JEE Main Exam 2023 - Session 1

30 Jan 2023 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. A car travels $4\ \mathrm{km}$ distance with a speed of $3\ \mathrm{km\ h^{-1}}$ and the next $4\ \mathrm{km}$ distance with a speed of $5\ \mathrm{km\ h^{-1}}$. Find the average speed of car.

A) $\frac{15}{2}\ \mathrm{km\ h^{-1}}$

B) $\frac{15}{4}\ \mathrm{km\ h^{-1}}$

C) $15\ \mathrm{km\ h^{-1}}$

D) $10\ \mathrm{km\ h^{-1}}$

Answer: $\frac{15}{4}\ \mathrm{km\ h^{-1}}$

Solution: Average speed is given by:

$v_{avg} = \frac{\text{Total distance}}{\text{time}}.$

Since the time taken for the first $4\ \mathrm{km}$ is $\frac{4}{3}\ \mathrm{h}$ and the time taken for the next $4\ \mathrm{km}$ is $\frac{4}{5}\ \mathrm{h}$.

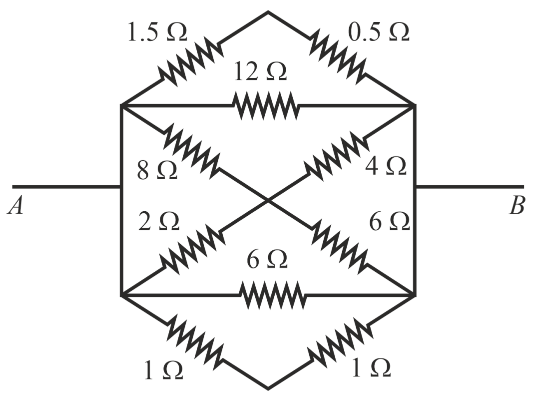
$$v_{\text{avg}} = \frac{8}{\frac{4}{3}+\frac{4}{5}}$$

$$= \frac{2\times3\times5}{3+5}$$

$$= \frac{30}{8} = \frac{15}{4}\ \mathrm{km\ h^{-1}}$$

Hence, option B is correct.

Q.2. In the given circuit, the equivalent resistance between terminals A and B is equal to



A) $2\ \Omega$

B) $\frac{3}{2}\ \Omega$

C) $\frac{2}{3}\ \Omega$

D) $6\ \Omega$

Answer: $\frac{2}{3}\ \Omega$

Solution: The equivalent resistance of the combination of $1.5\ \Omega$ and $0.5\ \Omega$ resistors is $2\ \Omega$ (since both are in series).

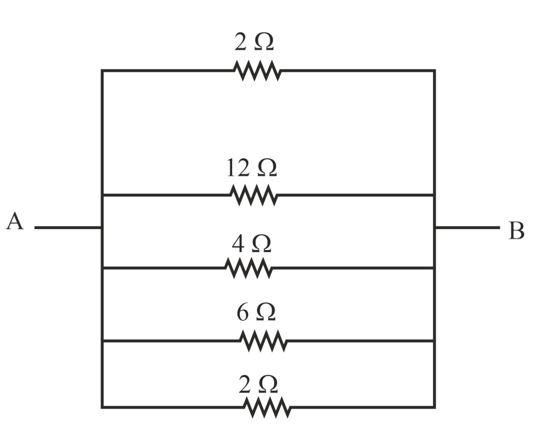
The equivalent resistance of the combination of both $1\ \Omega$ resistors is $2\ \Omega$ (since both are in series).

The equivalent resistance of the combination of $4\ \Omega$ and $6\ \Omega$ is, $\frac{4\times6}{4+6}=2.4\ \Omega$ (since both are in parallel).

The equivalent resistance of the combination of $8\ \Omega$ and $2\ \Omega$ resistors is, $\frac{8\times2}{8+2}=1.6\ \Omega$ (since both are in parallel).

The equivalent resistance of the combination of $1.6\ \Omega$ and $2.4\ \Omega$ combinations is $4\ \Omega$ (since both combinations are in series).

Thus, the circuit can be redrawn as:



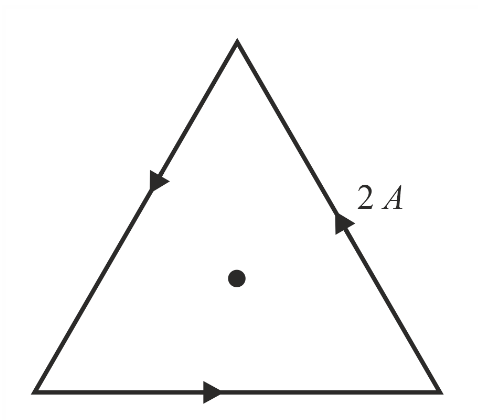
Since all the resistors of the redrawn circuit are in parallel,

$$\frac{1}{Req}=\frac{1}{2}+\frac{1}{12}+\frac{1}{4}+\frac{1}{6}+\frac{1}{2}=\frac{3}{2}$$

$$\Rightarrow R_{eq}=\frac{2}{3}\ \Omega$$

Hence, option C is correct.

Q.3. A current $2\ \mathrm{A}$ flowing through the sides of an equilateral triangular loop of side $4\sqrt{3}\ \mathrm{m}$ as shown. Find the magnetic field induction at the centroid of the triangle.



A) $3\sqrt{3}\times10^{-7}\ \mathrm{T}$

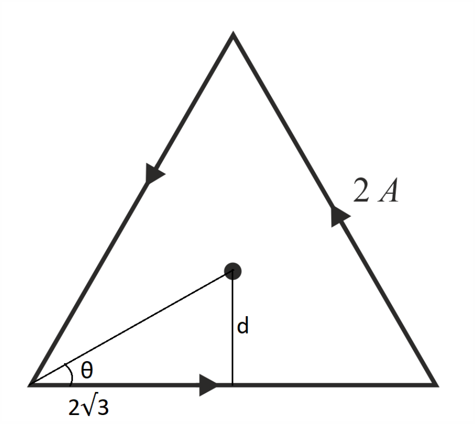
B) $\sqrt{3}\times10^{-7}\ \mathrm{T}$

C) $2\sqrt{3}\times10^{-7}\ \mathrm{T}$

D) $5\sqrt{3}\times10^{-7}\ \mathrm{T}$

Answer: $3\sqrt{3}\times10^{-7}\ \mathrm{T}$

Solution:



The distance d between the centre and side is

$d = 2\sqrt{3}\ \tan\ \theta = 2\sqrt{3}\ \tan\ 30^\circ = \ 2\ \text{m}$

The magnitude of magnetic field at the centre due to one side is:

$B = \frac{\mu_\circ i}{4\pi d}\left(\sin\ 60^\circ + \ \sin\ 60^\circ\right) = \sqrt{3} \times 10^{-7}\ \text{T}$

Applying right-hand rule for all sides shows that the magnetic field due to each side at the centre is in same direction and is equal due to symmetry. Therefore,

$B_{net} = 3\sqrt{3} \times 10^{-7}\ \text{T}$

Hence, option A is correct.

