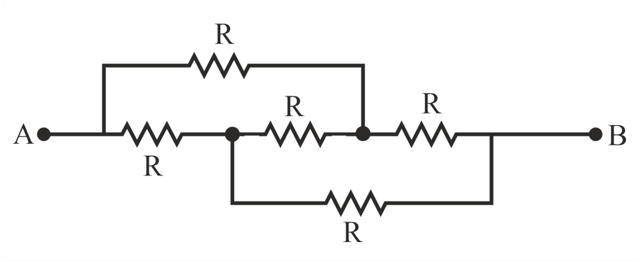
JEE Main Exam 2023 - Session 1

1 Feb 2023 - Shift 1 (Memory-Based Questions)

Section A: Physics

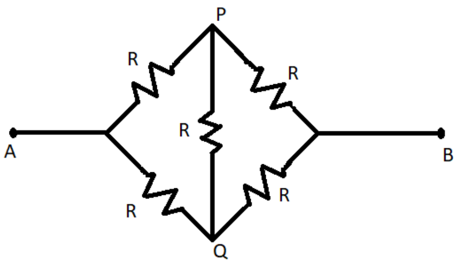
Q.1. Find the equivalent resistance between points A and B.



A) $\frac{3R}{2}$ B) $2R$ C) $4R$ D) R

Answer: R

Solution: The given circuit can be redrawn as shown below:



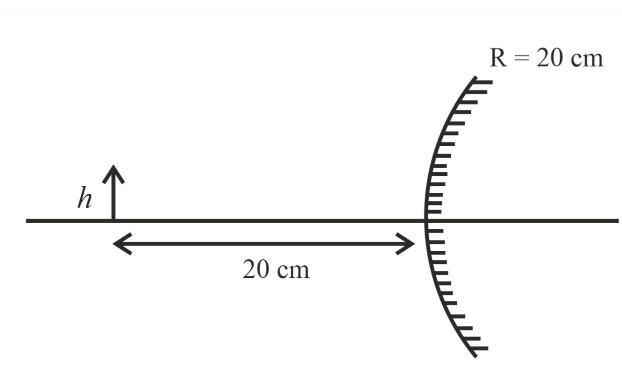
Since the circuit is in balanced wheatstone configuration, the capacitor and the resistor connected across points P and Q can be omitted. Now, the circuit has two arms having two resistors connected in series.

Therefore, the resistance of each arm is $2R$. The equivalent resistance will therefore be:

$$R_{eq} = \frac{(2R)^2}{2R+2R} = R$$

Hence, option D is correct.

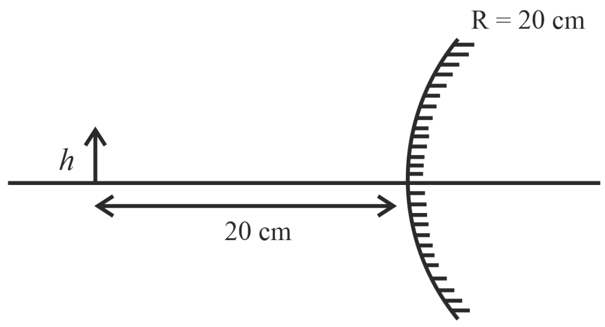
Q.2. An object of height h is placed at a distance of $20\ \text{cm}$ in front of a convex mirror of radius of curvature $R = 20\ \text{cm}$. Find the height of image.



A) $\frac{h}{2}$ B) $\frac{h}{3}$ C) $\frac{h}{6}$ D) $\frac{h}{4}$

Answer: $\frac{h}{3}$

Solution:



As per sign conventions, object distance $u=-20\ \text{cm}$, focal length $f=\frac{R}{2}=10\ \text{cm}$. From the mirror formula

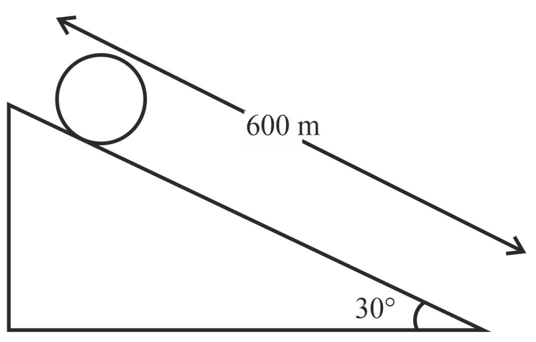
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$
$$\Rightarrow \frac{1}{v}+\frac{1}{-20}=\frac{1}{10}$$
$$\Rightarrow v=\frac{20}{3}\ \text{cm}$$

If h' is the height of image,

$$\frac{h'}{h}=\frac{-v}{u}=\frac{1}{3}$$
$$\Rightarrow h'=\frac{h}{3}$$

Hence, option B is correct.

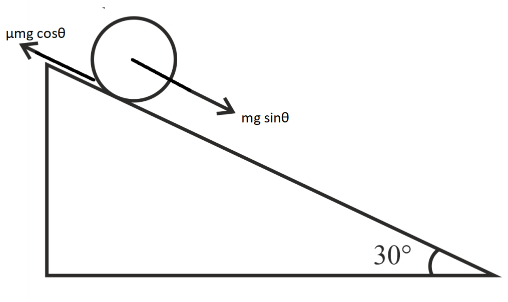
Q.3. In the diagram shown below, if a uniform solid cylinder is released from rest, find the time taken by it to reach the bottom of the inclined plane.



A) $60\ \text{s}$ B) $6\sqrt{10}\ \text{s}$ C) $3\sqrt{10}\ \text{s}$ D) $20\ \text{s}$

Answer: $6\sqrt{10}\ \text{s}$

Solution:



Taking torque about centre of mass

$fR = \frac{1}{2}MR^2\alpha$

$\Rightarrow f = \frac{1}{2}Ma \quad (\because a = R\alpha)$

Applying Newton's second law along the inclined plane

$Mg\ \sin\theta - f = Ma$

$\Rightarrow Mg\ \sin\theta - \frac{1}{2}Ma = Ma$

$\Rightarrow a = \frac{2}{3}g\ \sin\theta = \frac{10}{3}\ \mathrm{m\ s^{-2}}$

From the second equation of motion,

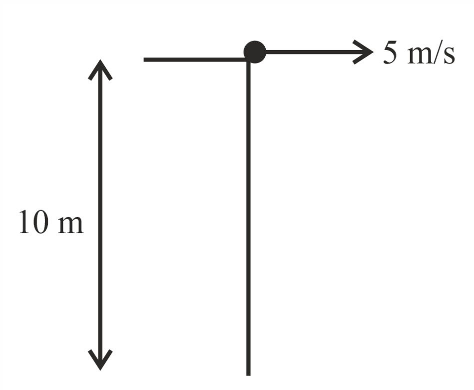
$s = \frac{1}{2}at^2$

$\Rightarrow 600 = \frac{1}{2} \times \frac{10}{3} \times t^2$

$\Rightarrow t = 6\sqrt{10}\ \mathrm{s}$

Hence, option B is correct.

Q.4. A particle is projected horizontally with speed of $5\ \mathrm{m\ s^{-1}}$ from a height of $10\ \mathrm{m}$. Find its speed as it hits the ground.



A) $15\ \mathrm{m\ s^{-1}}$ B) $5\ \mathrm{m\ s^{-1}}$ C) $10\ \mathrm{m\ s^{-1}}$ D) $20\ \mathrm{m\ s^{-1}}$

Answer: $15\ \mathrm{m\ s^{-1}}$

Solution: From conservation of mechanical energy,

$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2$

(where u is initial velocity at height h and v is the velocity of particle as it reaches the ground).

$\therefore \frac{1}{2}m \times 5^2 + m \times 10 \times 10 = \frac{1}{2}mv^2$

$\Rightarrow v^2 = 225$

$\Rightarrow v = 15\ \mathrm{m\ s^{-1}}$

Hence, option A is correct.

Q.5. Two moles of an ideal gas (adiabatic constant $= \frac{3}{2}$) undergoes adiabatic expansion. If the change of temperature is $-T$, find the work done by the gas.

