JEE Main Exam 2023 - Session 1

25 Jan 2023 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. A wire of resistance $5\ \Omega$ is redrawn such that its length becomes 5 times the original length. What is the final resistance of the wire?

A) $25\ \Omega$ B) $16\ \Omega$ C) $125\ \Omega$ D) $32\ \Omega$

Answer: $125\ \Omega$

Solution: The resistance of a wire (R) is related to its resistivity (ρ), length(l) and cross-sectional area (A) as:

$$R=\rho\frac{l}{A}$$

As the length of wire increases, the cross-sectional area decreases. The cross-sectional area in terms of volume (V) and length (l) is

$$A=\frac{V}{l}$$

$$\therefore\ R=\rho\frac{l^2}{V}$$

Taking the volume of wire constant,

$$R\propto l^2$$

So if R' is the resistance of wire after getting redrawn and R is the initial resistance of wire,

$$\therefore\ \frac{R'}{R}=\frac{\left(5l\right)^2}{l^2}=25$$

$$\Rightarrow R'=25\times R=125\ \Omega$$

Hence, option C is correct.

Q.2. Find the velocity of the particle at time $t=2\ \text{s}$, if position of the particle is given by $x=2t^2$ (where x is in m and t is in s).

A) $8\ \text{m s}^{-1}$ B) $4\ \text{m s}^{-1}$ C) $16\ \text{m s}^{-1}$ D) $32\ \text{m s}^{-1}$

Answer: $8\ \text{m s}^{-1}$

Solution: The velocity of the particle is

$$v=\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\left(2t^2\right)=4t$$

Therefore at $t=2\ \text{s}$,

$$v=8\ \text{m s}^{-1}$$

Hence, option A is correct.

Q.3. A particle performing an SHM with amplitude A, starts from $x=0$ and reaches $x=\frac{A}{2}$ in $2\ \text{s}$. Find the time required for the particle to move from $x=\frac{A}{2}$ to $x=A$?

A) $1.5\ \text{s}$ B) $4\ \text{s}$ C) $6\ \text{s}$ D) $1\ \text{s}$

Answer: $4\ \text{s}$

Solution: The position of a particle performing an SHM is given by:

$x = A\ \sin(\omega t + \phi)$

Since at $t = 0\ \text{s}$, the particle is at $x = 0$, therefore the initial phase $\phi = 0$. At time $t = 2\ \text{s}$, the particle is at $x = \frac{A}{2}$.

$\therefore\ \frac{A}{2} = A\ \sin(2\omega)$
$\Rightarrow \omega = \frac{\pi}{12}\ s^{-1}$

Therefore, time period

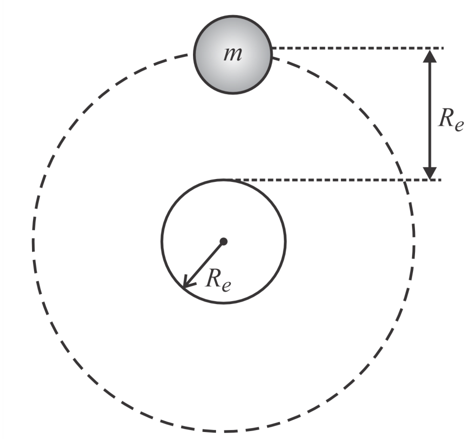
$T = \frac{2\pi}{\omega} = 24\ \text{s}$

The total time taken from $x = 0$ to $x = A$ is $\frac{T}{4} = 6\ \text{s}$.

Since the time taken from $x = 0$ to $x = \frac{A}{2}$ is $2\ \text{s}$, therefore the time taken from $x = \frac{A}{2}$ to $x = A$ is $4\ \text{s}$.

Hence, option B is correct.

Q.4. An object of mass m is placed at a height R_e from the surface of the earth. Find the increase in potential energy of the object, if the height of the object is increased to $2R_e$ from the surface. (R_e is the radius of Earth)



A) $\frac{1}{3}mgR_e$ B) $\frac{1}{6}mgR_e$ C) $\frac{1}{2}mgR_e$ D) $\frac{1}{4}mgR_e$

Answer: $\frac{1}{6}mgR_e$

Solution: The potential energy of a particle at a distance $r\ (>R_e)$ from the centre of Earth is,

$$U=-G\frac{Mem}{r}$$

Initially, the particle is at a height R_e from the surface. Therefore, $r=R_e+R_e=2R_e$.

$$\therefore U_i=-G\frac{Mem}{2Re}$$

When height is increased to $2R_e$ from surface, $r=2R_e+R_e=3R_e$

$$\therefore U_f=-G\frac{Mem}{3Re}$$

Thus, the change in potential energy is

$$\Delta U=U_f-U_i$$
$$=-G\frac{Mem}{3Re}+G\frac{Mem}{2Re}$$
$$=\frac{1}{6}G\frac{Mem}{Re}$$
$$=\frac{1}{6}G\frac{Me}{Re^2}mRe$$
$$=\frac{1}{6}mgRe\ \left(\text{as } g=G\frac{Me}{Re^2}\right)$$

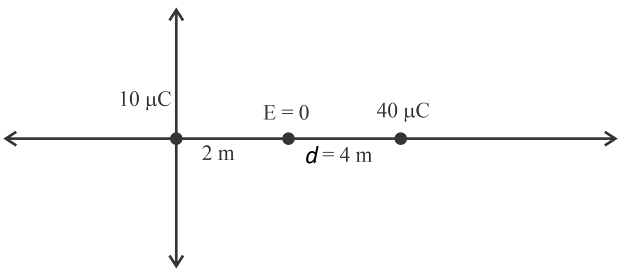
Hence, option B is correct.

Q.5. A charge of $10\mu\mathrm{C}$ is placed at origin. Where should a charge of $40\mu\mathrm{C}$ be placed on x-axis such that electric field is zero at $x=2$.

A) $x=-2$ B) $x=4$ C) $x=6$ D) $x=2$

Answer: $x=6$

Solution:



Let the charge of $40\ \mu C$ be placed at a distance d from the point $x=2$ as shown in the figure. For the electric field at $x=2$ to be 0,

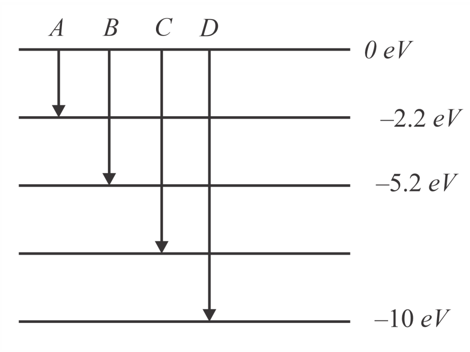
$$\frac{k\times 10\ \mu C}{2^2}=\frac{k\times 40\ \mu C}{d^2}$$

$$\Rightarrow d=4\ \mathrm{m}$$

Therefore, the particle is located at $x=d+2=6$

Hence, option C is correct.

Q.6. The diagram shown represents different transitions of electron $(A,\ B,\ C,\ D)$ between different energy levels with energies as shown. Among the shown transitions, which transition will generate a photon of wavelength $124.1\ \text{nm}$?
$(hc = 1242\ \text{eV}\ \ \text{nm})$



A) A

B) B

C) C

D) D

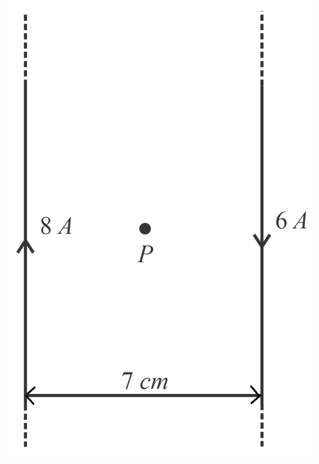
Answer: D

Solution: The change in energy is related to wavelength of photon emitted as:

$$\Delta E = \frac{hc}{\lambda}$$
$$= \frac{1242\ \text{eV}\ \ \text{nm}}{124.1\ \text{nm}}$$
$$= 10\ \text{eV}$$

Hence, option D is correct.