Q.4. A particle is released at a height equal to radius of earth above the surface of the earth. Its velocity when it hits the surface of earth is equal to
($M_e =$ mass of earth, $R_e =$ radius of earth)

A) $v = \sqrt{\left[\frac{2GMe}{Re}\right]}$

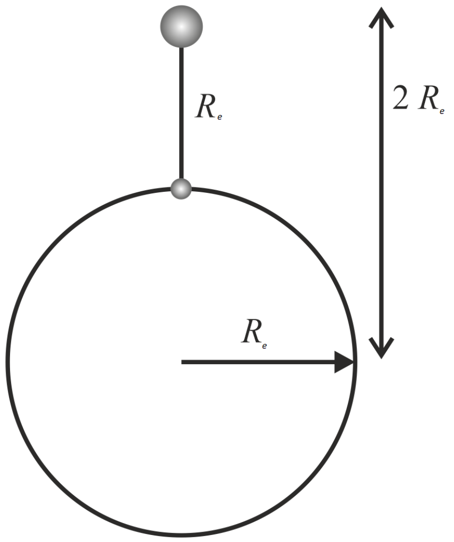
B) $v = \sqrt{\left[\frac{GMe}{2Re}\right]}$

C) $v = \sqrt{\left[\frac{GMe}{Re}\right]}$

D) $v = \sqrt{\left[\frac{2GMe}{3Re}\right]}$

Answer: $v = \sqrt{\left[\frac{GMe}{Re}\right]}$

Solution:



The potential of the particle at initial and final positions are $\frac{-GMem}{2Re}$ and $\frac{-GMem}{Re}$ respectively.

Applying conservation of energy,

$$\frac{-GMem}{2Re}+0=-\frac{GMem}{Re}+\frac{1}{2}mv^2$$

$$\Rightarrow \quad v=\sqrt{\frac{GMe}{Re}}$$

Hence, option C is correct.

Q.5. A faulty scale reads the melting point of water as $5\ ^{\circ}\mathrm{C}$ and the boiling point of water as $95\ ^{\circ}\mathrm{C}$. Find the actual temperature if this faulty scale reads $41\ ^{\circ}\mathrm{C}$.

A) $40^{\circ}\mathrm{C}$ B) $41^{\circ}\mathrm{C}$ C) $36^{\circ}\mathrm{C}$ D) $45^{\circ}\mathrm{C}$

Answer: $40^{\circ}\mathrm{C}$

Solution: As we know in case of a thermometer,

$$\frac{X-LFP}{UFP-LFP}=\text{constant.}$$

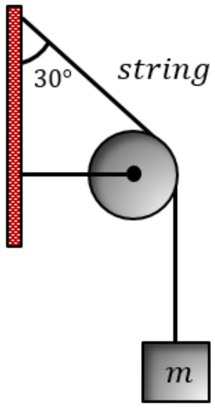
Therefore, comparing it with respect to a correct thermometer in Celsius scale, we get

$$\frac{41-5}{95-5}=\frac{T-0\ ^{\circ}\mathrm{C}}{100^{\circ}\mathrm{C}-0^{\circ}\mathrm{C}}$$

$$\Rightarrow T=40\ ^{\circ}\mathrm{C}$$

Hence, option A is correct.

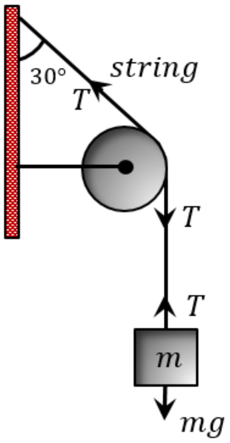
Q.6. The block in the figure below stays in equilibrium. Find the tension in the string if $m=\sqrt{3}\ \mathrm{kg}$.



A) $\sqrt{3}g\ \mathrm{N}$ B) $3g\ \mathrm{N}$ C) $\frac{g}{2}\ \mathrm{N}$ D) $\frac{g}{\sqrt{3}}\ \mathrm{N}$

Answer: $\sqrt{3}g\ \mathrm{N}$

Solution:



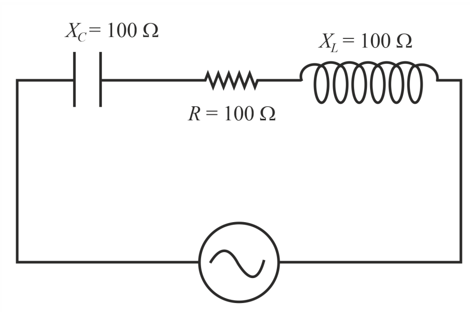
Applying Newton's second law for the block,

$T-mg=0$ (since the block is stationary)

$\Rightarrow T=mg=\sqrt{3}g\ \mathrm{N}$

Hence, option A is correct.

Q.7. In the AC circuit shown in the figure, the value of I_{rms} is equal to



A) $2\ \mathrm{A}$ B) $2\sqrt{2}\ \mathrm{A}$ C) $4\ \mathrm{A}$ D) $\sqrt{2}\ \mathrm{A}$

Answer: $2\ \mathrm{A}$

Solution: Since $V_{max} = 200\sqrt{2}$ V, $V_{rms} = \frac{V_{max}}{\sqrt{2}} = 200$ V

$$I_{rms} = \frac{V_{rms}}{\left[R^2 + \left(X_C - X_L\right)^2\right]^{\frac{1}{2}}} = \frac{200}{100} = 2\text{ A}$$

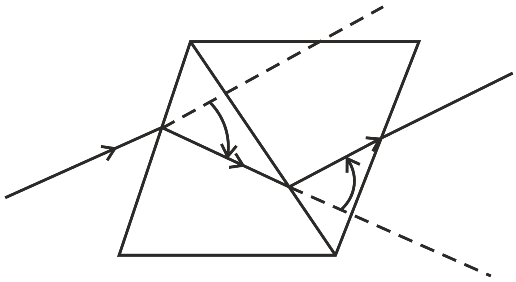
Hence, option A is correct.

Q.8. A prism has an angle of prism, $A_1 = 6^\circ$ and refractive index $\mu_1 = 1.54$. Another inverted prism has refractive index $\mu_2 = 1.72$. Find the angle of prism A_2 of the second prism such that the combination causes dispersion of light without any deviation.

A) 3.5° B) 3.0° C) 4.5° D) 5.0°

Answer: 4.5°

Solution:



For dispersion without deviation,

$A_1(\mu_1 - 1) = A_2(\mu_2 - 1)$

$\Rightarrow A_2 = A_1 \frac{(\mu_1 - 1)}{(\mu_2 - 1)} = 6^\circ \times \frac{0.54}{0.72} = 4.5^\circ$

Hence, option C is correct.

Q.9. Match the columns

a	Pressure gradient	p	$[M\ L\ T^{-1}]$
b	Impulse	q	$[M\ T^{-2}]$
c	Viscosity	r	$[M\ L^{-2}\ T^{-2}]$
d	Surface tension	s	$[M\ L^{-1}\ T^{-1}]$

A) a-p, b-r, c-q, d-s B) a-r, b-p, c-s, d-q C) a-p, b-s, c-q, d-r D) a-q, b-r, c-p, d-s

Answer: a-r, b-p, c-s, d-q

Solution: As we know, dimensions of force is given by,

$[F]=\left[\mathrm{M}^{1}\mathrm{L}^{1}\mathrm{T}^{-2}\right]$.