A) $3RT$ B) $2RT$ C) $4RT$ D) $-RT$

Answer: $4RT$

Solution: Work done by gas in adiabatic process is:

$$W = -\frac{(P_2V_2 - P_1V_1)}{\gamma - 1}$$

$$= -\frac{nR(T_2 - T_1)}{\gamma - 1}$$

$$= \frac{-2 \times R \times (-T)}{\frac{3}{2} - 1} \quad \text{(Since } T_2 - T_1 = -T\text{)}$$

$$= 4RT$$

Hence, option C is correct.

Q.6. Which of the following is not a frequency of Frequency Modulated (FM) signal?

A) $90\ \mathrm{MHz}$ B) $89\ \mathrm{MHz}$ C) $106\ \mathrm{MHz}$ D) $100\ \mathrm{kHz}$

Answer: $100\ \mathrm{kHz}$

Solution: Frequency of FM signal lies in the range of $30\ \mathrm{MHz}$ to $300\ \mathrm{MHz}$. Therefore, $100\ \mathrm{kHz}$ is not a frequency of FM signal.

Hence, option D is correct.

Q.7. Statement 1: Magnitude of acceleration due to gravity is same at all the points inside Earth assuming it to have a uniform density.

Statement 2: Magnitude of gravitational field increases as we go towards centre in a uniform spherical shell.

A) Both statement 1 and statement 2 are true

B) Statement 1 is true but statement 2 is false

C) Statement 1 is false but statement 2 is true

D) Both statement 1 and statement 2 are false

Answer: Both statement 1 and statement 2 are false

Solution: The magnitude of acceleration due to gravity varies with depth d as:

$$g' = g\left(1 - \frac{d}{R}\right)$$

(where g is the magnitude of acceleration due to gravity at Earth's surface and R is the radius of Earth).

Therefore, the magnitude of acceleration due to gravity decreases as d increases (i.e., as we go towards the centre of Earth).

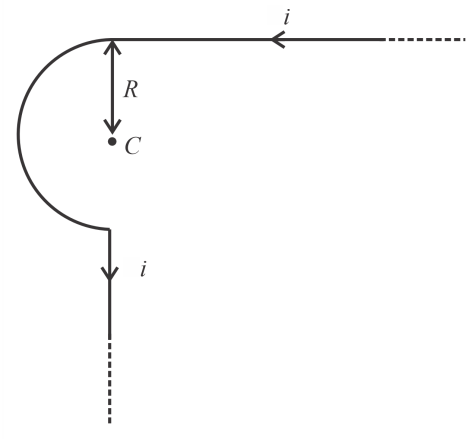
Also for a uniform solid sphere, the magnitude of gravitational field at a distance $x(< \text{the radius of sphere, } R)$ is

$$E = G\frac{m}{R^3}x$$

Which shows that the gravitational field decrease as the distance from centre decreases.

Hence, both the statements are false and option D is correct.

Q.8. An infinite wire is bent in the shape as shown. Find the magnetic field at point C.



A) $\frac{\mu_0 i}{4\pi R}(1+\pi)$

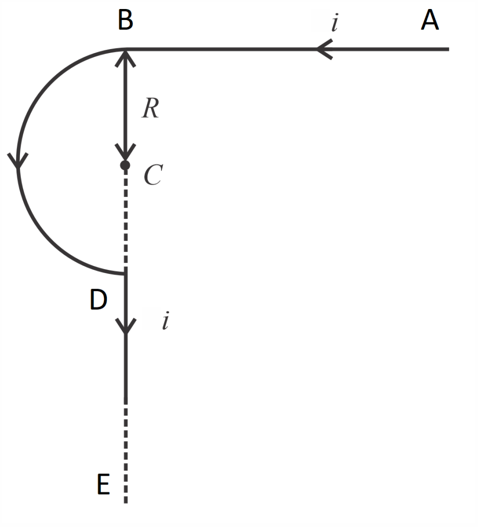
B) $\frac{\mu_0 i}{4\pi R}(2+\pi)$

C) $\frac{\mu_0 i}{2\pi R}(1+\pi)$

D) $\frac{\mu_0 i}{2R}$

Answer: $\frac{\mu_0 i}{4\pi R}(1+\pi)$

Solution:



Magnitude of magnetic field due to semi-infinite portion of wire AB at point C is

$B_1 = \frac{\mu_0 i}{4\pi R}$

Magnitude of magnetic field due to semicircular arc BD at point C is

$B_2 = \frac{\mu_0 i}{4R}$

Magnitude of magnetic field due to semi-infinite portion DE at point C is 0.

Using right-hand rule, it can be seen that directions of $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$ are both outwards from the plane.

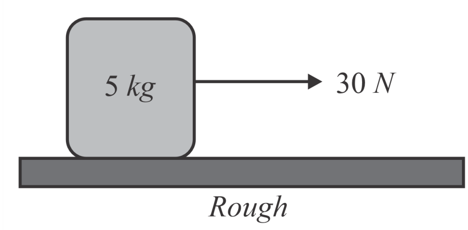
Therefore, the magnitude of net magnetic field at point C is

$B = B_1 + B_2$

$= \frac{\mu_0 i}{4\pi R}[1+\pi]$

Hence, option A is correct.

Q.9. A force of $30 \mathrm{~N}$ is applied on a block of mass $5 \mathrm{~kg}$. Starting from rest, the block travels a distance of $50 \mathrm{~m}$ in $10 \mathrm{~s}$. Find the coefficient of friction between the surface and the block.



A) 0.5

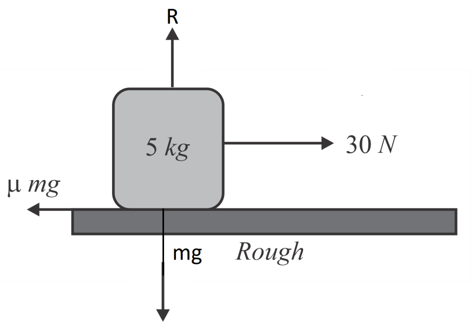
B) 0.7

C) 0.3

D) 0.8

Answer: 0.5

Solution:



From the second equation of motion,

$s=\frac{1}{2} a t^{2}$

$\Rightarrow 50=\frac{1}{2} a \times 10^{2}$

$\Rightarrow a=1 \mathrm{~m} \mathrm{~s}^{-2}$

Applying Newton's second law in vertical direction,

$R-m g=0$
$\Rightarrow R=m g$

(where R is the magnitude of normal reaction on the block due to the surface)

If μ is the coefficient of kinetic friction between the block and the surface, applying Newton's second law in horizontal direction,

$30-\mu R=m a$
$\Rightarrow 30-\mu m g=m a$
$\Rightarrow 30-50 \mu=5$
$\Rightarrow \mu=\frac{1}{2}$

Hence, option A is correct.

Q.10. A stone is thrown vertically up with speed v_{o} from a cliff of height H. Find the average speed of the ball till the moment it reaches ground. Given that $H=100 \mathrm{~m}, v_{o}=10 \mathrm{~m} \mathrm{~s}^{-1}$ & $g=10 \mathrm{~m} \mathrm{~s}^{-2}$.

