# JEE Main Exam 2023 - Session 1

## 31 Jan 2023 - Shift 2 (Memory-Based Questions)

## Section A: Physics

Q.1. Match the type of radiation listed in column-I with their uses listed in column-II correctly.

| Column-I | | Column_II | |
|---|---|---|---|
| A) | UV rays | P) | Physiotherapy |
| B) | Infrared rays | Q) | Treatment of cancer |
| C) | X-Rays | R) | Lasik eye surgery |
| D) | Microwave rays | S) | Aircraft navigation |

A) A-S, B-P, C-R, D-Q

B) A-R, B-P, C-Q, D-S

C) A-Q, B-P, C-S, D-R

D) A-R, B-P, C-S, D-Q

Answer: A-R, B-P, C-Q, D-S

Solution: Lasers emitting UV rays are used in Lasik surgery. These UV rays have high energy can shape the collagen tissues that make up the cornea without heating it.

Infrared rays are used in physiotherapy. These rays warm up muscle fibres and thus helps in pain relief.

X-rays are used in treatment of cancer. These high energy rays can pe focused on cancer cells to eliminate them.

Microwave rays are used for radars and help in aircraft navigation.

Hence, option B is correct.

Q.2. Two balls are projected with equal speed $\left(40 \text{ m s}^{-1}\right)$, one at an angle of projection of $30^\circ$ and other at an angle of projection of $60^\circ$. Find the ratio of maximum height of both the balls.

A) $\frac{1}{4}$

B) $\frac{3}{1}$

C) $\frac{1}{3}$

D) $\frac{4}{1}$

Answer: $\frac{1}{3}$

Solution: The maximum height of a projectile is given by

$$h=\frac{u^2 \sin^2\theta}{2g}.$$

If $h_1$ is the height of the projectile projected at an angle of projection of $30°$ and $h_2$ is the height of the projectile projected at an angle of projection of $60°$,

$$\frac{h_1}{h_2}=\frac{u^2\sin^2 30^\circ}{u^2\sin^2 60^\circ}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$$

Hence, option C is correct.

Q.3. During an adiabatic process performed on a diatomic gas, $725 \text{ J}$ of work is done on the gas. The change in internal energy of the gas is equal to

A) $495 \text{ J}$

B) $725 \text{ J}$

C) $225 \text{ J}$

D) Zero

Answer: $725 \text{ J}$

Solution: Since the process is adiabatic, the heat given to the gas, $Q=0$. From first law of thermodynamics,

$$Q=\Delta U+W$$

(where $\Delta U$ is change in internal energy of the gas and $W$ is work done by the gas).

Therefore,

$$0=\Delta U+W$$
$$\Rightarrow \Delta U=-W=-(-725 \text{ J})=725 \text{ J}$$

Hence, option B is correct.

Q.4. Find ionisation energy of 2nd exited state of $Li^{2+}$. It is given that ionisation energy of ground state of hydrogen atom is $13.6 \text{ eV}$.

| | | | |
|---|---|---|---|
| A) $240\ \text{eV}$ | B) $27.2\ \text{eV}$ | C) $6.8\ \text{eV}$ | D) $13.6\ \text{eV}$ |

Answer: $13.6\ \text{eV}$

Solution: The energy of an electron in $n^{th}$ orbit $E$ is related to atomic number $Z$ as

$$E=-E_{\circ}\frac{Z^2}{n^2}$$

(where $E_{\circ}=13.6\ \text{eV}$)

Thus, the energy required for ionisation of $Li^{2+}$ from its second excited state is

$$\Delta E=-E_o\times 3^2\left(\frac{1}{\infty^2}-\frac{1}{3^2}\right)=\frac{9}{9}E_{\circ}=13.6\ \text{eV}$$

Hence, option D is correct.

Q.5. Match the physical quantities given in Column-I with their dimensions in Column-II

| Column-I | | Column_II | |
|---|---|---|---|
| A) | Torque | P) | $\mathrm{ML^{-1}T^{-2}}$ |
| B) | Stress | Q) | $\mathrm{ML^{2}T^{-2}}$ |
| C) | Pressure Gradient | R) | $\mathrm{ML^{-2}T^{-2}}$ |
| D) | Angular momentum | S) | $\mathrm{ML^{2}T^{-1}}$ |

A) A-S, B-P, C-R, D-Q B) A-Q, B-P, C-R, D-S

C) A-P, B-S, C-R, D-Q D) A-Q, B-P, C-S, D-R

Answer: A-Q, B-P, C-R, D-S

Solution: The dimension of torque $\tau$ is

$$[\tau]=[F]\,[L]=\left[\mathrm{ML^2T^{-2}}\right]$$

The dimension of stress $\sigma$ is

$$[\sigma]=\frac{[F]}{[A]}=\left[\mathrm{ML^{-1}T^{-2}}\right]$$

The dimension of pressure gradient is

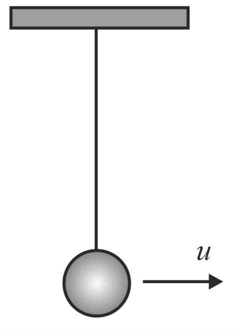
$$\frac{[P]}{[L]}=\left[\mathrm{ML^{-2}T^{-2}}\right]$$

The dimension of angular momentum is

$$[L]=[L]\,[P]=\left[\mathrm{ML^2T^{-1}}\right]$$

Hence, option B is correct.

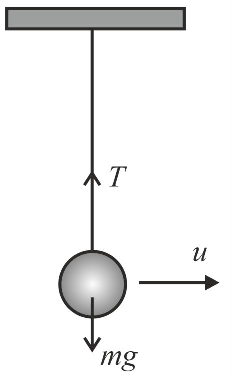
Q.6. A ball of mass $1\ \text{kg}$ is hanging from $1\ \text{m}$ long inextensible string which can withstand a maximum tension of $400\ \text{N}$. Find the maximum horizontal speed $u$ that can be given to the ball.




| | | | |
|---|---|---|---|
| A) $\sqrt{390}\ \text{m s}^{-1}$ | B) $\sqrt{410}\ \text{m s}^{-1}$ | C) $20\ \text{m s}^{-1}$ | D) $22\ \text{m s}^{-1}$ |

Answer: $\sqrt{390}\ \text{m s}^{-1}$

Solution:




Once speed $u$ is given to the ball in horizontal direction, it will start moving in a circular path of radius $r=1\ m$. Applying Newton's second law along vertical direction

$$T-mg=\frac{mu^2}{r}$$

(where $\frac{mu^2}{r}$ is centripetal force)

Since at maximum speed, the tension will be maximum, therefore

$$400-10=u_{max}{}^2$$
$$\Rightarrow u_{max}=\sqrt{390}\ \mathrm{m\ s}^{-1}$$

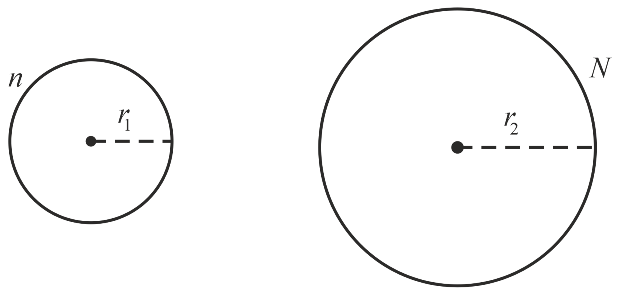
Hence, option A is correct.

