JEE Main Exam 2023 - Session 1

31 Jan 2023 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. The ratio of molar specific heat capacity at constant pressure (c_p) to the molar specific heat capacity at constant volume (c_v) for a given gas varies with temperature (T) as:

[Assume temperature to be low]

A) T^0 B) $T^{1/2}$ C) T^1 D) $T^{3/2}$

Answer: T^0

Solution: The ratio of molar specific heat capacity at constant pressure to the molar specific heat capacity at constant volume is

$\frac{cp}{cv} = \gamma$

where γ is constant for a given gas.

Therefore,

$\frac{cp}{cv} \propto T^0$

Hence, option A is correct.

Q.2. If n is number density of charge carriers, A is cross-sectional area of conductor, q is charge on each charge carrier and I is current through the conductor, then the expression of drift velocity is:

A) $\frac{nAq}{I}$ B) $\frac{I}{nAq}$ C) $nAqI$ D) $\frac{IA}{nq}$

Answer: $\frac{I}{nAq}$

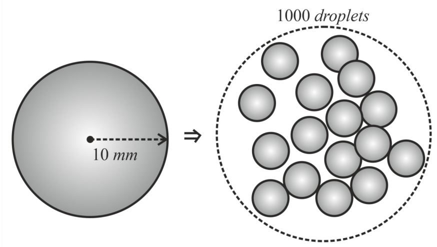
Solution: The current I flowing through a conductor depends on the number density of charge carriers n, area of cross-section A and charge on each carrrier q as

$I = nq\mathrm{A}v_d$

$\Rightarrow v_d = \frac{I}{nqA}$

Hence, option B is correct.

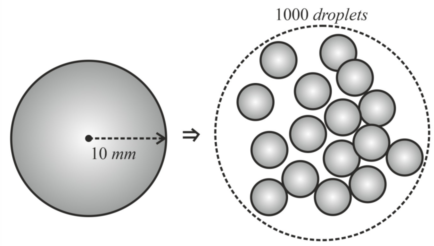
Q.3. A drop of water of $10\ \mathrm{mm}$ radius is divided into 1000 droplets. If surface tension of water surface is equal to $0.073\ \mathrm{J\ m^{-2}}$, then increment in surface energy while breaking down the bigger drop in small droplets as mentioned is equal to:



A) $8.25 \times 10^{-5}\ \mathrm{J}$ B) $9.17 \times 10^{-4}\ \mathrm{J}$ C) $9.17 \times 10^{-5}\ \mathrm{J}$ D) $8.25 \times 10^{-4}\ \mathrm{J}$

Answer: $8.25 \times 10^{-4}\ \mathrm{J}$

Solution:



Let the radius of small droplet is r, then by volume conservation:

$1000\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi(10)^3$

$\Rightarrow r=1\ \mathrm{mm}$

Final potential energy:

$U_f=1000\left(4\pi r^2 T\right)$

$=1000\times\left(4\pi\times10^{-6}\times0.073\right)$

$=9.17\times10^{-4}\ \mathrm{J}$

Initial potential energy:

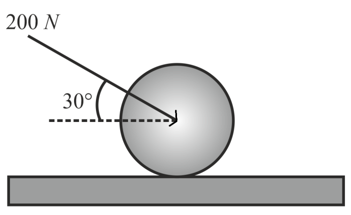
$U_i=4\pi\times\left(10^{-2}\right)^2 T$

$=9.17\times10^{-5}\ \mathrm{J}$

Therefore, change in potential energy,

$\Delta U=U_f-U_i=8.25\times10^{-4}\mathrm{J}$

Q.4. A force $200\ \mathrm{N}$ is exerted on a disc of mass $70\ \mathrm{kg}$ as shown. Find the normal reaction given by ground on the disc.



A) $200\ \mathrm{N}$

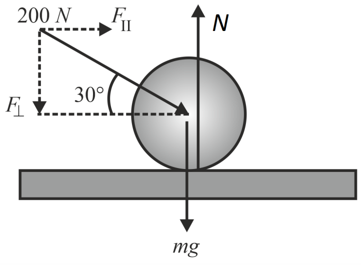
B) $600\ \mathrm{N}$

C) $800\ \mathrm{N}$

D) $\frac{200}{\sqrt{3}}\mathrm{N}$

Answer: $800\ \mathrm{N}$

Solution:



Component of the force perpendicular ro the ground,

$F\sin 30^\circ$.

Now for equilibrium along vertical direction, we get

$N=mg+F\sin 30^\circ$

$=70\times10+200\times\frac{1}{2}$

$=800\ \mathrm{N}$

Q.5. At depth d from the surface of Earth, the magnitude of acceleration due to gravity is same as its value at height d above the surface of Earth. If Earth is a sphere of radius $6400\ \mathrm{km}$, then the value of d is equal to

A) $2975\ \mathrm{km}$

B) $3955\ \mathrm{km}$

C) $2525\ \mathrm{km}$

D) $4915\ \mathrm{km}$

Answer: $3955\ \mathrm{km}$

Solution: The magnitude of acceleration due to gravity g_1 at a depth of d below the surface of Earth is

$$g_1 = G\frac{M}{d^3}(R-d) = g\left(1-\frac{d}{R}\right) \quad ....(1)$$

(where $g = G\frac{M}{R^2}$ is the acceleration due to gravity at Earth's surface)

The magnitude of acceleration due to gravity g_2 at a height d above the surface of Earth is

$$g_2 = G\frac{M}{(R+d)^2} = \frac{g}{\left(1+\frac{d}{R}\right)^2} \quad ....(2)$$

Since the magnitude of acceleration due to gravity at depth d below the surface of Earth is same as that at height d above the surface of Earth,

$$g\left(1-\frac{d}{R}\right) = \frac{g}{\left(1+\frac{d}{R}\right)^2}$$

Taking $\frac{d}{R} = x$,

$$(1+x)^2\left(1-x\right) = 1$$
$$\Rightarrow -x^3 - x^2 + x = 0$$
$$\Rightarrow -x\left(x^2+x-1\right) = 0$$

Omitting $x = 0$ as it corresponds to $d = 0$

$$\left(x^2+x-1\right) = 0$$
$$\Rightarrow x = \frac{d}{R} = \frac{-1\pm\sqrt{5}}{2}$$

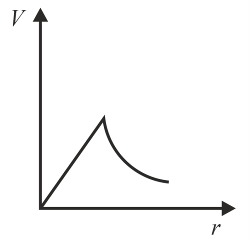
As d should be positive,

$$d = \left(\frac{-1\pm\sqrt{5}}{2}\right)R \approx 3955 \text{ km}$$

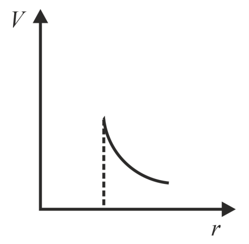
Hence, option B is correct.

Q.6. Which of the following graphs depicts the variation of electric potential with respect to radial distance from center of a conducting sphere charged with positive charge.

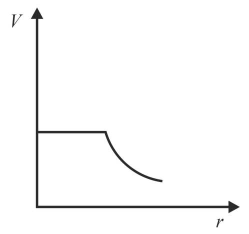
A)



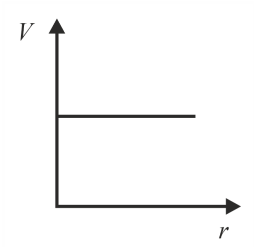
B)



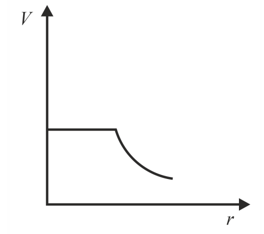
C)



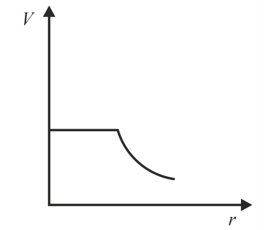
D)



Answer:



Solution:



As electric field inside the charged conducting sphere is zero, therefore potential will be constant for the inside part.

For outside part, electric potential decreases as $V=\frac{kq}{r}$ and the conducting sphere behaves like a point charge concentrated at the centre.

Q.7. In a sample of Hydrogen atoms, one atom goes through a transition $n=3 \rightarrow$ ground state with emitted wavelength λ_1. Another atom goes through a transition $n=2 \rightarrow$ ground state with emitted wavelength λ_2. Find $\frac{\lambda_1}{\lambda_2}$.

A) $\frac{6}{5}$ B) $\frac{5}{6}$ C) $\frac{27}{32}$ D) $\frac{32}{27}$

Answer: $\frac{27}{32}$

Solution: Wavelength corresponding to the photon emitted in transition $n=3\ to\ n=1$ is

$\frac{1}{\lambda_1}=R\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=\frac{8R}{9}$ ....(1)

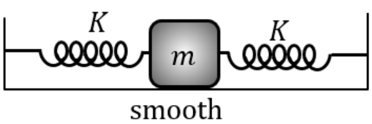
Wavelength corresponding to the photon emitted for transition in transition $n=2\ to\ n=1$ is

$\frac{1}{\lambda_2}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=\frac{3R}{4}$ ....(2)

$\frac{\lambda_1}{\lambda_2}=\frac{3R}{4}\times\frac{9}{8R}=\frac{27}{32}$

Hence, C is the correct option.