Q.7. Two straight wires placed parallel to each other are carrying currents as shown. P is equidistant from the wires. Find the magnetic field at point P.



A) $8 \times 10^{-5}\ \text{T}$

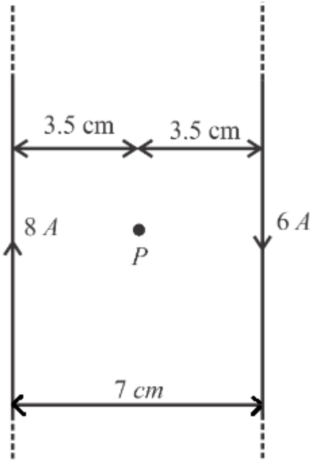
B) $8 \times 10^{-7}\ \text{T}$

C) $16 \times 10^{-5}\ \text{T}$

D) $2 \times 10^{-5}\ \text{T}$

Answer: $8 \times 10^{-5}\ \text{T}$

Solution:



Magnetic field due to both wires at point P will be inside the plane of the paper.

Therefore, magnitude of magnetic field can be written as,

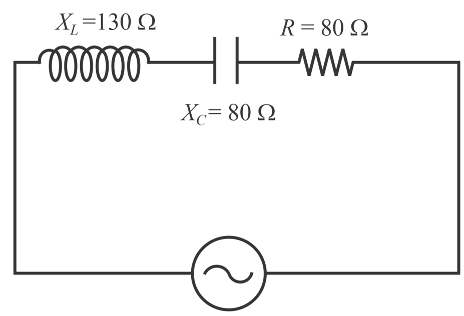
$B=\frac{\mu_0 i_1}{2\pi d}+\frac{\mu_0 i_2}{2\pi d}$

$=\frac{\mu_0}{2\pi d}[i_1+i_2]$

$=\frac{4\pi\times10^{-7}\times14}{2\pi\times3.5\times10^{-2}}$

$=8\times10^{-5}\ \mathrm{T}$

Q.8. For a LCR series circuit $\chi_L=130\ \Omega$, $\chi_C=80\ \Omega$ and $R=80\ \Omega$. The value of power factor of the circuit is equal to:



A) $\frac{\sqrt{54}}{9}$

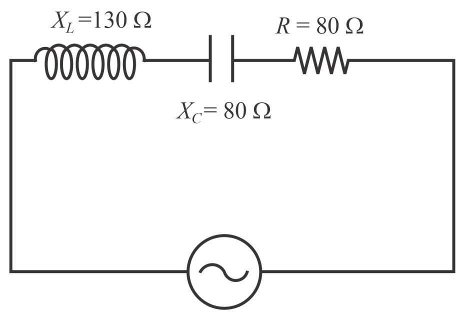
B) $\frac{8}{\sqrt{89}}$

C) $\frac{8}{13}$

D) $\frac{7}{9}$

Answer: $\frac{8}{\sqrt{89}}$

Solution:



Power factor in LCR circuit is given by,

$\cos\phi = \frac{R}{Z}$

$= \frac{R}{\sqrt{\left(x_L - x_C\right)^2 + R^2}}$

$= \frac{80}{\sqrt{\left(130-80\right)^2 + 80^2}}$

$= \frac{80}{\sqrt{2500+6400}}$

$= \frac{8}{\sqrt{89}}$

Q.9. What will be the molar specific heat capacity of an isochoric process of a diatomic gas if it has additional vibrational mode?

A) $\frac{5}{2}R$ B) $\frac{3}{2}R$ C) $\frac{7}{2}R$ D) $\frac{9}{2}R$

Answer: $\frac{7}{2}R$

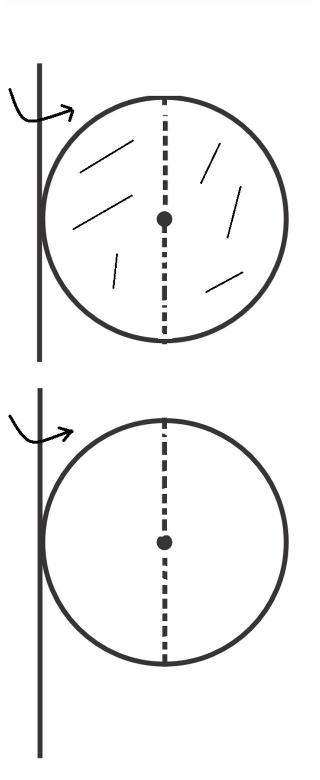
Solution: A diatomic gas has 5 degrees of freedom apart from vibrational degree of freedom. For each additional vibrational mode, degree of freedom is increased by 2. So new degree of freedom,

$f = 5 + 2 = 7$

Now, isochoric process is also called as constant-volume process. Therefore, we require molar specific heat capacity at constant volume, which is given by

$C_v = \frac{f}{2}R = \frac{7}{2}R$

Q.10. A disk & a solid sphere of same radius are rotated as shown in the figure. If mass of disk & solid sphere are $4 \mathrm{~kg}$ and $5 \mathrm{~kg}$ respectively, then $\frac{I_{\text{disc}}}{I_{\text{soild sphere}}}=$_________.



A) $\frac{7}{5}$

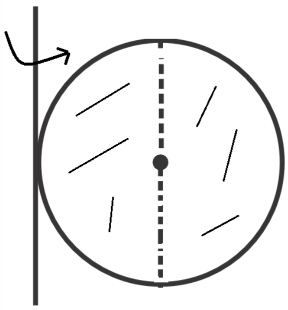
B) $\frac{25}{28}$

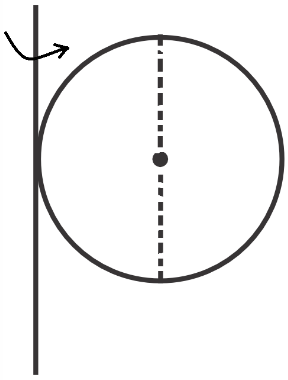
C) $\frac{5}{7}$

D) $\frac{28}{25}$

Answer: $\frac{5}{7}$

Solution:





Moment of inertia of the disc about the diameter is given by, $\frac{MR^2}{4}$.

Now using parallel axis theorem, we get

$I_{\text{disc}} = \frac{MR^2}{4} + MR^2$

$= \frac{5}{4}MR^2$

$= \frac{5}{4} \times 4R^2$

$= 5R^2$

Moment of inertia of the solid sphere about any diameter is given by, $\frac{2MR^2}{5}$.

Using parallel axis theorem, we get

$I_{\text{solid sphere}} = \frac{2}{5}MR^2 + MR^2$

$= \frac{7}{5}MR^2$

$= \frac{7}{5} \times 5R^2$

$= 7R^2$

Therefore,

$\frac{I_{\text{disc}}}{I_{\text{soild sphere}}} = \frac{5}{7}$

Q.11. Two projectiles are thrown at an angle of projection α and β with the horizontal. If $\alpha + \beta = 90^\circ$ then ratio of range of two projectiles on horizontal plane is equal to

A) $1:1$ B) $2:1$ C) $1:2$ D) $1:3$

Answer: $1:1$

Solution: Given: $\alpha+\beta=90^{\circ}$

Range of the first projectile:

$R_1=\frac{u^2\sin 2\alpha}{g}$

Range of the second projectile:

$R_2=\frac{u^2\sin 2\beta}{g}$

$=\frac{u^2\sin 2\left(90^{\circ}-\alpha\right)}{g}$

$=\frac{u^2\sin 2\alpha}{g}$

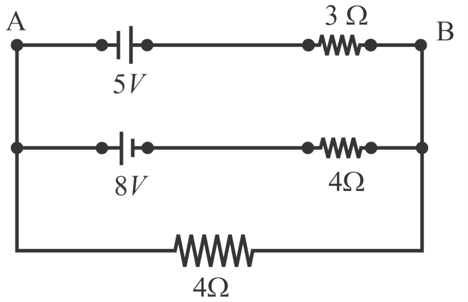
As we can see, both are equal.