A) $\frac{64}{1+\sqrt{21}} \mathrm{~m} \mathrm{~s}^{-1}$

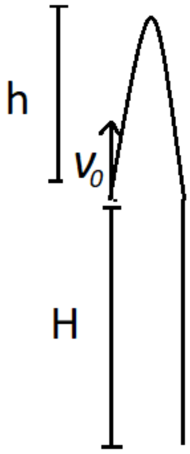
B) $55 \mathrm{~m} \mathrm{~s}^{-1}$

C) $110(1+\sqrt{21}) \mathrm{~m} \mathrm{~s}^{-1}$

D) $\frac{110}{1+\sqrt{21}} \mathrm{~m} \mathrm{~s}^{-1}$

Answer: $\frac{110}{1+\sqrt{21}} \mathrm{~m} \mathrm{~s}^{-1}$

Solution:



The maximum height reached above the point of projection,

$h = \frac{v_\circ^2}{2g} = 5 \text{ m}$

As shown in the figure, total distance $= 2h + 100 = 110\ m$

If t is the time of flight, from second equation of motion (taking upwards direction as positive)

$H = v_\circ t - \frac{1}{2} g t^2$

$\Rightarrow -100 = 10t - \frac{1}{2} \times 10 \times t^2$

$\Rightarrow t = 1 + \sqrt{21} \text{ s}$

$\Rightarrow$ Average speed $= \frac{\text{total distance}}{\text{time of flight}} = \frac{110}{1+\sqrt{21}} \text{ m s}^{-1}$

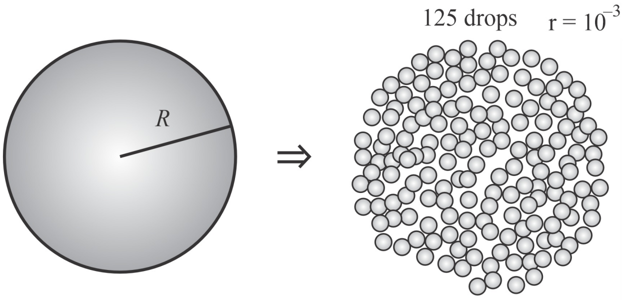
Hence, option D is correct.

Q.11. A drop of Mercury is divided into 125 drops of equal radius $10^{-3}\,\text{m}$ each. If surface tension of Mercury is equal to 0.45 N m^{-1}. Magnitude of change in surface energy is equal to nearly:

A) $1.14 \times 10^{-4} \text{ J}$ B) $7.06 \times 10^{-4} \text{ J}$ C) $8.47 \times 10^{-4} \text{ J}$ D) $5.65 \times 10^{-4} \text{ J}$

Answer: $5.65 \times 10^{-4} \text{ J}$

Solution:



As the volume of the mercury will remain same, therefore

$\frac{4}{3}\pi R^3 = 125\left(\frac{4}{3}\pi r^3\right)$
$\Rightarrow R = 5r$

Now, change in surface energy,

$\Delta U = 125 \times \left(S \times 4\pi r^2\right) - \left(S \times 4\pi R^2\right)$

$= 125 \times S \times 4\pi r^2 - S \times 4\pi \times 25r^2$

$= S \times 4\pi r^2 \times 100$
$= 0.45 \times 4 \times 3.14 \times 10^{-6} \times 100$
$= 5.65 \times 10^{-4}\ \mathrm{J}$

Q.12. A charged particle with charge $2 \times 10^{-6}\ \mathrm{C}$, at rest, is first accelerated through a potential difference of $100\ \mathrm{V}$ and then it is subjected to a transverse magnetic field of $4\ \mathrm{mT}$. In region of magnetic field it undergoes a circular path of radius $3\ \mathrm{cm}$. Mass of the particle is equal to

A) $1.44 \times 10^{-16}\ \mathrm{kg}$ B) $7.2 \times 10^{-16}\ \mathrm{kg}$ C) $1.44 \times 10^{-10}\ \mathrm{kg}$ D) $7.2 \times 10^{-10}\ \mathrm{kg}$

Answer: $1.44 \times 10^{-16}\ \mathrm{kg}$

Solution: As we know, radius of the circular path is given by,

$r = \frac{mv}{qB}$
$= \frac{\sqrt{2mK}}{qB}$
$= \frac{\sqrt{2 \times m \times qV}}{qB}$

$\Rightarrow r^2 = \frac{2mV}{qB^2}$

$\Rightarrow m = \frac{r^2 qB^2}{2V}$
$= \frac{\left(3 \times 10^{-2}\right)^2 \times \left(2 \times 10^{-6}\right) \times \left(4 \times 10^{-3}\right)^2}{2 \times 100}$
$= 144 \times 10^{-18}\ \mathrm{kg}$
$= 1.44 \times 10^{-16}\ \mathrm{kg}$

Q.13. A string of mass per unit length equal to $7 \times 10^{-3}\ \mathrm{kg\ m^{-1}}$ is subjected to a tension equal to $70\ \mathrm{N}$. The speed of transverse wave on this string is equal to

A) $10\ \mathrm{m\ s^{-1}}$ B) $50\ \mathrm{m\ s^{-1}}$ C) $100\ \mathrm{m\ s^{-1}}$ D) $200\ \mathrm{m\ s^{-1}}$

Answer: $100\ \mathrm{m\ s^{-1}}$

Solution: The speed of transverse wave on this string is given by,

$v = \sqrt{\frac{T}{k}}$
$= \sqrt{\frac{70}{7 \times 10^{-3}}}$
$= 100\ \mathrm{m\ s^{-1}}$

Q.14. In an series LCR circuit connected across $220\ \mathrm{V},\ 50\ \mathrm{Hz}$ AC supply, if the inductive reactance of the circuit is $79.6\ \Omega$. If the power delivered in the circuit is maximum, the capacitance of the circuit is $x\ \mu\mathrm{F}$. Find x

Answer: 40

Solution: Power delivered is maximum in case of the resonance, therefore

$$X_C = X_L$$

$$\Rightarrow \frac{1}{\omega C} = 79.6\Omega$$

$$\Rightarrow \quad C = \frac{1}{\omega \times 79.6}$$

$$= \frac{1}{2\pi f \times 79.6}$$

$$= \frac{1}{100\pi \times 79.6}$$

$$\approx 40\ \mu\mathrm{C}$$

Hence, $x = 40$.

Q.15. An alpha particle and a proton have same de$-$Broglie wavelengths. Ratio of kinetic energies of proton and alpha particle is _____ .

Answer: 4

Solution: As we know, de Broglie wavelength is given by,

$$\lambda = \frac{h}{p}$$

Now kinetic energy can be written as,

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Therefore,

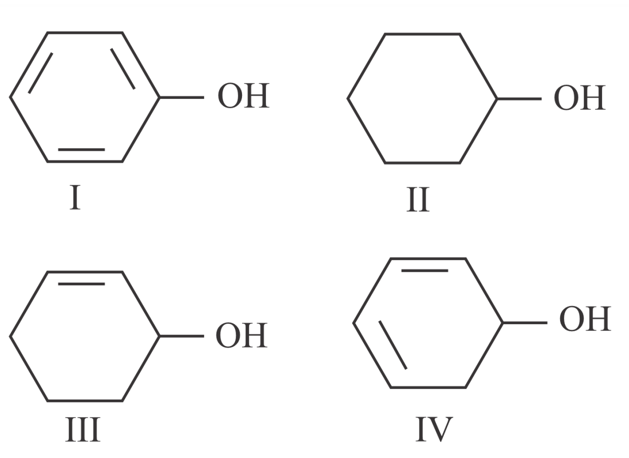
$$K \propto \frac{1}{m}$$

Hence, required ratio

$$\frac{Kp}{K\alpha} = \frac{m\alpha}{mp} = 4$$

Section B: Chemistry

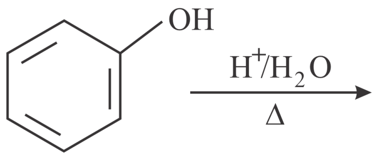
Q.1. Dehydration of the following in increasing order is _______.



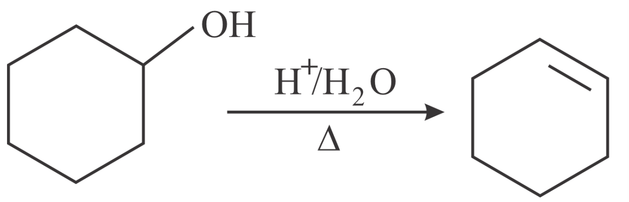
A) I < II < III < IV B) II < III < IV < I C) I < III < II < IV D) I < IV < II < III

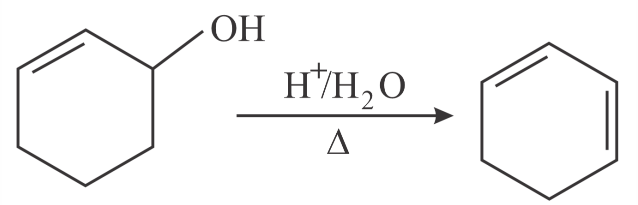
Answer: I < II < III < IV

Solution: The dehydration of alcohols takes place via formation of carbocation. Hence, rate of reaction depends on the stability of carbocation.