Q.7. In a circular coil carrying $i$ current, the turns changed from $n$ to $N$. Then the ratio of initial and final magnetic field at the center of the coil is-

A) $\frac{n}{N}$ B) $\frac{N}{n}$ C) $\frac{n^2}{N^2}$ D) $\frac{N^2}{n^2}$

Answer: $\frac{n^2}{N^2}$

Solution:




As total length of the wire will remain same,

$$L=2\pi r_1 n=2\pi r_2 N$$

Therefore, $\frac{r_2}{r_1}=\frac{n}{N}$

Now required ratio,

$$\frac{B_1}{B_2}=\frac{n\mu_0 i}{2r_1}\times\frac{2r_2}{N\mu_0 i}$$

$$=\frac{n}{N}\times\frac{r_2}{r_1}=\frac{n}{N}\times\frac{n}{N}=\frac{n^2}{N^2}$$

Q.8. In an LCR circuit connected to an AC source of frequency $100\ \mathrm{Hz}$. Find the inductive reactance for inductor of $5\ \mathrm{mH}$.

A) $1.57\ \Omega$ B) $3.14\ \Omega$ C) $6.28\ \Omega$ D) $9.42\ \Omega$

Answer: $3.14\ \Omega$

**Solution:** As we know,

$\omega = 2\pi f = 2 \times 3.14 \times 100 = 628 \text{ rad s}^{-1}$

Then, inductive reactance

$X_L = L\omega = 5 \times 10^{-3} \times 628$

$= 3.14\ \Omega$

Q.9. In an adiabatic process, pressure of an ideal gas becomes $\frac{16}{81}$ times of the initial pressure whereas its volume becomes $\frac{27}{8}$ times. Then its $\frac{Cp}{Cv}$ is

A) $\frac{7}{5}$ B) $\frac{4}{3}$ C) $\frac{5}{3}$ D) $2$

**Answer:** $\frac{4}{3}$

**Solution:** For adiabatic process, $PV^{\gamma} = \text{constant.}$

Therefore, we can write

$PV^{\gamma} = \left(\frac{16P}{81}\right)\left(\frac{27V}{8}\right)^{\gamma}$

$\Rightarrow 1 = \frac{16}{81}\left(\frac{27}{8}\right)^{\gamma}$

$\Rightarrow \left(\left(\frac{3}{2}\right)^{3}\right)^{\gamma} = \left(\frac{3}{2}\right)^{4}$

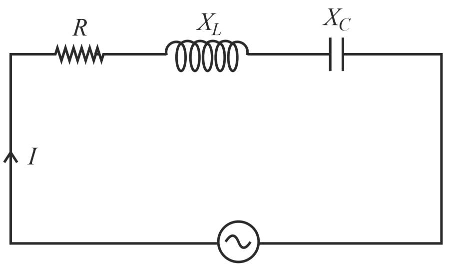
$\Rightarrow 3\gamma = 4$

$\Rightarrow \gamma = \frac{4}{3}$

Q.10. In a series $RLC$ circuit, $R = 80\ \Omega,\ \mathrm{X_L} = 100\ \Omega,\ \mathrm{X_C} = 40\ \Omega$. If the source voltage is given by $V = 2500 \cos(628t)\ \mathrm{V}$, find peak current (in $\mathrm{A}$).

**Answer:** 25

**Solution:**




Since voltage supply is $V = 2500 \cos(628t)\ \mathrm{V}$, the peak voltage $V_\circ = 2500\ \mathrm{V}$.

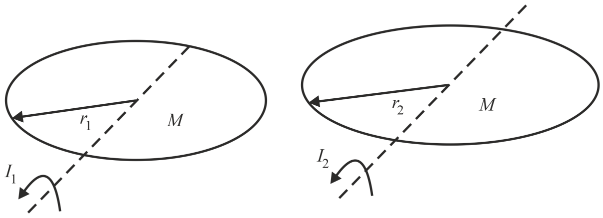
Therefore, the peak current $i_\circ$ is given by

$i_\circ = \frac{V_\circ}{Z} = \frac{V_\circ}{\left[R^2 + (Xc - X_L)^2\right]^{\frac{1}{2}}}$

$= \frac{2500}{\left[80^2 + (100-40)^2\right]^{\frac{1}{2}}} = 25\ \mathrm{A}$

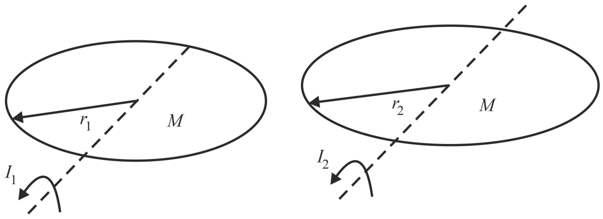
Hence, the correct answer is 25.

Q.11. Two discs of same mass, radii $r_1$ and $r_2$, thickness $1\ \text{mm}$ and $0.5\ \text{mm}$, have densities in ratio $3:1$ and the ratio of their moment of inertia about diameter is $1:x$. Find the value of $x$.




Answer: 6

Solution:




The moment of inertia of a disc about its diameter is:

$I=\frac{1}{4}Mr^2$

Let the densities of the two discs be $3\rho$ and $\rho$.

Since the mass of both discs are same,

$3\rho\times\pi \text{r}_1{}^2\times \text{t}_1=\rho\times\pi \text{r}_2{}^2\times \text{t}_2$ (where $t_1$ and $t_2$ are thickness of the discs)

$\Rightarrow \frac{r_1{}^2}{r_2{}^2}=\frac{t_2}{3t_1}=\frac{1}{6}$ ....(1)

Therefore, the ratio of moment of inertia of the two discs

$\frac{I_1}{I_2}=\frac{\frac{1}{4}Mr_1{}^2}{\frac{1}{4}Mr_2{}^2}=\frac{1}{6}$

Therefore, $x=6$

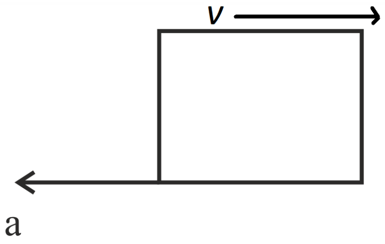
Hence, the correct answer is 6.