$R_1=R_2$

Therefore,

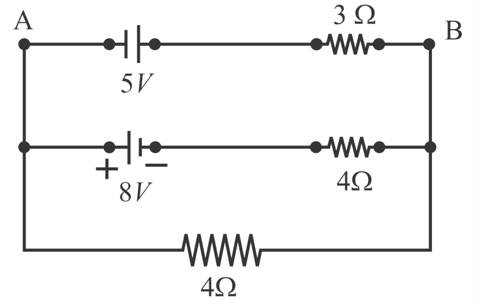
$R_1:R_2=1:1$

Q.12. In the circuit shown, the current (in A) through the $4\ \Omega$ resistor connected across A & B is $\frac{1}{n}$ Amperes. Find n



Answer: 10

Solution:



The equivalent EMF due to combination of cells of $5\ V$& $8\ V$ and resistors of $3\ \Omega$ & $4\ \Omega$ (in the upper arm) is

$$\varepsilon_{eq}=\frac{\varepsilon_1 r_2+\varepsilon_2 r_1}{r_1+r_2}$$

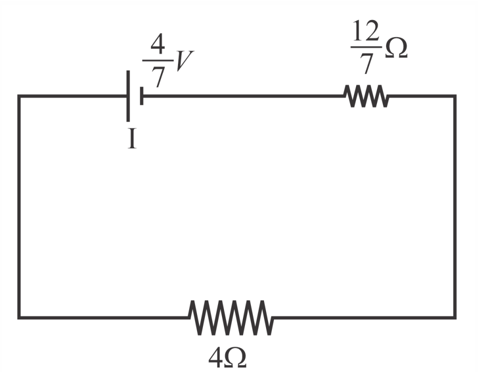
$$=\frac{8\times3-5\times4}{7}$$

$$=\frac{4}{7}\text{ V}$$

The equivalent resistance of resistors of $3\ \Omega$ and $4\ \Omega$ (in the upper arm) is

$$r_{eq}=\frac{3\times4}{3+4}=\frac{12}{7}\Omega$$

The simplified circuit is as shown below.



The net resistance of the whole circuit is

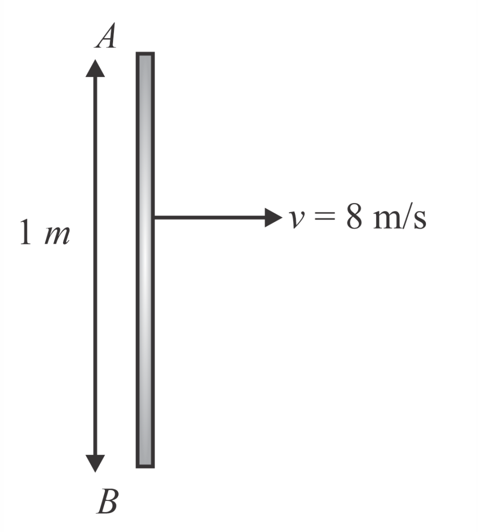
$$R_{eq}=r_{eq}+4=\frac{12}{7}+4=\frac{40}{7}\ \Omega$$

Therefor the current flowing in lower resistor of resistance $4\ \Omega$ is

$$i=\frac{\varepsilon_{eq}}{R_{eq}}=\frac{4}{7}\times\frac{7}{40}=\frac{1}{10}\text{ A}$$

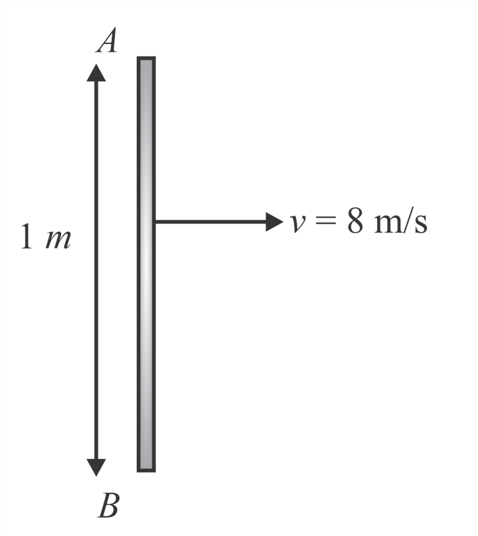
Therefore, $n=10$.

Q.13. A metal rod of length $1 \mathrm{~m}$ is moving perpendicular to its length with a velocity of $8 \mathrm{~m} \mathrm{~s}^{-1}$ along positive x-axis. If a magnetic field $B= 2 \mathrm{~T}$ exist perpendicular to the plane of motion. The emf induced between the two ends of rod is____.



Answer: 16

Solution:



Since the velocity of rod, the length of rod and the magnetic field are mutually perpendicular, the induced motional EMF is

$\varepsilon=v B l=8 \times 2 \times 1=16 \mathrm{~V}$

Section B: Chemistry

Q.1. If $\left[\mathrm{H}^{+}\right]$ ion concentration is increased by a factor of 1000 . Then pH is?

A) Decreased by 3 B) Increased by 3 C) No change D) Decreased by 1

Answer: Decreased by 3

Solution: $\mathrm{pH}=-\log \left[\mathrm{H}^{+}\right]$

$=-\log_{10}\left[10^{3}\right]$

$=-3$

Q.2. Select the correct match

A) Hexan-2-one & Hexan-3-one - position isomer

B) Pentan-3-one & Pentan-2-one - functional isomer

C) 2-pentene & 1-pentene - metamers

D) Pentanoic acid & Hexanoic acid - Functional isomers

Answer: Hexan-2-one & Hexan-3-one - position isomer

Solution:

Hexane – 2 – one

$$CH_3-CH_2-CH_2-CH_2-\overset{\overset{O}{||}}{C}-CH_2$$

Hexane – 3 – one

$$CH_3-CH_2-\overset{\overset{O}{||}}{C}-CH_2-CH_2-CH_3$$

They are position isomers and metamers as well.
Pent-1-ene and pent-2-ene are position isomers.
Pentanoic and hexanoic acid are not isomers at all as the molecular formula are different

Q.3. Which of the following has two chiral centres?

A) 2-Bromo-3-duetrobutane

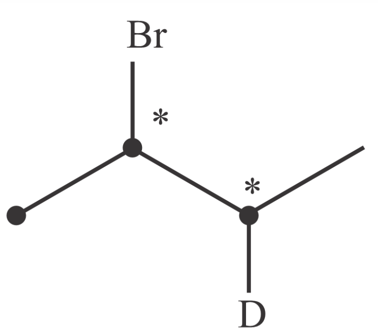
B) 1-Bromo-2-duetrobutane

C) 1-Bromo-3-duetrobutane

D) 1-Bromo-4-duetrobutane

Answer: 2-Bromo-3-duetrobutane

Solution: A carbon attached with four different groups are atom is called chiral carbon. Two chiral carbon is only present in 2-Bromo-3-duetrobutane. In 1-Bromo-2-duetrobutane and 1-Bromo-3-duetrobutane only one chiral carbon atom are present.



Q.4. Chloride salt of M is treated with the excess of $AgNO_3$. It forms curdy white precipitate 'A'. When 'A' is treated with NH_4OH, it forms a soluble salt B. Then 'A' and 'B' respectively are:

A) $AgCl, [Ag(NH_3)_2]^+$

B) $AgBr, [Ag(OH)_2]^-$

C) $AgCl, [Ag(OH)_4]^{2-}$

D) $AgBr, [Ag(OH)_4]^{2-}$

Answer: $AgCl, [Ag(NH_3)_2]^+$

Solution: $MCl + AgNO_3 \rightarrow AgCl\,(A) + MNO_3$

Silver chloride is curdy white precipitate.

$AgCl + 2NH_4OH \rightarrow [Ag(NH_3)_2]Cl + 2H_2O$

Silver bromide light yellow precipitate.