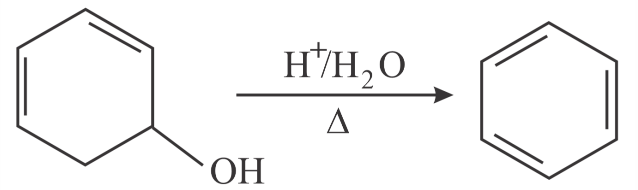


No dehydration is possible in phenol. Because carbocation formation is difficult in case of phenol.





Fast due to loss of H_2O and conjugation increasing stable of compound as compared to(ii)



Very fast because stable compound formed.

Q.2. Which of the following complexes have maximum splitting?

A) $\left[Fe\left(CN\right)_6\right]^{4-}$ B) $\left[Fe\,(NH_3)_6\right]^{2+}$ C) $\left[Fe\,(Cl)_6\right]^{4-}$ D) $\left[Fe\,(ox)_3\right]^{4-}$

Answer: $\left[Fe\left(CN\right)_6\right]^{4-}$

Solution: Some ligands are able to produce strong fields in which case, the splitting will be large whereas others produce weak fields and consequently result in small splitting of d orbitals.

In general, ligands can be arranged in a series called spectro-chemical series according to which maximum splitting is done by the strongest ligand which is cyanide ion in this case.

Q.3. Average kinetic energy of an ideal gas depends only on its _____

A) pressure. B) volume. C) the number of moles. D) temperature.

Answer: temperature.

Solution: The formula of average kinetic energy of ideal gas molecule is as follows:

$$\mathrm{KE}=\frac{3}{2}\mathrm{kT}$$

Here, k is the Boltzmann constant and T is the temperature.

Average kinetic energy depends only on temperature.

Q.4. Assertion: Hydrogen is an environment/eco-friendly fuel.

Reason: Hydrogen is the lightest element

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

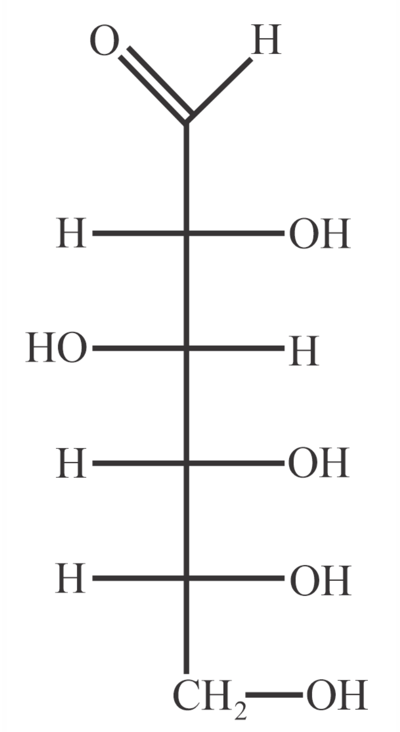
C) Assertion is true but Reason is false.

D) Assertion is false but Reason is true.

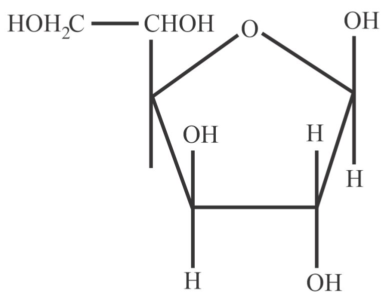
Answer: Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

Solution: Hydrogen can be produced from diverse domestic resources with the potential for near-zero greenhouse gas emissions. Once produced, hydrogen generates electrical power in a fuel cell, emitting only water vapour and warm air. Hydrogen, H, is the lightest element with the atomic number 1.

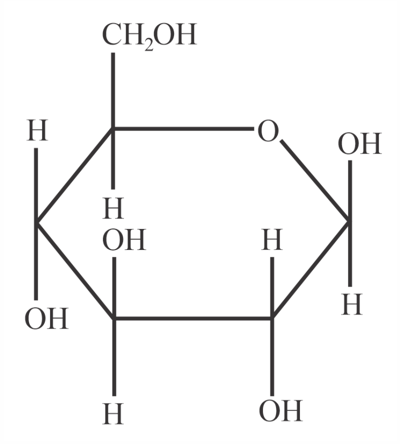
Q.5. Pyranose form of the given compound is?



A)

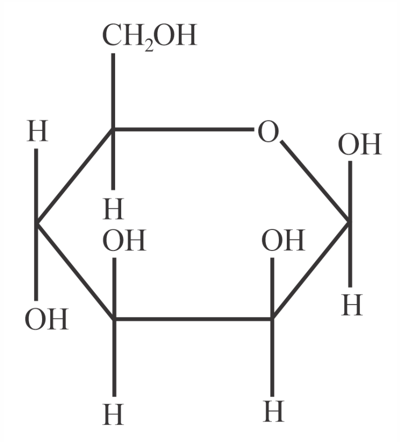


B)



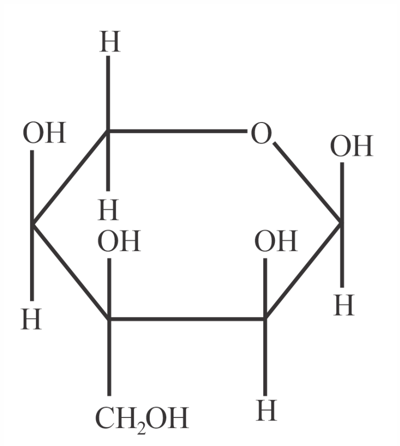
CH2OH
H
O
OH
H
OH
H
OH
H
H
OH
OH

C)



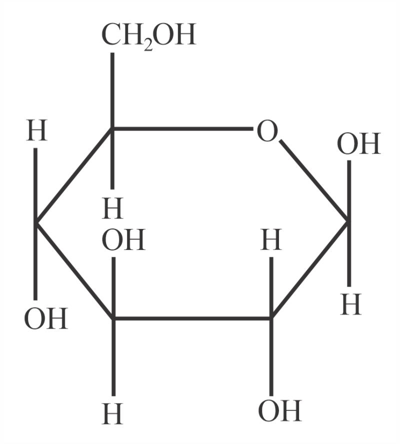
CH2OH
H
O
OH
H
OH
OH
H
OH
H
H

D)



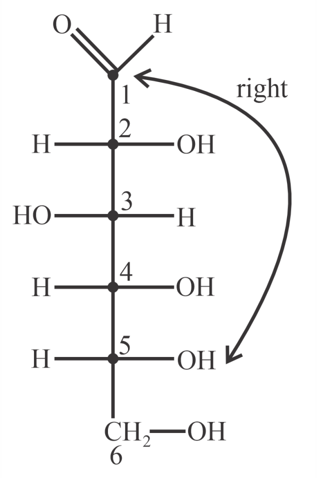
H
OH
O
OH
H
OH
OH
H
H
CH2OH
H

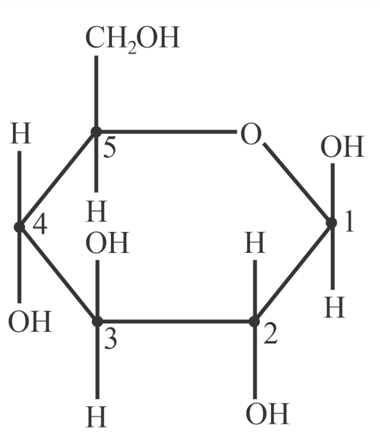
Answer:



Solution: In glucose, one of the —OH groups may add to the —CHO group and form a cyclic hemi-acetal structure of Glucopyranose.

In order to check the configuration of groups attached, the position of groups on 2[nd] and 4[th] carbon as marked must be similar which is only present in option B.





Q.6. $X(g) \rightleftharpoons 2Y(g); K_{P_1} - \text{(i)}$

$A(g) \rightleftharpoons B(g) + C(g); K_{P_2} - \text{(ii)}$

If degree of dissociation is same for both the reactions. Find out ratio of total pressure P_1 & P_2 respectively.

