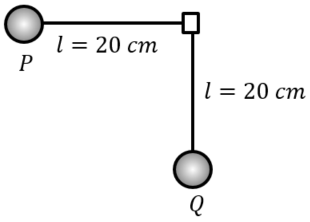
JEE Main Exam 2023 - Session 1

30 Jan 2023 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. Bob P is released from its initial position as shown in the figure. If it collides elastically with an identical bob Q hanging freely, then the velocity of Q just after collision is $\left(g=10\ \mathrm{m\ s^{-2}}\right)$



A) $1\ \mathrm{m\ s^{-1}}$

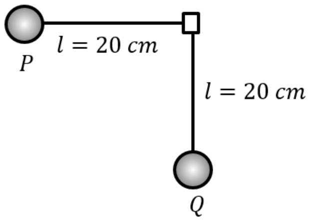
B) $4\ \mathrm{m\ s^{-1}}$

C) $2\ \mathrm{m\ s^{-1}}$

D) $8\ \mathrm{m\ s^{-1}}$

Answer: $2\ \mathrm{m\ s^{-1}}$

Solution:



Let v_P be the velocity of bob P just before collision. Therefore, applying conservation of energy for P (between the moment it is released and the moment just before collision),

$mgl+0=\frac{1}{2}mv_P{}^2$

$\Rightarrow v_P=\sqrt{2gl}$

$=\sqrt{2\times 10\times 0.2}$

$=2\ \mathrm{m\ s^{-1}}$

Since both pendulums have equal masses and the collision is perfectly elastic, the velocities of both the bobs will get interchanged.

Therefore, the velocity of bob Q just after the collision is $2\ \mathrm{m\ s^{-1}}$.

Hence, option C is correct.

Q.2. Two solid conducting spheres A and B are placed at a very large separation. Their charge densities and radii are shown in the figure below. Find the ratio of charge densities after the key K is closed.

A) $4:1$

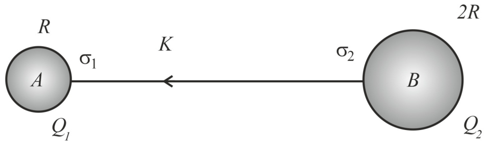
B) $1:2$

C) $2:1$

D) $1:4$

Answer: $2:1$

Solution:



Let the charges and the charge densities on both the spheres after the key is closed be $Q_1,\ Q_2,\ \sigma_1$ and σ_2 as shown in the figure.

Once the key is closed, charge on both the spheres will redistribute such that potentials at the surface of both spheres is the same.

$\therefore \frac{kQ_1}{R} = \frac{kQ_2}{2R}$

$\Rightarrow \frac{Q_1}{Q_2} = \frac{1}{2}$

$\Rightarrow \frac{\sigma_1 \times 4\pi\ R^2}{\sigma_2 \times 4\pi\ (2R)^2} = \frac{1}{2}$

$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{2}{1}$

Hence, option C is correct.

Q.3. Heat energy is supplied to the system undergoing isothermal process. Choose the option with correct statements.
1) Internal energy will increase.
2) Internal energy will decrease.
3) Work done by the system is positive.
4) Work done by the system is negative.
5) Internal energy remains constant.

A) $(1), (3), (5)$ B) $(2), (4)$ C) $(3),\ (5)$ D) $(1), (4), (5)$

Answer: $(3),\ (5)$

Solution: From the first law of thermodynamics, heat supplied to the system Q, change in internal energy of the system ΔU and work done by the system W are related as:

$Q = \Delta U + W$

Since the process is isothermal, the temperature of the system is constant and therefore $\Delta U = 0$ (i.e., **internal energy is constant**).

$\Rightarrow Q = W$

As heat is supplied to the system, Q is positive and therefore
W **is positive.**

Hence, option C is correct.

Q.4. Heat passing through the cross-section of a conductor, varies with time t as $Q\left(t\right) = \alpha t\ -\ \beta t^2 + \gamma t^3$ (where $\alpha,\ \beta$ and γ are positive constants). Minimum heat current through the conductor is

A) $\alpha - \frac{\beta^2}{2\gamma}$ B) $\alpha - \frac{\beta^2}{3\gamma}$ C) $\alpha - \frac{\beta^2}{\gamma}$ D) $\alpha - \frac{3\beta^2}{\gamma}$

Answer: $\alpha - \frac{\beta^2}{3\gamma}$

Solution: Heat current i_h is given by

$i_h = \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma t^2$ ....(1)

For Heat current to be minimum,

$\frac{di_h}{dt} = 0$
$\Rightarrow -2\beta + 6\gamma t = 0$

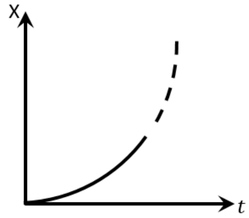
$\Rightarrow t = \frac{\beta}{3\gamma}$

Substituting in equation (1),

$i_{h_{min}} = \alpha - 2\beta \frac{\beta}{3\gamma} + 3\gamma \frac{\beta^2}{9\gamma^2} = \alpha - \frac{\beta^2}{3\gamma}$

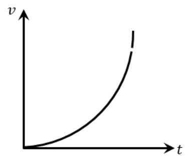
Hence, option B is correct.

Q.5. Position-time graph for a particle is parabolic and is shown below:

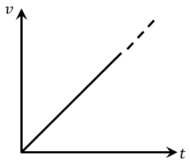


Choose the corresponding $v-t$ graph

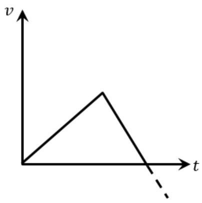
A)



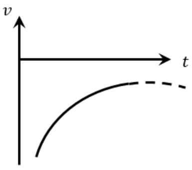
B)



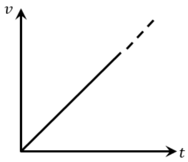
C)



D)



Answer:



Solution: For parabolic $x-t$ graph passing through origin,

$x=\alpha t^2$

$\therefore v=\frac{dx}{dt}=2\alpha t$

Which shows that $v-t$ graph will be linear with a slope 2α (i.e., velocity will be linearly increasing).

Hence, option B is correct.

Q.6. A particle, moving in unidirectional motion, travels half of the total distance with a constant speed of $15 \mathrm{~m~s}^{-1}$. It travels the first half of the remaining time at $10 \mathrm{~m~s}^{-1}$ and the second half of the remaining time at $5 \mathrm{~m~s}^{-1}$. The average speed of the particle for the complete journey is:

A) $12 \mathrm{~m~s}^{-1}$ B) $10 \mathrm{~m~s}^{-1}$ C) $7 \mathrm{~m~s}^{-1}$ D) $9 \mathrm{~m~s}^{-1}$

Answer: $10 \mathrm{~m~s}^{-1}$

Solution: Let the total distance covered be $2d$.

If time taken for the second half of journey is $2t$, the average speed during the second half is

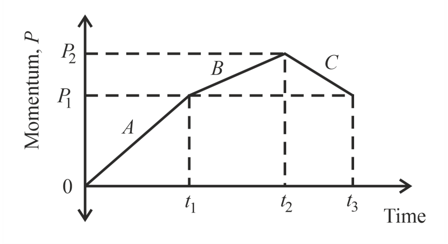
$$v_2 = \frac{10 \times \frac{t}{2} + 5 \times \frac{t}{2}}{t} = 7.5 \text{ m s}^{-1}$$

The average speed for the complete journey is

$$v = \frac{2d}{\frac{d}{v_1} + \frac{d}{v_2}}$$
$$= \frac{2v_1 v_2}{v_1 + v_2}$$
$$= \frac{2 \times 15 \times 7.5}{15 + 7.5}$$
$$= 10 \text{ m s}^{-1}$$

Hence, option B is correct.