Now pressure gradient is given by,

$$\left[\frac{\Delta P}{\Delta X}\right]=\frac{\left[\frac{\mathrm{M}^{1}\mathrm{L}^{-1}\mathrm{T}^{-2}}{\mathrm{L}^{2}}\right]}{\mathrm{L}^{1}}$$
$$=\left[\frac{\mathrm{M}^{1}\mathrm{L}^{-1}\mathrm{T}^{-2}}{\mathrm{L}^{1}}\right]$$
$$=\left[\mathrm{M}^{1}\mathrm{L}^{-2}\mathrm{T}^{-2}\right]$$

For impulse,

$$I=\Delta P=\Delta mv=\left[\mathrm{M}^{1}\mathrm{L}^{1}\mathrm{T}^{-1}\right]$$

For viscosity,

$$f=\eta A\frac{dV}{dx}$$
$$(Pa)-sec=[\eta]$$
$$\Rightarrow[\eta]=[Pa-sec]=\left[\mathrm{M}^{1}\mathrm{L}^{-1}\mathrm{T}^{-1}\right]$$

For surface tension, we can write

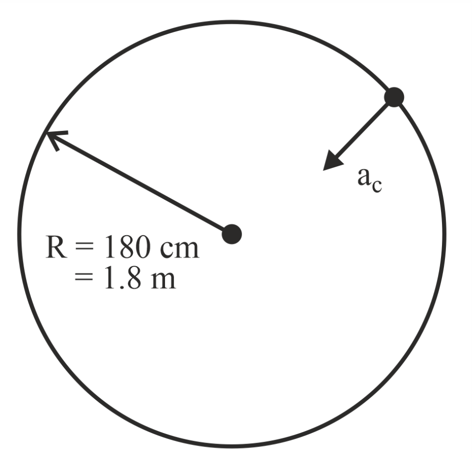
$$[T]=\frac{F}{l}$$
$$=\left[\mathrm{M}^{1}\mathrm{T}^{-2}\right]$$

Q.10. A stone of mass $1\ \mathrm{kg}$ tied to a string of length $180\ \mathrm{cm}$ is whirled in a horizontal circle with angular speed $\omega=1\ \mathrm{rad\ s}^{-1}$. The centripetal acceleration of the stone is about

A) $0.3\ \mathrm{m\ s}^{-2}$ B) $0.9\ \mathrm{m\ s}^{-2}$ C) $1.8\ \mathrm{m\ s}^{-2}$ D) $3.6\ \mathrm{m\ s}^{-2}$

Answer: $1.8\ \mathrm{m\ s}^{-2}$

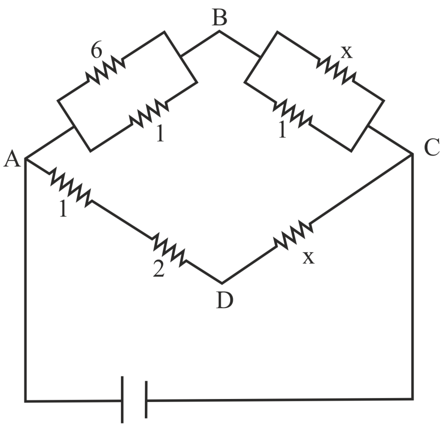
Solution:



Direction of centripetal acceleration would be towards the centre of the horizontal circle. Therefore,

$$a_c=\omega^2 r$$
$$=1^2\times1.8$$
$$=1.8\ \mathrm{m\ s}^{-2}$$

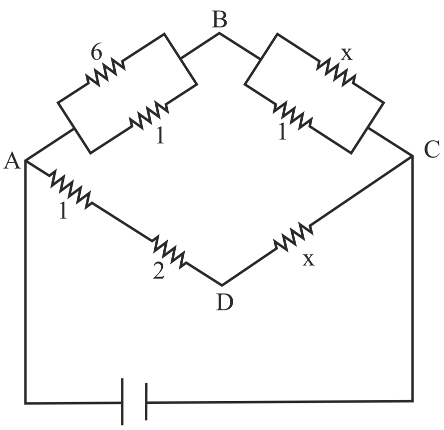
Q.11. If potential difference across B and D is zero, then find the value of x.



A) 1 B) 2.5 C) 5 D) 7.5

Answer: 2.5

Solution:



$\because$ PD across B and D is 0.

So this must be a balanced Wheatstone bridge. Therefore

$$\frac{\frac{6}{7}}{3}=\frac{\frac{x}{x+1}}{x}$$

$$\Rightarrow \frac{2}{7}=\frac{1}{x+1}$$

$$\Rightarrow 2x+2=7$$

$$\Rightarrow x=2.5\ \Omega$$

Q.12. A man of mass $10\ \mathrm{kg}$ shoots bullets of $0.02\ \mathrm{kg}$ at 180 bullets $1\ \mathrm{s}$ at $100\ \mathrm{m\ s^{-1}}$. Find impulse imparted to gun.

A) $400\ \mathrm{kg\ m\ s^{-1}}$ B) $250\ \mathrm{kg\ m\ s^{-1}}$ C) $300\ \mathrm{kg\ m\ s^{-1}}$ D) $360\ \mathrm{kg\ m\ s^{-1}}$

Answer: $360\ \mathrm{kg\ m\ s^{-1}}$

Solution: $I=\Delta p=N(mv-0)$

$$=180\times\left(\frac{2}{100}\times 100-0\right)$$

$$=360\ \mathrm{kg\ m\ s^{-1}}$$

Q.13. Two waves of same intensity from sources in phase are made to superimpose at a point. If path difference between these two coherent waves is zero then resultant intensity is I_0. If this path difference is $\frac{\lambda}{2}$, where λ is wavelength of these waves then resultant intensity is I_1 and if the difference is $\frac{\lambda}{4}$, then resultant Intensity is I_2. Value of $\frac{I_0}{I_1+I_2}$ is equal to

Answer: 2

Solution: As we know that a path difference of $\frac{\lambda}{2}$ is equivalent to phase difference of π.

Therefore, path difference of $\frac{\lambda}{4}$ is equivalent to phase difference of $\frac{\pi}{2}$.

Now resultant intensity is given by,

$I' = 4I\cos^2\left(\frac{\theta}{2}\right)$

Now, when path difference is zero, $I_0 = 4I \quad [\theta = 0]$.

When path difference is $\frac{\lambda}{2}$, $I_1 = 0 \quad [\theta = \pi]$

When path difference is $\frac{\lambda}{4}$, $I_2 = 4I\cos^2\left(\frac{\pi}{4}\right) = 4I \times \frac{1}{2} = 2I$

Hence, required value $= \frac{4I}{0+2I} = 2$

Section B: Chemistry

Q.1. The maximum number of electrons in $\mathrm{n} = 4$ shell:

A) 72 B) 50 C) 16 D) 32

Answer: 32

Solution: The maximum number of electrons that could be present in a shell is given by the rule $2\mathrm{n}^2$, where n is the shell number. That is, the first shell K shell can accommodate a maximum of 2 electrons $\left(2 \times 1^2 = 2\right)$, second shell L can accommodate a maximum of 8 electrons $\left(2 \times 2^2 = 8\right)$ likewise, the 4$^{\text{th}}$ shell named N can accommodate a maximum of 32 electrons $\left(2 \times 4^2 = 2 \times 16 = 32\right)$.

Q.2. BOD value of a water sample is $3\,\mathrm{ppm}$. Select the correct option about given sample of water:

A) It is highly polluted water B) It is clean water

C) Concentration of oxygen in the given sample is very less D) None of these

Answer: It is clean water

Solution: Drinking water has a BOD level of $1-2\ \mathrm{ppm}$. When the BOD value of water is in the range $3-5\ \mathrm{ppm}$, the water is moderately clean. Polluted water has a BOD value in the range of $6-9\ \mathrm{ppm}$.

Q.3. Which of the following chloride is more soluble in organic solvent ?

A) Be B) K C) Ca D) Mg

Answer: Be

Solution: Beryllium cation has a higher charge and smaller size, as a result have a higher charge density and covalent nature as per Fajan's rule.

Hence, its chloride will be more soluble in organic solvent

Q.4. Which one of the following is Nessler Reagent?

A) $\mathrm{K_2HgI_4}$ B) $\mathrm{KHgI_2}$ C) $\mathrm{NaHCO_3}$ D) $\mathrm{H_3PO_4}$

Answer: $\mathrm{K_2HgI_4}$

Solution: An alkaline solution of tetraiodomercurate (II) is known as Nessler's reagent. It is $\mathrm{K_2[HgI_4]}$. It is obtained by the reaction of $\mathrm{HgI_2}$ with excess KI.
$\mathrm{HgI_2 + KI(excess) \rightarrow K_2[HgI_4]}$.
Nessler's reagent gives brown precipitate (iodide of Million's base) with ammonia.