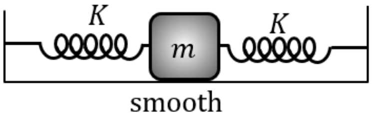
Q.8. A block of mass m is connected to two identical springs of force constant K as shown. Find the total number of oscillations of block per unit time.



A) $2\pi\sqrt{\left[\frac{2m}{K}\right]}$ B) $\frac{1}{2\pi}\sqrt{\left[\frac{K}{m}\right]}$ C) $2\pi\sqrt{\left[\frac{m}{2K}\right]}$ D) $\frac{1}{2\pi}\sqrt{\left[\frac{2K}{m}\right]}$

Answer: $\frac{1}{2\pi}\sqrt{\left[\frac{2K}{m}\right]}$

Solution:



Both the springs are connected in series, therefore

$K_{eq}=2K$

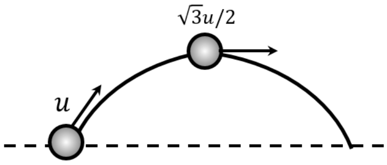
Now, time period of spring-block system is given by,

$T=2\pi\sqrt{\frac{m}{K_{eq}}}$

$=2\pi\sqrt{\frac{m}{2K}}$

$f=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{2K}{m}}$

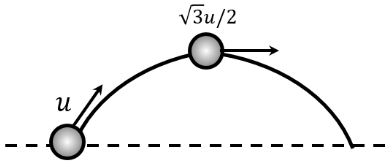
Q.9. A projectile is launched on horizontal surface that if thrown with initial velocity of u, has velocity of $\frac{\sqrt{3}u}{2}$ at maximum height. Then time of flight of the projectile is equal to:



A) $\frac{\sqrt{3}u}{g}$ B) $\frac{2u}{g}$ C) $\frac{u}{g}$ D) $\frac{u}{2g}$

Answer: $\frac{u}{g}$

Solution:



At maximum height, only horizontal velocity of the particle remains. Let the angle of project be θ. The horizontal velocity of the particle will remain same, therefore

$u\cos\theta=\frac{\sqrt{3}u}{2}$

$\Rightarrow\theta=30^{\circ}$

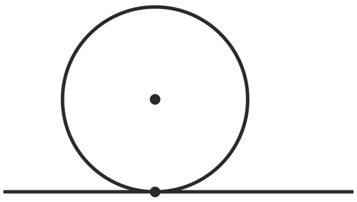
Now,

$T=\frac{2u\sin\theta}{g}=\frac{u}{g}$

Q.10. A solid sphere is rolling with kinetic energy $=7\times10^{-3}$ J. If mass of the sphere is $1\ \mathrm{kg}$, then find the speed of the centre of mass in cm/s. (consider pure rolling)

Answer: 10

Solution:



The total kinetic energy is

$K=\frac{1}{2}I_{cm}\omega^2+\frac{1}{2}mv_{cm}^2$

$\Rightarrow K=\frac{1}{2}\times\frac{2}{5}mr^2\omega^2+\frac{1}{2}mv_{cm}^2$ (for solid sphere, $I_{cm}=\frac{2}{5}mr^2$)

$\Rightarrow K=\frac{1}{2}\times\frac{2}{5}mr^2\left(\frac{v_{cm}}{r}\right)^2+\frac{1}{2}mv_{cm}^2$ (for pure rolling, $\omega=\frac{v_{cm}}{r}$)

$\Rightarrow K=\frac{7}{10}mv_{cm}{}^2$

$\Rightarrow 7\times10^{-3}=\frac{7}{10}\times1\times v_{cm}{}^2$

$\Rightarrow v_{cm}=10\ cm/s$

Hence, the correct answer is 10.

Q.11. A lift of mass $500\ kg$ starts moving downwards with initial speed $2\ m\ s^{-1}$ and accelerates at $2\ m\ s^{-2}$. The kinetic energy of the lift when it has moved a distance of $6\ m$ downwards is ______ kJ.

Answer: 7

Solution: From the third equation of motion,

$v^2 = u^2 + 2aS$

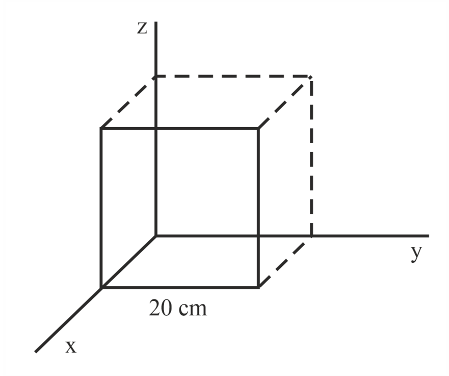
$= 4 + 2 \times 2 \times 6 = 28\ m^2\ s^{-2}$

Therefore, the kinetic energy of lift after moving $6\ m$ downwards is

$K = \frac{1}{2} m v^2 = \frac{1}{2} \times 500 \times 28\ = 7000\ J = 7\ kJ$

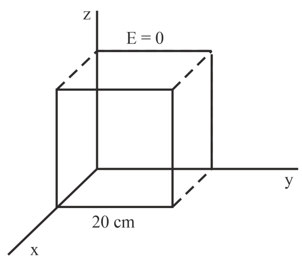
Hence, the correct answer is 7.

Q.12. The electric field in a region is given by $4000\ x^2\ \hat{i}\ \ N/C$. The flux through the cube shown below is $\frac{x}{5}\ N\ m^2\ C^{-1}$. Find the value of x.



Answer: 32

Solution:



Since the Electric field is along positive x-axis direction, flux through the faces of cube parallel to x-z plane and parallel to x-y plane is zero.

The flux through the face along y-z plane passing through origin is zero, since the electric field at all points of the face $E = 4000\ x^2\ \hat{i}\ \ N/C = E = 4000 \times\ \ 0^2\ \hat{i}\ \ N/C = \vec{0}$.

The flux through the other face parallel to y-z plane is,

$\phi = \left(4000\ a^2\ \hat{i}\right) \cdot \left(a^2\ \hat{i}\right)$ (where a is the length of the edge of the cube)

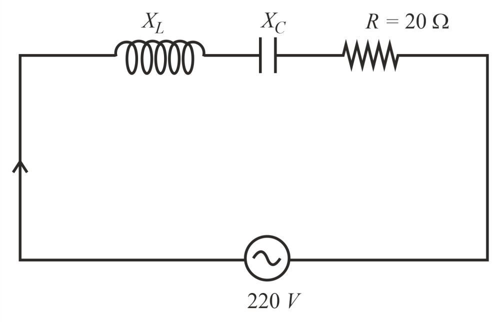
$\Rightarrow \phi = 4000 \times 0.2^2 \times 0.2^2 = 6.4 = \frac{32}{5}\ N\ m^2\ C^{-1}$

Hence, 32 is the correct answer.

Q.13. For a series *LCR* circuit across an *AC* source, the current and the voltage are in the same phase. Given the resistance is of $20\ \Omega$ and the RMS voltage of the source is $220\ V$. Find the RMS current (in A) in the circuit

Answer: 11

Solution:



Since the phase difference (ϕ) between instantaneous current (i) and instantaneous voltage (V) is given by:

$\phi = \tan^{-1}\left(\frac{Xc - X_L}{R}\right)$

Since, for the given circuit, $\phi = 0$,

$\tan^{-1}\left(\frac{Xc - X_L}{R}\right) = 0$

$\Rightarrow Xc = X_L$

Therefore, the RMS current in the circuit is

$i_{RMS} = \frac{V_{RMS}}{Z} = \frac{V_{RMS}}{\left[R^2 + (Xc - X_L)^2\right]^{\frac{1}{2}}}$

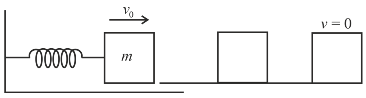
$= \frac{V_{RMS}}{R} = \frac{220}{20} = 11\ A$

Hence, the correct answer is 11.

Q.14. For a particle performing SHM, maximum potential energy is $25\ J$. The kinetic energy of particle (in J) at the point located at the distance of half of the amplitude from the mean position is $\frac{n}{4}\ J$. Find n.