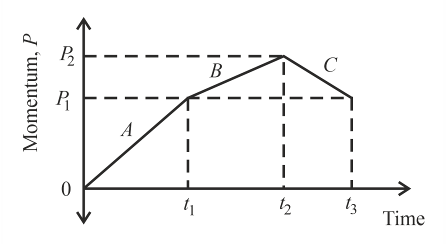
Q.7. Momentum-time graph of an object moving along a straight line is shown in the figure. If $(P_2 - P_1) < P_1$ and $(t_2 - t_1) = t_1 < (t_3 - t_2)$, then at which points among A, B and C, the magnitude of force experienced by the object is maximum and minimum respectively.



A) A, B B) A, C C) B, C D) B, A

Answer: A, C

Solution:



Magnitude of forces during different intervals are given by:

$$|F_A| = \frac{P_1}{t_1}$$

$$|F_B| = \frac{P_2 - P_1}{t_2 - t_1}$$

Since $(p_2 - p_1) < p_1$ & $(t_2 - t_1) = t_1$

$|F_A| > |F_B|$

$$|Fc| = \left|\frac{P_1 - P_2}{t_3 - t_2}\right| = \frac{P_2 - P_1}{t_3 - t_2}$$

Since $(t_2 - t_1) < (t_3 - t_2)$

$\Rightarrow |F_B| > |Fc|$

$\therefore |F_A| > |F_B| > |Fc|$

Hence, option B is correct.

Q.8. A beam of Electromagnetic wave of power 20 mW is incident on a perfectly absorbing body for 300 ns. The total momentum transferred by the beam to the body is equal to

A) 2×10^{-17} N s B) 1×10^{-17} N s C) 3×10^{-17} N s D) 5×10^{-17} N s

Answer: 2×10^{-17} N s

Solution: From de-Broglie's equation,

$p_{wave} = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{E}{c}$ (since $E = \frac{hc}{\lambda}$)

Where p_{wave} is momentum and E is energy.

$\therefore p_{wave} = \frac{Pt}{c}$

Where P is the power transferred by the beam. Since the momentum beam is transferred to the body, from conservation of linear momentum,

$\Delta p_{body} = \frac{E}{c} = \frac{Pt}{c}$

$\Delta p_{body} = \frac{20\times10^{-3}\times300\times10^{-9}}{3\times10^{8}}$

$= 2 \times 10^{-17}\ \mathrm{N\ s}$

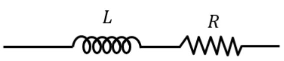
Hence, option A is correct.

Q.9. If an insulator with inductive reactance $\chi_L = R$ is connected in series with resistance R across an AC voltage, power factor comes out to be P_1. Now, if a capacitor with capacitive reactance $\chi_C = R$ is also connected in series with the inductor and resistor in the same circuit, power factor becomes P_2. Find $\frac{P_1}{P_2}$.

A) $\sqrt{2}:1$ B) $1:\sqrt{2}$ C) $1:1$ D) $1:2$

Answer: $1:\sqrt{2}$

Solution: In case of LR circuit:



Given: $\chi_L = R$

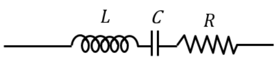
$P_1 = \cos\phi$

$= \frac{R}{Z}$

$= \frac{R}{\sqrt{R^2+(\chi_L-\chi_C)^2}}$

$= \frac{1}{\sqrt{2}}$

In case of LCR circuit:



$P_2 = \cos\phi$

$= \frac{R}{Z}$

$= \frac{R}{\sqrt{R^2+(\chi_L-\chi_C)^2}}$

$= 1$

Therefore,

$\frac{P_1}{P_2} = 1:\sqrt{2}$

Q.10. The velocity of an electron in the seventh orbit of a hydrogen like atom is $3.6 \times 10^6\ \mathrm{m\ s^{-1}}$. Find the velocity of the electron in the 3^{rd} orbit.

A) $4.2 \times 10^6\ \mathrm{m\ s^{-1}}$ B) $8.4 \times 10^6\ \mathrm{m\ s^{-1}}$ C) $2.1 \times 10^6\ \mathrm{m\ s^{-1}}$ D) $3.6 \times 10^6\ \mathrm{m\ s^{-1}}$

Answer: $8.4 \times 10^6\ \mathrm{m\ s^{-1}}$

Solution: For hydrogen like atom, the velocity of electron in n^{th} orbit is given by

$v = \frac{z^2e^2}{2\varepsilon_\circ hn}$

$\therefore v \propto \frac{1}{n}$

$\Rightarrow \frac{v_1}{v_2} = \frac{n_2}{n_1}$

$\Rightarrow \frac{3.6\times10^6}{v_2} = \frac{3}{7}$

$v_2 = 8.4 \times 10^6\ \mathrm{m\ s^{-1}}$

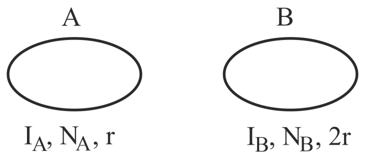
Hence, option B is correct.

Q.11. Coil A of radius $10\ \mathrm{cm}$ has N_A number of turns and I_A current is flowing through it. Coil B of radius $20\ \mathrm{cm}$ has N_B number of turns and I_B current is flowing through it. If magnetic dipole moment of both the coils is same then

A) $I_AN_A = 4I_BN_B$ B) $I_AN_A = \frac{1}{4}I_BN_B$ C) $I_AN_A = 2I_BN_B$ D) $I_AN_A = \frac{1}{2}I_BN_B$

Answer: $I_A N_A = 4 I_B N_B$

Solution:



As magnetic moment of both the coil is the same,

$\mu_1 = \mu_2$

$\Rightarrow N_A \left[I_A \left(\pi r^2\right)\right] = N_B \left[I_B \pi (2r)^2\right]$

$\Rightarrow I_A N_A = 4 I_B N_B$

Q.12. An ideal gas undergoes a thermodynamic process following the relation $PT^2 =$ constant. Assuming symbols have their usual meaning, then volume expansion coefficient of the gas is equal to:

A) $\frac{2}{T}$ B) $\frac{3}{T}$ C) $\frac{1}{2T}$ D) $\frac{1}{T}$

Answer: $\frac{3}{T}$

Solution: The equation of thermodynamic process is given as

$PT^2 = \text{constant} \left(\text{say } c\right)$ ...(1)

From ideal gas equation,

$PV = nRT$ ....(2)

Dividing equation (2) by equation (1)

$V = \frac{nR}{c} T^3$ ....(3)

If γ is the coefficient of volume expansion,

$dV = \gamma V dT$ ....(4)

From equation (3)

$dV = \frac{3nR}{c} T^2 dT$ ....(5)

From equations (3), (4) and (5),

$\frac{3nR}{c} T^2 dT = \gamma \frac{nR}{c} T^3 dT$

$\Rightarrow \gamma = \frac{3}{T}$

Hence, option B is correct.

Q.13. If the height of capillary rise is 5 cm for a liquid. What is the rise in height if the surface tension and density is doubled.

A) 10 cm B) 5 cm C) 2.5 cm D) 20 cm

Answer: 5 cm

Solution: The height h through which a liquid will rise in a capillary tube of radius r is given by,

$h = \frac{2T \cos\theta}{r \rho g} \propto \frac{T}{\rho} \propto \frac{2T}{2\rho}$

As we can in both cases, value of height will remain same. Therefore,

$h' = h = 5 \text{ cm}$

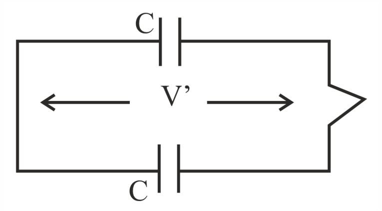
Q.14. Capacitor of $400 \mu\text{F}$ is connected to a 100 V battery. Now battery is removed and identical capacitor is connected. Find change is PE of the system.