Q.12. A body moving horizontally has an initial speed of $20\ \text{m s}^{-1}$. Due to friction, body stops after $5\ \text{s}$. If mass of body is $5\ \text{kg}$, coefficient of friction is $\frac{x}{5}$. Find $x$.

(Take $g=10\ \text{m s}^{-2}$)

Answer: 2

Solution:




If the block is moving towards right, then direction of retardation would be towards left as shown in the figure.

Now,

$a=\frac{f}{m}=\mu g$

Applying equation of motion for constant acceleration, we get

$\Rightarrow v=u-at$

$\Rightarrow \mu g=\frac{20-0}{5}$

$\Rightarrow \ \mu\times 10=4$

$\Rightarrow \ \ \mu=\frac{2}{5}$

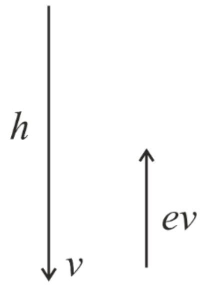
Therefore, $x=2$.

Q.13. A ball was dropped from $20\ \mathrm{m}$ height from ground. Find the height (in $\mathrm{m}$) up to which it rises after the collision.

(Use $e=\frac{1}{2}$, $g=10\ \mathrm{m\ s^{-2}}$)

Answer: 5

Solution:




Magnitude of the velocity after the collision would be, $v^1=ev$.

Therefore, height covered after collision,

$H=\frac{(v')^2}{2g}=\frac{e^2v^2}{2g}$

$\Rightarrow H=e^2h$

$=\frac{1}{4}\times 20$

$=5\ \mathrm{m}$

## Section B: Chemistry

Q.1. For a given hydrocarbon, $11$ moles of $O_2$ is used and produces $4$ moles of $H_2O$. Then the formula of hydrocarbon is:

A) $C_{11}H_8$ B) $C_9H_8$ C) $C_{11}H_{16}$ D) $C_6H_{14}$

Answer: $C_9H_8$

Solution: Balanced equation for combustion is

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \left(\frac{y}{2}\right)H_2O$$

Given moles of water produced is 4, so

$$\frac{y}{2} = 4$$

$$y = 8$$

Moles of oxygen used is 11, so

$$x + \frac{y}{4} = 11$$

$$\Rightarrow\ x = 9$$

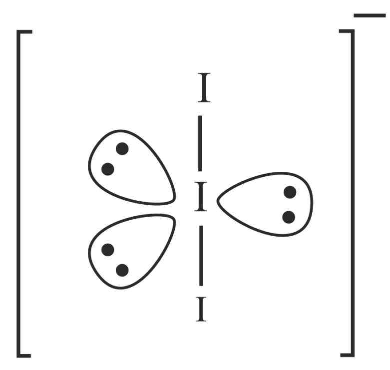
Hence, formula becomes $C_9H_8$

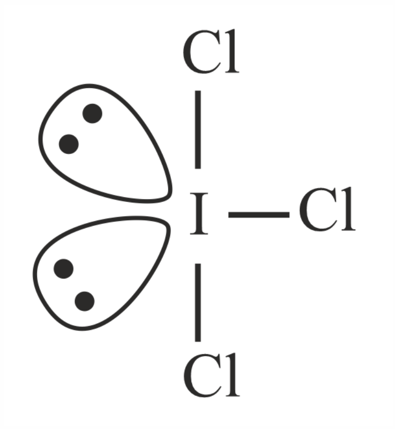
Q.2. Which one of the following species is linear in shape?

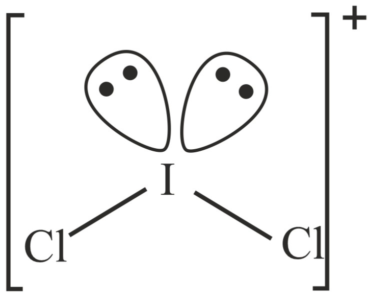
A) $I_3^-$ B) $I_3^+$ C) $ICl_3$ D) $ICl_2^+$

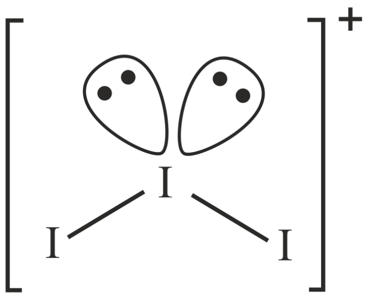
Answer: $I_3^-$

Solution: $I_3^-$ is linear as shown






$I_3^+$ and $ICl_2^+$ are bent as shown above while $ICl_3$ is T-shaped.

Q.3. Which one of the following have important role in neuromuscular functions.

A) Ca B) Mg C) Be D) Li

Answer: Ca

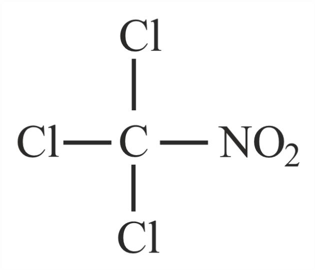
Solution: Calcium plays an important role in neuromuscular functions, interneuronal transmissions, cell membrane integrity and blood coagulation.

Q.4. Which of the following compounds contain maximum number of chlorine atoms?

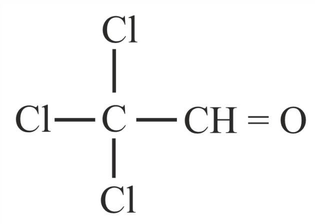
A) Chloropicrin B) Chloral C) Gammexane D) Freon-12

Answer: Gammexane

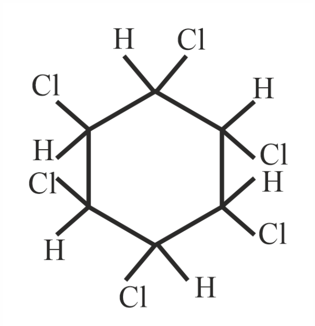
Solution: Chloropicrin




Chloral




Gammexane




Freon-12: $CF_2Cl_2$

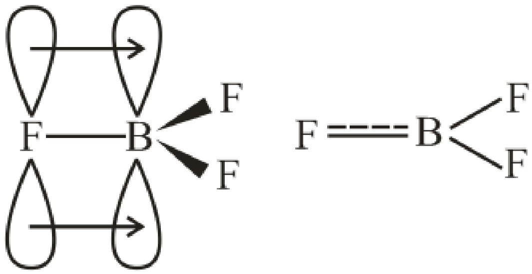
Q.5. The order of acidic strength of boron trihalides

A) $BF_3 < BCl_3 < BBr_3 < BI_3$ B) $BI_3 < BBr_3 < BCl_3 < BF_3$ C) $BCl_3 < BBr_3 < BI_3 < BF_3$ D) $BBr_3 < BCl_3 < BF_3 < BI_3$

Answer: $BF_3 < BCl_3 < BBr_3 < BI_3$

Solution: The strength of acidic character of boron trihalides depends upon $p\pi - p\pi$ back bonding.