Answer: 75

Solution:



The particle possess maximum potential energy (which is $\frac{1}{2}kA^2 = 25$ J) at $x = A$ where the kinetic energy is zero. Hence, the total energy of the particle can be taken as:

$E = \frac{1}{2}kA^2$ (where k is the spring constant)

The potential energy of the particle as it passes through the point located at a distance of half of the amplitude $\left(at\ x = \frac{A}{2}\right)$ from the mean position is $\frac{1}{2}k\left(\frac{A}{2}\right)^2$. Since the total energy is conserved during SHM,

$\frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{A}{2}\right)^2 + \frac{n}{4}$ ....(1)

(where $\frac{n}{4}$ is the kinetic energy of particle at $x = \frac{A}{2}$). Since the maximum potential energy, $\frac{1}{2}kA^2 = 25\ J$,

$25 = \frac{25}{4} + \frac{n}{4}$ (substituting $\frac{1}{2}kA^2$ and $\frac{1}{2}k\left(\frac{A}{2}\right)^2$ from equation 1)

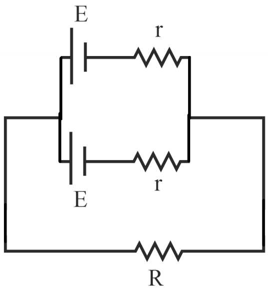
$\Rightarrow n = 75$

Hence, the correct answer is 75.

Q.15. The current through a $5\ \Omega$ resistance remains the same, irrespective of its connection across series or parallel combination of two identical cells. Find the internal resistance (in Ω) of the cell.

Answer: 5

Solution:



When cells are in parallel combination, as shown above, the equivalent EMF

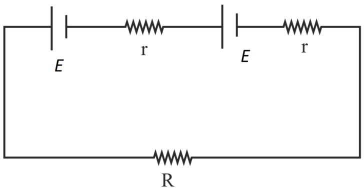
$E_1 = \frac{\frac{2E}{r}}{\frac{2}{r}} = E$

and the equivalent internal resistance

$r_1 = \frac{r^2}{2r} = \frac{r}{2}$

Therefore, the current through the resistance R is

$i_1 = \frac{E}{\frac{r}{2}+R}$ ....(1)



When cells are in series, the equivalent EMF $E_2 = 2E$ and the equivalent internal resistance $r_2 = 2r$. Therefore, the current through resistance R is

$i_2 = \frac{2E}{2r+R}$

Since it is given that the current through resistance R is same in both cases,

$i_1 = i_2$
$\Rightarrow \frac{E}{\frac{r}{2}+R} = \frac{2E}{2r+R}$

$\Rightarrow 2r + R = r + 2R$
$\Rightarrow r = R = 5\ \Omega$

Hence, the correct answer is $5\ \Omega$

Section B: Chemistry

Q.1. The electronic configuration of Nd^{2+} is given by

A) $4f^2$ B) $4f^3$ C) $4f^4$ D) $4f^5$

Answer: $4f^4$

Solution: The atomic number of Neodymium(Nd) is 60. Its electronic configuration is $[\mathrm{Xe}]\ 4f^4\ 6s^2$. Now , by removing two electrons from outermost shell $6s$, Nd^{2+} ion is formed. Hence, electronic configuration of Nd^{2+} is $[\mathrm{Xe}]\ 4f^4$.

Q.2. Basic strength of oxides of V:

V_2O_3, V_2O_5, V_2O_4

A) $V_2O_3 < V_2O_5 < V_2O_4$ B) $V_2O_3 < V_2O_4 < V_2O_5$ C) $V_2O_3 > V_2O_4 > V_2O_5$ D) $V_2O_3 = V_2O_4 = V_2O_5$

Answer: $V_2O_3 > V_2O_4 > V_2O_5$

Solution: V_2O_3

- In V_2O_3, the oxidation state of Vanadium (V) is as follows

$2(x)+3(-2)=0$

$2x-6=0$

$x=\frac{6}{2}=3$

- The oxidation state of V in V_2O_3 is $+3$.
- The Vanadium is present in its lowest oxidation state.
- The Vanadium tends to donate the pair of electrons.
- Hence, it is basic oxide.

V_2O_5

- In V_2O_5, the oxidation state of Vanadium (V) is as follows:

$2(x)+5(-2)=0$

$2x-10=0$

$X=\frac{10}{2}=5$

- The oxidation state of V in V_2O_5 is $+5$.
- The Vanadium tends to donate and accept the pair of electrons.
- It is amphoteric oxide.

V_2O_4

- In V_2O_4, the oxidation state of Vanadium (V) is as follows

$2(x)+4(-2)=0$

$2x-8=0$

$x=\frac{8}{2}=4$

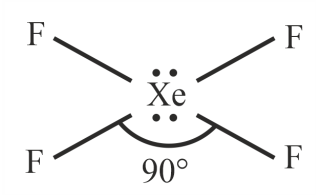
- The oxidation state of V in V_2O_4 is $+4$.
- The Vanadium is present in its other low oxidation state.
- The Vanadium tends to donate and accept the pair of electrons.
- It is amphoteric oxide and less basic than V_2O_3, more basic than V_2O_5

Q.3. The hybridization of XeF_4, SF_4 and $BrCl_3$ are respectively given as:

A) sp^3, sp^3, sp^3 B) dsp^2, sp^3, sp^3 C) sp^3d^2, sp^3d, sp^3d D) d^2sp^2, sp^3d, sp^3d

Answer: sp^3d^2, sp^3d, sp^3d

Solution:



Square Plannar Shape.

$\Rightarrow \frac{1}{2}\left[\begin{array}{c}\text{Number of valence } e^{-} \\ \text{on central atom}\end{array} + \begin{array}{c}\text{Number of monovalent} \\ \text{atom}\end{array} + \text{Charge}\right]$

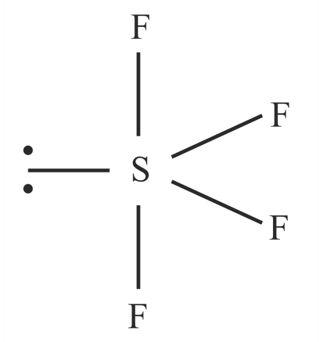
$\Rightarrow \frac{1}{2}[8+4+0]=6$

$=\mathrm{sp}^{3}\mathrm{d}^{2}$

or, $\text{Bond pair} + \text{lone pair}$

$=4+2=6$

$=\mathrm{sp}^{3}\mathrm{d}^{2}$.



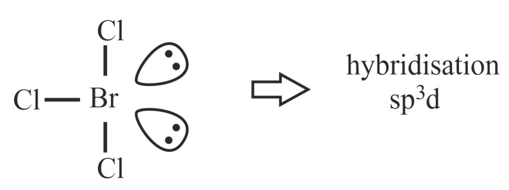
By the formula,

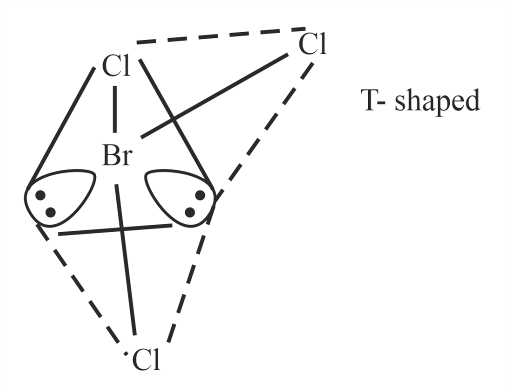
$\frac{1}{2}(6+4)$

$\frac{10}{2}=5$

$\therefore$ Hybritisation is $\mathrm{sp}^{3}\mathrm{d}$ with trigonal bipyramidal geometry.

In BrCl_3, Bond pair + Lone pair = 3+2=5





Q.4. Following values of K (rate constants) are given at different temperatures, find out $\mathrm{E_a}$ (activation energy).

$\mathrm{T}_1=200K;\ \mathrm{K}_1=0.03$

$\mathrm{T}_2=300K\ ;\ \mathrm{K}_2=0.05$

A) $2.303\,\mathrm{kJ}$ B) $11.488\,\mathrm{kJ}$ C) $1.106\,\mathrm{kJ}$ D) $51.437\,\mathrm{kJ}$

Answer: $2.303\,\mathrm{kJ}$

Solution: Arrhenius equation is given as:

$K = Ae^{-Ea/RT}$

or $\ln K = \ln A - \frac{Ea}{RT}$

At temperatures T_1 and T_2, rate equation with rate constants K_1 and K_2 .