A) 2 J B) 5 J C) 1 J D) 4 J

Answer: 1 J

Solution: Initial charge in the first case will be,

$q=CV$.



Now, the capacitor is connected to an identical capacitor, let the final potential difference across the capacitor be V'.

As total charge on the system will remain same.

$q=CV=2C\cdot V'$

$\Rightarrow V'=\frac{V}{2}$

Now, change in potential energy will be

$\Delta U=\frac{1}{2}CV^2-2\left[\frac{1}{2}C\left(\frac{V}{2}\right)^2\right]$

$=\frac{1}{2}CV^2-\frac{1}{4}CV^2$

$=\frac{1}{4}CV^2$

Therefore,

$\Delta U=\frac{1}{4}\times\left(400\times10^{-6}\right)\times10^4=\frac{4}{4}=1\ \mathrm{J}$

Q.15. What is the correct relation between Young's Modulus (Y), modulus of rigidity (η), and poisson ratio (σ)?

A) $Y=2\eta(1+\sigma)$ B) $Y=\eta(1-2\sigma)$ C) $Y=2\eta(1+2\sigma)$ D) $\mathrm{Y}=2\eta(1-\sigma)$

Answer: $Y=2\eta(1+\sigma)$

Solution: The correct relation between Young's Modulus (Y), modulus of rigidity (η), and poisson ratio (σ) is $Y=2\eta(1+\sigma)$.

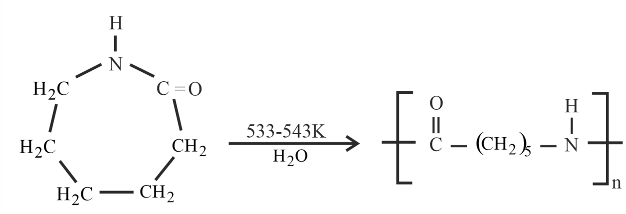
Section B: Chemistry

Q.1. Caprolactam when heated at high temperature gives:

A) Nylon 6,6 B) Dacron C) Teflon D) Nylon 6

Answer: Nylon 6

Solution: Nylon-6 is the synthetic polymer prepared by using caprolactam. It is obtained by heating caprolactam with water at a high temperature. The reaction is shown below.



Q.2. Molarity of CO_2 in soft drink is $0.01\mathrm{M}$. The volume of soft drink is $300\ \mathrm{mL}$. Mass of CO_2 in soft drink is:

A) $0.132\ \mathrm{g}$ B) $0.481\ \mathrm{g}$ C) $0.312\ \mathrm{g}$ D) $0.290\ \mathrm{g}$

Answer: $0.132\ \mathrm{g}$

Solution: Number of moles $=\frac{\text{Mass of substance(W)}}{\text{Molar mass of substance(MM)}}$

$\mathrm{M}=\frac{\mathrm{W}}{\mathrm{MM}\times\mathrm{V(L)}}$

$\mathrm{M}=$ Molarity

$\mathrm{V}=$Volume of solution in L

$0.01=\frac{\mathrm{W}}{44\times\frac{300}{1000}}$

$=0.01\times44\times\frac{300}{1000}=0.132\ \mathrm{g}$

Q.3. During the qualitative analysis of SO_3^{2-} using dilute H_2SO_4, SO_2 gas evolved which turns $K_2Cr_2O_7$ solution

A) Green B) Black C) Blue D) Red

Answer: Green

Solution: Sulphite reacts with dilute Sulphuric acid, it gives Sulphur dioxide which is a colourless gas with burning smell of sulphur as its product. This is acidic in nature. Potassium dichromate can be used to test for sulphur dioxide, as it turns distinctively from orange to green.

$$SO_3{}^{2-} + 2H^+ \longrightarrow H_2SO_3 \longrightarrow H_2O + SO_2 \uparrow$$

$$\underset{}{SO_2(g)} + \underset{\text{(orange)}}{K_2Cr_2O_7} \xrightarrow{H^+} \underset{\text{green}}{Cr_2(SO_4)_3} + K_2SO_4 + H_2O$$

Q.4. Order of strength of ligand

$S^{2-}, CO, en, C_2O_4^{2-}, NH_3$

A) $CO > en > NH_3 > C_2O_4^{2-} > S^{-2}$

B) $en > CO > NH_3 > S^{-2} > C_2O_4^{2-}$

C) $NH_3 > en > CO > S^{-2} > C_2O_4^{2-}$

D) $NH_3 > en > CO > C_2O_4^{2-} > S^{-2}$

Answer: $CO > en > NH_3 > C_2O_4^{2-} > S^{-2}$

Solution: A spectrochemical series is a list of ligands ordered by ligand "strength". The spectrochemical series showing the strength of various ligands is given as:

$$I^- < Br^- < SCN^- < Cl^- < S^{2-} < F^- < OH^- < C_2O_4{}^{2-} < H_2O < NCS^- < edta^{4-} < NH_3 < en < CN^- < CO$$

Q.5. Match the following no. of lone pair of central atom

Column I		Column II	
(A)	IF_7	(p)	0
(B)	ICl_4^-	(q)	1
(C)	XeF_2	(r)	2
(D)	XeF_6	(s)	3

A) A – p; B – q; C – r; D – s

B) A – p; B – r; C – s; D – q

C) A – r; B – s; C – p; D – q

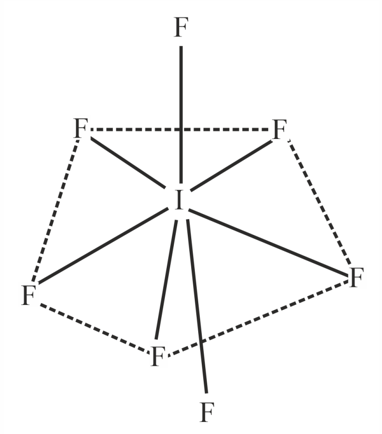
D) A – s; B – r; C – q; D – p

Answer: A – p; B – r; C – s; D – q

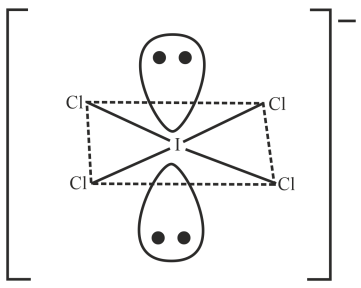
Solution: The number of lone pairs on the central atom = number of hybrid orbitals - Number of bond pairs.

IF_7 undergoes sp^3d^3 hybridisation and has pentagonal bipyramidal geometry.

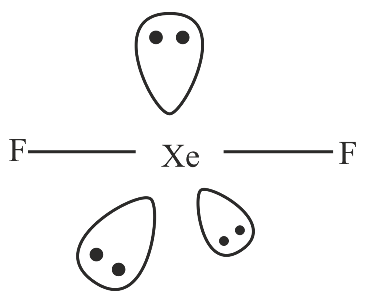
The number of lone pairs = 7-7 =0



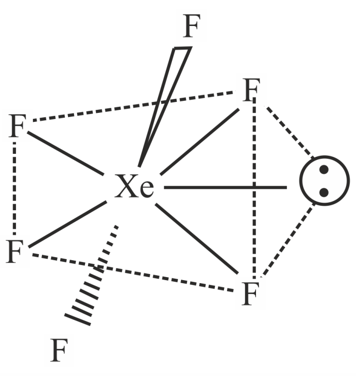
ICl_4^- undergoes sp^3d^2 hybridisation and has square planar geometry. The number of lonepairs = 6-4 =2



XeF_2 undergoes sp^3d hybridisation and has linear geometry. The number of lone pairs =5-2 =3



XeF_6 undergoes sp^3d^3 hybridisation and has distorted octahedral. The number of lone pairs = 7-6=1



Q.6. Which of the following is water-soluble?

a) $BeSO_4$ b) $MgSO_4$ c) $CaSO_4$ d) $SrSO_4$ e) $RaSO_4$

A) Only a and b B) Only a, b, and c C) Only d and e D) Only a and e

Answer: Only a and b

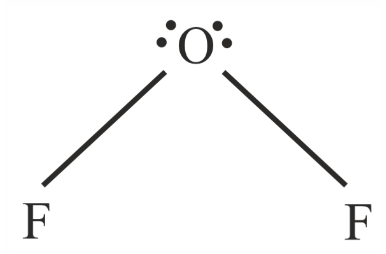
Solution: Sulphates of Be and Mg are readily soluble in water but sulphates of Ca, Sr, Ba, etc., are insoluble. It is due to increase in lattice energy of sulphate down the group which predominates over hydration energy.