Q.5. Match List I with List II

Amine (List I)		pk_b (aqueous medium) (List II)	
(A)	Aniline	(1)	9.0
(B)	Ethanamine	(2)	3.29
(C)	N-ethylethanamine	(3)	3.25
(D)	N,N-diethylethanamine	(4)	3.0

A) A-1 B-2 C-4 D-3

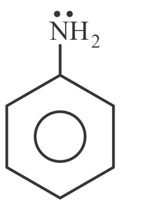
B) A-1 B-4 C-3 D-2

C) A-1 B-2 C-3 D-4

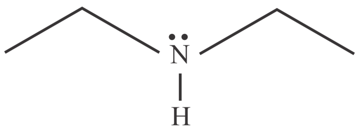
D) A-1 B-3 C-4 D-2

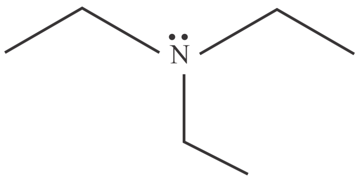
Answer: A-1 B-2 C-4 D-3

Solution: Stronger base has a smaller value of pK_b and the order of basic nature is 2 with aromatic amine being weakest among alkyl amines due to resonance of its lone pair electrons



$C_2H_5\ddot{N}H_2$





Basic nature order: $C > D > B > A$

Q.6. Find the change in oxidation number of Cr when $K_2Cr_2O_7$ is used in acidic medium during titration.

A) 6 B) 2 C) 3 D) 4

Answer: 6

Solution: $Cr_2O_7^{2-} + 14H^{+} + 6e^{-} \longrightarrow 2\,Cr^{3+} + 7H_2O$

The oxidation state of Cr in dichromate is +6 and it is converting to +3. Two cromium atoms are present in one dichromate molecule.

. Hence, change in oxidation state of Cr in dichromate is 6.

Q.7. Match the following

Column I		Column II	
(i)	Neoprene	(a)	Synthetic wool
(ii)	Acrolein	(b)	Paint
(iii)	LDP	(c)	Flexible pipes
(iv)	Glyptal	(d)	Gaskets

A) ii-(d), iv-(b), iii-(a), i-(c)

B) ii-(c),iv-(d), iii-(a), i-(c)

C) ii-(a),iv-(b), iii-(c), i-(d)

D) ii-(b),iv-(c), iii-(d), i-(a)

Answer: ii-(a),iv-(b), iii-(c), i-(d)

Solution: Neoprene is used in gaskets, acrolein is used as synthetic wool, LDP is used to make flexible pipes and glyptal is used in paint industry

Q.8. Which is the correct order of reducing power for: $\mathrm{Li, Na, K, Rb, Cs}$

A) $\mathrm{Li < Rb < Cs < K < Na}$ B) $\mathrm{Na < K < Cs < Rb < Li}$ C) $\mathrm{Li < Na < K < Rb < Cs}$ D) $\mathrm{Cs < Rb < K < Na < Li}$

Answer: $\mathrm{Na < K < Cs < Rb < Li}$

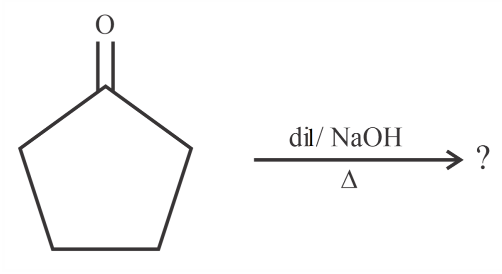
Solution: Reducing power is inversely related to the Standard Reduction Potential (SRP) values

As SRP Values of alkali metals has the following order:

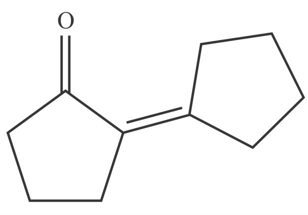
$\mathrm{Li < Rb < Cs < K < Na}$

Hence, order of reducing power will be: $\mathrm{Na < K < Cs < Rb < Li}$

Q.9.



A)



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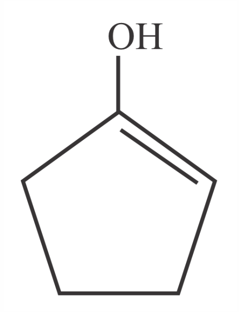
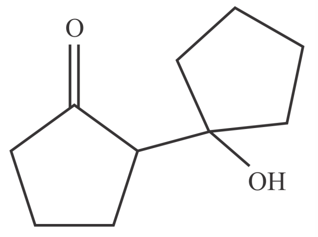
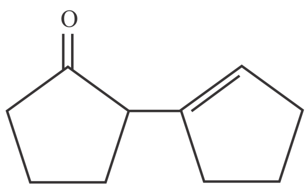


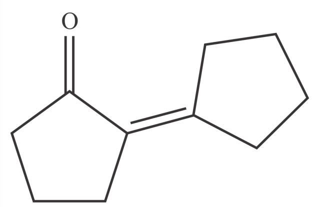
C)



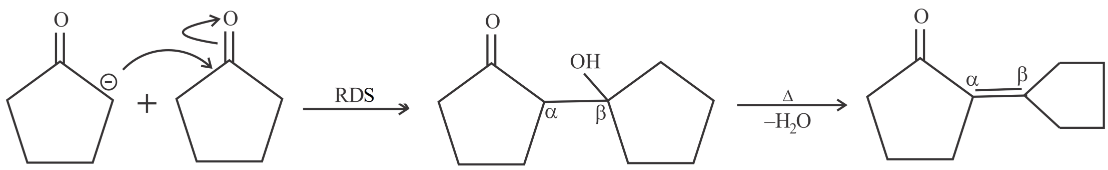
D)



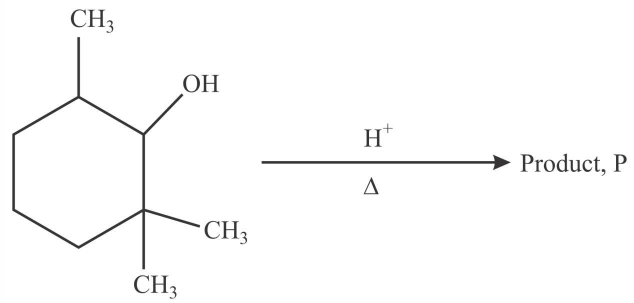
Answer:



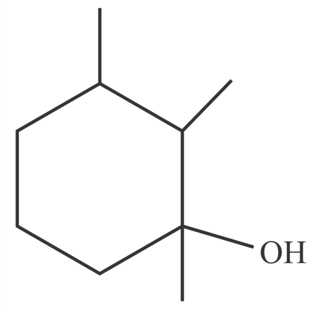
Solution: The given reaction is an example of aldol condensation reaction.



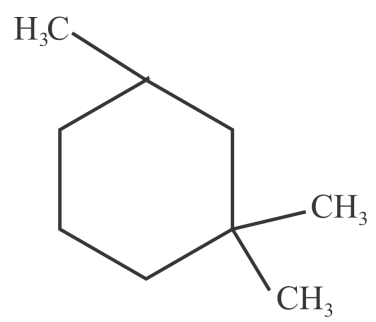
Q.10. Consider the following reaction. The correct product 'P' is given by



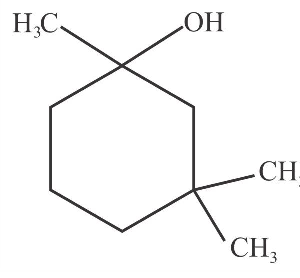
A)



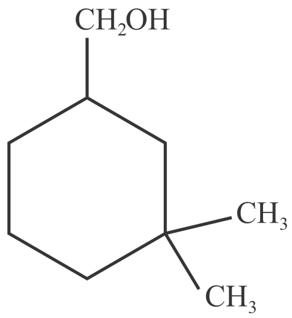
B)



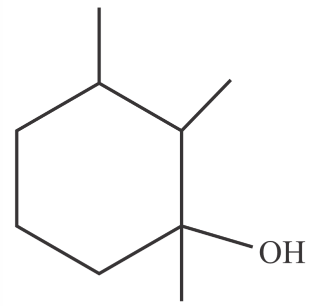
C)