$\ln K_2 - \ln K_1 = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$\log K_2 - \log K_1 = \frac{Ea}{2.303(R)}\left[\frac{1}{200} - \frac{1}{300}\right]$

Taking the given values of rate constants, we get,

$0.7 - 0.5 = \frac{Ea}{\frac{100}{12}\times 2.303}\left[\frac{100}{300\times 200}\right]$

$E_a = 0.2 \times \frac{100}{12} \times 2.3\frac{\times 300\times 200}{100}$

Approximately, $E_a = 2.303\,\mathrm{kJ}$

Q.5. $Cu^{2+} + I^- \rightarrow A \rightarrow B + C$
B and C are

A) I_2, Cu_2I_2 B) $[CuI_4]$ C) $[CuI_3]^-$ D) I^-, CuI_2

Answer: I_2, Cu_2I_2

Solution: $2\,Cu^{2+}(aq) + 4I^-(aq) \rightarrow 2\,CuI_2(A) \rightarrow Cu_2I_2(s)\,(B) + I_2(s)\,(C)$
The oxidation number of Cu changes from $+2$ to $+1$.
Total decrease in the oxidation number of $Cu = 1$.
The oxidation number of iodine increases from -1 to 0.
Increase in oxidation number of one iodine atom $= 1$.
Total increase in oxidation number of two iodine atoms $= 2$.

Q.6. Choose the correct information regarding the products obtained on electrolysis of brine solution

A) Cl_2 at cathode B) O_2 at cathode C) H_2 at cathode D) OH^- at anode

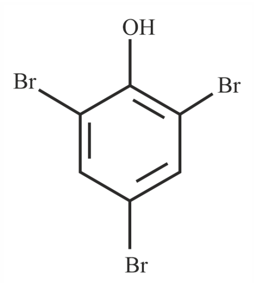
Answer: H_2 at cathode

Solution: When electricity is passed through a concentrated solution of $NaCl$, which is called Brine solution, it decomposes and results in the formation of sodium hydroxide ($NaOH$) solution formed near the cathode, Chlorine gas (Cl_2) at anode, and Hydrogen gas (H_2) at cathode.

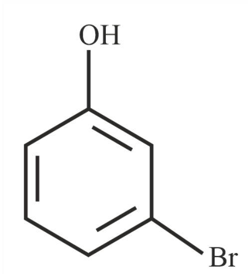
- $2\,NaCl\,(aq) + 2H_2O\,(l) \rightarrow 2\,NaOH\,(aq) + Cl_2\,(g) + H_2\,(g)$

Q.7. When phenol reacts with Br in low polarity solvent, which of the following will be the major product.

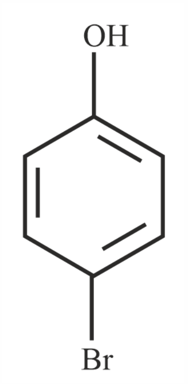
A)



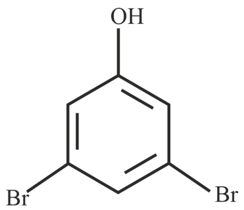
B)



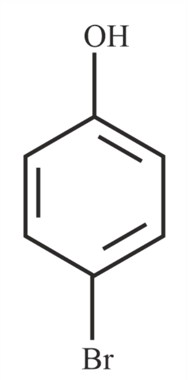
C)



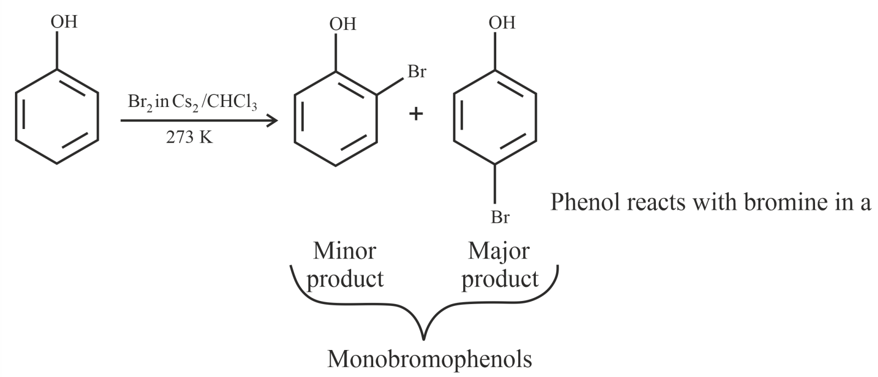
D)



Answer:



Solution: Phenol reacts with bromine in a presence of low polarity solvent like carbon disulphide to form a mixture of o-bromophenol and p-bromophenol.



Among them p-bromophenol is major. In CS_2 ionisation is not facilitated that much, as it is a low polar solvent. Also - OH group is moderately $\mathrm{o, p}$-directing.

Q.8. $SO_2 + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

If $K_p = 2 \times 10^{12}$ and $K_c = \mathrm{a} \times 10^{13}$, the value of a in terms of RT will be:

A) $\frac{\sqrt{\mathrm{RT}}}{4}$

B) $\frac{\sqrt{\mathrm{RT}}}{5}$

C) $\frac{\sqrt{\mathrm{RT}}}{10}$

D) $10\sqrt{\mathrm{RT}}$

Answer: $\frac{\sqrt{\mathrm{RT}}}{5}$

Solution: The relation between equilibrium constant in terms of pressures K_p and equilibrium constant in terms of concentration K_c is

$$K_p = K_c(RT)^{\Delta ng}$$

$$2 \times 10^{12} = a \times 10^{13}(RT)^{\frac{-1}{2}}$$

$$a = \frac{2\times10^{12}}{10^{13}}\sqrt{RT}$$

$$a = \frac{\sqrt{RT}}{5}$$

Q.9. Which of the following is the compound with highest sweetening value?

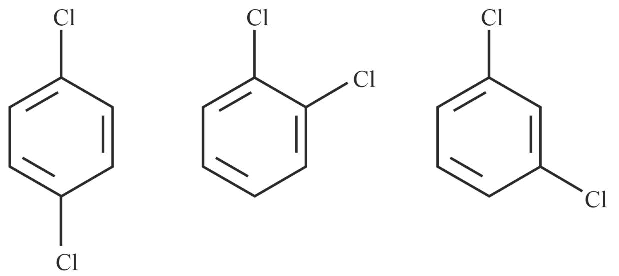
A) Aspartame B) Saccharin C) Sucralose D) Alitame

Answer: Alitame

Solution:

Artificial Sweetener	Sweetness value in comparison to cane sugar
Aspartame	100
Saccharin	550
Sucralose	600
Alitame	2000

Q.10. Find the correct order of melting points of the given compounds.

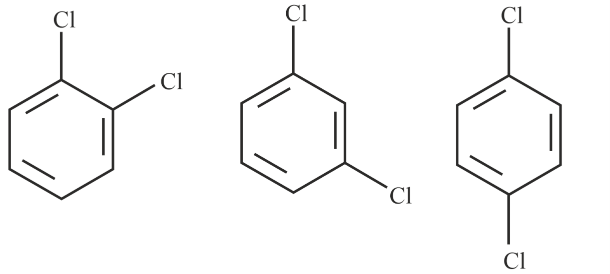


A B C

A) $A > B > C$ B) $C > A > B$ C) $B > A > C$ D) $A > C > B$

Answer: $A > B > C$

Solution:



o-dichlorobenzene *m*-dichlorobenzene *p*-dichlorobenzene

Out of the above three isomers of dichlorobenzene, the p-isomer is more symmetrical than other two isomers, So, it has more closely packed arrangement of molecules in its crystal lattice. So, p-dichlorobenzene has a higher melting point as compared to ortho and meta Isomers.

Q.11. In which of the following reactions H_2O_2 acts as a reducing agent

A) $H_2O_2 + Mn^{2+} \longrightarrow MnO_2 + H_2O$ B) $NaOCl + H_2O_2 \longrightarrow NaCl + O_2$ C) $Fe^{2+} + H_2O_2 \longrightarrow Fe^{3+} + H_2O$ D) $PbS + H_2O_2 \longrightarrow PbSO_4 + H_2O$

Answer: $NaOCl + H_2O_2 \longrightarrow NaCl + O_2$

Solution: When Hydrogen peroxide act as reducing agent, it gets oxidised to oxygen gas as

$$2H_2O_2 \xrightarrow{\text{Oxidises}} 2H_2O + O_2$$

Q.12. Arrange the following ions in the increasing order of their ionic radii

S^{2-}, Cl^-, K^+ and Ca^{2+}

A) $S^2 < Cl < K^+ < Ca^{2+}$ B) $Cl^- < S^{2-} < K^+ < Ca^{2+}$ C) $K^+ < Ca^{2+} < Cl^- < S^{2-}$ D) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

Answer: $Ca^{2+} < K^{+} < Cl^{-} < S^{2-}$

Solution: For isoelectronic species, as nuclear charge increases radius decreases. Greater the positive charge, lesser the size of ion. Greater the negative charge, larger the size of ion.

$\therefore\ S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

Q.13. Which of the following method is not a concentration method of ore?

A) Electrolysis B) Leaching C) Froth floatation D) Hydraulic washing

Answer: Electrolysis

Solution: Electrolysis is not a concentration method.

Electrolytic refining is a process of refining a metal (mainly copper) by the process of electrolysis.

The other three methods are concentration methods of ores.