A) $\frac{K_{P_1}}{K_{P_2}}$ B) $\frac{4K_{P_1}}{K_{P_2}}$ C) $\frac{K_{P_1}}{4K_{P_2}}$ D) $\frac{K_{P_1}}{2K_{P_2}}$

Answer: $\frac{K_{P_1}}{4K_{P_2}}$

Solution:

	X	$\rightleftharpoons$	2Y
initial moles	1		–
mole at equilibrium	$1-\alpha$		2α
mole fractions at equilibrium	$\frac{1-\alpha}{1+\alpha}$		$\frac{2\alpha}{1+\alpha}$
partial pressure at equilibrium	$\left(\frac{1-\alpha}{1+\alpha}\right)P_1$		$\left(\frac{2\alpha}{1+\alpha}\right)P_1$

Similarly, we can write initial moles, moles at equilibrium, mole fractions and partial pressures for the second equilibrium as follows,

A	$\rightleftharpoons$	B	+	C
1		–		–
$1-\alpha$		α		α
$\frac{1-\alpha}{1+\alpha}$		$\frac{\alpha}{1+\alpha}$		$\frac{\alpha}{1+\alpha}$
$\left(\frac{1-\alpha}{1+\alpha}\right)P_2$		$\left(\frac{\alpha}{1+\alpha}\right)P_2$		$\left(\frac{\alpha}{1+\alpha}\right)P_2$

Now, the ratio of equilibrium constants can be written as follows,

$$\frac{K_{P_1}}{K_{P_2}}=\frac{\left[\left(\frac{2\alpha}{1+\alpha}\right)P_1\right]^2}{\left(\frac{1-\alpha}{1+\alpha}\right)P_1}\div\frac{\left[\left(\frac{\alpha}{1+\alpha}\right)P_2\right]^2}{\left(\frac{1-\alpha}{1+\alpha}\right)P_2}$$

$$\frac{4\alpha^2P_1}{\alpha^2P_2}=\frac{K_{P_1}}{K_{P_2}}$$

$$\Rightarrow\frac{K_{P_1}}{K_{P_2}}=\frac{4P_1}{P_2}$$

$$\Rightarrow\frac{P_1}{P_2}=\frac{K_{P_1}}{4K_{P_2}}$$

Q.7. Which of the following is not correctly matched

Column I		Column II	
(a)	Antibiotic	(p)	Penicillin
(b)	Antiseptic	(q)	Chloroxylenol
(c)	Tranquilizer	(r)	Erythromycin
(d)	Analgesic	(s)	Aspirin

A) (a) is not correctly matched with (p)

B) (b) is not correctly matched with (q)

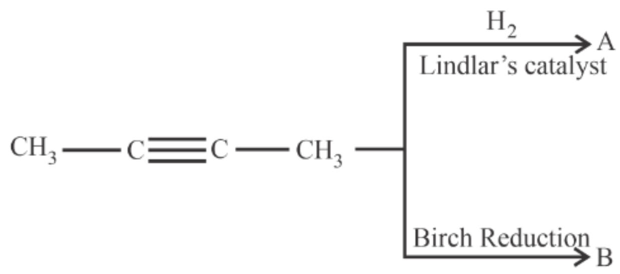
C) (c) is not correctly matched with (r)

D) (d) is not correctly matched with (s)

Answer: (c) is not correctly matched with (r)

Solution: Erythromycin is an antibiotic not a tranquilizer, while other options are correctly matched

Q.8. In the given reaction, identify the correct statement on physical properties of (A) and (B).



A) Melting point: A>B
Boiling point: A>B

B) Melting point: A<B
Boiling point: A>B

C) Melting point: A>B
Boiling point: A<B

D) Melting point: A<B
Boiling point: A<B

Answer: Melting point: A<B
Boiling point: A>B

Solution: When 2-butyne is treated with H_2/Lindlar's catalyst, compound A (cis-2-butene) is produced as the major product; and when treated with liq NH_3 it produce B (trans-2-butene) as the major product.

Cis-isomers have higher dipole moment than trans because in cis isomers similar groups are at same direction whereas in trans they are at opposite direction.

Due to higher dipole moment, cis isomer will have higher boiling point than trans.

Trans isomer is more cosely packed than cis isomer, hence, it has more melting point.

Q.9. Which of the following options contain the correct match

Column I		Column II	
(a)	Caustic Soda	(p)	$CaSO_4$
(b)	Washing Soda	(q)	$Na_2CO_3.10H_2O$
(c)	Dead burnt plaster	(r)	$Ca(OH)_2$
(d)	Slaked lime	(s)	$NaOH$

A) $A-S, B-Q, C-P, D-R$ B) $A-P, B-Q, C-R, D-S$ C) $A-S, B-P, C-Q, D-R$ D) $A-R, B-S, C-Q, D-P$

Answer: $A-S, B-Q, C-P, D-R$

Solution: Caustic soda is $NaOH$

Washing soda is $Na_2CO_3.10H_2O$

Dead burnt plaster is $CaSO_4$

Slaked lime is $Ca(OH)_2$

Q.10. When $K_4[Fe(CN)_6]$ is added to $FeCl_3$, the Prussian blue complex compound formed is:

A) $Fe_3[Fe(CN)_6]_4$ B) $Fe_4[Fe(CN)_6]_3$ C) $K_2Fe[Fe(CN)_6]$ D) $K_2Fe_3[Fe(CN)_6]_2$

Answer: $Fe_4[Fe(CN)_6]_3$

Solution: Addition of $FeCl_3$ to $K_4Fe(CN)_6$ in dilute and cold solution gives a solution, Prussian blue solution, $Fe_4[Fe(CN)_6]$
The reaction is as follows:

$$4FeCl_3 + 3K_4[Fe(CN)_6] \rightarrow Fe_4[Fe(CN)_6]_3 + 12KCl$$

Q.11. How photochemical smog can be controlled in automobiles?

A) Using catalytic converters which will increase the emission of nitrogen oxides

B) Using catalytic converters which will decrease the emission of nitrogen oxides

C) By increasing sulphur content in fuel

D) By decreasing sulphur content in fuel

Answer: Using catalytic converters which will decrease the emission of nitrogen oxides

Solution: Photochemical smog is generally produced by nitrogenous oxides and long passage containing catalytic converters can be used to decrease nitrogen oxides emission in automobiles

Q.12. Electron are emitted in cathode ray tube with a velocity of $1000\ m/s$. Select the correct statement

A) De broglie wavelength of e^- is $666.67\ nm$

B) Cathode ray travel from cathode to anode.

C) The characteristics of e depends on the metal used in cathode

D) The characteristics of e^{-} depends on the gas filled inside the cathode tube

Answer: Cathode ray travel from cathode to anode.

Solution: Given velocity of electron $= 1000\ \mathrm{m/s}$

De broglie wavelength will be

$\therefore\ \lambda_{db} = \frac{h}{mv}$

$= \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 1000}$

$= 728\ \mathrm{nm}$

Characteristic of electrons is independent of the metal and the gas used in the experiment.

Q.13. Match the tests given in Column - I with the compounds given in Column- II

Column-I	Column-II
(A) Schiff's Test	1. Carbohydrate
(B) Carbylamine test	2. Peptide
(C) Molish Test	3. Aldehyde
(D) Biuret test	4. 1° amine

A) A-1, B-2, C-4, D-3 B) A-2, B-4, C-3, D-1 C) A-4, B-3, C-2, D-1 D) A-3, B-4, C-1, D-2

Answer: A-3, B-4, C-1, D-2

Solution: Molisch's test is a chemical test which is used to check for the presence of carbohydrates in a given analyte.

The biuret test is a chemical test that can be used to see if an analyte has peptide bonds or not.

Schiff's test is a chemical test used to check for the presence of aldehydes in a given analyte.

The carbylamine reaction, also known as Hofmann's isocyanide test is a chemical test for the detection of primary amines.