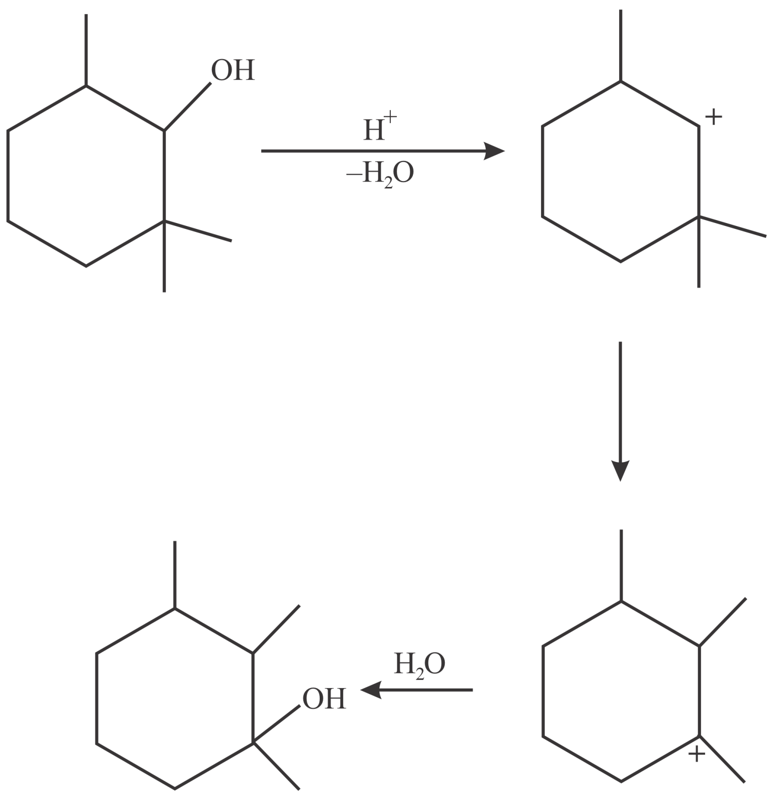
D)



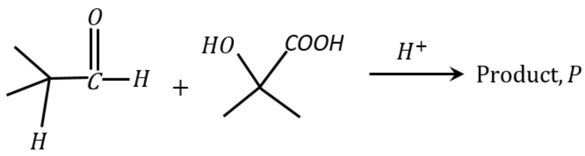
Answer:



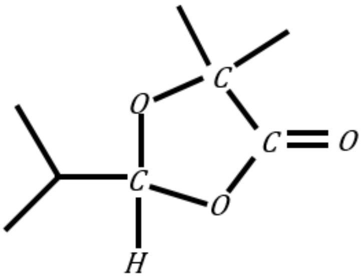
Solution: In the first step protonation of alcohol takes place which leads to the formation of carbocation intermediate. The carbocation undergo rearrangement by 1,2-methyl shift to get stability. Finally water is added to get alcohol.



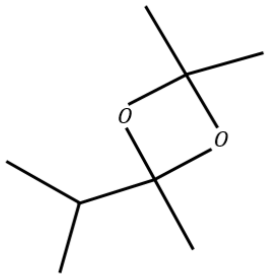
Q.11. Consider the following reaction. Find the product, P



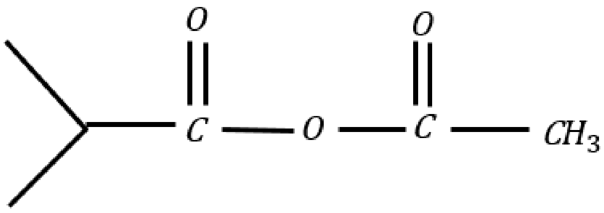
A)



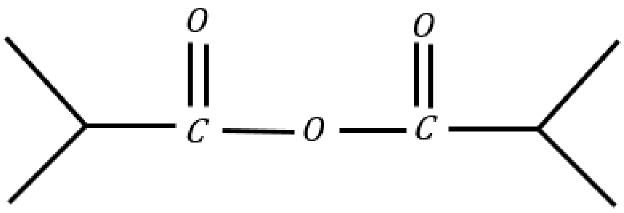
B)



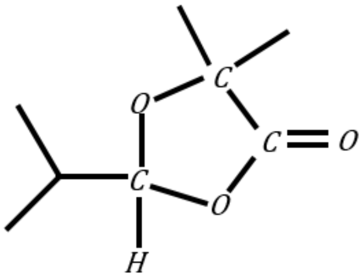
C)



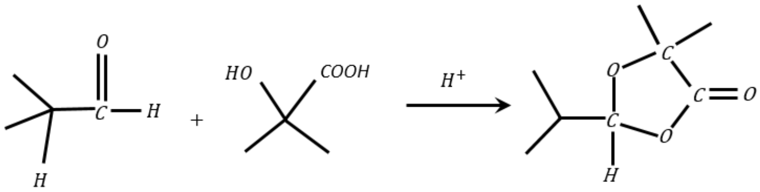
D)



Answer:



Solution: Hydroxyl group attacks the carbonyl group by nucleophilic addition reaction. The hemiacetal formed undergo the reaction similar to esterfication with carboxylic acid to get the final product as shown below.



Q.12. Find out mass ratio of ethylene glycol $(62\ \mathrm{g})$ required to make $500\,\mathrm{ml}, 0.25\mathrm{M}$ and $250\,\mathrm{ml}, 0.25\mathrm{M}$

A) 1:1
B) 1:2
C) 2:1
D) 4:1

Answer: 2:1

Solution: millimoles of 1st case = 500×0.25

millimoles of 2nd case = 250×0.25

Moles ratio is equal to mass ratio.

Hence, the ratio $= 2:1$

Q.13. Assertion: BHA is added to butter to increase self-life.
Reason: BHA reacts with oxygen more than butter.

A) Assertion is correct
Reason is correct

B) Assertion is correct
Reason is incorrect

C) Assertion is incorrect
Reason is correct

D) Assertion is incorrect Reason is incorrect

Answer: Assertion is correct
Reason is correct

Solution: BHA is added to butter because it acts as preservative by lowering the rate of self-oxidation of butter.

Q.14. Match the following

Column I		Column II	
(a)	$[\mathrm{Co(CN)_6}]^{3-}$	(p)	$535\,\mathrm{mm}$
(b)	$[\mathrm{Co(NH_3)_6}]^{3+}$	(q)	$375\,\mathrm{nm}$
(c)	$[\mathrm{Co(NH_3)_5Cl}]^{2+}$	(s)	$600\,\mathrm{nm}$

A) a-(s), b-(p), c-(q)
B) a-(p), b-(q), c-(s)
C) a-(q), b-(p), c-(s)
D) a-(s), b-(q), c-(p)

Answer: a-(q), b-(p), c-(s)

Solution: Wavelength of light absorbed is inversely related to splitting energy in complex compounds

Splitting energy is directly related to ligand strength

Order of ligand strength: $CN^- > NH_3 > Cl^-$

Q.15. Which of the following option contains the correct match.

List I		List II	
(a)	Adiabatic	(1)	$\Delta T = 0$
(b)	Isothermal	(2)	Heat exchange $= 0$
(c)	Isochoric	(3)	$\Delta P = 0$
(d)	Isobaric	(4)	Work done $= 0$

A) a-2,b-1,c-4,d-3 B) a-1,b-2,c-3,d-4 C) a-4,b-3,c-2,d-1 D) a-1,b-3,c-4,d-1

Answer: a-2,b-1,c-4,d-3

Solution:

List I	List II
Adiabatic	Heat exchange = 0
Isothermal	$\Delta T = 0$
Isochoric	Work done = 0
Isobaric	$\Delta P = 0$

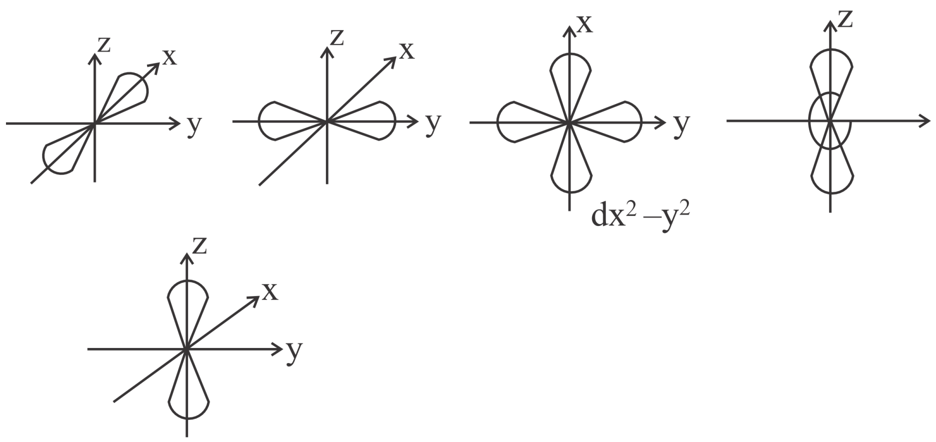
Q.16. How many of the following orbitals is considered as axial orbital(s)