Leaching is used when the ore is soluble in a solvent. The powdered ore is dissolved in a chemical, usually a strong solution of $NaOH$. The chemical solution dissolves the metal in the ore, and it can be extracted and separated from the gangue by extracting the chemical solution. Froth flotation is a process for selectively separating hydrophobic materials from hydrophilic. Hydraulic washing is a technique used when the impurities are lighter and the ore particles are heavier. The lighter impurities are removed by washing in current of water. As gold particles are heavier than the impurities like sand, we can use hydraulic washing for the concentration of ores of gold.

Q.14. Which of the following transition emits the same wavelength as that for $(n = 4 \rightarrow n = 2)$ for He^{+} ion :

A) $H\ (n = 3 \rightarrow n = 1)$ B) $Li^{2+}(n = 4 \rightarrow n = 3)$ C) $H\ (n = 2 \rightarrow n = 1)$ D) $He^{+}(n = 2 \rightarrow n = 1)$

Answer: $H\ (n = 2 \rightarrow n = 1)$

Solution: For He^{+} ion, the wave number $(\bar{v})$ associated with the Balmer transition, $n = 4$ to $n = 2$ is given by:

$$\bar{v} = \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where

$n_1 = 2$

$n_2 = 4$

$Z =$atomic number of helium

$$\bar{v} = \frac{1}{\lambda} = R(2)^2\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$= 4R\left(\frac{4-1}{16}\right)$$

$$= \bar{v} = \frac{1}{\lambda} = \frac{3R}{4}$$

$$\Rightarrow\quad \lambda = \frac{4}{3R}$$

According to the question, the desired transition for hydrogen will have the same wavelength as that of He^{+}.

$$\rightarrow R\left(Z\right)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{3R}{4}$$

$$Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{3R}{4} \quad \ldots\left(1\right)$$

By hit and trail method, the equality given by equation (1) is is true only when $n_1 = 1$ and $n_2 = 2$.

$\therefore$ The transition for $n = 2$ to $n = 1$ in hydrogen spectrum would have the same wavelength as Balmer transition $n = 4$ to $n = 2$.

Q.15. What is the volume of hydrogen gas produced (lit) when $11.2\ g$ of Zn metal reacts with excess of dil. HCl. (Closest integer)
Given, Molar volume of $H_2 = 22.7\ L/mol$
Molar mass of $Zn = 65\ g/mol$

Answer: 4

Solution: $1\,mol$ of Zn has mass of $65\ g$.

The amount of Zn is $\frac{11.2\ g}{65\ g/mol} = 0.17\ mol.$

The amount of H_2 produced is the same as the amount of Zn consumed $(0.17\,mol)$

$1\,mol$ of ideal gas will occupy $22.7\ L$ at STP according to question.

The H_2 will occupy $0.17 mol \times 22.7 L/mol = 3.859 L.$

Q.16. A complex compound of CO "x" is pink color in water. On reaction with conc. HCl forms "y" of deep blue color and has geometry "z". Identify x, y, z

A) $\left[Co\left(H_2O\right)_6\right]^{2+}, \left[CoCl_6\right]^{3-}$, Octahedral

B) $\left[Co\left(H_2O\right)_6\right]^{3+}, \left[CoCl_4\right]^{2-}$, square planar

C) $\left[Co\left(H_2O\right)_6\right]^{2+}, \left[CoCl_4\right]^{2-}$, Tetrahedral

D) $\left[Co\left(H_2O\right)_6\right]^{3+}, \cdot\left[CoCl_6\right]^{3-}$, Octahedral

Answer: $\left[Co\left(H_2O\right)_6\right]^{2+}, \left[CoCl_4\right]^{2-}$, Tetrahedral

Solution: The Co^{2+} forms pink colour octahedral complex with water. When this complex is treated with $conc.HCl$, it will form the equilibrium with $[CoCl_4]^{2-}$, which is blue colour complex and has tetrahedral shape.

$$\underset{\text{pink}}{[Co(H_2O)_6]^{2+}(aq)} + \underset{\text{colourless}}{4Cl^-(aq)} \rightleftharpoons \underset{\text{blue}}{[CoCl_4]^{2-}(aq)} + 6H_2O(l)$$

Q.17. Match the following

Column I		Column-II	
(a)	XeF_4	(p)	T shape
(b)	SF_4	(q)	see-saw
(c)	NH_4^+	(r)	Square planar
(d)	BrF_3	(s)	Tetrahedral

A) $a-r,\ b-q,\ c-s,\ d-p$ B) $a-p,\ b-q,\ c-s,\ d-r$ C) $a-r,\ b-s,\ c-q,\ d-p$ D) $a-r,\ b-q,\ c-p,\ d-s$

Answer: $a-r,\ b-q,\ c-s,\ d-p$

Solution: XeF_4 has sp^3d^2 hybridisation. Hence, it will have octahedral geometry. When we arrange all the four F atoms along with the 2 lone pairs in that geometry, it is found to have a square planar shape. There are four pairs of bonding electrons and two lone pairs in the molecule.

SF_4 molecular geometry is see-saw with one pair of valence electrons. The nature of the molecule is polar. These atoms form a trigonal bipyramidal shape.

In NH_4^+, being polar covalent bonds, all four $N-H$ bonds are equivalent and has a tetrahedral structure.

BrF_3 molecular geometry is said to be T-shaped with a bond angle of 86.2° which is slightly smaller than the usual 90°.

Q.18. The value of logarithm of the equilibrium constant of the following reaction is $\frac{x}{3}$. Find the value of 'x'.

$Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$

Given: $Pd^{2+} + 2e^- \longrightarrow Pd \quad E^0 = 0.83\ V$

$PdCl_4^{2-} + 2e^- \rightarrow Pd + 4Cl^- E^0 = 0.63\ V$

$\frac{2.303RT}{F} = 0.06$

A) 20 B) 10 C) 30 D) 15

Answer: 20

Solution: $Pd + 4Cl^- \longrightarrow PdCl_4^{2-} + 2e^-\ E^0 = -0\cdot 63,\ \Delta G_1 = +2F \times 0.63$

$Pd^{2+} + 2e^- \rightarrow Pd\ E^0 = 0.83V\ \Delta G_2 = -2F \times 0.83$

$\Delta G = \Delta G_1 + \Delta G_2$

$= -2 \times F \times 0.83 + 2 \times F \times 0.63$

$= -2 \times 0.2 \times F$

$\Delta G = -RT\ln K$

$-RT\ \ln K = -2 \times 0.2 \times F$

$-2.303RT\log K = -0.4F$

$\log K = \frac{0.4F}{2.303RT} = \frac{0.4}{0.06} = \frac{20}{3} = \frac{x}{3}$

$\Rightarrow x = 20$

Q.19. $2.56\ g$ of a non-electrolyte solute is dissolved in one litre of a solution, it has osmotic pressure equal to $4\ bar\ at\ 300\ K$ temperature. Then, find the molar mass of the compound.

Given, $R = 0.083\ bar$, round off to the nearest integer

Answer: 16

Solution: The osmotic pressure of non-electrolytic solution can be calculated as follows,

$\pi = CRT$

$\pi = \text{osmotic pressure}$

$C = \text{molarity}$

$C = \frac{\text{mass}}{\text{Molar mass (M)}} \times \frac{1}{\text{V in L}}$

$4 = \frac{2.56}{M} \times 0.083 \times 300$

$M = \frac{2.56}{4 \times 12} \times 300$

$M = 16$

Q.20. Weight of an organic compound is 0.492g. When the hydrocarbon undergoes combustion, it produces 0.792 g of CO_2. Find the % of carbon in the given hydrocarbon.

(Round off to the nearest integer)

Answer: 44

Solution: 44 grams of CO_2 contain 12 grams of carbon. Hence, one gram of carbon dioxide contain $\frac{12}{44}$g of carbon.

