \frac{\rho l}{A}$.

Clearly, $R \propto l$, therefore new rod parts will have resistance $\frac{R}{2}$.

For parallel connection, $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{\frac{R}{2} \times \frac{R}{2}}{\frac{R}{2} + \frac{R}{2}} = \frac{R}{4}$

Q.40. Statement 1: Linear momentum and moment of force have the same dimensions

Statement 2: Planck's constant and angular momentum have the same dimensions

A) Statement 1 is true while statement 2 is false

B) Statement 1 is false while statement 2 is true

C) Both statements are true

D) Both statements are false

Answer: Statement 1 is false while statement 2 is true

Solution: Linear momentum $= mv$. Hence, its dimensions would be $\left[\mathrm{M^1 L^1 T^{-1}}\right]$.

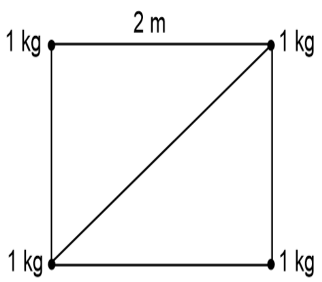
Moment of force $= Fd$. Hence, its dimensions would be $\left[\mathrm{M^1 L^2 T^{-2}}\right]$.

Planck's constant $= \frac{E}{\nu}$. Hence, its dimensions would be $\left[\mathrm{M^1 L^2 T^{-1}}\right]$.

Angular momentum $= rp$. Hence, its dimensions would be $\left[\mathrm{M^1 L^2 T^{-1}}\right]$

Clearly, Statement 1 is false while statement 2 is true.

Q.41. Consider the square design shown below. Find the moment of inertia of the object about the diagonal.



A) 2 kg m^2

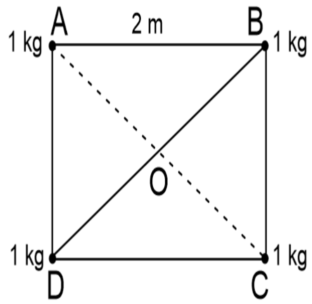
B) 1 kg m^2

C) 6 kg m^2

D) 4 kg m^2

Answer: 4 kg m^2

Solution: Let's consider the following diagram:



As both $1\ \mathrm{kg}$ masses at D and B, lie on the diagonal, their contribution to the net moment of inertia will be zero.

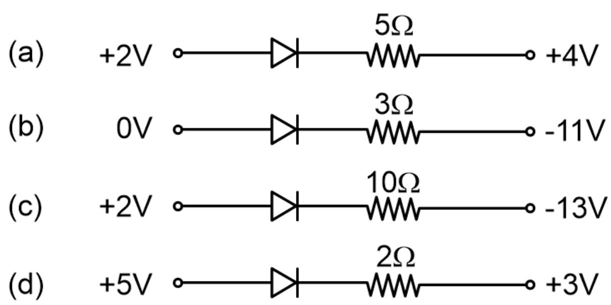
The distance (r) of the cross-diagonal masses from the centre O can be calculated as follows:

$$r = \frac{\sqrt{2^2+2^2}}{2}\ \mathrm{m}$$
$$= \sqrt{2}\ \mathrm{m}$$

Hence, the net moment of inertia of the object about the diagonal can be written as

$$I = \sum_i m_i {r_i}^2$$
$$= 1\ \mathrm{kg} \times \left(\sqrt{2}\ \mathrm{m}\right)^2 + 1\ \mathrm{kg} \times \left(\sqrt{2}\ \mathrm{m}\right)^2$$
$$= 4\ \mathrm{kg\ m^2}$$

Q.42. In which of the following circuits, the diode is in reverse bias?



A) (a) only
B) (b) only
C) (a) and (d) only
D) (d) only

Answer: (a) only

Solution: The reverse biasing of a PN-junction diode involves applying an external voltage in a way that the P-side is connected to the lower potential, and the N-side is connected to the higher potential.

Reverse biasing is a crucial operation in diodes because it allows the diode to block current flow in one direction and enables its use in rectification circuits and other electronic applications.

From the given figures, it is clear that only in configuration (a), the P-type is put at a lower potential than the N-type.

Hence, this is the correct option.