$p_x, p_y, p_y, d_x, d_y, d_{xz}, d_{x^2-y^2}, d_{z^2}$

Answer: 5

Solution: Out of $p_x, p_y, p_z, d_{xy}, d_{yz}, d_{xz}, d_{x^2-y^2}, d_{z^2}$ orbitals

$p_x, p_y, p_z, d_{x^2-y^2} d_{z^2}$ are axial orbitals. 5



Q.17. Arrange the following elements in increasing order of metallic character Si, K, Mg and Be

A) $Si < Mg < Be < K$ B) $Be < Mg < Si < K$ C) $Si < Be < Mg < K$ D) $K < Mg < Si < Be$

Answer: $Si < Be < Mg < K$

Solution: Metallic character decreases from left to right in the periods periodic table and increases from top to bottom in groups. Silicon is a metalloid.

Hence, the correct order is

$Si < Be < Mg < K$

Q.18. Assertion [A]: Carbon form two oxides CO and CO_2, where CO is neutral, while CO_2 is acidic

Reason [R]: CO_2 soluble in water to give carbonic acid and CO is almost insoluble in water.

A) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A)

B) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A)

C) Assertion (A) is true and Reason (R) is false

D) Assertion (A) is false and Reason (R) is true

Answer: Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A)

Solution: Carbon monoxide is neutral and almost insoluble in water.

Carbon dioxide is acidic as carbon in +4 oxidation state is acidic.

Acidic carbon dioxide reacts with water to form carbonic acid

$$H_2O + CO_2 \rightarrow H_2CO_3$$

Q.19. Consider the following cell

$$\underset{(1\,\text{bar})}{\text{Pt}|\text{H}_2}\underset{(1\,\text{M})}{|\text{H}^+}\,\Big\|\,\text{M}^{3+}|\,\text{M}^+$$

Then value of $\frac{[\text{M}^+]}{[\text{M}^{3+}]}$ is 10^x, then find the value of 'x'

(given: $E^0_{\text{M}^{3+}/\text{M}^+} = 2\text{V}$ and $E_{\text{cell}} = 1.1\ \text{V}$)

Answer: 30

Solution: $\text{H}_2 \rightarrow 2\text{H}^+ + 2\text{e}^-$

$\text{M}^{3+} + 2\text{e} \rightarrow \text{M}^+$

$$E = E^0 - \frac{0.06}{2}\log_{10}\frac{[M^+]}{[M^{3+}]}$$

$$1.1 = \left(2 - 0\right) - 0.03\log_{10}\frac{[M^+]}{[M^{3+}]}$$

$$-0.9 = -0.03\log_{10}\frac{[M^+]}{[M^{3+}]}$$

$$\left[\frac{\text{M}^+}{\text{M}^{3+}}\right] = 10^{30}$$

$x = 30$

Section C: Mathematics

Q.1. Find the value of $\sum_{r=0}^{6} {}^{51-k}C_3$

A) ${}^{51}C_4 - {}^{45}C_4$

B) ${}^{52}C_4 - {}^{45}C_4$

C) ${}^{52}C_4 - {}^{45}C_3$

D) ${}^{51}C_4 + {}^{45}C_4$

Answer: ${}^{52}C_4 - {}^{45}C_4$

Solution: Given,

$\sum_{r=0}^{6} {}^{51-k}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3$

Now we know that ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$ or ${}^{n}C_r = {}^{n+1}C_{r+1} - {}^{n}C_{r+1}$

Now using the above formula in given expression we get,

$\sum_{r=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + \overbrace{\quad\quad}^{{}^{46}C_3 + {}^{46}C_4} - {}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + \overbrace{\quad\quad}^{{}^{47}C_3 + {}^{47}C_4} - {}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4 - {}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4 - {}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4 - {}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{51}C_4 - {}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6} {}^{51-k}C_3 = {}^{52}C_4 - {}^{45}C_4.$

Q.2. If $f(x) = 2x^n + \lambda$ and $f(4) = 133$, $f(5) = 255$, then sum of positive integral divisors of $f(3) - f(2)$ is

A) 60 B) 22 C) 40 D) 6

Answer: 22

Solution: Given:

$f(x) = 2x^n + \lambda$

And,

$f(4) = 133$

$\Rightarrow 2 \cdot 4^n + \lambda = 133 \quad \ldots.(1)$

And,

$f(5) = 255$

$\Rightarrow 2 \cdot 5^n + \lambda = 255 \quad \ldots.(2)$

Subtracting (1) & (2), we get

$2 \cdot 5^n - 2 \cdot 4^n = 122$

$\Rightarrow 5^n - 4^n = 61$

Now, $5^3 - 4^3 = 125 - 64 = 61$, so

$n = 3$

Hence, $\lambda = 133 - 128 = 5$

So,

$f(x) = 2x^3 + 5$

$\therefore f(3) - f(2) = 54 - 16$

$\Rightarrow f(3) - f(2) = 38 = 1 \times 2 \times 19$

Hence, sum of positive integral divisors of $f(3) - f(2)$ is

$= 1 + 2 + 19 = 22$

Q.3. If $\left|\frac{z+2i}{z-i}\right| = 2$ is a circle, then the centre of the circle will be,

A) $(0,0)$ B) $(2,0)$ C) $(0,2)$ D) $(-2,0)$

Answer: $(0,2)$

Solution: Given,

$\left|\frac{z+2i}{z-i}\right|=2$

Now let $z=x+iy$, now putting in the above equation we get,

$\left|\frac{x+iy+2i}{x+iy-i}\right|=2$

$\Rightarrow\left|\frac{x+i(y+2)}{x+i(y-1)}\right|=2$

$\Rightarrow\frac{\sqrt{x^2+(y+2)^2}}{\sqrt{x^2+(y-1)^2}}=2$

$\Rightarrow x^2+(y+2)^2=4\left(x^2+(y-1)^2\right)$

$\Rightarrow x^2+y^2+4y+4=4\left(x^2+y^2-2y+1\right)$

$\Rightarrow 3x^2+3y^2-12y=0$

$\Rightarrow x^2+y^2-4y=0$

Now comparing with $x^2+y^2+2gx+2fy+c=0$ whose centre is $(-g,-f)$, the centre will be $(0,2)$

Q.4. Find the number of numbers between 5000 & 10000 by using the digits $1,3,5,7,9$ without repetition

A) 6 B) 12 C) 72 D) 120

Answer: 72

Solution: Given,

Numbers $1,3,5,7,9$

Now to form number between $5000-10000$

$\underbrace{\_}_{5,7,9}\ \underbrace{\_}_{4\text{ ways}}\ \underbrace{\_}_{3\text{ ways}}\ \underbrace{\_}_{2\text{ ways}}$

Here at 1000's place should be taken by $5,\ 7\ \&\ 9$

So for 1000's place there will be 3 ways,

Now for hundredth place there will be 4 ways,

For ten's place there will be 3 ways,

And for unit place there will be 2 ways,

So total ways will be $3\times4\times3\times2=72$ ways.