Q.5. Assertion: Antihistamines does not affect secretion of acid in stomach

Reason: Anti-allergic and antacids attack on different receptors.

A) Both assertion and reason are correct
B) Assertion is correct but reason is wrong
C) Both assertion and reason are wrong
D) Assertion is wrong but reason is correct

Answer: Both assertion and reason are correct

Solution: Antihistamines do not affect the secretion of acid in the stomach because they act on different receptors. The receptors present in the stomach do not interact with antihistamines.

Q.6. Find the correct order of bond strength for the following compounds

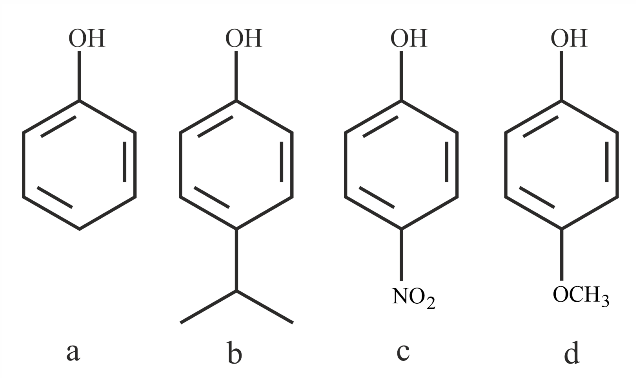
$H_2O,\ H_2S,\ H_2Se,\ H_2Te$

A) $H_2O > H_2S > H_2Se > H_2Te$
B) $H_2S > H_2O > H_2Se > H_2Te$
C) $H_2Te > H_2Se > H_2S > H_2O$
D) $H_2Te > H_2S > H_2O > H_2Se$

Answer: $H_2O > H_2S > H_2Se > H_2Te$

Solution: Bond strength is inversely proportional to the bond length of $\mathrm{E-H}$ bond of hydrides of 16th group hydrides. The bond length increases down the group. Hence, bond dissociation enthalpy(refers bond strength) decreases down the group.

Q.7. The correct order of the acidic strength of the following compounds is given as:



A) $\mathrm{a > b > c > d}$
B) $\mathrm{c > a > b > d}$
C) $\mathrm{d > c > b > a}$
D) $\mathrm{c > b > a > d}$

Answer: $\mathrm{c > a > b > d}$

Solution: Electron withdrawing groups increases the acidic nature of phenol. Nitro group is electron withdrawing group, hence, it is the more acidic compound among the given phenols.

Electron donating groups decreases the acidic nature where methoxy group is more electron donating, hence less acidic than iso-propyl group which donates electron by hyperconjugation effect.

The overall acidic nature order is

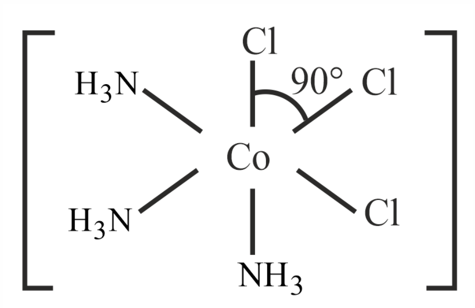
Methoxyphenol<isopropylphenol<phenol<nitrophenol

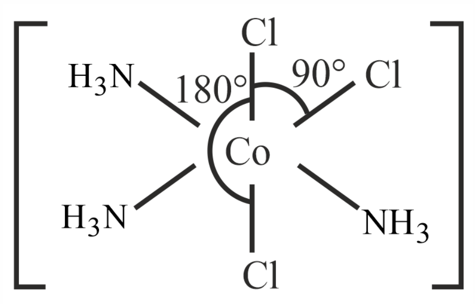
Q.8. What is $\mathrm{Cl-Co-Cl}$ bond angle in $[\mathrm{CO}\,(\mathrm{NH_3})\,\mathrm{Cl_3}]$

A) 120° and 90°
B) 90° and 180°
C) 90°
D) 180°

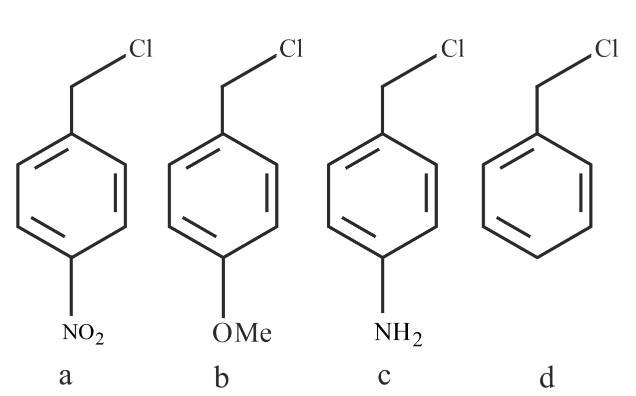
Answer: 90° and 180°

Solution: The complex $[CO(NH_3)Cl_3]$ exhibits two geometrical isomers called facial and meridional isomers. The possible structures of the complex are





Q.9. Find the orret order of S_N1 reaction for the following compounds.

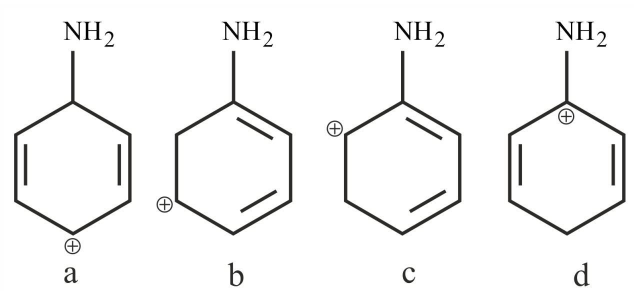


A) $a > b > c > d$ B) $c > b > d > a$ C) $c > a > b > d$ D) $d > a > b > c$

Answer: $c > b > d > a$

Solution: The relative reactivity alkyl halides in the S_N1 reaction depends on the stability of carbocation. Electron withdrawing groups decreases the stability and electron releasing groups increases the stability of carbocations. $-NO_2$ group is an electron withdrawing group by $-M$ effect. $-OMe$ and $-NH_2$ are electron releasing groups by $+M$ effect. Hence, the overall order is $c > b > d > a$

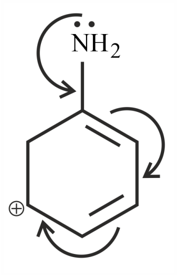
Q.10. Find the correct order of decreasing stability of following compounds



A) $a > b > c > d$ B) $d > b > c > a$ C) $b > d > a > c$ D) $b > a > d > c$

Answer: $b > d > a > c$

Solution: Maximum resonating structures are possible in b structure



Next in d structure the carbocation is stabilised by lone pair on nitrogen as well as two double bonds in the ring. In a and c structure lone pair of nitrogen is not involved in conjugation to stabilise positive charge. So NH_2 group will show $-I$ effect. So C will be the least stable molecule.

Q.11. Lead storage battery have $38\%(w/w) H_2SO_4$ find the temperature at which the liquid of battery will freeze $\left(i = 2.67; k_f \text{ of water} = 1.86\ K \cdot \frac{kg}{mol}\right)$

A) $-3.1^\circ C$ B) $-31^\circ C$ C) $-0.31^\circ C$ D) $-0.031^\circ C$

Answer: $-31^\circ C$

Solution: Freezing point of the solution $T_f = ?$

Now the expression for depression in freezing point is

$\Delta T_f = T_f^0 - T_f = i \times K_f \times m$

Mass of solute $= 38$

Mass of solvent $= 62$

molality $= \frac{\text{mass of solute}}{\text{Molar mass of solute}} \times \frac{1000}{\text{mass of solvent in grams}}$

molalily $= \frac{38 \times 1000}{98 \times 62} = 6.25$

$\Delta T_f = 2.67 \times 1.86 \times 6.25$

$T_f^0 - T_f = 31.06^\circ C$

$T_f = -31.06^\circ C$

Q.12. $KMnO_4$ oxidizes I^- in acidic and neutral medium to which forms respectively.