In boron trihalides, $p\pi - p\pi$ -back bonding occurs due to empty orbital of boron and filled orbitals of halogens.




Phenomenon of back bonding in $BF_3$ molecule

The $p\pi - p\pi$ back bonding is shown maximum by $BF_3$, as the size of $B$ and $F$ are small and comparatively same. Due to this effect tendency of accepting lone-pair of electrons of boron decreases as size of halogen increases. The order of size of halogens are $F < Cl < Br < I$. Thus, acidic nature is in order

$BF_3 < BCl_3 < BBr_3 < BI_3$

Q.6. Which of the following elements of $f$-block have half-filled $f$-subshell?

1. Samarium (Sm)
2. Gadolinium (Gd)
3. Europium (Eu)
4. Terbium (Tb)

[Atomic Numbers : $Sm = 62, Eu = 63, Gd = 64,\ Tb = 65$]

A) 1 and 2 B) 2 and 3 C) 1 and 3 D) 2 and 4

Answer: 2 and 3

Solution: $f-$subshell contain seven orbitals in it. Hence, if seven electrons are present in these orbitals then it said to be half-filled.

1. Samarium (Sm) 62 $\left[Xe\right] 4f^6 5s^2$
2. Gadolinium (Gd) 64 $[Xe]\ 4\ f^7\ 5d^1\ 6s^2$
3. Europium (Eu) 63 $[Xe]4f^7 6s^2$
4. Terbium (Tb) 65 $[Xe]4f^9 6s^2$

Hence, Gadolinium and Europium are having half-filled f-subshell.

Q.7. $pH$ of acid rain is $5.6$. Which of the following reaction is involved in acid rain.

A) $H_2O + SO_2 + O_2 \longrightarrow H_2SO_4$ B) $N_2 + O_2 + H_2O \longrightarrow HNO_3$ C) $N_2O + O_2 + H_2O \longrightarrow HNO_3$ D) None of these

Answer: $H_2O + SO_2 + O_2 \longrightarrow H_2SO_4$

Solution:

$$2\,SO_2 + O_2 \\ \downarrow \\ 2\,SO_3$$

$$SO_3 + H_2O \\ \downarrow \\ H_2SO_4$$

Dissolved sulphur oxides makes rain acidic

Q.8. If ionization energy of $H$-atom is $13.6\,eV$. Find out ionization energy of $Li^{2+}$ ions.

A) $54.4\,eV$ B) $122.4\,eV$ C) $13.6\,eV$ D) $3.4\,eV$

Answer: $122.4\,eV$

**Solution:** ionisation energy $= -$ (Energy of $n^{th}$ orbit)

Energy of $1^{st}$ orbit of Hydrogen $= -13 \cdot 6\,\text{eV}$

$\therefore$ Energy of $1^{st}$ orbit of $Li^{2+} = -13.6 \times (3)^2$

$= -13.6 \times 9$

$= -122.4\,\text{eV}$

ionisation energy $= -(-122 \cdot 4)$

$= 122.4\,\text{eV}$

Q.9. A reaction follows $1^{st}$ order kinetics with rate constant $(k) = 20\ \text{min}^{-1}$. Calculate the time required to reach the concentration to $\frac{1}{32}$ times of initial concentration.

A) 0.17325 min  B) 1.7325 min  C) 17.325 min  D) 173.25 min

**Answer:** 0.17325 min

**Solution:** Using First order kinetics equation and using k=20 $\text{min}^{-1}$ and $a_0$ as initial concentration

$$t = \frac{2.303}{20}\log_{10}\left(\frac{a_0}{\frac{a_0}{32}}\right)$$

$$= \frac{2.303}{20}\log_{10}(32)$$

$$= \frac{2.303}{20} \times 5 \times 0.3010$$

$$= \frac{0.693}{4}$$

t = 0.17325

Q.10. Which of the following is not a disinfectant?

A) Chloroxylenol  B) Biothionol  C) Terineol  D) Peracetic acid

**Answer:** Peracetic acid

**Solution:** Antiseptics and disinfectants are also the chemicals which either kill or prevent the growth of microorganisms. Antiseptics are applied to the living tissues such as wounds, cuts, ulcers and diseased skin surfaces. Examples are furacine, soframicine, etc. These are not ingested like antibiotics. Commonly used antiseptic, dettol is a mixture of chloroxylenol and terpineol. Bithionol (the compound is also called bithional) is added to soaps to impart antiseptic properties. Peracetic acid is used mainly in the food industry, where it is applied as a cleanser and as a disinfectant.

Q.11. If solubility of $AgCl$ in aqueous solution is $1.434 \times 10^{-3}M$ then find the value of $[-\log K_{sp}]$ where $K_{sp}$ is the solubility product of $AgCl$

A) $3.7$  B) $5.7$  C) $6.7$  D) $7.7$

**Answer:** $5.7$

**Solution:** Assume $S$ is the solubility of silver chloride in water. The equilibrium reaction can be written as follows,

$$\begin{array}{ccccc} AgCl & \rightleftharpoons & Ag^{+} & + & Cl^{-} \\ & & S & & S \end{array}$$

$S = 1.434 \times 10^{-3}$

$K_{sp} = S^2 = (1.434 \times 10^{-3})^2 = 2 \times 10^{-6}$

$-\log(K_{sp}) = -\log 2 + 6$

$= -0.3010 + 6$

$= 5.7$

Q.12. Consider the following combination of $n, l$, and $m$ values.

(i) $n = 3$ ; $l = 0$ ; $m = 0$
(ii) $n = 4; l = 0; m = 0$
(iii) $n = 3; l = 1; m = 0$
(iv) $n = 3$ ; $1 = 2$ ; $m = 0$

The correct order of energy of the corresponding orbitals for multielectron species.

A) (ii) > (i) > (iv) > (iii)

B) (iv) > (ii) > (iii) > (i)

C) (i) > (iii) > (iv) > (ii)

D) (iv) > (iii) > (i) > (ii)

Answer: (iv) > (ii) > (iii) > (i)

Solution: Using (n+l) rule, the energy values of given orbitals are:

$3s = 3 + 0$ = 3

$4s = 4 + 0$ = 4

$3p = 3 + 1$ = 4

$3d = 3 + 2$ = 5

For 4s and 3p, orbital with higher value of n has higher energy.