Q.5. If $\vec{a}=-\hat{i}-\hat{j}+\hat{k}$ and $\vec{a}\times\vec{b}=\hat{i}-\hat{j}$ and $\vec{a}\cdot\vec{b}=1$, then $\vec{a}-6\vec{b}$ is equal to

A) $3\hat{i}+3\hat{j}+3\hat{k}$ B) $3\hat{i}-3\hat{j}+3\hat{k}$ C) $-3\hat{i}+3\hat{j}+3\hat{k}$ D) $3\hat{i}+3\hat{j}-3\hat{k}$

Answer: $3\hat{i}+3\hat{j}+3\hat{k}$

Solution: Let

$\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

Then,

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ x & y & z \end{vmatrix}$

$\Rightarrow \vec{a} \times \vec{b} = -(y+z)\hat{i} + (x+z)\hat{j} + (x-y)\hat{k}$

So,

$-(y+z)\hat{i} + (x+z)\hat{j} + (x-y)\hat{k} = \hat{i} - \hat{j} + 0\hat{k}$

Hence,

$y + z = -1$
$x + z = -1$
$x - y = 0 \Rightarrow x = y$

Also,

$\vec{a} \cdot \vec{b} = 1$

$\Rightarrow -x - y + z = 1$

$\Rightarrow -2x + z = 1$

$\Rightarrow z = 2x + 1$

$\Rightarrow -1 - x = 2x + 1$

$\Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$

So, $y = -\frac{2}{3}$; $z = -\frac{1}{3}$

Hence,

$\vec{b} = -\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

$\Rightarrow 6\vec{b} = -4\hat{i} - 4\hat{j} - 2\hat{k}$

So,

$\vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Q.6. We have 8 oranges, 7 red apples and 5 white apples. In how many ways we can select 5 fruits containing atleast 2 oranges, 1 red apple and 1 white apple?

Answer: 6860

Solution: We have 8 oranges, 7 red apples and 5 white apples.

Number of ways in which we can select 5 fruits containing atleast 2 oranges, 1 red apple and 1 white apple is

$= {}^{8}C_3 \times {}^{7}C_1 \times {}^{5}C_1 + {}^{8}C_2 \times {}^{7}C_2 \times {}^{5}C_1 + {}^{8}C_2 \times {}^{7}C_1 \times {}^{5}C_2$

$= (56 \times 7 \times 5) + (28 \times 21 \times 5) + (28 \times 7 \times 10)$

$= 1960 + 2940 + 1960 = 6860$

Q.7. If $f(x) = \log_{\sqrt{m}}\left(\sqrt{2}(\sin x - \cos x) + m - 2\right)$ and the range of $f(x)$ is $[0, 2]$, then m is

A) 3 B) 4 C) 5 D) None of these

Answer: 5

Solution: Given:

$f(x) = \log_{\sqrt{m}}\left(\sqrt{2}(\sin x - \cos x) + m - 2\right)$

We know that

$(\sin x - \cos x) \in \left[-\sqrt{2}, \sqrt{2}\right]$

$\left[\because a \sin x + b \cos x \in \left[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}\right]\right]$

So,

$-2 \leq \sqrt{2}(\sin x - \cos x) \leq 2$

$\Rightarrow m - 2 \leq \sqrt{2}(\sin x - \cos x) + m \leq m + 2$

$\Rightarrow m - 4 \leq \sqrt{2}(\sin x - \cos x) + m - 2 \leq m$

$\Rightarrow \log_{\sqrt{m}}(m-4) \leq \left[\sqrt{2}(\sin x - \cos x) + m - 2\right] \leq \log_{\sqrt{m}} m$

So,

$\log_{\sqrt{m}}(m-4) = 0 \;\&\; \log_{\sqrt{m}} m = 2$

$\Rightarrow m - 4 = 1 \Rightarrow m = 5$

Q.8. If $\frac{dy}{dt} + \alpha y = \gamma \cdot e^{-\beta t}$, then the value of limit $\lim_{t \to \infty} y(t)$, where $\alpha > 0,\ \beta > 0,\ \gamma > 0,\ \alpha \neq \beta,$ is

A) 0

B) 1

C) Does not exist

D) $\alpha\beta$

Answer: 0

Solution: Given,

$\frac{dy}{dt} + \alpha y = \gamma \cdot e^{-\beta t}$

Which is a linear differential equation,

So, integrating factor will be $I.F = e^{\int \alpha dt} = e^{\alpha t}$

So, the solution of differential equation is given by

$y \times e^{\alpha t} = \gamma \int e^{-\beta t} \times e^{\alpha t}$

$\Rightarrow y \times e^{\alpha t} = \gamma \int e^{(\alpha - \beta)t}$

$\Rightarrow y \times e^{\alpha t} = \gamma \frac{e^{(\alpha-\beta)t}}{(\alpha-\beta)} + c$

$\Rightarrow y = \gamma \frac{e^{-\beta t}}{(\alpha-\beta)} + c e^{-\alpha t}$

Now finding $\lim_{t \to \infty} y$ we get,

$\lim_{t \to \infty} y = \lim_{t \to \infty}\left(\gamma \frac{e^{-\beta t}}{(\alpha-\beta)} + c e^{-\alpha t}\right)$

$\Rightarrow \lim_{t \to \infty} y = \gamma \times 0 + c \times 0 = 0 \;\{\text{as } e^{-\infty} \to 0\}$

Q.9. If A be a symmetric matrix and B & C are skew-symmetric matrices of same order, then

A) $A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$ is a skew symmetric matrix

B) $AC - A$ is a symmetric matrix

C) $A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$ is a symmetric matrix

D) $AC - A$ is a skew-symmetric matrix

Answer: $A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$ is a skew symmetric matrix

Solution: Since, A be a symmetric matrix, so A^{13} is a symmetric matrix and B & C are skew-symmetric matrices so B^{26} and C^{26} are symmetric matrices.

Now,

$\left(A^{13}\cdot B^{26}-B^{26}\cdot A^{13}\right)^T=\left(A^{13}\cdot B^{26}\right)^T-\left(B^{26}\cdot A^{13}\right)^T$

$\Rightarrow\left(A^{13}\cdot B^{26}-B^{26}\cdot A^{13}\right)^T=\left(B^{26}\right)^T\left(A^{13}\right)^T-\left(A^{13}\right)^T\left(B^{26}\right)^T$

$\Rightarrow\left(A^{13}\cdot B^{26}-B^{26}\cdot A^{13}\right)^T=\left(B^{26}\right)\left(A^{13}\right)-\left(A^{13}\right)\left(B^{26}\right)$

$\Rightarrow\left(A^{13}\cdot B^{26}-B^{26}\cdot A^{13}\right)^T=-\left(A^{13}B^{26}-B^{26}A^{13}\right)$

So, $A^{13}\cdot B^{26}-B^{26}\cdot A^{13}$ is a skew symmetric matrix.

Now,

$(AC-A)^T=(AC)^T-A^T$

$\Rightarrow(AC-A)^T=C^TA^T-A^T$

$\Rightarrow(AC-A)^T=-CA-A$

$\Rightarrow(AC-A)^T=-(CA+A)$

So, $AC-A$ is neither symmetric nor skew symmetric.

Q.10. If $a, b, \frac{1}{18}$ are in GP and $\frac{1}{10}, \frac{1}{a}, \frac{1}{b}$ are in AP, then the value of $a+180b$ is

Answer: 20

Solution: Since, $a, b, \frac{1}{18}$ are in GP, so

$b^2=\frac{a}{18} \quad \ldots(1)$

Also, $\frac{1}{10}, \frac{1}{a}, \frac{1}{b}$ are in AP, so

$\frac{2}{a}=\frac{1}{10}+\frac{1}{b}$

$\Rightarrow\frac{2}{a}=\frac{10+b}{10b}$

$\Rightarrow\frac{2}{18b^2}=\frac{10+b}{10b}$

$\Rightarrow\frac{1}{9b^2}=\frac{10+b}{10b}$

$\Rightarrow 90b^2+9b^3=10b$

$\Rightarrow b\left(9b^2+90b-10\right)=0$

$\Rightarrow 9b^2+90b-10=0$

Since, $b\neq 0$.

$\Rightarrow 9\left(\frac{a}{18}\right)+90b=10$

$\Rightarrow a+180b=20$

Q.11.