Q.14. Choose correct statement from the following
A: Beryllium oxide is an acidic oxide
B: Beryllium sulphate is soluble in aqueous medium
C: Beryllium carbonate is thermally stable
D: Beryllium shows anomalous behaviour in comparison to other Group 2 elements

A) A & B B) B & C C) B & D D) C & D

Answer: B & D

Solution: The reaction of BeO with acid:

$\mathrm{BeO\,(s) + 2\,HCl\,(l) + H_2O\,(l) \rightarrow BeCl_2\,(s) + 2H_2O\,(l)}$

The reaction of BeO with base:

$\mathrm{BeO\,(s) + 2\,NaOH\,(s) + H_2O\,(l) \rightarrow Na_2Be\,(OH)_4\,(s)}$

Therefore, it is evident that it produces salt when interacting with both acid and base, indicating that it behaves as a base when reacting with acid and acic when reacting with base.
As a result, it is also an amphoteric oxide.

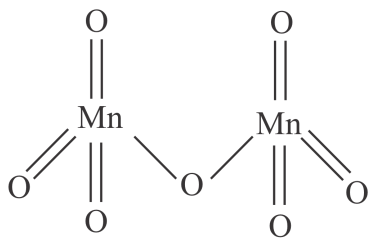
We know that beryllium is small as compared to the sulphate ion. Thus, it has very low lattice energy. Also, due to small size, beryllium has high charge density and thus higher hydration. So it will be freely soluble in water.
Beryllium carbonate is unstable and decomposes to give beryllium oxide and carbon dioxide.

Due to its small size, beryllium shows anomalous behaviour in comparison to other Group 2 elements.

Q.15. Consider structure of $\mathrm{Mn_2O_7}$
X: Number of $\mathrm{Mn-Mn}$ bonds
Y: Number of $\mathrm{Mn-O-Mn}$ bonds
Find out $(\mathrm{X+Y})$

Answer: 1

Solution: Each Mn is tetrahedrally surrounded with oxygen atoms. In addition, there is a bridging two-coordinated oxygen atom between two tetrahedral units($Mn - O - Mn$). The bridged $Mn - O$ has different bond length than the tetrahedrally surrounded $Mn - O$ bond lengths.



Mn_2O_7

Here, X = 0 and Y = 1

Q.16. X is oxidation number of Br in Bromic acid.
Y is oxidation number of Br in perbromic acid.
Find out $(X + Y)$

Answer: 12

Solution: Let, X be the oxidation state of bromine in $HBrO_3$.

$+1 + X + 3(-2) = 0$

$1 + X - 6 = 0$

$X - 5 = 0$

$X = +5$

Hence, the oxidation state of Br in $HBrO_3$ is $+5$.

Let, Y be the oxidation state of bromine in $HBrO_4$.

$+1 + Y + 4(-2) = 0$

$1 + Y - 8 = 0$

$Y - 7 = 0$

$Y = +7$

Hence, the oxidation state of Br in $HBrO_4$ is $+7$.

$X + Y = 5 + 7 = 12$

Q.17. The density of a $3\,MNaCl$ solution is $1\ g/mol$. The molality of the solution is 'X'. Then find $2X$.
(Round off to nearest integer)

Answer: 7

Solution: 3 Molar solution means there are 3 moles of $NaCl$ salt in 1 Liter.

Molecular weight of $NaCl = 58.5$. Hence, there are $3 \times 58.5\ gms$ in 1 Litre of water.

Density $=$ mass/volume

Mass of 1 litre of solution $= 1\ gms/ml \times 1000ml = 1000\ gms$
V= volume of water added to make the solution.
Mass of solute $+$ Mass of solvent$=$ Mass of solution
$175.5\ gms+$ mass of solvent$= 1000\ gm$
mass of solvent $= 1000 - 175.5 = 824.5$

So $824.5\ gms$ of water is mixed with $3\ moles$ of $NaCl$ to make the $3M$ solution.
Molality $=$ number of moles / mass of solvent in kg $= 3/0.8245\ kg$
$= 3.638\ m = X$

$\Rightarrow 2X = 7.276$

Q.18. H-atom in ground state absorbs $12.75\,eV$ of energy. The orbital angular momentum of the electron becomes $\frac{nh}{2\pi}$, the value of n is:

Answer: 4

Solution: In the orbital angular momentum $\frac{\mathrm{nh}}{2\pi}$, n is principal quantum number.

Transition energy for Hydrogen atom is given by:

$\Delta\mathrm{E}=-\mathrm{R}\left(1/\mathrm{n}_2^2-1/\mathrm{n}_1^2\right)$

where $\mathrm{R}=13.6\,\mathrm{eV}, \mathrm{n}_1=1, \mathrm{n}_2=\mathrm{n}$

$12.75=-13.6\left(1/\mathrm{n}^2-1\right)$

$-0.9375+1=1/n^2$

$\mathrm{n}^2=16$

$\mathrm{n}=4$

Section C: Mathematics

Q.1. $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots\frac{1}{2n}\right)=$

A) $\ln\frac{2}{3}$ B) $\ln 2$ C) $-\ln 2$ D) 0

Answer: $\ln 2$

Solution: $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots\frac{1}{n+n}\right)$

$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r}$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\frac{r}{n}}$

$=\int_0^1\frac{1}{1+x}dx=\left[\ln(1+x)\right]_0^1$

$=\ln 2$

Q.2. If $\frac{dy}{dx}+y\tan x=\sec x$ and $y(0)=1$, then $y\left(\frac{\pi}{6}\right)=$ ____

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1+\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) $-\frac{\sqrt{3}}{2}$

Answer: $\frac{1+\sqrt{3}}{2}$

Solution: Given:

$\frac{dy}{dx}+y\tan x=\sec x$

This is a linear differential equation.

$\mathrm{I.F.}=e^{\int\tan x\,dx}=e^{\ln|\sec x|}=\sec x$

So, solution is

$y\sec x=\int\sec^2 x\,dx$

$\Rightarrow y\sec x=\tan x+C$

Also, $y(0)=1\Rightarrow C=1$, so

$y\sec x=\tan x+1$

$\Rightarrow y=\sin x+\cos x$

$\Rightarrow y\left(\frac{\pi}{6}\right)=\frac{1+\sqrt{3}}{2}$

Q.3. $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\ldots.\infty=$

A) 0

B) 1

C) $\frac{1}{2}$

D) 2

Answer: $\frac{1}{2}$

Solution: $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\ldots.\infty$

$=\sum_{r=1}^{\infty}\frac{r}{1+r^2+r^4}$

$=\frac{1}{2}\sum_{r=1}^{\infty}\frac{\left(r^2+r+1\right)-\left(r^2-r+1\right)}{\left(r^2+r+1\right)\left(r^2-r+1\right)}$

$=\frac{1}{2}\left[\sum_{r=1}^{\infty}\frac{1}{\left(r^2-r+1\right)}-\sum_{r=1}^{\infty}\frac{1}{\left(r^2+r+1\right)}\right]$

$=\frac{1}{2}(1-0)=\frac{1}{2}$

Q.4. If $f(x)+f'(x)=\int_0^2 f(t)dt$ and $f(0)=e^{-2}$, then the value of $f(2)-2f(0)$ is:

A) 0

B) 1

C) -1

D) 2

Answer: -1

Solution: Let

$\int_0^2 f(t)dt = k$

$y = f(x)$

Now, we have

$f(x) + f'(x) = \int_0^2 f(t)dt$

$\Rightarrow \frac{dy}{dx} + y = k$

This is a linear differential equation.