Q.13. Two metals are given:

Metal -1 : Work function $4.8\ eV$

Metal -2 : Work function $2.8\ eV$

Photons of wavelength $350\ nm$ are incident on both metals separately.

Which metal will eject electrons at this wavelength?

A) Metal-1 only

B) Metal-2 only

C) Both metal-1 and metal-2

D) None of the metal-1 and metal-2

Answer: Metal-2 only

Solution: To eject electrons the required condition is

Radiation energy(incident light energy) > Work function.

The radiation energy in eV can be calculated using the following formula.

$$E_{photon} = \frac{12400}{\lambda(A^{\circ})} eV$$

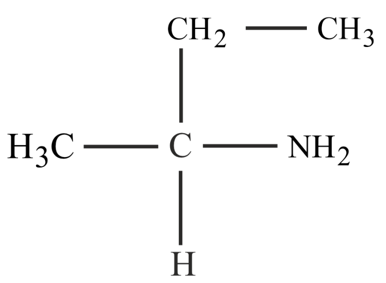
$$\frac{12400}{3500} = 3.54\,eV$$

Hence, the radiation energy is greater than metal-2 and less than metal-1.

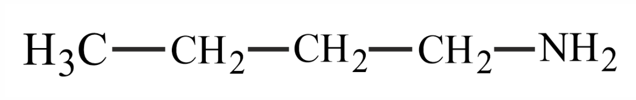
Hence, electron only ejected from metal-2.

Q.14. What is the structural formula of compound $C_4H_{11}N$, which reacts with $HNO_2$ and is optically active?

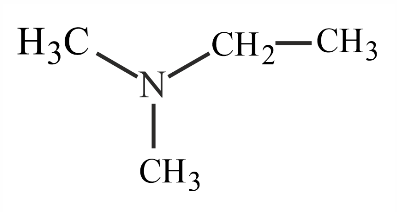
A)



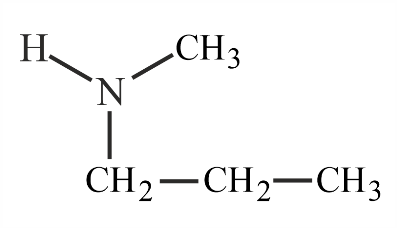
B)



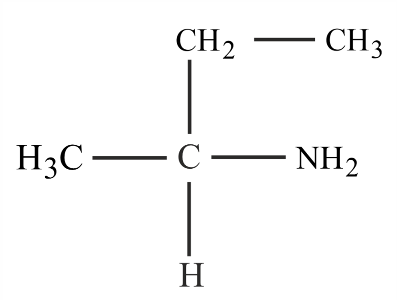
C)



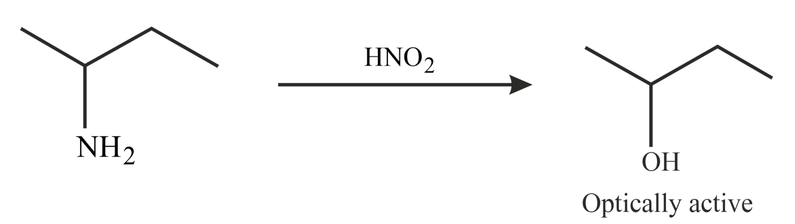
D)



Answer:



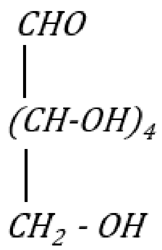
Solution: Primary amines when reacts with nitrous acid give alcohol. The chiral primary amine produce chiral alcohol. The molecular formula $C_4H_{11}N$ exhibits four primary amines. Out of them 2-aminobutane is chiral(optically active). Hence, it produces optically active alcohol.



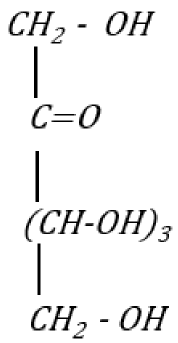

Q.15. A biomolecule gives the following observations

(I) With $Br_2/H_2O$, it gives mono carboxylic acid
(II) With acetate, it gives tetraacetate
(III) With $HI$/Red $P$, it gives isopentane
The correct structure of biomolecule is:

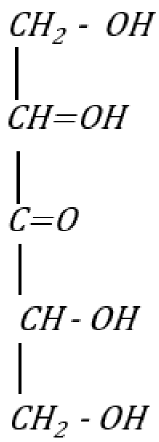
A)


CHO
(CH-OH)4
CH2 - OH

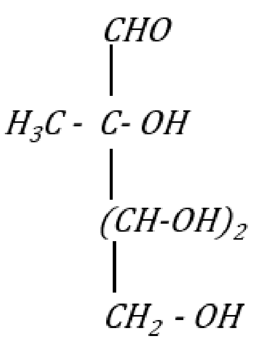
B)


CH2 - OH
C=O
(CH-OH)3
CH2 - OH

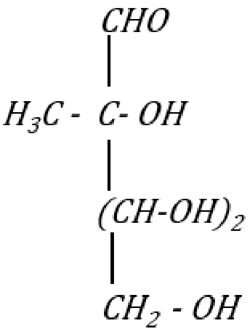
C)


CH2 - OH
CH=OH
C=O
CH - OH
CH2 - OH

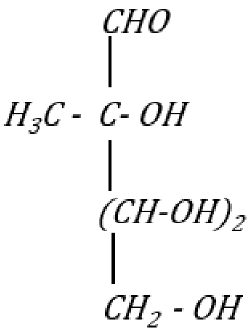
D)


CHO
H3C - C- OH
(CH-OH)2
CH2 - OH

Answer:




Solution:




Above compound with $\mathrm{Br_2/H_2}O$ --CHO becomes - $\mathrm{COOH}$ giving mono-carboxylic acid.

With $\mathrm{(CH_3COO)_2O}$, it gives tetra acetate due to the presence of $4\,(-\mathrm{OH})$ groups.

With, $\mathrm{HI/P_4}$, it gives $\mathrm{CH_3-\underset{|}{\overset{CH_3}{CH_3}}-CH_3-CH_3-CH_3}$

Q.16. $C+O_2 \rightarrow CO_2(g)$

12g of $C$ is reacted with 48g of $O_2$. If volume of $CO_2$ gas produced at STP is $t$L. Find out $2t$.

Answer: 45

Solution: $C+O_2 \rightarrow CO_2$

Given : $12g \quad 48g \quad tL$

Mole : $\frac{12}{12} \quad \frac{48}{32}$

$= 1 \text{ mole} \quad 1.5 \text{ moles}$

$\because$ 1 mole of carbon requires 1 mole of $O_2$ but it is given in excess, so product will be formed as per carbon as limiting reagent.

$\therefore$moles of $CO_2$ produced = 1 mole.