A) $IO_3^-,\ IO^-$ B) $IO_3^-,\ IO_3^-$ C) $IO_3^-,\ I_3^-$ D) I_2, IO_3^-

Answer: I_2, IO_3^-

Solution: In the acidic medium, MnO_4^- convert to Mn^{2+} and I^- convert to I_2.

$2MnO_4^{\ominus} + 10I^{\ominus} + 16H^+ \rightarrow 5I_2 + 2Mn^{2+} + 8H_2O$

In the neutral medium MnO_4^- convert to MnO_2 and I^- convert to IO_3^-.

$H_2O + 2MnO_4^- + I^{\ominus} \rightarrow 2MnO_2 + IO_3^{\ominus} + 2OH^-$

Q.13. Which of the following equation is correct?

A) $LiNO_3 \longrightarrow Li + NO_2 + O_2$ B) $LiNO_3 \longrightarrow LiNO_2 + O_2$ C) $LiNO_3 \longrightarrow Li_2O + NO_2 + O_2$ D) $LiNO_3 \longrightarrow Li_2O + N_2O_4 + O_2$

Answer: $LiNO_3 \longrightarrow Li_2O + NO_2 + O_2$

Solution: Lithium nitrate when decomposed, the product formed are $Li_2O,\ NO_2$ and O_2. The remaining alkali metal nitrates decomposes to nitrite of metals and oxygen gas. It is due to the lithium monoxide is stable than other monoxides.

Q.14. The option containing the correct match is given as:

List -I		List -II	
A	$Ni(CO)_4$	(i)	sp^3
B.	$[Ni(CN)_4]^{2-}$	(ii)	sp^3d^2
C.	$[Cu(H_2O)_6]^{2+}$	(iii)	d^2sp^3
D.	$[Fe(CN)_6]^{4-}$	(iv)	dsp^2

A) (A)-(i) : (B)(iv) : (C)-(ii) : (D)-(iii)

B) (A)-(iii) : (B)(ii) : (C)-(iv) : (D)-(i)

C) (A)-(ii) : (B)(iii) : (C)-(iv) : (D)-(i)

D) (A)-(iv) : (B)(ii) : (C)-(i) : (D)-(iii)

Answer: (A)-(i) : (B)(iv) : (C)-(ii) : (D)-(iii)

Solution: Ni is in zero oxidation state in $\mathrm{Ni(CO)_4}$ so the electronic configuration of Ni is $3d^84s^2$. As CO is a strong ligand, it pushes all the electrons in the $3d$ orbital, therefore the hybridisation of $\mathrm{Ni(CO)_4}$ is sp^3 and it has tetrahedral geometry. It is diamagnetic due to the absence of unpaired electrons.

In $\left[\mathrm{Ni(CN)_4}\right]^{2-}$, there is $\mathrm{Ni^{2+}}$ ion for which the electronic configuration in the valence shell is $3d^84s^0$.

In presence of strong field $\mathrm{CN^-}$ ions, all the electrons are paired up.

The empty, $3d$, $3s$ and two $4p$ Orbitals undergo dsp^2 hybridization to make bonds with $\mathrm{CN^-}$ ligands in square planar geometry.

The atomic number of Cu is 29 and its valence shell electronic configuration is $3d^{10}4s^1$.

Cu is in $+2$ oxidation state in the complex $\left[\mathrm{Cu(H_2O)_6}\right]^{2+}$.

$\left[\mathrm{Cu(H_2O)_6}\right]^{2+}$ is sp^3d^2 hybridized and it is octahedral in shape.

Another $\mathrm{Fe^{2+}}$ complex, $\left[\mathrm{Fe(CN)_6}\right]^{4-}$, is diamagnetic and has no unpaired electrons. The hybrid orbitals used to form this complex are d^2sp^3.

Q.15. Statement 1: During Hall-Heroult process, mixing $\mathrm{CaF_2}$ and$\mathrm{Na_3AlF_6}$ decreases the melting point.of $\mathrm{Al_2O_3}$

Statement 2: During electrolytic refining, anode is pure and cathode is impure.

A) Both the statements are correct

B) statement-1 is correct, while statement-2 is incorrect

C) Both the statements are incorrect

D) statement-1 is incorrect, while statement-2 is correct

Answer: statement-1 is correct, while statement-2 is incorrect

Solution: Fluorspar ($\mathrm{CaF_2}$) is added in small quantity in the electrolytic reduction of alumina dissolved in fused cryolite ($\mathrm{Na_3AlF_6}$). Addition of cryolite and fluorspar increases the electrical conductivity of alumina and lowers the fusion temperature to around $1140\ \mathrm{K}$. Impure metal is anode and pure metal acts as cathode in the electrolytic refining.

Q.16. For given E_{cell}

$$\mathrm{X|X^{2+}(0.001M)||Y^{2+}(0.01)|Y\ \ at\ \ 298K}$$

$$\mathrm{E^0}\left(\mathrm{X^{2+}|X}\right) = -0.76$$

$$\mathrm{E^0}\left(\mathrm{Y^{2+}|Y}\right) = +0.34$$

$$\frac{2.303\,\mathrm{RT}}{\mathrm{F}} = 0.06$$

If $\mathrm{E_{cell}} = \mathrm{a}$, find $5\mathrm{a}$ (closest integer)

Answer: 6

Solution: The cell reaction can be written as:

$X \rightarrow X^{2+} + 2e^{-}$

$Y^{2+} + 2e^{-} \rightarrow Y$

The overall reaction is

$X + Y^{2+} \rightarrow X^{2+} + Y$

The standard cell potential can be calculated as follows,

$$E^{0} = 0.34 - (-0.76)$$
$$= 1.10V$$

Using Nernst equation, the cell potential can be written as,

$$E_{cell} = 1.10 - \frac{0.06}{2}\log_{10}10^{-1}$$

$$a = 1.10 + 0.03 = 1.13$$

$$5a = 5 \times 1.13$$
$$= 5.65 \simeq 6$$

Q.17. $2SO_2 + O_2 = 2SO_3 + \Delta$

A. Increasing temperature
B. increasing pressure
C. increasing SO_2
D. increasing O_2
E. Adding catalyst
How many factors are responsible for getting more products

Answer: 3

Solution: Increasing temperature will shift the equilibrium backward as the reaction is exothermic and addition of catalyst has no effect on equilibrium.

Increasing pressure will shift the equilibrium forward as the equilibrium will try to decrease the pressure and will move to forward direction as number of gaseous moles are decreasing on moving from reactants to products.

Increasing SO_2 and O_2 will also shift the equilibrium forward as both are acting as a reactant in the given equilibrium reaction

Q.18. In the logarithmic equation Freundlich adsorption isotherm, the slope is at 45°, intercept $= 0.6020$, Pressure $= 0.4$ atm find x/m(The nearest integer).

Answer: 2

Solution: x/m is known as extent of adsorption.

The equation Freundlich adsorption isotherm is

$$\frac{x}{m} = kP^{1/n}$$

The logarithmic equation is

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

$$y = c + mx = 4$$

$$\log = \frac{x}{m} = \log 4 + \log 0.4$$

So, $\log\left(\frac{x}{m}\right) = \log\left(4 \times 0.4\right)$

$$\frac{x}{m} = 1.6$$

Q.19. Volume strength of a sample of $H_2O_2 = 50$. Find the molarity(The nearest integer).