Q.43. A prism has a refractive index $\cot\left(\frac{A}{2}\right)$, where A is the refractive angle of the prism. The minimum deviation due to this prism is

A) $\pi - 2A$
B) $\frac{A}{2}$
C) $\pi - 3A$
D) A

Answer: $\pi - 2A$

Solution: The formula to calculate the refractive index of the material of the prism can be written as

$$\mu=\frac{\sin\frac{A+\delta m}{2}}{\sin\frac{A}{2}}\quad\ldots(1)$$

Simplify equation (1) to obtain the expression for the minimum angle of deviation for the prism.

$$\mu\sin\frac{A}{2}=\sin\left(\frac{A+\delta m}{2}\right)$$
$$\Rightarrow\frac{A+\delta m}{2}=\sin^{-1}\left(\mu\sin\frac{A}{2}\right)$$
$$\Rightarrow\delta_m=2\sin^{-1}\left(\mu\sin\frac{A}{2}\right)-A\quad\ldots(2)$$

Substitute the given expression for the refractive index into equation (2) and simplify to obtain the required minimum angle of deviation for the prism.

$$\begin{aligned}\delta_m&=2\sin^{-1}\left(\cot\frac{A}{2}\sin\frac{A}{2}\right)-A\\&=2\sin^{-1}\left(\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}\sin\frac{A}{2}\right)-A\\&=2\sin^{-1}\left(\cos\frac{A}{2}\right)-A\\&=2\sin^{-1}\left(\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)\right)-A\\&=2\left(\frac{\pi}{2}-\frac{A}{2}\right)-A\\&=\pi-2A\end{aligned}$$

Q.44. If displacement of the particle is $s=3t^2+4t+5$, then velocity(in $\mathrm{m\ s^{-1}}$) at $t=\ 5\ \mathrm{s}$ is

Answer: 34

Solution: As we know, velocity is given by $\frac{ds}{dt}$.

Therefore, $v=6t+4=(6\times5)+4=34\ \mathrm{m\ s^{-1}}$

Q.45. The velocity at the mean position is $10\ \mathrm{cm\ s^{-1}}$ for a SHM with $A=4\ \mathrm{cm}$. The magnitude of the displacement(in cm), when velocity is $5\ \mathrm{cm\ s^{-1}}$ is $\alpha\sqrt{3}$. Find the value of α.

Answer: 2

Solution: The formula to calculate the velocity of the particle at the mean position is given by

$$v_m=A\omega\quad\ldots(1)$$

Substituting the values of the known parameters into equation (1), we have

$$10=4\omega$$
$$\Rightarrow\omega=2.5\ \mathrm{rad\ s^{-1}}$$

The formula to calculate the velocity of the particle at any distance x from the mean position is given by

$$v=\omega\sqrt{A^2-x^2}\quad\ldots(2)$$

From equation (2), it follows that

$$5=2.5\times\sqrt{4^2-x^2}$$
$$\Rightarrow\sqrt{16-x^2}=2$$
$$\Rightarrow x^2=12$$
$$\Rightarrow x=\sqrt{12}$$
$$=\pm2\sqrt{3}\ \mathrm{cm}$$

Q.46. In a hydrogen like atom if the radius of the 3^{rd} orbit is r, then the radius of the 4^{th} orbit is $\frac{p}{9}r$. Find the value of p.

Answer: 16

Solution: The formula to calculate the radius of the n^{th} orbit is given by

$$r_n = 0.52\frac{n^2}{Z}\ \mathring{A} \quad \ldots(1)$$

Thus, the ratio of the radius of the third and the fourth orbit can be written as

$$\frac{r_3}{r_4} = \frac{0.53\frac{3^2}{Z}}{0.53\frac{4^2}{Z}}$$

$$= \frac{9}{16}$$

$$\Rightarrow r_4 = \frac{16}{9}r_3$$

$$= \frac{16}{9}r$$

Hence, $p = 16$.

Q.47. If the electric field component of an EM wave is given by $E = E_0 \sin(\omega t - kx)$, then the intensity of the EM wave is given by $I = \frac{1}{2}\varepsilon_0(E_0)^n c$. Find the value of n.

Answer: 2

Solution: The intensity (I) of an electromagnetic field in terms of the peak value (E_0) of the electric field is given by the formula:

$$I = \frac{1}{2}c\varepsilon_0 {E_0}^2$$

This formula is derived from the relationship between the intensity and the electric field in an electromagnetic wave. The intensity is proportional to the square of the electric field amplitude, and the factor of $\frac{1}{2}$ is introduced to account for the time-averaged power in the wave.

Hence, $n = 2$.