$\therefore V_{CO_2}$ at STP,

Moles of carbon-dioxide produced $= \frac{V_{CO_2}}{22.4}$

or $1 = \frac{t}{22.4}$

$\therefore t = 22.4L$

$\therefore 2t = 44.8L \approx 45L$

## Section C: Mathematics

Q.1. If the eccentricity of hyperbola is $\sqrt{2}$ having foci at $\left(1 \pm \sqrt{2}, 0\right)$, then the length of latus rectum is

A) $2$ B) $4$ C) $1$ D) $\sqrt{2}$

Answer: $2$

Solution: Distance between foci of hyperbola is

$2ae=2\sqrt{2}$

$\Rightarrow ae=\sqrt{2}$

$\Rightarrow a\sqrt{2}=\sqrt{2}$

$\Rightarrow a=1$

Also for hyperbola,

$e=\sqrt{1+\frac{b^2}{a^2}}$

$\Rightarrow \sqrt{2}=\sqrt{1+\frac{b^2}{1^2}}$

$\Rightarrow b^2=1$

So, length of latus rectum is$=\frac{2b^2}{a}=2$

Q.2. If $\vec{a}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$, $\vec{b}=\hat{\imath}-\hat{\jmath}+3\hat{k}$ and $\vec{c}=\hat{\imath}+2\hat{\jmath}+2\hat{k}$. Also, $\vec{u}\times\vec{a}=\vec{b}\times\vec{c}$ and $\vec{u}\cdot\vec{a}=0$, then $25|\vec{u}|^2=$ _____

A) $\frac{925}{7}$ B) $\frac{925}{6}$ C) $\frac{825}{7}$ D) $\frac{924}{7}$

Answer: $\frac{925}{7}$

Solution: We have,

$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & 3\\ 1 & 2 & 2\end{vmatrix}$

$\Rightarrow \vec{b}\times\vec{c}=-8\hat{\imath}+\hat{\jmath}+3\hat{k}$

$\Rightarrow |\vec{b}\times\vec{c}|^2=64+1+9$

$\Rightarrow |\vec{b}\times\vec{c}|^2=74$

We know that,

$|\vec{u}\times\vec{a}|^2+|\vec{u}\cdot\vec{a}|^2=|\vec{u}|^2|\vec{a}|^2$

$\Rightarrow |\vec{u}\times\vec{a}|^2=|\vec{u}|^2|\vec{a}|^2$

$\Rightarrow |\vec{b}\times\vec{c}|^2=|\vec{u}|^2(1+4+9)$

$\Rightarrow 74=14|\vec{u}|^2$

$\Rightarrow |\vec{u}|^2=\frac{74}{14}$

$\Rightarrow 25|\vec{u}|^2=\frac{925}{7}$

Q.3. The range of $y=\frac{x^2+2x+1}{x^2+8x+1}$ is $(x\in R)$ is

A) $\left(-\infty,-\frac{2}{3}\right]\cup\left[2,\infty\right)$ B) $\left(-\infty,0\right]\cup\left[\frac{2}{5},\infty\right)$ C) $(-\infty,\infty)$ D) $\left(-\infty,-\frac{2}{5}\right]\cup\left[1,\infty\right)$

Answer: $\left(-\infty,0\right]\cup\left[\frac{2}{5},\infty\right)$

**Solution:** Let

$y=\frac{x^2+2x+1}{x^2+8x+1};\ x^2+8x+1\neq 0$

$\Rightarrow yx^2+8xy+y=x^2+2x+1$

$\Rightarrow (y-1)x^2+(8y-2)x+y-1=0$

So,

$D\geq 0$

$\Rightarrow 4(4y-1)^2-4(y-1)^2\geq 0$

$\Rightarrow (4y-1)^2-(y-1)^2\geq 0$

$\Rightarrow 15y^2-6y\geq 0$

$\Rightarrow y(5y-2)\geq 0$

So, $y\in\left(-\infty,0\right]\cup\left[\frac{2}{5},\infty\right)$

Q.4. If $\int\frac{x}{\sqrt{x^2+x+2}}dx=Af(x)+Bg(x)+C$, where $C$ is constant of integration, then $A+2B$ is

A) $1$ B) $0$ C) $-1$ D) $-2$

**Answer:** $0$

**Solution:** Let

$I=\int\frac{x}{\sqrt{x^2+x+2}}dx$

$\Rightarrow I=\frac{1}{2}\int\frac{2x+1-1}{\sqrt{x^2+x+2}}dx$

$\Rightarrow I=\frac{1}{2}\int\left[\frac{2x+1}{\sqrt{x^2+x+2}}-\frac{1}{\sqrt{x^2+x+2}}\right]dx$

$\Rightarrow I=\frac{1}{2}\int\frac{d\left(x^2+x+2\right)}{\sqrt{x^2+x+2}}-\frac{1}{2}\int\frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{7}{4}}}dx$

$\Rightarrow I=\sqrt{x^2+x+2}-\frac{1}{2}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+2}\right|+C$

So, $A=1,\ B=-\frac{1}{2}\Rightarrow A+2B=0$

Q.5. If $a,\ b\ \in\ I$ and relation $R_1$ is defined as $a^2-b^2\in I$ and relation $R_2$ is defined as $2+\frac{a}{b}>0$, then

A) $R_2$ is symmetric but $R_1$ is not

B) $R_1$ is symmetric but $R_2$ is not

C) $R_1$ & $R_2$ both are symmetric

D) $R_1$ & $R_2$ both are transitive

**Answer:** $R_1$ is symmetric but $R_2$ is not

**Solution:** Given,

$a,\ b\ \in\ I$ and relation $R_1$ is defined as $a^2 - b^2 \in I$

Now if $a^2 - b^2 \in I$ so $b^2 - a^2$ will also be integer, for example if $5^2 - 4^2 = 9$ is integer then $4^2 - 5^2 = -9$ is also a integer,

Now checking transitive,

If $4^2 - 3^2 = 7$ is an integer, $3^2 - 2^2 = 5$ is an integer then $4^2 - 2^2 = 12$ is also an integer, hence we can say that $R_1$ is symmetric and transitive,

Now checking relation $R_2$ which is defined as $2 + \frac{a}{b} > 0$,

So, if we replace $\frac{a}{b}$ by $\frac{-1}{9}$ then $2 + \frac{a}{b} > 0$ is true but we take $\frac{b}{a} = -9$ for symmetric we get $2 + \frac{b}{a} = 2 - 9 = -7 \not> 0$ hence, the relation is not symmetric,

Now checking transitive, now if $2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2$ and $\frac{b}{c} > -2$ then we cannot say that $\frac{a}{c} > -2$,

For example if we take $\frac{4}{1} > -2,\ \frac{1}{-1} > -2$ then $\frac{4}{-1} \not> -2$, hence it is not transitive,

Hence, we can say that $R_1$ is symmetric and $R_2$ is not.