Answer: 4

Solution: Volume strength represents

$1 \mathrm{~L}$ solution of $\mathrm{H_2O_2}$ will are $50 \mathrm{~L}$ of $\mathrm{O_2}$ gas at STP.

$\mathrm{H_2O_2} \longrightarrow \mathrm{H_2O} + \frac{1}{2}\mathrm{O_2}$

i.e., $34 \mathrm{~gH_2O_2} \rightarrow \frac{1}{2}$ mole $\mathrm{O_2}$ gas at STP

or $34\mathrm{g}\ \mathrm{H_2O_2} \longrightarrow 11.2\mathrm{L}\ \mathrm{O_2}$

$\therefore$ $50 \mathrm{~L}\ \mathrm{O_2}$ gas is produced by $\frac{34}{11.2} \times 50 \mathrm{~g}$ of $\mathrm{H_2O_2}$

$$\therefore \mathrm{M} = \frac{\text{The mass of } \mathrm{H_2O_2}}{\text{Molar Mass of } \mathrm{H_2O_2} \times \mathrm{V_{Solution}} \text{ in L}} == \frac{34\times50}{11.2\times34\times1} = 4.46$$

Section C: Mathematics

Q.1. If 50^{th} root of x is 12 and 50^{th} root of y is 18, then find the remainder when $x+y$ is divided by 25

A) 23 B) 22 C) 2 D) 1

Answer: 23

Solution: Given:

$$x^{\frac{1}{50}} = 12 \Rightarrow x = 12^{50}$$

$$y^{\frac{1}{50}} = 18 \Rightarrow y = 18^{50}$$

So,

$$x + y = 12^{50} + 18^{50}$$

$$\Rightarrow \frac{x+y}{25} = \frac{12^{50}}{25} + \frac{18^{50}}{25}$$

$$\Rightarrow \frac{x+y}{25} = \frac{\left(12^3\right)^{16} 12^2}{25} + \frac{\left(18^2\right)^{25}}{25}$$

$$\Rightarrow \frac{x+y}{25} = \frac{(1728)^{16} 12^2}{25} + \frac{(325-1)^{25}}{25}$$

$$\Rightarrow \frac{x+y}{25} = \frac{(1725+3)^{16} 12^2}{25} + \frac{(325-1)^{25}}{25}$$

So, remainder is

$$\frac{3^{16} 12^2}{25} - 1 = \frac{\left(3^3\right)^5 \times 3 \times 144}{25} - 1$$

$$= \frac{(25+2)^5 \times 3 \times 144}{25} - 1$$

$$= \frac{(25+2)^5 \times 3 \times (150-6)}{25} - 1$$

$$= \frac{2^5 \times 3 \times (-6)}{25} - 1$$

$$= \frac{(25+7) \times 3 \times (-6)}{25} - 1$$

$$= \frac{(21) \times (-6)}{25} - 1$$

$$= \frac{(-4) \times (-6)}{25} - 1$$

$$= 24 - 1 = 23$$

Q.2. If $\left|\vec{a}\right| = 1$, $\left|\vec{b}\right| = 2$ and $\vec{a} \cdot \vec{b} = 4$ and $\vec{c} = 2\left(\vec{a} \times \vec{b}\right) - 3\vec{b}$, then $\vec{b} \cdot \vec{c} = ?$

A) -12 B) 12 C) 14 D) 10

Answer: -12

Solution: We have,

$$\vec{b} \cdot \vec{c} = \vec{b} \cdot \left(2\left(\vec{a} \times \vec{b}\right) - 3\vec{b}\right)$$

$$\Rightarrow \vec{b} \cdot \vec{c} = 2\left(\vec{b} \cdot \left(\vec{a} \times \vec{b}\right)\right) - 3\left|\vec{b}\right|^2$$

$$\Rightarrow \vec{b} \cdot \vec{c} = 2\left[\vec{b} \quad \vec{a} \quad \vec{b}\right] - 3 \times 4$$

$$\Rightarrow \vec{b} \cdot \vec{c} = -12$$

Q.3. $\lim\limits_{n \to \infty} \frac{3}{n}\left[4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \ldots + \left(3 - \frac{1}{n}\right)^2\right]$ is

A) 19 B) 21 C) -19 D) 0

Answer: 19

Solution: Let

$L=\lim_{n\to\infty}\frac{3}{n}\left[4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots+\left(3-\frac{1}{n}\right)^2\right]$

$\Rightarrow L=\lim_{n\to\infty}\frac{3}{n}\left[\left(2+\frac{0}{n}\right)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots+\left(2+\left(1-\frac{1}{n}\right)\right)^2\right]$

$\Rightarrow L=3\lim_{n\to\infty}\frac{1}{n}\left[\sum_{r=0}^{n-1}\left(2+\frac{r}{n}\right)^2\right]$

$\Rightarrow L=3\int_0^1(2+x)^2dx$

$\Rightarrow L=\left[(2+x)^3\right]_0^1$

$\Rightarrow L=27-8$

$\Rightarrow L=19$

Q.4. Two A.P's are given as $3,7,11,15,\ldots$ and $1,6,11,16,\ldots,$ then 8^{th} common term appearing in the both the series is

A) 151 B) 160 C) 140 D) 120

Answer: 151

Solution: Two A.P's are given as

$3,7,11,15,\ldots$

First term$(a_1)=3$

Common difference$(d_1)=4$

And,

$1,6,11,16,\ldots,$

First term$(a_2)=1$

Common difference $(d_2)=5$

Now, common difference of the series of the common terms of the given A.P's is $d=\text{LCM}\left(d_1,d_2\right)=\text{LCM}\left(4,5\right)=20$

Now, series of common term is

$11,31,51,71,\ldots$

So, 8^{th} common term appearing in the both the series is

$a_8=11+(8-1)d$

$\Rightarrow a_8=11+7\times20=151$

Q.5. If $f(x)=\begin{cases}\frac{x}{|x|} & x\neq0\\ 1 & x=0\end{cases}$, $g(x)=\begin{cases}\frac{\sin(x+1)}{x+1} & x\neq-1\\ 1 & x=-1\end{cases}$ and $h(x)=2[x]+f(x)$ where $[.]$ represents greatest integer function, then $\lim_{x\to1}g(h(x-1))=$

A) $\sin2$ B) $\frac{\sin2}{2}$ C) $\sin1$ D) $\frac{\sin1}{2}$

Answer: $\frac{\sin2}{2}$

Solution: Given $f(x)=\begin{cases}\frac{x}{|x|} & x\neq 0\\ 1 & x=0\end{cases}$, $g(x)=\begin{cases}\frac{\sin(x+1)}{x+1} & x\neq -1\\ 1 & x=-1\end{cases}$ and $h(x)=2[x]+f(x)$

Here $\lim\limits_{x\to 1^+} h(x-1)=2\lim\limits_{x\to 1^+}[x-1]+\lim\limits_{x\to 1^+} f(x-1)$

$\lim\limits_{x\to 1^+} h(x-1)=0+1=1$

and $\lim\limits_{x\to 1^-} h(x-1)=2\lim\limits_{x\to 1^-}[x-1]+\lim\limits_{x\to 1^-} f(x-1)$

$\lim\limits_{x\to 1^-} h(x-1)=-2-1=-3$

Now, $g\left(\lim\limits_{x\to 1^+} h(x-1)\right)=g(1)=\frac{\sin 2}{2}$

and $g\left(\lim\limits_{x\to 1^-} h(x-1)\right)=g(-3)=\frac{\sin(-2)}{-2}=\frac{\sin 2}{2}$

Hence, $\lim\limits_{x\to 1} g(h(x-1))=\frac{\sin 2}{2}$

Q.6. Find the value of $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)\ldots\ldots\ldots\ldots\ldots\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$, if $a_1=1$ and a_i are consecutive natural numbers.