$0.792\text{g of } CO_2 \text{ contain } \frac{12}{44} \times 0.792\text{g of Carbon}$

Percentage of carbon = $\frac{\text{mass of carbon}}{\text{mass of organic compound}} \times 100$

= $\frac{12}{44} \times \frac{0.792}{0.492} \times 100 = 44\%$

Section C: Mathematics

Q.1. If $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$, then $12f(8) =$

A) 17 B) 20 C) 30 D) 40

Answer: 17

Solution: Given:

$f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$

$\Rightarrow f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$

Put $y = f(x)$, then

$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{2\sqrt{x+1}}$

So, $\text{I.F.} = e^{\int \frac{dx}{x}} = e^{\ln|x|} = x$

Hence, solution is

$xy = \frac{1}{2} \int \frac{x}{\sqrt{x+1}} dx$

$\Rightarrow xy = \frac{1}{2} \int \left(\frac{x+1-1}{\sqrt{x+1}}\right) dx$

$\Rightarrow xy = \frac{1}{2} \int \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right) dx$

$\Rightarrow xy = \frac{1}{2} \left(\frac{2}{3}(x+1)^{\frac{3}{2}} - 2\sqrt{x+1}\right) + C$

$\Rightarrow xy = \frac{1}{3}(x+1)^{\frac{3}{2}} - \sqrt{x+1} + C$

Put $x = 3$, then $f(3) = 2$

So,

$6 = \frac{8}{3} - 2 + C \Rightarrow C = \frac{16}{3}$

Hence,

$xy = \frac{1}{3}(x+1)^{\frac{3}{2}} - \sqrt{x+1} + \frac{16}{3}$

So, put $x = 8$, then

$8f(8) = \frac{27}{3} - 3 + \frac{16}{3}$

$\Rightarrow f(8) = \frac{34}{3\times 8}$

$\Rightarrow 12f(8) = 17$

Q.2. The product and sum of first four terms of G.P. is 1296 and 126 respectively, then the sum of the possible values of the common ratio is

A) 14 B) $\frac{10}{3}$ C) $\frac{7}{2}$ D) 3

Answer: 3

Solution: Let the four terms of G.P be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$, so

$\frac{a}{r^3} \times \frac{a}{r} \times ar \times ar^3 = 1296$

$\Rightarrow a^4 = 1296$

$\Rightarrow a = 6$

Also,

$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$

$\Rightarrow \frac{1}{r^3} + \frac{1}{r} + r + r^3 = 21$

$\Rightarrow \left(r + \frac{1}{r}\right) + \left(r^3 + \frac{1}{r^3}\right) = 21$

$\Rightarrow \left(r + \frac{1}{r}\right) + \left(r + \frac{1}{r}\right)^3 - 3\left(r + \frac{1}{r}\right) = 21$

Put $\left(r + \frac{1}{r}\right) = t$, then

$t^3 - 3t - 21 = 0$

$\Rightarrow t = 3$

So,

$r + \frac{1}{r} = 3$

$\Rightarrow r^2 - 3r + 1 = 0$

So, required sum is $= -\frac{(-3)}{1} = 3$

Q.3. If $B = \ln(1-a)$ and $P(a) = \left(a + \frac{a^2}{2} + \frac{a^3}{3} + \ldots\ldots + \frac{a^{50}}{50}\right)$, then $\int_0^a \frac{t^{50}}{1-t} dt$ is

A) $-(B + P(a))$ B) $-B + P(a)$ C) $B - P(a)$ D) $B + P(a)$

Answer: $-(B + P(a))$

Solution: Let

$I = \int_0^a \frac{t^{50}}{1-t} dt$

$\Rightarrow I = -\int_0^a \left(\frac{1-t^{50}-1}{1-t}\right) dt$

$\Rightarrow I = -\int_0^a \left(\frac{1-t^{50}}{1-t}\right) dt + \int_0^a \left(\frac{1}{1-t}\right) dt$

$\Rightarrow I = -\int_0^a \left(1 + t + t^2 + \ldots + t^{49}\right) dt - [\ln(1-t)]_0^a$

$\Rightarrow I = -\left[t + \frac{t^2}{2} + \frac{t^3}{3} + \ldots + \frac{t^{50}}{50}\right]_0^a - \ln(1-a)$

$\Rightarrow I = -\left[a + \frac{a^2}{2} + \frac{a^3}{3} + \ldots + \frac{a^{50}}{50}\right] - \ln(1-a)$

$\Rightarrow I = -(B + P(a))$

Q.4. The direction ratios of two lines which are parallel are given by $<2, 1, -1>$ and $<\alpha + \beta, 1 + \beta, 2>$, then the value of $|2\alpha + 3\beta|$ is

Answer: 11

Solution: Two lines are parallel if their direction ratios are proportional, so

$\frac{\alpha+\beta}{2} = \frac{1+\beta}{1} = \frac{2}{-1}$

So,

$\alpha + \beta = -4 \quad \ldots.(i)$
$1 + \beta = -2 \quad \ldots.(ii)$

On solving, we get

$\alpha = -1, \ \beta = -3$

Hence,

$|2\alpha + 3\beta| = |-2 - 9| = 11$

Q.5. Find the value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2+3\sin x}{\sin x(1+\cos x)} dx$

A) $\ln\left(\sqrt{3}+2\right) - \frac{\ln 3}{2} + 6\sqrt{3} - \frac{28}{3}$ B) $\ln\left(\sqrt{3}+2\right) - \frac{\ln 3}{2}$ C) $\ln\left(\sqrt{3}+2\right) - \frac{\ln 3}{2} - \frac{28}{3}$ D) $6\sqrt{3} - \frac{28}{3}$

Answer: $\ln\left(\sqrt{3}+2\right)-\frac{\ln 3}{2}+6\sqrt{3}-\frac{28}{3}$

Solution:

Let, $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2+3\sin x}{\sin x(1+\cos x)}\mathrm{d}x$

Now rearranging the above integral we get,

$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2}{\sin x(1+\cos x)}\mathrm{d}x+\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{3}{(1+\cos x)}\mathrm{d}x$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2\sin x}{\sin^2 x(1+\cos x)}\mathrm{d}x+\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{3}{\left(2\cos^2\frac{x}{2}\right)}\mathrm{d}x$$

$$\Rightarrow I=\underbrace{\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2\sin x}{\sin^2 x(1+\cos x)}\mathrm{d}x}_{I_1}+\frac{3}{2}\underbrace{\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sec^2\frac{x}{2}\mathrm{d}x}_{I_2}$$

$$\Rightarrow I=\underbrace{\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2\sin x}{\sin^2 x(1+\cos x)}\mathrm{d}x}_{I_1}+\frac{3}{2}\times 2\underbrace{\left[\tan\frac{x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}}_{I_2}$$

$$\Rightarrow I=\underbrace{\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2\sin x}{\sin^2 x(1+\cos x)}\mathrm{d}x}_{I_1}+3\left[\frac{1}{\sqrt{3}}-\left(2-\sqrt{3}\right)\right]$$

Now solving I_1 we get,

$$I_1=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{2\sin x}{\sin^2 x(1+\cos x)}dx$$

Let $\cos x=t\Rightarrow-\sin x dx=dt$

$$\Rightarrow I_1=-\int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}\frac{2dt}{\left(1-t^2\right)(1+t)}$$

$$\Rightarrow I_1=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\frac{2dt}{\left(1-t^2\right)(1+t)}$$

$$\Rightarrow I_1=2\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left[\frac{1}{4}\left(\frac{1}{t+1}\right)+\frac{1}{2}\left(\frac{1}{(t+1)^2}\right)+\frac{1}{4}\left(\frac{1}{1-t}\right)\right]dt$$

$$\Rightarrow I_1=2\left[\frac{1}{4}\ln(t+1)-\frac{1}{2}\left(\frac{1}{t+1}\right)-\frac{1}{4}\ln|t-1|\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow I_1=2\left[\frac{1}{4}\ln\left(\frac{\sqrt{3}}{2}+1\right)-\frac{1}{2}\left(\frac{1}{\frac{\sqrt{3}}{2}+1}\right)-\frac{1}{4}\ln\left|\frac{\sqrt{3}}{2}-1\right|-\left(\frac{1}{4}\ln\left(\frac{1}{2}+1\right)-\frac{1}{2}\left(\frac{1}{\frac{1}{2}+1}\right)-\frac{1}{4}\ln\left|\frac{1}{2}-1\right|\right)\right]$$

$$\Rightarrow I_1=2\left[\frac{1}{4}\ln\left(\frac{\sqrt{3}+2}{2}\right)-\left(\frac{1}{\sqrt{3}+2}\right)-\frac{1}{4}\ln\left|\frac{\sqrt{3}-2}{2}\right|-\left(\frac{1}{4}\ln\left(\frac{3}{2}\right)-\left(\frac{1}{3}\right)-\frac{1}{4}\ln\left|\frac{1}{2}\right|\right)\right]$$

$$\Rightarrow I_1=2\left[\frac{1}{4}\ln\left|\frac{\sqrt{3}+2}{\sqrt{3}-2}\right|-\left(\frac{1}{\sqrt{3}+2}\right)-\left(\frac{1}{4}\ln 3-\left(\frac{1}{3}\right)\right)\right]$$

$$\Rightarrow I_1=2\left[\frac{1}{4}\ln\left|\frac{\left(\sqrt{3}+2\right)^2}{1}\right|-\left(\frac{1}{\sqrt{3}+2}\right)-\left(\frac{1}{4}\ln 3-\left(\frac{1}{3}\right)\right)\right]$$

$$\Rightarrow I_1=\left[\ln\left|\sqrt{3}+2\right|-\left(\frac{2}{\sqrt{3}+2}\right)-\left(\frac{1}{2}\ln 3-\left(\frac{2}{3}\right)\right)\right]$$