Q.6. Foot of perpendicular from origin$(O)$ to a plane which cuts the coordinate axes at $A,\ B,\ C$ is $(2, a, 4)$. Area of tetrahedron $OABC$ is $144\ m^2$. Which of the following point does not lie on the plane?

A) $(2, 2, 4)$ B) $(0, 3, 4)$ C) $(1, 1, 5)$ D) $(5, 5, 1)$

**Answer:** $(0, 3, 4)$

**Solution:** Foot of perpendicular from origin to a plane which cuts the coordinate axes at $A,\ B,\ C$ is $(2, a, 4)$. So, point on the plane is $(2, a, 4)$ and direction ratios of normal of the plane is $<2, a, 4>$.

Hence,

$2(x-2) + a(y-a) + 4(z-4) = 0$

$\Rightarrow 2x + ay + 4z = 20 + a^2 \quad ....(1)$

So,

$$A\left(\frac{20+a^2}{2}, 0, 0\right);\ B\left(0, \frac{20+a^2}{a}, 0\right);\ C\left(0, 0, \frac{20+a^2}{4}\right)$$

Area of tetrahedron $OABC$ is $144\ m^2$, so

$$\Rightarrow \frac{1}{6}\left(\frac{20+a^2}{2}\right)\left(\frac{20+a^2}{a}\right)\left(\frac{20+a^2}{4}\right) = 144$$

$$\Rightarrow \left(20 + a^2\right)^3 = 144 \times 48a$$

$\Rightarrow a = 2$

So, equation of plane is

$x + y + 2z = 12$

So, point $(0, 3, 4)$ does not lie on the plane.

Q.7. Given that $\theta \in [0, 2\pi]$, then the largest interval of values of $\theta$ satisfying the inequation $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) \geq 0$ is:

A) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ B) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ C) $[0, \pi]$ D) $\left[\frac{\pi}{2}, \frac{5\pi}{4}\right]$

**Answer:** $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

Solution: Given:

$\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) \geq 0$

$\Rightarrow \sin^{-1}(\sin\theta) - \left(\frac{\pi}{2} - \sin^{-1}(\sin\theta)\right) \geq 0$

$\Rightarrow 2\sin^{-1}(\sin\theta) \geq \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(\sin\theta) \geq \frac{\pi}{4}$

So,

$\frac{\pi}{4} \leq \sin^{-1}(\sin\theta) \leq \frac{\pi}{2}$

$\Rightarrow \sin\left(\frac{\pi}{4}\right) \leq \sin\theta \leq \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{1}{\sqrt{2}} \leq \sin\theta \leq 1$

Since, $\theta \in [0, 2\pi]$ so largest interval $\theta$ can take from options will be $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

Q.8. Find the value of limit $\lim_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$

A) $27$ B) $\frac{27}{2}$ C) $18$ D) $6$

Answer: $27$

Solution: Let,

$$L = \lim_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$$

$$= \lim_{x\to\infty} \frac{\left(x\sqrt{3+\frac{1}{x^2}}+x\sqrt{3-\frac{1}{x^2}}\right)^6}{x^6\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+x^6\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6}$$

$$= \lim_{x\to\infty} \frac{x^6\left(\sqrt{3+\frac{1}{x^2}}+\sqrt{3-\frac{1}{x^2}}\right)^6}{x^6\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+x^6\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{3+\frac{1}{x^2}}+\sqrt{3-\frac{1}{x^2}}\right)^6}{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6}$$

Now putting the value of limit we get,

$$L = \frac{(\sqrt{3+0}+\sqrt{3-0})^6}{(1+\sqrt{1-0})^6+(1-\sqrt{1-0})^6}$$

$$\Rightarrow L = \frac{\left(\sqrt{3}+\sqrt{3}\right)^6}{(1+1)^6+(1-1)^6}$$

$$\Rightarrow L = \frac{2^6\left(\sqrt{3}\right)^6}{2^6} = 27.$$

Q.9. If $z=\frac{-1+i}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$ then $z=$

A) $\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$ B) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$ C) $\frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$ D) $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

Answer: $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

Solution: Given $z=\frac{-1+i}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$

i.e. $z=\frac{-1+i}{\frac{1}{2}+i\frac{\sqrt{3}}{2}}$

$=2\left(\frac{-1+i}{1+i\sqrt{3}}\right)$

$=2\left(\frac{-1+i}{1+i\sqrt{3}}\right)\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$=2\left(\frac{-1+i+i\sqrt{3}+\sqrt{3}}{4}\right)=\frac{\sqrt{3}-1+i\left(1+\sqrt{3}\right)}{2}$

$=\sqrt{2}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}+i\frac{1+\sqrt{3}}{2\sqrt{2}}\right)$

$=\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

Q.10. If $((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$ is tautology, where $r\in\{p,q,\sim p,\sim q\}$, then number of values of $r$ is/are:

A) 1 B) 2 C) 3 D) 4

Answer: 2

Solution: Given,

$((p\wedge q)\Rightarrow(r\vee q))\wedge((p\wedge r)\Rightarrow q)$

$=(\sim(p\wedge q)\vee(r\vee q))\wedge(\sim(p\wedge r)\vee q)$

$=((\sim p\vee\sim q)\vee(r\vee q))\wedge((\sim p\vee\sim r)\vee q)$

$=((\sim p\vee r)\vee(\sim q\vee q))\wedge((\sim p\vee\sim r)\vee q)$

$=((\sim p\vee r)\vee \mathrm{U})\wedge((\sim p\vee\sim r)\vee q)$

$=\mathrm{U}\wedge((\sim p\vee\sim r)\vee q)$

$=(\sim p\vee\sim r)\vee q$

Now given $\sim p\vee q\vee\sim r$ is tautology,

Now for $\sim p\vee q\vee\sim r$ to be tautology $r$ can take value as $\sim p$ or $q$,

As for $r=\sim p$ expression $(\sim p\vee\sim r)\vee q$ becomes $=(\sim p\vee p)\vee q=\mathrm{U}\vee q=\mathrm{U}$ and for $r=q$ the expression $(\sim p\vee\sim r)\vee q$ becomes $\sim p\vee\sim q\vee q=\sim p\vee\mathrm{U}=\mathrm{U}$,

Hence, $r$ can take two values.