Q.7. Shape of OF_2 molecule is?

A) Bent B) Linear C) Tetrahedral D) T-shaped

Answer: Bent

Solution: The central atom is oxygen and is surrounded by two fluorine atoms.The number of hybrid orbitals(H) = Number of lone pair+ Number of atoms attached. $H = 2 + 2 = 4$. Hence, the hybridisation of oxygen is sp^3. The shape of the molecule is bent due to the presence of two ,one pairs.

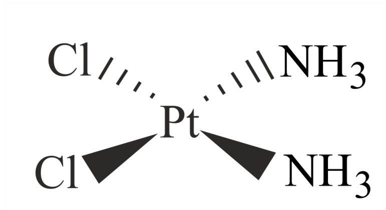


Q.8. Which of the following compounds acts as an inhibitor for cancer growth.

A) Cisplatin B) EDTA C) Cobalt D) Ethane 1,2-diamine

Answer: Cisplatin

Solution: Cisplatin is a chemotherapy medication used to treat a number of cancers. It is given by injection into a vein. Cisplatin binds to the $N7$ reactive center on purine residues and as such can cause deoxyribonucleic acid (DNA) damage in cancer cells, blocking cell division and resulting in apoptotic cell death. The structure of the cisplatin is shown below.



Q.9. Speed of electron in 7^{th} orbit is $3.6 \times 10^6 ms^{-1}$, then find speed in 3^{rd} orbit

A) $3.6 \times 10^6\ m/s$ B) $8.4 \times 10^6\ m/s$ C) $7.5 \times 10^6\ m/s$ D) $1.8 \times 10^6\ m/s$

Answer: $8.4 \times 10^6\ m/s$

Solution: The speed of the electron in n^{th} orbit is given by

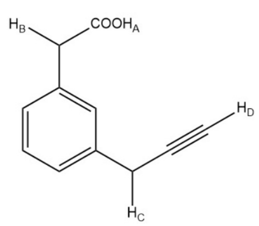
$$V = \frac{V_0 Z}{n}$$

$$3.6 \times 10^6 = \frac{V_0 Z}{7}$$

$$V_0 Z = 7 \times 3.6 \times 10^6 = 25.2 \times 10^6$$

Now, speed of electron in 3rd orbit $= \frac{25.2 \times 10^6}{3} = 8.4 \times 10^6\ m/s$

Q.10. Consider the following molecule

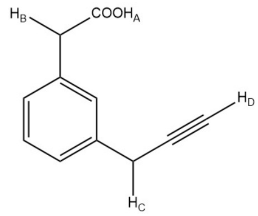


Select the correct order of acidic strength

A) $H_A > H_B > H_C > H_D$ B) $H_B > H_A > H_C > H_D$ C) $H_A > H_C > H_B > H_D$ D) $H_A > H_D > H_B > H_C$

Answer: $H_A > H_D > H_B > H_C$

Solution:



In the above molecule the hydrogen of carboxylic acid is more acidic because carboxylate ion is resonance stabilised. The hydrogen attached to sp carbon is more acidc than sp^3 carbon because sp carbon is more electronegative. Now, amongst B and C, H_B is more acidic hydrogen as it is close to electron withdrawing group.

Q.11. Match the following

Atomic no			
(a)	52	(p)	s block
(b)	37	(q)	p block
(c)	65	(r)	d block
(d)	74	(s)	f block

A) $(i)-q,\ (ii)-p,\ (iii)-r,\ (iv)-s$

B) $(i)-q,\ (ii)-p,\ (iii)-s,\ (iv)-r$

C) $(i)-s,\ (ii)-r,\ (iii)-p,\ (iv)-q$

D) $(i)-r,\ (ii)-p,\ (iii)-q,\ (iv)-s$

Answer: $(i)-q,\ (ii)-p,\ (iii)-s,\ (iv)-r$

Solution: The atomic number 52 represents the element tellurium. It is a halogen and its electronic configuration is [Kr] $4d^{10}5s^25p^4$. It is a p-block element.

The atomic number 37 represents the element Rubidium. It is an alkali metal and its electronic configuration is [Kr] $5s^1$. It is a s-block element.

The atomic number 65 represents the element Terbium. It is a metal and its electronic configuration is [Xe] $4f^85d^16s^2$. It is a f-block element.

The atomic number 74 represents the element Tungsten. It is a metal and its electronic configuration is [Xe] $4f^{14}5d^46s^2$. It is a d-block element.

Q.12. Consider the following reactions

$NO_2 \xrightarrow{UV} A + B$

$A + O_2 \rightarrow C$

$B + C \rightarrow NO_2 + O_2$

Find $A,\ B$ and C respectively

A) $O,\ NO,\ O_3$

B) $NO,\ O,\ O_3$

C) $NO,\ O_3,\ O$

D) $O,\ O_3,\ NO$

Answer: $O,\ NO,\ O_3$

Solution: Nitrogen dioxide breaks into nitric oxide and nascent oxygen in the presence of sunlight. The nascent oxygen combine with oxygen molecule forms ozone. Ozone and nitric oxide combines to give nitrogen dioxide and oxygen molecule.

$$NO_2 \xrightarrow{UV} NO + O\ (\text{atomic})$$

$$O + O_2 \rightarrow O_3$$

$$NO + O_3 \rightarrow NO_2 + O_2$$

Q.13. Assertion: ketos gives seliwanoff test.

Reason : ketos undergoes β-elimination to form furfural

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

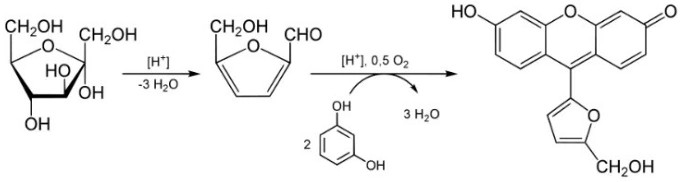
B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

C) Assertion is true but Reason is false.

D) Assertion is false but Reason is true.

Answer: Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Solution: **Seliwanoff's test**
Ketoses dehydrate very rapidly under acidic conditions to give furfural, which reacts with resorcinol to give a coloured product.



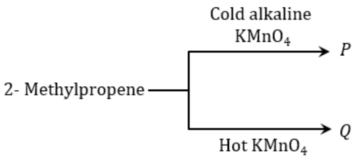
Q.14. Which of the following is an antacid?

A) Ranitidine B) Prontosil C) Norethindrone D) Codeine

Answer: Ranitidine

Solution: Ranitidine is a medicine that reduces the amount of acid in our stomach. It was used for indigestion, heartburn and acid reflux, gastro-oesophageal reflux disease and to prevent and treat stomach ulcers.

Q.15. Consider the following reactions



The product P & Q respectively are?