Now putting the value of I_1 in I we get,

$$\Rightarrow I=\left[\ln\left|\sqrt{3}+2\right|-\left(\frac{2}{\sqrt{3}+2}\right)-\left(\frac{1}{2}\ln 3-\left(\frac{2}{3}\right)\right)\right]+3\left[\frac{1}{\sqrt{3}}-\left(2-\sqrt{3}\right)\right]$$

$$\Rightarrow I=\ln\left|\sqrt{3}+2\right|-\left(\frac{2}{\sqrt{3}+2}\right)-\frac{\ln 3}{2}+\frac{2}{3}+\frac{3}{\sqrt{3}}-6+3\sqrt{3}$$

$$\Rightarrow I=\ln\left|\sqrt{3}+2\right|-\left(\frac{2\left(2-\sqrt{3}\right)}{1}\right)-\frac{\ln 3}{2}+\frac{2}{3}+\frac{3}{\sqrt{3}}-6+3\sqrt{3}$$

$\Rightarrow I = \ln|\sqrt{3}+2| - \frac{\ln 3}{2} + \frac{2}{3} - 10 + \frac{3}{\sqrt{3}} + 5\sqrt{3}$

$\Rightarrow I = \ln|\sqrt{3}+2| - \frac{\ln 3}{2} - \frac{28}{3} + \frac{18}{\sqrt{3}}$

$\Rightarrow I = \ln|\sqrt{3}+2| - \frac{\ln 3}{2} - \frac{28}{3} + 6\sqrt{3}$

Q.6. Bag containing 6 balls of unknown colours. two balls are drawn at random and found to be black. Find the probability that the bag contains 5 black balls.

A) $\frac{5}{7}$ B) $\frac{4}{7}$ C) $\frac{3}{7}$ D) $\frac{1}{7}$

Answer: $\frac{5}{7}$

Solution: Let the bag contains black balls(B) and non-black balls $(\overline{B})$, then we can have the following possibilities:

$\{(6B, 0\overline{B}),\ (5B, 1\overline{B}),\ (4B, 2\overline{B}),\ (3B, 3\overline{B}),$
$(2B, 4\overline{B})\}$

All the events are equally likely, i.e., $\frac{1}{5}$.

Required probability

$$= \frac{\frac{1}{5}\times\frac{{}^6C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^5C_2}{{}^6C_2}}{\frac{1}{5}\times\frac{{}^6C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^5C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^4C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^3C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^2C_2}{{}^6C_2}}$$

$= \frac{25}{35} = \frac{5}{7}$

Q.7. If maximum distance of a normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$ from $(0, 0)$ is 1, then the eccentricity of the ellipse is given by

A) $\frac{\sqrt{3}}{4}$ B) $\frac{1}{\sqrt{2}}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$

Answer: $\frac{\sqrt{3}}{2}$

Solution: Given:

$\frac{x^2}{4} + \frac{y^2}{b^2} = 1$

Equation of normal to the ellipse is

$2x\sec\theta - by\,\mathrm{cosec}\,\theta = 4 - b^2$

Distance of normal from origin is

$d = \left|\frac{4-b^2}{\sqrt{4\sec^2\theta + b^2\,\mathrm{cosec}^2\theta}}\right|$

For maximum distance, denominator must be minimum.

$\sqrt{4\sec^2\theta + b^2\,\mathrm{cosec}^2\theta}$

$= \sqrt{4 + 4\tan^2\theta + b^2 + b^2\cot^2\theta}$

Now,

$\frac{4\tan^2\theta + b^2\cot^2\theta}{2} \geq \sqrt{4\tan^2\theta \times b^2\cot^2\theta}$

$\Rightarrow 4\tan^2\theta + b^2\cot^2\theta \geq 4b$

$\Rightarrow 4 + b^2 + 4\tan^2\theta + b^2\cot^2\theta \geq 4 + b^2 + 4b$

$\Rightarrow 4 + b^2 + 4\tan^2\theta + b^2\cot^2\theta \geq (2+b)^2$

So, minimum value of $\sqrt{4\sec^2\theta + b^2\,\mathrm{cosec}^2\theta}$ is $\sqrt{(2+b)^2} = b + 2$

So,

$d = \left|\frac{4-b^2}{2+b}\right| = 1$

$\Rightarrow |2 - b| = 1$

$\Rightarrow 2 - b = \pm 1$

$\Rightarrow b = 1,\ 3$

But $b < 2$, so $b = 1$, hence

$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

Q.8. Let $C_1: \; |z| = 4$ and $C_2: \; \left|z + \frac{\bar{z}}{4}\right| = \frac{15}{4}$. Then

A) C_1 lies inside C_2

B) C_2 lies inside C_1

C) C_1 & C_2 has two points of intersection

D) C_1 & C_2 has four points of intersection

Answer: C_1 & C_2 has four points of intersection

Solution: $C_1: \; |z| = 4$ represents a circle with centre at origin and radius 4 units

$C_2: \; \left|z + \frac{\bar{z}}{4}\right| = \frac{15}{4}$

$\Rightarrow \; \left|\frac{5x}{4} + i\frac{3y}{4}\right| = \frac{15}{4}$

$\Rightarrow \; \left(\frac{5x}{4}\right)^2 + \left(\frac{3y}{4}\right)^2 = \left(\frac{15}{4}\right)^2$

$\Rightarrow \; \frac{x^2}{9} + \frac{y^2}{25} = 1$ represents an ellipse with major axis as $y-$axis

The end point of major axis are $(0, \pm 5)$ and end points of minor axis are $(\pm 3, 0)$

So, the circle cuts the ellipse at four distinct points.

Q.9. If $\sin^{-1}\left(\frac{a}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{77}{36}\right) = 0$ then the value of $\sin^{-1}(\sin a) + \cos^{-1}(\cos a)$ will be

A) 0

B) $16 - 2\pi$

C) π

D) 5π

Answer: π

Solution: Given,

$\sin^{-1}\left(\frac{a}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{77}{36}\right) = 0$

$\Rightarrow \sin^{-1}\left(\frac{a}{17}\right) = \tan^{-1}\left(\frac{77}{36}\right) - \cos^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow \sin^{-1}\left(\frac{a}{17}\right) = \tan^{-1}\left(\frac{77}{36}\right) - \tan^{-1}\left(\frac{3}{4}\right)$

$\Rightarrow \sin^{-1}\left(\frac{a}{17}\right) = \tan^{-1}\left(\frac{\frac{77}{36} - \frac{3}{4}}{1 + \frac{77}{36} \times \frac{3}{4}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{a}{17}\right) = \tan^{-1}\left(\frac{8}{15}\right)$

$\Rightarrow \sin^{-1}\left(\frac{a}{17}\right) = \sin^{-1}\left(\frac{8}{17}\right)$

$\Rightarrow a = 8$

Now solving $\sin^{-1}(\sin a) + \cos^{-1}(\cos a) = \sin^{-1}(\sin 8) + \cos^{-1}(\cos 8)$

$\Rightarrow \sin^{-1}(\sin 8) + \cos^{-1}(\cos 8) = 3\pi - 8 + 8 - 2\pi = \pi$

Q.10. If $f(x) = \sin^3\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right\}\right]$, then $f'(1)$ is

A) $\frac{3\pi^2}{8}$

B) $\frac{3\pi^2}{4}$

C) $\frac{3\pi^2}{16}$

D) $\frac{\pi^2}{2}$

Answer: $\frac{3\pi^2}{16}$

Solution: Given:

$$f(x)=\sin^3\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]$$

$$\Rightarrow f'(x)=3\sin^2\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\cos\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\frac{d}{dx}\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]$$

$$\Rightarrow f'(x)=3\sin^2\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\cos\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\left[-\frac{\pi}{3}\sin\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\frac{d}{dx}\left[\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)\right]^{\frac{3}{2}}$$

$$\Rightarrow f'(x)=3\sin^2\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\cos\left[\frac{\pi}{3}\cos\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\left[-\frac{\pi}{3}\sin\left\{\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right\}\right]\left[\frac{\pi}{2\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{1}{2}}\left(-12x^2+10x\right)\right]$$

Put $x=1$, then we get

$$f'(1)=3\sin^2\left[\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right]\cos\left\{\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right\}\left[-\frac{\pi}{3}\sin\left(\frac{2\pi}{3}\right)\right](-\pi)$$

$$\Rightarrow f'(1)=3\sin^2\left[-\frac{\pi}{3}\cos\left(\frac{\pi}{3}\right)\right]\cos\left[-\frac{\pi}{3}\cos\left(\frac{\pi}{3}\right)\right]\left[-\frac{\pi}{3}\sin\left(\frac{\pi}{3}\right)\right](-\pi)$$

$$\Rightarrow f'(1)=3\sin^2\left(-\frac{\pi}{6}\right)\cos\left(-\frac{\pi}{6}\right)\left[\left(\frac{\pi^2\sqrt{3}}{6}\right)\right]$$

$$\Rightarrow f'(1)=\frac{3\sqrt{3}}{8}\left(\frac{\pi^2\sqrt{3}}{6}\right)$$

$$\Rightarrow f'(1)=\frac{3\pi^2}{16}$$

Q.11. Find the number of real solution of the expression $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$