A) $\frac{\pi}{4}-\cot^{-1}2021$ B) $\frac{\pi}{4}-\cot^{-1}2022$ C) $\frac{\pi}{4}-\tan^{-1}2021$ D) $\frac{\pi}{4}-\tan^{-1}2022$

Answer: $\frac{\pi}{4}-\cot^{-1}2022$

Solution: Given,

$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)\ldots\ldots\ldots\ldots\ldots\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$

$=\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)\ldots\ldots\ldots\ldots\ldots\tan^{-1}\left(\frac{a_{2022}-a_{2021}}{1+a_{2021}a_{2022}}\right)$

$=\tan^{-1}a_2-\tan^{-1}a_1+\tan^{-1}a_3-\tan^{-1}a_2\ldots\ldots\ldots+\tan^{-1}a_{2022}-\tan^{-1}a_{2021}$

$\left\{\text{using the formula }\ \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right\}$

$=\tan^{-1}a_{2022}-\tan^{-1}a_1$

$=\tan^{-1}a_{2022}-\tan^{-1}1\ \{\text{as given } a_1=1\}$

$=\tan^{-1}a_{2022}-\frac{\pi}{4}$

$=\frac{\pi}{2}-\cot^{-1}a_{2022}-\frac{\pi}{4}$

$=\frac{\pi}{4}-\cot^{-1}a_{2022}$

$=\frac{\pi}{4}-\cot^{-1}2022$

As $a_1=1$, $a_2=2$ .... and so on $a_{2022}=2022$

Q.7. If $f(x)=\sqrt{3-x}+\sqrt{x+2}$, then range of $f(x)$ is

A) $\left(2\sqrt{2},\sqrt{10}\right)$ B) $\left(\sqrt{5},\sqrt{10}\right)$ C) $\left(\sqrt{2},\sqrt{7}\right)$ D) $\left(\sqrt{7},\sqrt{10}\right)$

Answer: $\left(\sqrt{5},\sqrt{10}\right)$

Solution: Given:

$f(x) = \sqrt{3-x} + \sqrt{x+2}$

$\Rightarrow f'(x) = -\frac{1}{2\sqrt{3-x}} + \frac{1}{2\sqrt{x+2}}$

$\Rightarrow f''(x) = -\frac{1}{4(3-x)^{3/2}} - \frac{1}{4(x+2)^{3/2}}$

For critical points

$f'(x) = 0$

$\Rightarrow -\frac{1}{2\sqrt{3-x}} + \frac{1}{2\sqrt{x+2}} = 0$

$\Rightarrow \frac{1}{\sqrt{3-x}} = \frac{1}{\sqrt{x+2}}$

$\Rightarrow 3 - x = x + 2$

$\Rightarrow x = \frac{1}{2}$

Also, for domain of $f(x)$, we must have

$3 - x \geq 0 \Rightarrow x \leq 3$

And, $x + 2 \geq 0 \Rightarrow x \geq -2$

So, domain is $[-2, 3]$.

Now, $f''\left(\frac{1}{2}\right) < 0$, so $x = \frac{1}{2}$ is the point of maxima.

Now,

$f(-2) = \sqrt{5}$
$f(3) = \sqrt{5}$
$f\left(\frac{1}{2}\right) = \sqrt{\frac{5}{2}} + \sqrt{\frac{5}{2}} = \sqrt{10}$

So, range is $\left(\sqrt{5}, \sqrt{10}\right)$.

Q.8. q is the maximum value of P lying in the interval $[0, 10]$ and the equation $x^2 - Px + \frac{5P}{4} = 0$ have rational roots, then the area (in sq. units) of the region $S : \left\{0 \leq y \leq (x-q)^2\right\}$ is equal to

A) 243 B) 723 C) 81 D) 3

Answer: 243

Solution: Since, equation $x^2 - Px + \frac{5P}{4} = 0$ have rational roots, so

$D = \text{perfect square}$

$\Rightarrow P^2 - 5P = \text{perfect square}$

$\Rightarrow P(P-5) = \text{perfect square}$

So, $P > 5$ and $P \in [0, 10]$, so $P = 6, 7, 8, 9, 10$

Now,

$10(10-5) = 50 = \text{not perfect square}$

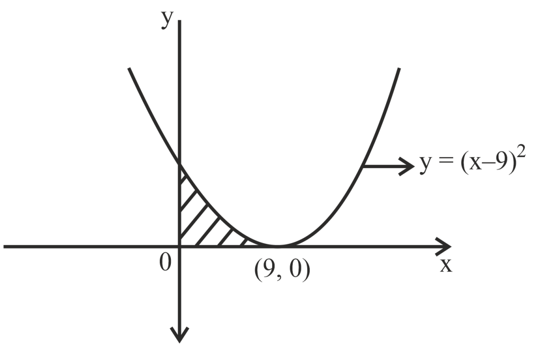
$9(9-5) = 36 = \text{perfect square}$

So, $P = 9$

Hence, $\max(P) = q = 9$

So,

$S : \left\{0 \le y \le (x-9)^2\right\}$



So,

$S = \int_0^9 (x-9)^2 dx$

$\Rightarrow S = \left[\frac{(x-9)^3}{3}\right]_0^9$

$\Rightarrow S = 243$ sq. units

Q.9. If p : i am well, q : i will not take rest and r : i will not sleep properly then "if i am not well then i will not take rest and i will not sleep properly" is logically equivalent which of the following:

A) $(\sim p \to q) \vee r$ B) $\sim p \to (q \wedge r)$ C) $(\sim p \wedge q) \to r$ D) $(\sim p \vee q) \to r$

Answer: $\sim p \to (q \wedge r)$

Solution: Given,

p : i am well, q : i will not take rest and r : i will not sleep properly

Now $\sim p$:i am not well, then operator will be used as $\to$, And operator will be used as $\wedge$ between q & r

So, "if i am not well then i will not take rest and i will not sleep properly" is logically equivalent to $\sim p \to (q \wedge r)$

Q.10. If $\frac{dy}{dx} = -\frac{3x^2+y^2}{3y^2+x^2}$; $y(1) = 0$, then $y = f(x)$ is

A) $(x+y)^2\left(3y^2 - 2xy + 3x^2\right) = 3$ B) $(x+y)\left(3y^2 - 2xy + 3x^2\right) = 3$ C) $\log(x+y) + \frac{2xy}{(x+y)^2} = 0$ D) $\log(x+y) - \frac{2xy}{(x+y)^2} = 0$

Answer: $(x+y)^2\left(3y^2 - 2xy + 3x^2\right) = 3$

Solution: Given:

$$\frac{dy}{dx}=-\frac{3x^2+y^2}{3y^2+x^2}$$

This is a homogenous differential equation.