$\mathrm{I.F.} = e^{\int dx} = e^x$

Solution of the given differential equation is

$ye^x = k\int e^x dx$

$\Rightarrow ye^x = ke^x + C$

Now, $f(0) = e^{-2}$, so

$C = e^{-2} - k$

Hence,

$ye^x = ke^x + \left(e^{-2} - k\right)$

$\Rightarrow y = f(x) = k + \left(e^{-2} - k\right)e^{-x} \quad \ldots.(\mathrm{i})$

Now,

$k = \int_0^2 f(t)dt$

$\Rightarrow k = \int_0^2 \left[k + \left(e^{-2} - k\right)e^{-t}\right]dt$

$\Rightarrow k = \left[kt - \left(e^{-2} - k\right)e^{-t}\right]_0^2$

$\Rightarrow k = 2k - \left(e^{-2} - k\right)\left(e^{-2} - 1\right)$

$\Rightarrow \left(e^{-2} - k\right)\left(e^{-2} - 1\right) = k$

$\Rightarrow k = \left(e^{-2} - 1\right)$

So,

$f(x) = \left(e^{-2} - 1\right) + e^{-x}$

$\therefore f(2) = 2e^{-2} - 1$

$f(0) = e^{-2}$

So,

$f(2) - 2f(0) = -1$

Q.5. If set $S = \left\{x : \left(\sqrt{3} + \sqrt{2}\right)^{x^2-4} + \left(\sqrt{3} - \sqrt{2}\right)^{x^2-4} = 10\right\}$, then $n(S) =$

A) 2 B) 3 C) 4 D) 6

Answer: 4

Solution: Given:

$$S=\left\{x:\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\right\}$$

Now,

$$\sqrt{3}-\sqrt{2}=\frac{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}}=\frac{1}{\sqrt{3}+\sqrt{2}}$$

So,

$$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$$

$$\Rightarrow\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^{x^2-4}=10$$

Put $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=u$, then

$$u+\frac{1}{u}=10$$

$$\Rightarrow u^2-10u+1=0$$

$$\Rightarrow u=\frac{10\pm\sqrt{100-4}}{2}$$

$$\Rightarrow u=5\pm\sqrt{24}$$

$$\Rightarrow u=5\pm2\sqrt{3}\sqrt{2}$$

$$\Rightarrow u=\left(\sqrt{3}\pm\sqrt{2}\right)^2$$

$$\Rightarrow\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}\pm\sqrt{2}\right)^2$$

So, $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}+\sqrt{2}\right)^2$ and $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^2$

Therefore,

$$x^2-4=2\ \&\ x^2-4=-2$$

$$\Rightarrow x=\pm\sqrt{6}\ \&\ x=\pm\sqrt{2}$$

So, $S=\left\{\sqrt{6},\ \sqrt{2},\ -\sqrt{6},\ -\sqrt{2}\right\}$

Hence, $n(S)=4$

Q.6. Sum of the series $\frac{1}{1!\cdot50!}+\frac{1}{3!\cdot48!}+\frac{1}{5!\cdot46!}+\ldots+\frac{1}{51!\cdot0!}$ is

A) $\frac{2^{51}}{51!}$

B) 2^{51}

C) $5!\cdot2^{51}$

D) $\frac{2^{50}}{51!}$

Answer: $\frac{2^{50}}{51!}$

Solution: Let

$$S=\frac{1}{1!\cdot 50!}+\frac{1}{3!\cdot 48!}+\frac{1}{5!\cdot 46!}+\ldots+\frac{1}{51!\cdot 0!}$$

$$\Rightarrow S=\frac{1}{51!}\left[\frac{51!}{1!\cdot 50!}+\frac{51!}{3!\cdot 48!}+\frac{51!}{5!\cdot 46!}+\ldots+\frac{51!}{51!\cdot 0!}\right]$$

$$\Rightarrow S=\frac{1}{51!}\left[{}^{51}C_1+{}^{51}C_3+{}^{51}C_5+\ldots+{}^{51}C_{51}\right]$$

$$\Rightarrow S=\frac{2^{51-1}}{51!}$$

$$\Rightarrow S=\frac{2^{50}}{51!}$$

Q.7. Let $R=\left\{(a,b):3a-3b+\sqrt{7}\text{ is irrational}\right\}$

A) R is an equivalence relation

B) R is symmetric but not reflexive

C) R is reflexive but not symmetric

D) R is reflexive and symmetric but not transitive

Answer: R is reflexive but not symmetric

Solution: Given:

$R=\left\{(a,b):3a-3b+\sqrt{7}\text{ is irrational}\right\}$

Reflexive:

$3a-3a+\sqrt{7}=\sqrt{7}$ is irrational.

So, $(a,a)\in R$, hence R is reflexive.

Given:

$R=\left\{(a,b):3a-3b+\sqrt{7}\text{ is irrational}\right\}$

Symmetric:

Let $(a,b)\in R$, then

$3a-3b+\sqrt{7}$ is irrational.

So, $3b-3a+\sqrt{7}$ is not necessarily irrational, since if $3a=\sqrt{7}$ and $3b\in Z$, then $3a-3b+\sqrt{7}$ is irrational., but $3b-3a+\sqrt{7}$ is not irrational.

So, $(a,b)\in R \not\Rightarrow (b,a)\in R$, hence R is not symmetric

Q.8. Which of the following will be the negation of the statement $p\vee(p\wedge {\sim}q)$

A) p

B) ${\sim}p$

C) q

D) ${\sim}q$

Answer: ${\sim}p$

Solution: Given,

$p\vee(p\wedge{\sim}q)$

Now negation of $p\vee(p\wedge{\sim}q)$ will be,

$={\sim}\,[p\vee(p\wedge{\sim}q)]$

$={\sim}p\wedge{\sim}\,(p\wedge{\sim}q)$

$={\sim}p\wedge({\sim}p\vee q)$

$={\sim}p\ \{\text{as } A\cap(A\cup B)=A\}$

Q.9. If S be the solution set of values of x satisfying $\cos^{-1}2x+\cos^{-1}\sqrt{1-x^2}=\pi$, then the value of $\sum_{x\in S}2\sin^{-1}\left(x^2-1\right)$ will be

A) 0

B) $-\sin^{-1}\left(\frac{24}{25}\right)$

C) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

D) $\pi-\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Answer: $-\sin^{-1}\left(\frac{24}{25}\right)$

Solution: Given,

$\cos^{-1}2x+\cos^{-1}\sqrt{1-x^2}=\pi$

$\Rightarrow \cos^{-1}\sqrt{1-\mathrm{x}^2}=\pi-\cos^{-1}2x$

$\Rightarrow \sqrt{1-\mathrm{x}^2}=\cos\left(\pi-\cos^{-1}2x\right)$

$\Rightarrow \sqrt{1-\mathrm{x}^2}=-\cos\left(\cos^{-1}2x\right)$

$\Rightarrow \sqrt{1-\mathrm{x}^2}=-2x\ \{\text{for } x<0\}$

$\Rightarrow 1-\mathrm{x}^2=4x^2$

$\Rightarrow 5\mathrm{x}^2=1$

$\Rightarrow x=\frac{-1}{\sqrt{5}}\ \{\text{as } x<0\}$

Now solving $\sum_{x\in S}2\sin^{-1}\left(x^2-1\right)=2\sin^{-1}\left(\frac{1}{5}-1\right)=2\sin^{-1}\left(\frac{-4}{5}\right)$

$\Rightarrow \sum_{x\in S}2\sin^{-1}\left(x^2-1\right)=-\sin^{-1}\left(2\times\frac{4}{5}\times\sqrt{1-\left(\frac{4}{5}\right)^2}\right)$

$\Rightarrow \sum_{x\in S}2\sin^{-1}\left(x^2-1\right)=-\sin^{-1}\left(\frac{8}{5}\times\frac{3}{5}\right)=-\sin^{-1}\left(\frac{24}{25}\right)$

Q.10. A triangle ABC be such that $\cos 2A+\cos 2B+\cos 2C$ is minimum. If inradius of the triangle is 3, then which of the following is correct?

A) Area of triangle is $3\sqrt{3}$ sq. units

B) Perimeter of triangle is $18\sqrt{3}$ units

C) Area of triangle is $2\sqrt{3}$ sq. units

D) Perimeter of triangle is $9\sqrt{3}$ units

Answer: Perimeter of triangle is $18\sqrt{3}$ units

Solution: We know that,

$\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$

i.e., $\min(\cos 2A + \cos 2B + \cos 2C) = -\frac{3}{2}$

So,

$\cos 2A = \cos 2B = \cos 2C = -\frac{1}{2}$

$\Rightarrow A = B = C = 60^\circ$

So, triangle ABC is an equilateral triangle.

Let the side of triangle be a.

Also,

$r = \frac{\Delta}{s} = 3$

$\Rightarrow \frac{2\Delta}{2s} = 3$

$\Rightarrow \frac{2\left(\frac{\sqrt{3}}{4}a^2\right)}{3a} = 3$

$\Rightarrow a = \frac{18}{\sqrt{3}}$

So, perimeter is $= 3a = 18\sqrt{3}$ units.