Q.11. If $[\alpha\ \ \beta\ \ \gamma]\begin{bmatrix}5&6&8\\6&3&8\\-1&3&0\end{bmatrix}=[0\ \ 0\ \ 0]$ where, point $(\alpha,\beta,\gamma)$ lie on the plane $2x+5y+3z=5$, then $6\alpha+5\beta+9\gamma=$

A) 20 B) $\frac{20}{3}$ C) 21 D) 7

Answer: $\frac{20}{3}$

Solution: Given:

$$[\alpha \quad \beta \quad \gamma]\begin{bmatrix} 5 & 6 & 8 \\ 6 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix} = [0 \quad 0 \quad 0]$$

So,
$5\alpha + 6\beta - \gamma = 0$
$6\alpha + 3\beta + 3\gamma = 0$
$8\alpha + 8\beta = 0 \Rightarrow \alpha = -\beta$

Hence,

$\beta = \gamma$

So,

$(\alpha, \beta, \gamma) \equiv (-\gamma, \gamma, \gamma)$

It lies on plane $2x + 5y + 3z = 5$, so

$-2\gamma + 5\gamma + 3\gamma = 5$

$\Rightarrow \gamma = \frac{5}{6}$

So, point is $(\alpha, \beta, \gamma) \equiv \left(-\frac{5}{6}, \frac{5}{6}, \frac{5}{6}\right)$

Hence,

$6\alpha + 5\beta + 9\gamma = -5 + \frac{25}{6} + \frac{45}{6} = \frac{20}{3}$

Q.12. Find the coefficient of $x^{-6}$ in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$

Answer: 5040

Solution: Given,

$\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$

Now $r^{th}$ term of the expression is given by,

$T_{r+1} = {}^9C_r\left(\frac{4x}{5}\right)^{9-r}\left(\frac{5}{2x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^9C_r\left(\frac{4}{5}\right)^{9-r}\left(\frac{5}{2}\right)^r x^{9-r-2r}$

Now equating $9 - r - 2r = -6 \Rightarrow r = 5$

So, coefficient of $x^{-6}$ will be,

$= {}^9C_5\left(\frac{4}{5}\right)^{9-5}\left(\frac{5}{2}\right)^5$

$= \frac{9\times8\times7\times6}{4\times3\times2\times1} \times \left(\frac{4}{5}\right)^4\left(\frac{5}{2}\right)^5$

$= 5040$

Q.13. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$ then find the value of $n^2 + n + 15$

Answer: 45

**Solution:** Given,

$$\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_{n}} = \frac{11}{21}$$

$$\Rightarrow \frac{\frac{(2n+1)!}{(n+2)!}}{\frac{(2n-1)!}{(n-1)!}} = \frac{11}{21}$$

$$\Rightarrow 21 \times \frac{(2n+1)2n(2n-1)!}{(n+2)(n+1)n(n-1)!} = 11 \times \frac{(2n-1)!}{(n-1)!}$$

$$\Rightarrow 21 \times 2\,(2n+1) = 11\,(n+2)\,(n+1)$$

$$\Rightarrow \frac{(2n+1)}{(n+2)(n+1)} = \frac{11}{42}$$

Now on comparing both side we get, $n = 5$

So, the value of $n^2 + n + 15 = 5^2 + 5 + 15 = 45$

Q.14. Find the value of $[S]$, if $S = 1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 \ldots\ldots\ldots\ldots + 15 \cdot 29^2$

where $[.]$ represents greatest integer function

**Answer:** 6952

**Solution:** Given,

$S = 1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 \ldots\ldots\ldots\ldots + 15 \cdot 29^2$

Now rewriting the above expression we get,

$$S = \underbrace{1^2 + 2 \cdot 3^2 + 3 \cdot 5^2 \ldots\ldots\ldots\ldots + 15 \cdot 29^2}_{S_1} - 2\underbrace{\left[2 \cdot 3^2 + 4 \cdot 7^2 \ldots\ldots\ldots\ldots + 14 \cdot 27^2\right]}_{S_2}$$

Now solving $S_1 = 1^2 + 2 \cdot 3^2 + 3 \cdot 5^2 \ldots\ldots\ldots\ldots + 15 \cdot 29^2$

$$\Rightarrow S_1 = \sum_{r=1}^{15} r \cdot (2r-1)^2$$

$$\Rightarrow S_1 = \sum_{r=1}^{15} r \cdot \left(4r^2 + 1 - 4r\right)$$

$$\Rightarrow S_1 = \sum_{r=1}^{15} \left(4r^3 + r - 4r^2\right)$$

$$\Rightarrow S_1 = \left(4\sum_{r=1}^{15} r^3 + \sum_{r=1}^{15} r - 4\sum_{r=1}^{15} r^2\right)$$

$$\Rightarrow S_1 = \left(4\left(\frac{15\times16}{2}\right)^2 + \frac{15\times16}{2} - 4\frac{15\times16\times31}{6}\right)$$

$$\Rightarrow S_1 = \left(4(120)^2 + 120 - 4960\right)$$

$$\Rightarrow S_1 = 52760$$

Now solving $S_2 = 2 \cdot 3^2 + 4 \cdot 7^2 \ldots\ldots\ldots\ldots + 14 \cdot 27^2$

$$\Rightarrow S_2 = \sum_{r=1}^{7} (2r)(4r-1)^2$$

$$\Rightarrow S_2 = \sum_{r=1}^{7} (2r)\left(16r^2 + 1 - 8r\right)$$

$$\Rightarrow S_2 = 32\sum_{r=1}^{7} r^3 + 2\sum_{r=1}^{7} r - 16\sum_{r=1}^{7} r^2$$

$$\Rightarrow S_2 = 32\left(\frac{7\times8}{2}\right)^2 + 2\left(\frac{7\times8}{2}\right) - 16\left(\frac{7\times8\times15}{6}\right)$$

$$\Rightarrow S_2 = 32 \times 784 + 56 - 16 \times 140 = 22904$$

Hence, $S = S_1 - 2S_2 = 52760 - 2 \times 22904 = 6952$

Q.15. If the minimum value of $\left|x^2-x+1\right|+\left[x^2-x+1\right]$ $\{\text{where } [.] \text{ denotes the greatest integer function}\}$ for $x\in[-1,2]$ is $k$, then value of $4k$ is

Answer: 3

Solution: Given,

Function $g(x)=\left|x^2-x+1\right|+\left[x^2-x+1\right]$

Now solving $f(x)=x^2-x+1=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$

So, $f(x)_{\text{min}}=\frac{3}{4}$ at $x=\frac{1}{2}$

Now putting the value of $f(x)$ in given function $g(x)$ we get,

$g(x)=\left|\frac{3}{4}\right|+\left[\frac{3}{4}\right]$

$\Rightarrow g(x)=\left|\frac{3}{4}\right|+[0.75]$

$\Rightarrow g(x)=\left|\frac{3}{4}\right|+0=\frac{3}{4}$

Hence, minimum value of $\left|x^2-x+1\right|+\left[x^2-x+1\right]=\frac{3}{4}=k$,

Hence, $4k=3$