A) $CH_2(OH)-C(OH)(CH_3)-CH_3$ & $CH_3-CH(OH)-CH_3$

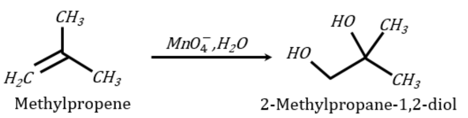
B) $CH_2(OH)-C(OH)(CH_3)-CH_3$ & $CH_3-C(=O)-CH_3$

C) $CH_3-C(=O)-CH_3$ & $HCOOH$

D) $HCOOH$ & $CH_3-CH(OH)-CH_3$

Answer: $CH_2(OH)-C(OH)(CH_3)-CH_3$ & $CH_3-C(=O)-CH_3$

Solution: Cold $KMnO_4$ coverts the alkene to 1,2-diol as shwon below.



The hot $KMnO_4$ oxidise the methylpropene to acetone and carbon dioxide. The products are shown below.

$$(CH_3)_2C=CH_2 \xrightarrow{KMnO_4/H^+} (CH_3)_2C=O + CO_2 + H_2O$$

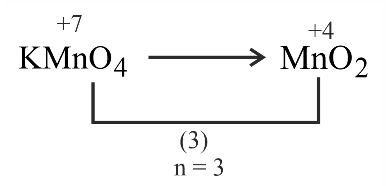
2-Methyl propene
Unsymmetric alkene

Propan-2-one

Q.16. How many moles of electron are required to reduce 1 mole of permanganate ion into manganese dioxide

Answer: 3

Solution: The oxidation state of manganese in potassium permanganate is +7 and in manganese dioxide is +4.



The change in oxidation state per molecule of potassium permanganate is 3. Hence, the number of moles electrons involved to reduce one mole of permanganate is 3.

Q.17. A solution of $2\ g$ of a solute and $20\ g$ water has boiling point $373.52\ K$. Then find the molecular mass of solute?
[given: $K_b = 0.52\, Kkg / mole$ and solute is non-electrolyte]

Answer: 100

Solution: The elevation in boiling point of non-electrolytic solution is the product of molal elevation constant and molality(m).

$\Delta T_B = k_b \times m$

molality(m)$= \frac{\text{Number of moles of solute}}{\text{mass of the solvent in kg}}$

Number of moles of solute $= \frac{\text{mass of solute}}{\text{Molar mass of solute(MM)}}$

$373.52 - 373 = 0.52 \times \frac{2}{MM \times 0.020}$

$0.52 = 0.52 \times \frac{2}{MM \times 0.020}$

$MM = 100\ g$

Q.18. For first order kinetic rate constant $2.303 \times 10^{-3} sec^{-1}$. The time taken for the decomposition of substance from $7\ g$ to $2\ g$ will be:

(Use log $7 = 0.845$ and $\log 2 = 0.301$)

Answer: 544

Solution: The integrated equation for first order kinetics is

$t = \frac{2.303}{k} \log \frac{[A_0]}{[A_t]}$

t =time taken

k =rate constant

$[A_0]$=initial concentration

$[A_t]$=concentration at time t.

$t = \frac{2.303}{2.303 \times 10^{-3}} \log \frac{7}{2}$

$= 1000\,(\log 7 - \log 2)$

$1000 \times .544 = 544\, sec$

Q.19. $600\, mL$ of $0.04\, M HCl$ is mixed with $400\, mL$ of $0.02\, M H_2SO_4$. The pH of the resulting solution is $\frac{x}{10}$. The value of x is

Answer: 14

Solution: Milli moles = Molarity $\times$ Volume in mL

milli moles of $H^+$from $HCl = 0.04 \times 600 = 24$

milli moles of $H^+$from $H_2SO_4 = 0.02 \times 2 \times 400 = 16$

Total milli moles of $H^+ = 24 + 16 = 40$

Final volume of solution $= 1000\, ml$

$[H^+] = \frac{40}{100} = 0.04M$

$pH = \log(0.04) = 1.4 = \frac{x}{10}$

$\Rightarrow x = 14$

Section C: Mathematics

Q.1. $\lim_{x \to 0} \left\{ \frac{48 \int_0^x \left(\frac{t^3}{1+t^6} \right) dt}{x^4} \right\} =$

A) 12 B) 16 C) 18 D) 14

Answer: 12

Solution: Let

$$L = \lim_{x \to 0} \left\{ \frac{48 \int_0^x \left(\frac{t^3}{1+t^6} \right) dt}{x^4} \right\} \to \frac{0}{0}$$

Applying L'hospital rule, we get

$$L = \lim_{x \to 0} \left\{ \frac{\left(\frac{48x^3}{1+x^6} \right)}{4x^3} \right\}$$

$$\Rightarrow L = \lim_{x \to 0} \left(\frac{12}{1+x^6} \right)$$

$$\Rightarrow L = 12$$

Q.2. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then $a + \frac{1}{a}$ is

A) 4 B) 2 C) 8 D) 1

Answer: 4

Solution: Given:

$$2a = \tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ$$

$$\Rightarrow 2a = \tan 15^\circ + \frac{1}{\tan(90^\circ - 15^\circ)} + \frac{1}{\tan(90^\circ + 15^\circ)} + \tan(180^\circ + 15^\circ)$$

$$\Rightarrow 2a = \tan 15^\circ + \frac{1}{\cot 15^\circ} - \frac{1}{\cot 15^\circ} + \tan 15^\circ$$

$$\Rightarrow 2a = 2 \tan 15^\circ$$

$$\Rightarrow a = \tan 15^\circ$$

$$\Rightarrow a = \tan(45^\circ - 30^\circ)$$

$$\Rightarrow a = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\Rightarrow a = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

So,

$$a + \frac{1}{a} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} + \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\Rightarrow a + \frac{1}{a} = \frac{\left(\sqrt{3} - 1\right)^2 + \left(\sqrt{3} + 1\right)^2}{2}$$

$$\Rightarrow a + \frac{1}{a} = \frac{8}{2} = 4$$

Q.3. Find the coefficient of x^{301} in $x^0(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} \ldots\ldots\ldots + x^{500}$

A) ${}^{500}C_{301}$ B) ${}^{501}C_{301}$ C) ${}^{502}C_{301}$ D) ${}^{501}C_{300}$

Answer: ${}^{501}C_{301}$

Solution: Let,

$S = x^0(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} \ldots\ldots\ldots + x^{500}$

Now the above expression is in G.P with common ratio $r = \frac{x}{1+x}$

So, the sum of given G.P will be,

$$S = (1+x)^{500}\left[\frac{\frac{x^{501}}{(1+x)^{501}} - 1}{\frac{x}{(1+x)} - 1}\right]$$

$$\Rightarrow S = (1+x)^{500}\left[\frac{x^{501} - (1+x)^{501}}{(1+x)^{501}\left(\frac{x-1-x}{1+x}\right)}\right]$$

$$\Rightarrow S = \left[\frac{x^{501} - (1+x)^{501}}{-1}\right]$$

$$\Rightarrow S = (1+x)^{501} - x^{501}$$

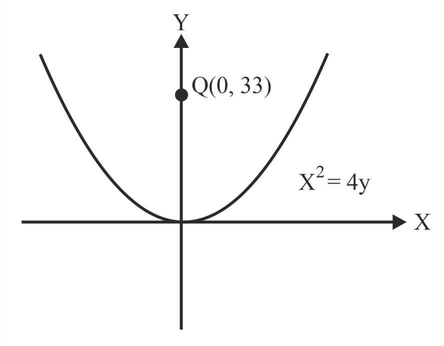
So, coefficient of x^{301} in the expansion of $(1+x)^{501}$ will be ${}^{501}C_{301}$

Q.4. Let $P(h,k)$ be any two points on $x^2 = 4y$ which is at shortest distance from $Q(0,33)$, then difference of distance of $P(h,k)$ from directrix of $y^2 = 4(x+y)$ is

A) 2 B) 4 C) 6 D) 8

Answer: 4

Solution: Plotting the diagram of the given data we have,



Now taking parametric point $P\left(2t, t^2\right)$, so the equation of normal at point $P\left(2t, t^2\right)$ will be $x + yt = 2t + t^3$ $\left\{\text{here in } x^2 = 4y\ a = 1\right\}$

Now normal is passing through $Q(0,33)$,

So the equation of normal becomes $0 + 33t = 2t + t^3$

$\Rightarrow t\left(t^2 - 31\right) = 0$

$\Rightarrow t = 0 \text{ or } t = \pm\sqrt{31}$

So, the points at which normal are passing will be $A(0,0)$, $B\left(2\sqrt{31}, 31\right)$ & $C\left(-2\sqrt{31}, 31\right)$

So, the shortest distance will be normal points.