A) 1 B) 2 C) 3 D) 4

Answer: 1

Solution: Given,

$$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$$

$$\Rightarrow \sqrt{(x-3)(x-1)}+\sqrt{(x-3)(x+3)}=\sqrt{(x-3)(4x-2)}$$

$$\Rightarrow \sqrt{(x-3)}\sqrt{(x-1)}+\sqrt{(x-3)}\sqrt{(x+3)}-\sqrt{(x-3)}\sqrt{(4x-2)}=0$$

$$\Rightarrow \sqrt{(x-3)}=0 \text{ or } \left[\sqrt{(x-1)}+\sqrt{(x+3)}-\sqrt{(4x-2)}\right]=0$$

So, $x=3$ is one solution and also $x\geq 3$

Now solving $\left[\sqrt{(x-1)}+\sqrt{(x+3)}-\sqrt{(4x-2)}\right]=0$

$$\Rightarrow \sqrt{(x-1)}+\sqrt{(x+3)}=\sqrt{(4x-2)}$$

Now squaring both side we get,

$$\Rightarrow (x-1)+(x+3)+2\sqrt{(x-1)(x+3)}=(4x-2)$$

$$\Rightarrow 2\sqrt{(x-1)(x+3)}=(2x-4)$$

$$\Rightarrow (x-1)(x+3)=(x-2)^2$$

$$\Rightarrow x^2+2x-3=x^2-4x+4$$

$\Rightarrow 6x=7\Rightarrow x=\frac{7}{6}$ which does not satisfy the given expression and $x\geq 3$

So, there is only one solution which is $x=3$

Q.12. If $A=\begin{bmatrix}2&0&0\\0&3&0\\0&0&1\end{bmatrix}$, then find sum of diagonal elements of $(A-I)^{11}$.

A) 4096 B) 4097 C) 2048 D) 2049

Answer: 2049

Solution: Given:

$$A=\begin{bmatrix}2&0&0\\0&3&0\\0&0&1\end{bmatrix}$$

So,

$$A-I=\begin{bmatrix}1&0&0\\0&2&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow (A-I)^2=\begin{bmatrix}1&0&0\\0&2&0\\0&0&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&2&0\\0&0&0\end{bmatrix}=\begin{bmatrix}1&0&0\\0&2^2&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow (A-I)^4=\begin{bmatrix}1&0&0\\0&2^4&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow (A-I)^8=\begin{bmatrix}1&0&0\\0&2^8&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow (A-I)^{11}=\begin{bmatrix}1&0&0\\0&2^{11}&0\\0&0&0\end{bmatrix}$$

So, sum of diagonal elements is

$=1+2^{11}+0=2049$

Q.13. If $\vec{a}, \vec{b}$ & $\vec{c}$ be three non-zero vector such that $\left|\vec{a}+\vec{b}+\vec{c}\right|=\left|\vec{a}+\vec{b}-\vec{c}\right|$ and $\vec{b}\cdot\vec{c}=0$ then:

Statement$-1:$ $\left|\vec{a}+\lambda\vec{c}\right|\geq 0$ for all $\lambda\in R$

Statement-2 : $\vec{a}$ is always parallel to vector $\vec{c}$

A) Statement-1 is true and statement-2 is false

B) Statement-1 is true and statement-2 is true

C) Statement-1 is false and statement-2 is true

D) Statement-1 is false and statement-2 is false

Answer: Statement-1 is true and statement-2 is false

Solution: Given,

$$\left|\vec{a}+\vec{b}+\vec{c}\right|=\left|\vec{a}+\vec{b}-\vec{c}\right|$$

Now squaring both side we get,

$$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left|\vec{a}+\vec{b}-\vec{c}\right|^2$$

$$\Rightarrow \left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{c}-2\vec{c}\cdot\vec{a}$$

$$\Rightarrow 2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}=-2\vec{b}\cdot\vec{c}-2\vec{c}\cdot\vec{a}$$

Now here given $\vec{b}\cdot\vec{c}=0,$

So, $0+2\vec{c}\cdot\vec{a}=0-2\vec{c}\cdot\vec{a}$

$\Rightarrow \vec{c}\cdot\vec{a}=0$ which means $\vec{c}$ & $\vec{a}$ are perpendicular vector, so statement -2 is false,

Now statement$-1:$ $\left|\vec{a}+\lambda\vec{c}\right|\geq 0$ is always true because modulus of any function is always greater than or equal to zero,

Hence, we can say that statement-1 is true and statement-2 is false.

Q.14. A relation $(a,b)R(c,d)$ be defined such that $ab(d-c)=cd(a-b)$. Then R is

A) Reflixive only

B) Symmetric only

C) Transitive but not symmetric

D) Reflexive and Symmetric but not Transitive

Answer: Symmetric only

Solution: For Reflexive

$(a,b)R(a,b)$

$\Rightarrow ab(b-a)\neq ab(a-b)$

So, the relation is not reflexive.

For symmetric

$(a,b)R(c,d)\equiv ab(d-c)=cd(a-b)$

$(c,d)R(a,b)\equiv cd(b-a)=ab(c-d)$

As both the above equations are same so the given relation is symmetric.

Q.15. If $\left|\vec{a}\right|=\sqrt{14},\ \left|\vec{b}\right|=\sqrt{6}\ \&\ \left|\vec{a}\times\vec{b}\right|=\sqrt{46}$, then find the value of $\left(\vec{a}\cdot\vec{b}\right)^2$

Answer: 38

Solution: Given,

$\left|\vec{a}\right|=\sqrt{14},\ \left|\vec{b}\right|=\sqrt{6}\ \&\ \left|\vec{a}\times\vec{b}\right|=\sqrt{46}$

Now we know that,

$\left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}\cdot\vec{b}\right|^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2\sin^2\theta+\left|\vec{a}\right|^2\left|\vec{b}\right|^2\cos^2\theta=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$

$\left\{\text{as } \cos^2\theta+\sin^2\theta=1\right\}$

$\Rightarrow 46+\left|\vec{a}\cdot\vec{b}\right|^2=14\times 6$

$\Rightarrow \left|\vec{a}\cdot\vec{b}\right|^2=84-46=38.$

Q.16. Find the number of 4 digit numbers using the digits $0,\ 3,\ 4,\ 7\ \&\ 9$ given that repetition is allowed:

Answer: 500

Solution: Given,

Digits $0,\ 3,\ 4,\ 7\ \&\ 9,$

Now we have form $4-$digit number in which repetition is allowed,

So thousand's place can be filled in 4 ways as 0 cannot take that place,

Now hundred's, ten's and unit place can be filled in 5 ways each,

So total number of ways will be $4\times 5\times 5\times 5=500$.

Q.17. If $y=f(x)$ is a parabola with focus $\left(-\frac{1}{2},0\right)$ and directrix $y=-\frac{1}{2}$ such that $\tan^{-1}\sqrt{f(x)}+\sin^{-1}\sqrt{f(x)+1}=\frac{\pi}{2}$, then the number of solutions for x is

Answer: 2

Solution: Given $y=f(x)$ is a parabola with focus $\left(-\frac{1}{2},0\right)$ and directrix $y=-\frac{1}{2}$

i.e. equation of parabola is $\left(x+\frac{1}{2}\right)^2+y^2=\left(y+\frac{1}{2}\right)^2$

$\Rightarrow y=x^2+x$

Now, $\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$\Rightarrow \cos^{-1}\frac{1}{\sqrt{x^2+x+1}}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$\Rightarrow \frac{1}{\sqrt{x^2+x+1}}=\sqrt{x^2+x+1}$

$\Rightarrow x^2+x+1=1 \Rightarrow\Rightarrow x^2+x=0$

$\Rightarrow x=0,-1$

Hence, two solutions.

Q.18. Find the remainder when 5^{99} is divided by 11

Answer: 9

Solution: Now to divide 5^{99} by 11 we need to check the cyclicity of power of 5 when divided by 11,

So remainder when $\frac{5^1}{11}\rightarrow 5 \text{ as remainder}$

Similarly $\frac{5^2}{11}=\frac{25}{11}\rightarrow 3 \text{ as remainder}$

$\frac{5^3}{11}=\frac{125}{11}\rightarrow 4 \text{ as remainder}$

$\frac{5^4}{11}=\frac{625}{11}\rightarrow 9 \text{ as remainder}$

And $\frac{5^5}{11}=\frac{3125}{11}\rightarrow 1 \text{ as remainder}$

So, rewriting the expression we get, $\left(5^5\right)^{19}\times 5^4$

Now when $\frac{\left(5^5\right)^{19}\times 5^4}{11}\rightarrow 1\times\frac{5^4}{11}$

Now again simplifying we get, $\frac{5^4}{11}\rightarrow 9 \text{ as remainder.}$