Put

$y=vx$

$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$\frac{dy}{dx}=-\frac{3x^2+y^2}{3y^2+x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=-\frac{3x^2+v^2x^2}{3v^2x^2+x^2}$$

$$\Rightarrow x\frac{dv}{dx}=-\frac{3+v^2}{3v^2+1}-v$$

$$\Rightarrow x\frac{dv}{dx}=\frac{-3-v^2-3v^3-v}{3v^2+1}$$

$$\Rightarrow \int\left(\frac{3v^2+1}{3+v^2+3v^3+v}\right)dv=-\int\frac{dx}{x}$$

$$\Rightarrow \int\left(\frac{3v^2+1}{(v+1)\left(3v^2-2v+3\right)}\right)dv=-\int\frac{dx}{x}$$

$$\Rightarrow \int\left(\frac{\frac{1}{2}}{(v+1)}+\frac{\frac{1}{4}(6v-2)}{\left(3v^2-2v+3\right)}\right)dv=-\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\log|v+1|+\frac{1}{4}\log\left|\left(3v^2-2v+3\right)\right|=-\log|x|+\log C$$

$$\Rightarrow \frac{1}{2}\log|x+y|-\frac{1}{2}\log|x|+\frac{1}{4}\log\left|\left(3y^2-2xy+3x^2\right)\right|-\frac{1}{2}\log|x|=-\log|x|+\log C$$

$$\Rightarrow \frac{1}{2}\log|x+y|+\frac{1}{4}\log\left|\left(3y^2-2xy+3x^2\right)\right|=\log C$$

$$\Rightarrow (x+y)^2\left(3y^2-2xy+3x^2\right)=C_1$$

Now,

$y(0)=1$

So,

$C_1=3$

Hence,

$$(x+y)^2\left(3y^2-2xy+3x^2\right)=3$$

Q.11. If $P=\left(8\sqrt{3}+13\right)^{13}$, $Q=\left(6\sqrt{2}+9\right)^{9}$ then which of the following is true:

where $[.]$ represents greatest integer function

A) $[P]=\text{odd},\ [Q]=\text{even}$ B) $[P]=\text{even},\ [Q]=\text{odd}$ C) $[P]=\text{odd},\ [Q]=\text{odd}$ D) $[P]+[Q]=\text{even}$

Answer: $[P]=\text{even},\ [Q]=\text{odd}$

Solution: Given,

$P=\left(8\sqrt{3}+13\right)^{13}$ & $Q=\left(6\sqrt{2}+9\right)^{9}$

Now taking $P=I_1+f_1$ and $f_1{}'=\left(8\sqrt{3}-13\right)^{13}$ and $0<f_1,\ f_1{}'<1$

So, the value of $P=I_1+f_1-f_1{}'=\left(8\sqrt{3}-13\right)^{13}-\left(8\sqrt{3}-13\right)^{13}$

$\Rightarrow I_1+f_1-f_1{}'=2\left[{}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)+{}^{13}C_3\left(8\sqrt{3}\right)^{10}(13)^3\ldots\ldots{}^{13}C_{13}\left(8\sqrt{3}\right)^{0}(13)^{13}\right]$

$\Rightarrow I_1+f_1-f_1{}'=2k$

Now we know that the value of $f_1-f_1{}'$ will lie between $-1<f_1-f_1{}'<1$, but $I_1+f_1-f_1{}'=2k$ (even integer), so $f_1-f_1{}'=0$

So, we can say that $[P]=\text{even}$

Now taking $Q=I_2+f_2$ where $0<f_2<1$ and let $f_2{}'=\left(9-6\sqrt{2}\right)^{9}$

Now solving $Q=I_2+f_2+f_2{}'=\left(9+6\sqrt{2}\right)^{9}+\left(9-6\sqrt{2}\right)^{9}$

$\Rightarrow I_2+f_2+f_2{}'=2\left[{}^{9}C_0(9)^9\left(6\sqrt{2}\right)^{0}+{}^{9}C_2(9)^7\left(6\sqrt{2}\right)^{2}+\ldots\ldots\ldots{}^{9}C_8(9)^1\left(6\sqrt{2}\right)^{8}\right]$

$\Rightarrow I_2+f_2+f_2{}'=2k_2$

Now the value of $f_2+f_2{}'$ will lie between $0<f_2+f_2{}'<2$ but $I_2+f_2+f_2{}'=2k_2\,(\text{even interger})$, so $f_2+f_2{}'=1$,

So $I_2+1=2k_2$

$\Rightarrow I_2=\text{odd}$, hence we can say that $[Q]=\text{odd}$

Q.12. Find the number of 7 digit odd number formed by using the seven digits $1,2,2,2,3,3,5$

Answer: 240

Solution: To find the number of 7 digit odd number formed by using the seven digits $1,2,2,2,3,3,5$

We will take following cases,

Case $1-$ fixing 1 as unit place,

So total number of ways will be $\frac{6!}{3!2!}$ $\{\text{as } 2 \text{ is repeating three times and } 3 \text{ is repeating two times}\}$

Case $2-$ fixing 3 unit place,

So total number of ways will be $\frac{6!}{3!}$

Case $3-$ fixing 5 as unit place,

So total number of ways will be $\frac{6!}{3!2!}$

Now adding all the cases we will get, total number of ways as $\frac{6!}{3!2!}+\frac{6!}{3!}+\frac{6!}{3!2!}=60+120+60=240$

Q.13. If common tangent is drawn to $y^2=16x$ & $x^2+y^2=8$, then find the square of distance between point of contact of common tangent of both curves.

Answer: 72

Solution: Given,

Equation of curves $y^2 = 16x$ & $x^2 + y^2 = 8$

Now we know that,

Equation of tangent to parabola is given by $y = mx + \frac{a}{m}$

$\Rightarrow y = mx + \frac{4}{m}$

Now this tangent is also touching the circle $x^2 + y^2 = 8$, so its distance from centre will be radius,

So, $2\sqrt{2} = \left|\frac{0-0-\frac{4}{m}}{\sqrt{1^2+m^2}}\right|$

$\Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow m = \pm 1$

So, equation of tangent will be $y = x + 4 \ \{\text{taking } m = 1\}$

Now point of intersection of circle $x^2 + y^2 = 8$ and tangent $y = x + 4$ will be $(-2, 2)$,

And point of intersection of parabola and tangent $y^2 = 16x$ & $y = x + 4$ respectively will be $(4, 8)$

So, the distance between the point of intersection will be $\sqrt{(-2-4)^2 + (2-8)^2} = \sqrt{72}$

So, its square will be $\left(\sqrt{72}\right)^2 = 72$

Q.14. If $A = \{2, 4, 6, 8, 10\}$, then the total number of functions defined on A satisfying $f(m \times n) = f(m) \times f(n),\ m,\ n \in A$ are

Answer: 25

Solution: Given $f(m \times n) = f(m) \times f(n),\ m,\ n \in A$

$f(x) = x^k\ \forall k \in R$

Now, $f(2) = 2^k$ so $k = 1,\ 2,\ \log_2 6,\ 3,\ \log_2 10$

i.e 5 ways

Similarly for each element their would be five ways.

So, total their would be $5 \times 5 = 25$ functions.

Q.15. Let $a = \{1, 3, 5, \ldots, 99\}$ and $b = \{2, 4, 6, \ldots, 100\}$, then the number of ordered pairs (a, b) such that $a + b$ when divided by 23 leaves remainder 2 is

Answer: 108

Solution: Let $a=\{1,3,5,\ldots,99\}$ and $b=\{2,4,6,\ldots,100\}$

Let

$a+b=23\lambda+2$

If $\lambda=1$, then

$a+b=25$

So, $\{(1,24),(3,22),\ldots.,(23,2)\}$

Total ordered pairs$=12$

If $\lambda=2$, then

$a+b=48$

No ordered pair possible.

If $\lambda=3$, then

$a+b=71$

So,

$\{(1,70),\ (3,68),\ (5,66),\ldots.,(61,10),\ (63,8),\ (65,6),\ (67,4),\ (69,2)\}$

Total 35 ordered pairs.

If $\lambda=5$, then

$a+b=117$

$\{(17,100),\ (19,98),\ \ldots,\ (99,18)\}$

Total 42 ordered pairs.

If $\lambda=7$, then

$a+b=163$

So,

$\{(63,100),\ (65,63),\ldots.\ (99,64)\}$

Total 19 pairs.

No further case possible.

So, required number is

$=12+35+42+19=108$