Now given parabola is $y^2 = 4(x+y)$

$\Rightarrow (y-2)^2 = 4(x+1)$

So, the equation of directrix is $x = -2$

Now the distance of point B from line $x = -2$ will be $D_1 = 2 + 2\sqrt{31}$

And distance of point C from the line $x = -2$ will be $D_2 = -2 + 2\sqrt{31}$

So, the difference of the distance will be $|D_1 - D_2| = \left|2 + 2\sqrt{31} - \left(-2 + 2\sqrt{31}\right)\right| = 4$

Q.5. Find the greater area enclosed by $y^2 = 8x;\ y = x$ and $x = 2$.

A) $\frac{22}{3}$ B) $\frac{23}{3}$ C) $\frac{26}{3}$ D) $\frac{31}{3}$

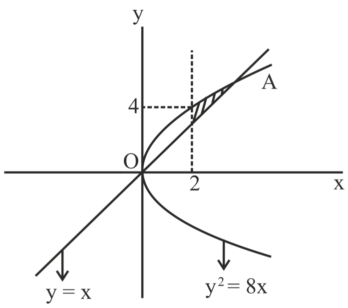
Answer: $\frac{22}{3}$

Solution: We have,

$y^2 = 8x$

$y = x$

Point of intersection of curve and a line is $A(8, 8)$.



Required area

$= \int_2^8 \left(\sqrt{8x} - x\right)dx$

$= \left[\frac{4\sqrt{2}x^{\frac{3}{2}}}{3} - \frac{x^2}{2}\right]_2^8$

$= \left[\left(\frac{4\sqrt{2}\left(2\sqrt{2}\right)^3}{3} - \frac{64}{2}\right) - \left(\frac{4\sqrt{2}\left(\sqrt{2}\right)^3}{3} - \frac{4}{2}\right)\right]$

$= \frac{112}{3} - 30$

$= \frac{22}{3}$

Q.6. Let $S = \{1, 2, 3, 4, 5\}$, then number of one-one function from S to $P(S)$ is

A) ${}^{32}C_5 \times 5!$ B) ${}^{32}C_6 \times 5!$ C) ${}^{32}C_5 \times 4!$ D) ${}^{32}C_6 \times 6!$

Answer: ${}^{32}C_5 \times 5!$

Solution: We have,

$S = \{1, 2, 3, 4, 5\}$

So,

$n(P(S)) = 2^5 = 32$

Hence, number of one-one function from S to $P(S)$ is

$= {}^{32}P_5 = {}^{32}C_5 \times 5!$

Q.7. Let $A = \{a, b, c, d\}$ and relation $R : A \rightarrow A$ be $R = \{(a, b), (b, c)\}$, then the minimum number of elements required to make R symmetric and transitive?

A) 2 B) 3 C) 4 D) 7

Answer: 7

Solution: Given:

$A = \{a, b, c, d\}$

And,

$R = \{(a, b), (b, c)\}$

For symmetric relation, we must add $(b, a),\ (c, b)$.

So, $R = \{(a, b), (b, a),\ (b, c), (c, b)\}$.

For transitive relation, we must add

$(a, c),\ (b, b),\ (c, a),\ (a, a),\ (c, c)$

Hence,

$R = \{(b, a),\ (c, b),\ (a, c),\ (b, b),\ (c, a),\ (a, a),\ (c, c),\ (a, b),\ (b, c)\}$

So, number of new elements added$= 7$

Q.8. The shortest distance between the line $\frac{x+4}{2} = \frac{y+6}{-1} = \frac{z-0}{2}$ and the line passing through $(2, 6, 2)$ and perpendicular to the plane $2x - 3y + z = 0$ is

A) $\frac{46}{\sqrt{45}}$ B) $\frac{42}{\sqrt{45}}$ C) $\frac{43}{\sqrt{45}}$ D) $\frac{41}{\sqrt{45}}$

Answer: $\frac{46}{\sqrt{45}}$

Solution: Given:

$\frac{x+4}{2}=\frac{y+6}{-1}=\frac{z-0}{2}$

Equation of the line passing through $(2,6,2)$ and perpendicular to the plane $2x-3y+z=0$ is

$\frac{x-2}{2}=\frac{y-6}{-3}=\frac{z-2}{1}$

So,

$\vec{a}_1=-4\hat{i}-6\hat{j}+0\hat{k}$

$\vec{b}_1=2\hat{i}-\hat{j}+2\hat{k}$

And,

$\vec{a}_2=2\hat{i}+6\hat{j}+2\hat{k}$

$\vec{b}_2=2\hat{i}-3\hat{j}+\hat{k}$

Now,

$\vec{a}_2-\vec{a}_1=6\hat{i}+12\hat{j}+2\hat{k}$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 2\\ 2 & -3 & 1\end{vmatrix}$

$\Rightarrow \vec{b}_1\times\vec{b}_2=5\hat{i}+2\hat{j}-4\hat{k}$

So,

$\Rightarrow \left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{25+4+16}=\sqrt{45}$

We know that the shortest distance between two skew lines is $\frac{\left|\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$=\left|\frac{\left(6\hat{i}+12\hat{j}+2\hat{k}\right)\cdot\left(5\hat{i}+2\hat{j}-4\hat{k}\right)}{\sqrt{45}}\right|$

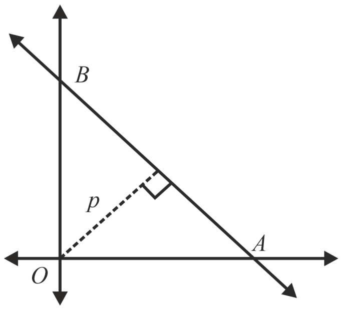
$=\frac{30+24-8}{\sqrt{45}}=\frac{46}{\sqrt{45}}$ units

Q.9. If a line is cutting $x-$axis and $y-$axis at two points A & B respectively, where $OA=a,\ OB=b$ and a perpendicular is drawn from O (origin) to AB at an angle of $\frac{\pi}{6}$ from positive $x-$axis and if area of triangle $OAB=\frac{98\sqrt{3}}{3}$ sq. units, then $\sqrt{3}a+b$ is equal to

A) 28 B) 14 C) 12 D) 7

Answer: 28

Solution: Plotting the diagram of the given data we get,



Here given $OA = a$ & $OB = b$, so equation of line AB in intercept form will be $\frac{x}{a} + \frac{y}{b} = 1 \ldots\ldots(1)$

And given perpendicular line of length p is making $\frac{\pi}{6}$ angle with the $x-$axis,

So, the equation of line in normal form will be $x\cos\frac{\pi}{6} + y\sin\frac{\pi}{6} = p$

$\Rightarrow \frac{x\sqrt{3}}{2p} + \frac{y}{2p} = 1$

So, on comparing with the equation (1) we get, $a = \frac{2p}{\sqrt{3}}$ & $b = 2p$