Area$= \frac{\sqrt{3}}{4}a^2 = \frac{81}{\sqrt{3}} = 27\sqrt{3}$ sq. units

Q.11. Find the remainder when $19^{200} + 23^{200}$ is divided by 49 ?

Answer: 29

Solution: Solving $19^{200} + 23^{200} = (21-2)^{200} + (21+2)^{200}$

$= 2\left\{{}^{200}C_0(21)^{200} \times 2^0 + {}^{200}C_2(21)^{198} \times 2^2 \ldots {}^{200}C_{200}(21)^0 \times 2^{200}\right\}$

Now we know that $21^2 = 441$ is divisible by 49,

So rest of the terms will be divisible by 49

So lets consider for $2\left\{{}^{200}C_{200}(21)^0 \times 2^{200}\right\} = 2^{201}$

Now rewriting the term $2^{201} = \left(2^3\right)^{67} = (7+1)^{67}$

Now $(1+7)^{67} = {}^{67}C_0 \times 1 + {}^{67}C_1 7 + {}^{67}C_2 7^2 \ldots\ldots\ldots$

$= 1 + 67 \times 7 + 49k$

Now when dividing $(1+7)^{67}$ by 49 we consider $1 + 67 \times 7$ as $49k$ is divisible by 49

Now $1 + 67 \times 7 = 470$ which when divided by 49 leaves remainder as 29.

Q.12. Find the number of 3 digit numbers which are divisible by 2 or 3 but not 7

Answer: 515

Solution: We know that total number of three-digit number will be 900,

Now let $n(A)$ be number of three-digit number which are divisible by 2,

$n(B)$ be number of three-digit number which are divisible by 3,

And $n(C)$ be number of three-digit number which are divisible by 7,

Now we have to find $n(A \cup B \cup C) - n(C)$

$\Rightarrow n(A \cup B \cup C) - n(C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) - n(C)$

$\Rightarrow n(A \cup B \cup C) - n(C) = n(A) + n(B) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$\Rightarrow n(A \cup B \cup C) - n(C) = \left[\frac{900}{2}\right] + \left[\frac{900}{3}\right] - \left[\frac{900}{6}\right] - \left[\frac{900}{21}\right] - \left[\frac{900}{14}\right] + \left[\frac{900}{42}\right]$

Where $[.]$ denotes the greatest integer function,

Now $n(A \cup B \cup C) - n(C) = 450 + 300 - 150 - 42 - 64 + 21$

$\Rightarrow n(A \cup B \cup C) - n(C) = 515$

Hence, 515 three-digit number are there which are divisible by $2 \text{ or } 3 \text{ but not divisible by } 7$.

Q.13. Given that $1,\ 3,\ 5,\ x,\ y$ are five observations. Mean of the observations is 5 and variance is 8. Find $x^3 + y^3$.

Answer: 1072

Solution: Given $\frac{1+3+5+x+y}{5} = 5 \Rightarrow x + y = 16$

Also $\frac{1^2+3^2+5^2+x^2+y^2}{5} - 5^2 = 8$

$\Rightarrow 35 + x^2 + y^2 = 165$

$\Rightarrow x^2 + y^2 = 130$

So possible values of $x\ \&\ y$ are $7\ \&\ 9$

Hence, $x^3 + y^3 = 7^3 + 9^3 = 1072$

Q.14. In how many ways the word $\mathrm{ASSASSINATION}$ can be arranged so vowels always come together

Answer: 50400

Solution: In the word $\mathrm{ASSASSINATION}$, we will treat all the vowels that is AAAIIO as one letter only.

Now, we have $(\mathrm{AAAIIO})\mathrm{SSSSNNT}$

This has $8 = (7\ +\ 1)$ letters of which S occurs 4 times, and N occur 2 times,

Thus, the number of ways of arranging these letters $= \frac{8!}{4!2!} = 840$.

Now, 6 vowels in which A occurs 3 times, and I occur two times, they can be arranged in $\frac{6!}{3!2!} = 60 \text{ ways}$.

Therefore, required number of ways$=\ (840\ \times\ 60)\ =\ 50400$.

Q.15. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$, then $f(4) - g(4)$ is equal to

Answer: 12

Solution: Given $f(x)=x^2+g'(1)x+g''(2)$

$\Rightarrow f'(x)=2x+g'(1)$

Also $g(x)=2x+f'(1)$

i.e. $g'(x)=2$ and $g''(x)=0$

So, $g'(1)=2,\ f'(1)=4$

$f(x)=x^2+2x$

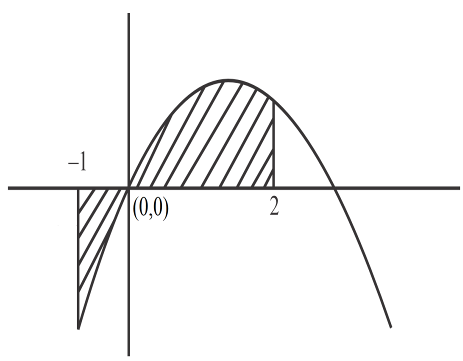
and $g(x)=2x+4$

Hence, $f(4)-g(4)=16-4=12$

Q.16. If the area bounded by $y=x|x-3|$ and $x-$axis between $x=-1$ and $x=2$ is A, then the value of $12A$ is

Answer: 62

Solution: Plotting the $y=x|x-3|$ and $x-$axis between $x=-1$ and $x=2$ we get,



Now area of the above region is given by $A=\left|\int_{-1}^{0}3x-x^2\,\mathrm{d}x\right|+\int_0^2 3x-x^2\,\mathrm{d}x$

$\Rightarrow A=\left|\left[3\times\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{0}\right|+\left[3\times\frac{x^2}{2}-\frac{x^3}{3}\right]_0^2$

$\Rightarrow A=\frac{11}{6}+\frac{10}{3}=\frac{11+20}{6}=\frac{31}{6}$

Hence, the value of $12A=62$

Q.17. If $8,\ a_1,\ a_2,\ a_3.........a_n$ are terms of an $A.P$ whose sum of first four terms is 50 and last four terms of series is 170 then the product of middle terms of the series is

Answer: 754

Solution: Given,

Series $8,\ a_1,\ a_2,\ a_3.........a_n$ whose sum of first four term is 50,

So by formula of sum of $A.P$ we get,

$\frac{4}{2}[2 \times 8 + 3d] = 50$

$\Rightarrow 16 + 3d = 25$

$\Rightarrow d = 3$

Also given sum of last four terms of the series is 170

Again by sum of $A.P$ formula by taking last term as first term we get,

$\frac{4}{2}[2a_n + 3d] = 170$

$\Rightarrow \frac{4}{2}[2(8 + (n-4)d) + 3d] = 170$

$\Rightarrow [2(8 + (n-4)3) + 3 \times 3] = 85$

$\Rightarrow 6(n-4) + 25 = 85$

$\Rightarrow (n-4) = 10 \Rightarrow n = 14$

Hence, T_7 & T_8 will be middle term,

So product of terms will be $(8 + 6 \times 3)(8 + 7 \times 3) = 26 \times 29 = 754$.

Q.18. For some values of λ, system of equations

$\lambda x + y + z = 1$
$x + \lambda y + z = 1$
$x + y + \lambda z = 1$

has no solution, then $\sum\left(|\lambda|^2 + |\lambda|\right)$ is

Answer: 6

Solution: System of equations

$\lambda x + y + z = 1$
$x + \lambda y + z = 1$
$x + y + \lambda z = 1$

has no solution, so

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \lambda+2 & 2+\lambda & 2+\lambda \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow (2+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow (2+\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 1 & \lambda-1 & 0 \\ 1 & 0 & \lambda-1 \end{vmatrix} = 0$$

$\Rightarrow (2+\lambda)(\lambda-1)^2 = 0$

$\Rightarrow \lambda = 1,\ -2$

For $\lambda = 1$, we get infinite solutions, since equation will be identical.

So,

$\sum\left(|\lambda|^2 + |\lambda|\right) = 4 + 2 = 6$