Now area of the triangle will be $\frac{1}{2} \times a \times b = \frac{98\sqrt{3}}{3}$

$\Rightarrow \frac{1}{2} \times \frac{2p}{\sqrt{3}} \times 2p = \frac{98\sqrt{3}}{3}$

$\Rightarrow p = 7$

So, $a = \frac{2p}{\sqrt{3}} = \frac{2\times7}{\sqrt{3}} = \frac{14}{\sqrt{3}}$ & $b = 14$

Hence, the value of $\sqrt{3}a + b = \frac{14\sqrt{3}}{\sqrt{3}} + 14 = 14 + 14 = 28$

Q.10. If $f(x+y) = f(x)\cdot f(y)$ and $f(2) = 3$ then find the value of $\sum_{n=0}^{5} f(n)$

A) $\frac{21}{\sqrt{3}-1}$ B) $\frac{26}{\sqrt{3}-1}$ C) $\frac{26}{\sqrt{3}+1}$ D) $\frac{21}{\sqrt{3}+1}$

Answer: $\frac{26}{\sqrt{3}-1}$

Solution: We know that, $f(x+y) = f(x)\cdot f(y)$ is a functional equation of function $f(x) = a^x$

So, using the given $f(2) = 3$ we get, $a^2 = 3 \Rightarrow a = \sqrt{3}$

Now finding the value of $\sum_{n=0}^{5} f(n)$

$= f(0) + f(1) + \ldots\ldots\ldots f(5)$

$= \left(\sqrt{3}\right)^0 + \left(\sqrt{3}\right)^1 + \left(\sqrt{3}\right)^2 \ldots\ldots\ldots + \left(\sqrt{3}\right)^5$

$= \frac{\left(\sqrt{3}\right)^6 - 1}{\sqrt{3}-1}$ {using the sum of G.P formula}

$= \frac{26}{\sqrt{3}-1}$

Hence, $\sum_{n=0}^{5} f(n) == \frac{26}{\sqrt{3}-1}$

Q.11. If $z = 1 + i$ & $z_1 = \frac{i+\bar{z}(1-i)}{z(1-z)}$, then the value of $\frac{12}{\pi}\arg(z_1)$ will be

Answer: 3

Solution: Given,

$z=1+i$ & $z_1=\frac{i+\bar{z}(1-i)}{z(1-z)}$

Now solving $z_1=\frac{i+\bar{z}(1-i)}{z(1-z)}$

$\Rightarrow z_1=\frac{i+(1-i)(1-i)}{(1-i)(1-(1+i))}$

$\Rightarrow z_1=\frac{i+(1-1-2i)}{(1-i)(-i)}$

$\Rightarrow z_1=\frac{-i}{(1-i)(-i)}$

$\Rightarrow z_1=\frac{1}{(1-i)}\times\frac{(1+i)}{(1+i)}$

$\Rightarrow z_1=\frac{(1+i)}{2}$

So, $\arg(z_1)=\tan^{-1}\left|\frac{\frac{1}{2}}{\frac{1}{2}}\right|=\tan^{-1}1=\frac{\pi}{4}$

So, $\frac{12}{\pi}\arg(z_1)=\frac{12}{\pi}\times\frac{\pi}{4}=3$

Q.12. Find number of four-digit number by using the digits $1,\ 2,\ 3$ & 5 which are divisible by 15

Answer: 21

Solution: Given,

We have to form four-digit number using the digits $1,\ 2,\ 3$ & 5

Now for number to be divisible by 15 we need to fix the last digit as 5 as $\_\ \_\ \_\ 5$, so that number can be divisible by 5 and for divisibility by 3 the addition of all number should be divisible by 3,

So making cases where sum is divisible by 3 we get,

Case $1-\ \{1,1,2\}\ \{\text{as } 1,1,2\ \&\ 5 \text{ sum is divisible by } 3\}$

So, total ways for case 1 is 3 ways,

Case $2-\{5,1,1\}\rightarrow 3\text{ ways},\ \{3,3,1\}\rightarrow 3\text{ ways}\ \&\ \{3,2,2\}\rightarrow 3\text{ ways}$

So, total ways for case 2 is 9 ways.

Case $3-\ \{5,3,2\}\rightarrow 6\text{ ways}$

Case $4-\ \{5,5,3\}\rightarrow 3\text{ ways}$

So, adding all cases we get, $3+9+6+3=21$ ways.

Q.13. If $a_n=-\frac{2}{4n^2-16n+15}$ and $a_1+a_2+\ldots+a_{15}=\frac{m}{n}$ where m and n are co-prime, then the value of $m+n$ is_____

Answer: 191

Solution: We have,

$a_n = -\frac{2}{4n^2-16n+15}$

$\Rightarrow a_n = -\frac{2}{(2n-5)(2n-3)}$

$\Rightarrow a_n = -\frac{1}{2}\left[\frac{1}{\left(n-\frac{5}{2}\right)\left(n-\frac{3}{2}\right)}\right]$

$\Rightarrow a_n = -\frac{1}{2}\left[\frac{1}{\left(n-\frac{5}{2}\right)} - \frac{1}{\left(n-\frac{3}{2}\right)}\right]$

$\Rightarrow a_n = \frac{1}{2}\left[\frac{1}{\left(n-\frac{3}{2}\right)} - \frac{1}{\left(n-\frac{5}{2}\right)}\right]$

$\Rightarrow \sum_{n=1}^{25} a_n = \frac{1}{2}\sum_{n=1}^{25}\left[\frac{1}{\left(n-\frac{3}{2}\right)} - \frac{1}{\left(n-\frac{5}{2}\right)}\right]$

$\Rightarrow \sum_{n=1}^{25} a_n = \frac{1}{2}\left[\left\{\frac{1}{\left(-\frac{1}{2}\right)} - \frac{1}{\left(-\frac{3}{2}\right)}\right\} + \left\{\frac{1}{\left(\frac{1}{2}\right)} - \frac{1}{\left(-\frac{1}{2}\right)}\right\} + \left\{\frac{1}{\left(\frac{3}{2}\right)} - \frac{1}{\left(\frac{1}{2}\right)}\right\} + \left\{\frac{1}{\left(\frac{5}{2}\right)} - \frac{1}{\left(\frac{3}{2}\right)}\right\} + \left\{\frac{1}{\left(\frac{7}{2}\right)} - \frac{1}{\left(\frac{5}{2}\right)}\right\} + \ldots + \left\{\frac{1}{\left(\frac{45}{2}\right)} - \frac{1}{\left(\frac{43}{2}\right)}\right\} + \left\{\frac{1}{\left(\frac{47}{2}\right)} - \frac{1}{\left(\frac{45}{2}\right)}\right\}\right]$

$\Rightarrow \sum_{n=1}^{25} a_n = \frac{1}{2}\left(\frac{2}{3} + \frac{2}{47}\right)$

$\Rightarrow \sum_{n=1}^{25} a_n = \left(\frac{1}{3} + \frac{1}{47}\right)$

$\Rightarrow \sum_{n=1}^{25} a_n = \frac{50}{141}$

So, $m = 50,\ n = 141 \Rightarrow m + n = 191$

Q.14. If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to coefficient of x^{-15} in the expansion of $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$, then find the value of $|ab - 5|$

Answer: 4

Solution: Given,

Coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to coefficient of x^{-15} in the expansion of $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$,

Now r^{th} term of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ will be $T_{r+1} = {}^{15}C_r\left(ax^3\right)^{15-r}\left(\frac{1}{bx^{\frac{1}{3}}}\right)^r = {}^{15}C_r\left(\frac{a^{15-r}}{b^r}\right)\cdot x^{3(15-r)-\frac{r}{3}} = {}^{15}C_r\left(\frac{a^{15-r}}{b^r}\right)\cdot x^{45-\frac{10r}{3}}$

Now for x^{15}, $45 - \frac{10r}{3} = 15 \Rightarrow r = 9$, so term will be T_{10}

And for $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$, $T_{r+1} = {}^{15}C_r\left(ax^{\frac{1}{3}}\right)^{15-r}\left(\frac{1}{bx^3}\right)^r = {}^{15}C_r\left(\frac{a^{15-r}}{b^r}\right)\cdot x^{\frac{(15-r)}{3}-3r} = {}^{15}C_r\left(\frac{a^{15-r}}{b^r}\right)\cdot x^{\frac{15-10r}{3}}$

So for x^{-15}, $\frac{15-10r}{3} = -15 \Rightarrow r = 6$, so term will be T_7

Now equating them we get,

${}^{15}C_9\left(\frac{a^{15-9}}{b^9}\right) = {}^{15}C_6\left(\frac{a^{15-6}}{b^6}\right) \Rightarrow a^3b^3 = 1 \Rightarrow ab = 1$

So, the value of $|ab - 5| = |1 - 5| = 4$

Q.15. If mean and variance of 7 observation is given as 8 & 16 respectively and if one of the observation 14 is removed and and the new mean and variance is given by a & b respectively then find the value of $a + 3b - 5$

Answer: 37

Solution: Given,

Mean and variance of 7 observation is given as 8 & 16 respectively,

So using the formula of mean we get, $\bar{x} = \frac{S}{7} \Rightarrow S = 8 \times 7 = 56$

Now if we remove 14 sum will be $S = 56 - 14 = 42$ and new mean will be $a = \frac{42}{6} = 7$

Now using the formula of variance we get,

$$\frac{\sum_{i=1}^{6} x_i + 14^2}{7} - (\bar{x})^2 = 16$$

$$\Rightarrow \frac{\sum_{i=1}^{6} x_i + 14^2}{7} - 8^2 = 16$$

$$\Rightarrow \frac{\sum_{i=1}^{6} x_i + 14^2}{7} = 80$$

$$\Rightarrow \sum_{i=1}^{6} x_i + 14^2 = 560$$

$$\Rightarrow \sum_{i=1}^{6} x_i = 364$$

Now new variance $b = \frac{\sum_{i=1}^{6} x_i}{6} - a^2 = \frac{364}{6} - 7^2 = \frac{35}{3}$

Hence, the value of $a + 3b - 5 = 7 + 35 - 5 = 37$

Q.16. If a die with points $\{2, 1, 0, -1, -2, 3\}$ is thrown 5 times, then probability that the product of outcomes on all throws is positive is given by $\frac{k}{2592}$ then value of k is

Answer: 521

Solution: Given,

Points on the die are $\{2, 1, 0, -1, -2, 3\}$

Now probability of positive outcomes is $\frac{3}{6} = \frac{1}{2}$ $\{\text{as only 3 points are positive}\}$

And probability of negative outcomes is $\frac{2}{6} = \frac{1}{3}$

Now product of all can be positive for the following cases,

Case $1-$ when all throws are positive so, probability will be ${}^5C_5\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

Case $2-$ when 3 are positive and 2 are negative, so probability will be ${}^5C_3\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{3}\right)^2 = \frac{5}{36}$

Case $3-$ when 1 is positive and rest all 4 are negative, so probability will be ${}^5C_1\left(\frac{1}{2}\right)^1 \times \left(\frac{1}{3}\right)^4 = \frac{5}{81 \times 2}$

So adding all the cases we get, $P = \frac{1}{32} + \frac{5}{36} + \frac{5}{162} = \frac{521}{2592}$

Hence, the value of $k = 521$

Q.17. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-zero vectors such that $\hat{n} \perp \vec{c}$, $\vec{a} = \alpha\vec{b} - \hat{n}$ where, $\alpha \neq 0$ and $\vec{b} \cdot \vec{c} = 12$, then $\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right| =$

Answer: 12

Solution: Given:

$\vec{a} = \alpha\vec{b} - \hat{n}$ where, $\alpha \neq 0$

$\Rightarrow \vec{a}\cdot\vec{c} = \alpha\left(\vec{b}\cdot\vec{c}\right) - \left(\hat{n}\cdot\vec{c}\right)$

$\Rightarrow \vec{a}\cdot\vec{c} = 12\alpha - 0$

Since, $\hat{n} \perp \vec{c} \Rightarrow \left(\hat{n}\cdot\vec{c}\right) = 0.$

Now,

$\vec{c}\times\left(\vec{a}\times\vec{b}\right) = \left(\vec{c}\cdot\vec{b}\right)\vec{a} - \left(\vec{c}\cdot\vec{a}\right)\vec{b}$

$\Rightarrow \vec{c}\times\left(\vec{a}\times\vec{b}\right) = 12\vec{a} - 12\alpha\vec{b}$

$\Rightarrow \vec{c}\times\left(\vec{a}\times\vec{b}\right) = 12\left(\vec{a} - \alpha\vec{b}\right)$

$\Rightarrow \vec{c}\times\left(\vec{a}\times\vec{b}\right) = 12\left(-\hat{n}\right)$

$\Rightarrow \left|\vec{c}\times\left(\vec{a}\times\vec{b}\right)\right| = \left|12\left(-\hat{n}\right)\right|$

$\Rightarrow \left|\vec{c}\times\left(\vec{a}\times\vec{b}\right)\right| = 12\left|\left(-\hat{n}\right)\right|$

$\Rightarrow \left|\vec{c}\times\left(\vec{a}\times\vec{b}\right)\right| = 12$

Q.18. If $\sum_{n=0}^{\infty}\left[\frac{n^3\{(2n)!\}+(2n-1)n!}{n!\times(2n)!}\right] = ae + \frac{b}{e} + c$, where $e = \sum_{n=0}^{\infty}\frac{1}{n!}$, then find $a^2 - b + c$.

Answer: 26

Solution: Given:

$\sum_{n=0}^{\infty}\left[\frac{n^3\{(2n)!\}+(2n-1)n!}{n!\times(2n)!}\right]$

$= \sum_{n=0}^{\infty}\left[\frac{n^3\{(2n)!\}}{n!\times(2n)!}\right] + \sum_{n=0}^{\infty}\left[\frac{(2n-1)n!}{n!\times(2n)!}\right]$

$= \sum_{n=0}^{\infty}\left[\frac{n^3}{n!}\right] + \sum_{n=0}^{\infty}\left[\frac{(2n-1)}{(2n)!}\right]$

$= \sum_{n=0}^{\infty}\left[\frac{n(n-1)(n-2)+3n(n-1)+n}{n!}\right] + \sum_{n=1}^{\infty}\left[\frac{1}{(2n-1)!}\right] - \sum_{n=0}^{\infty}\left[\frac{1}{(2n)!}\right]$

$= \sum_{n=0}^{\infty}\left[\frac{n(n-1)(n-2)}{n!}\right] + \sum_{n=0}^{\infty}\left[\frac{3n(n-1)}{n!}\right] + \sum_{n=0}^{\infty}\left[\frac{n}{n!}\right] + \left(\frac{e-e^{-1}}{2}\right) - \left(\frac{e+e^{-1}}{2}\right)$

$= \sum_{n=3}^{\infty}\left[\frac{1}{(n-3)!}\right] + 3\sum_{n=2}^{\infty}\left[\frac{1}{(n-2)!}\right] + \sum_{n=1}^{\infty}\left[\frac{1}{(n-1)!}\right] - \frac{1}{e}$

$= 5e - \frac{1}{e}$

So, $a = 5,\ b = -1,\ c = 0$

Hence, $a^2 - b + c = 25 + 1 = 26$