Given the function, $f(x)=\begin{cases}(1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & x<\frac{\pi}{2}\\ \mu, & x=\frac{\pi}{2}\\ e^{\frac{\cot 6x}{\cot 4x}}, & x>\frac{\pi}{2}\end{cases}$ which is continuos at $x=\frac{\pi}{2}$, then find the value of λ & μ

A) $\lambda=\frac{2}{3}\ \&\ \mu=e^{\frac{2}{3}}$ B) $\mu=\frac{2}{3}\ \&\ \lambda=e^{\frac{2}{3}}$ C) $\lambda=\frac{3}{2}\ \&\ \mu=e^{\frac{3}{2}}$ D) $\mu=\frac{3}{2}\ \&\ \lambda=e^{\frac{3}{2}}$

Answer: $\lambda=\frac{2}{3}\ \&\ \mu=e^{\frac{2}{3}}$

Solution: Given,

The function $f(x)=\begin{cases}(1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & x<\frac{\pi}{2}\\ \mu, & x=\frac{\pi}{2}\\ e^{\frac{\cot 6x}{\cot 4x}}, & x>\frac{\pi}{2}\end{cases}$ is continuos at $x=\frac{\pi}{2}$, so $f\left(\frac{\pi}{2}^{-}\right)=f\left(\frac{\pi}{2}\right)=f\left(\frac{\pi}{2}^{+}\right)$

Now finding $\lim_{x\to\frac{\pi}{2}^{-}}(1+|\cos x|)^{\frac{\lambda}{|\cos x|}}$ which is in the form of 1^{∞},

So $\lim_{x\to\frac{\pi}{2}^{-}}(1+|\cos x|)^{\frac{\lambda}{|\cos x|}}=e^{\lim_{x\to\frac{\pi}{2}^{-}}\frac{\lambda}{|\cos x|}\times|\cos x|}=e^{\lambda}$

Now finding $\lim_{x\to\frac{\pi}{2}^{+}}e^{\frac{\cot 6x}{\cot 4x}}=e^{\lim_{x\to\frac{\pi}{2}^{+}}\frac{\cot 6x}{\cot 4x}}=e^{\lim_{h\to 0}\frac{\cot 6h}{\cot 4h}}=e^{\lim_{h\to 0}\frac{\tan 4h}{\tan 6h}}=e^{\frac{4}{6}}=e^{\frac{2}{3}}$

Hence, $e^{\lambda}=e^{\frac{2}{3}}\Rightarrow\lambda=\frac{2}{3}\ \&\ \mu=e^{\frac{2}{3}}$

Q.12. $\int_{\frac{1}{3}}^{3}|\ln x|dx=\frac{m}{n}\ln\left(\frac{n^2}{e}\right)$, then the value of m^2+n^2-5 is ________

Answer: 20

Solution: Let

$I=\int_{\frac{1}{3}}^{3}|\ln x|dx$

$\Rightarrow I=\int_{\frac{1}{3}}^{1}|\ln x|dx+\int_{1}^{3}|\ln x|dx$

$\Rightarrow I=-\int_{\frac{1}{3}}^{1}\ln x dx+\int_{1}^{3}\ln x dx$

Since, $\ln x<0$ for $x\in(0,1)$.

$\Rightarrow I=-[x\ln x-x]_{\frac{1}{3}}^{1}+[x\ln x-x]_{1}^{3}$

$\Rightarrow I=-\left[(0-1)-\left(-\frac{1}{3}\ln 3-\frac{1}{3}\right)\right]+[3\ln 3-3-(0-1)]$

$\Rightarrow I=\frac{2}{3}-\frac{1}{3}\ln 3+3\ln 3-2$

$\Rightarrow I=\frac{8}{3}\ln 3-\frac{4}{3}$

$\Rightarrow I=\frac{4}{3}(2\ln 3-1)$

$\Rightarrow I=\frac{4}{3}\left(\ln 3^2-\ln e\right)$

$\Rightarrow I=\frac{4}{3}\ln\left(\frac{3^2}{e}\right)$

So, $m=4,\ n=3$

Hence, $m^2+n^2-5=16+9-5=20$

Q.13. Find the remainder when 2023^{2023} is divided by 35

Answer: 7

Solution: To find the remainder when 2023^{2023} is divided by 35 we will use binomial expansion,

Now rewriting above expression we get,

$2023^{2023} = (2030 - 7)^{2023} \{\text{as } 2030 \text{ is multiple of } 35\}$

$(2030 - 7)^{2023} = {}^{2023}C_0 2030^{2023} - {}^{2023}C_1 2030 \times 7 + \ldots\ldots - 7^{2023}$

$\Rightarrow (2030 - 7)^{2023} = 35k - 7^{2023}$

Now remainder will be -7^{2023},

Now rewriting -7^{2023} we get,

$-7^{2023} = -\left(7^3\right)^{674} \times 7$

$\Rightarrow -\left(7^3\right)^{674} \times 7 = -(343)^{674} \times 7$

$\Rightarrow -(343)^{674} \times 7 = -(350 - 7)^{674} \times 7$

$\Rightarrow -(343)^{674} \times 7 = \left(35k_1 + 7^{674}\right) \times (-7)$

So again using binomial we get remainder as -7×7^{674}

Again rewriting the expression as $-7 \times 7^{674} = -7^{675}$

$\Rightarrow -7^{675} = -\left(7^3\right)^{225}$

Which again can be written as $-(-7)^{225} = 7^{225} = \left(7^3\right)^{75} = (350 - 7)^{75}$

Again using binomial we get remainder as $(350 - 7)^{75} = (-7)^{75}$

Which can be written as $(-7)^{75} = -\left(7^3\right)^{25} = -(350 - 7)^{25}$

Again using binomial we get remainder as 7^{25}

Now again rewriting $\left(7^3\right)^8 \times 7 = (350 - 7)^8 \times 7$

Using binomial remainder will be 7^9 which can be written as $\left(7^3\right)^3 = (350 - 7)^3$,

Using binomial remainder will be -7^3 which can be written as -343, so remainder will be $350 - 343 = 7$,

Hence, the remainder when 2023^{2023} when divided by 35 is 7.

Q.14. If $I = \int_1^2 \frac{dx}{x^3\left(x^2+2\right)^2}$, and the value of $16I = \frac{11}{k} - 2\ln 2$, then the value of k is

Answer: 6

Solution: $I=\int_1^2 \frac{dx}{x^3\left(x^2+2\right)^2}$

$I=\int_1^2 \frac{dx}{x^3 x^4\left(1+\frac{2}{x^2}\right)^2}$

Let $\frac{2}{x^2}=t,\ -\frac{4}{x^3}dx=dt \Rightarrow x^2=\frac{2}{t}$

$\Rightarrow I=-\frac{1}{4}\int_2^{\frac{1}{2}} \frac{dt}{\frac{4}{t^2}(1+t)^2}=\frac{1}{16}\int_{\frac{1}{2}}^2 \frac{t^2 dt}{(1+t)^2}$

$\Rightarrow I=\frac{1}{16}\int_{\frac{1}{2}}^2 \frac{t^2-1+1}{(1+t)^2}dt$

$\Rightarrow I=\frac{1}{16}\int_{\frac{1}{2}}^2 \left[\frac{(t-1)(t+1)}{(1+t)^2}+\frac{1}{(1+t)^2}\right]dt$

$\Rightarrow I=\frac{1}{16}\int_{\frac{1}{2}}^2 \left[\frac{(t+1-2)}{(1+t)}+\frac{1}{(1+t)^2}\right]dt$

$\Rightarrow I=\frac{1}{16}\int_{\frac{1}{2}}^2 dt-\frac{1}{16}\int_{\frac{1}{2}}^2 \frac{2}{1+t}dt+\frac{1}{16}\int_{\frac{1}{2}}^2 \frac{1}{(1+t)^2}dt$

$\Rightarrow I=\frac{1}{16}\left(2-\frac{1}{2}\right)-\frac{1}{8}[\ln(1+t)]_{\frac{1}{2}}^2+\frac{1}{16}\left[-\frac{1}{1+t}\right]_{\frac{1}{2}}^2$

$\Rightarrow I=\frac{1}{16}\left(\frac{3}{2}\right)-\frac{1}{8}\left[\ln 3-\ln\frac{3}{2}\right]-\frac{1}{16}\left[\frac{1}{3}-\frac{2}{3}\right]$

$\Rightarrow I=\frac{3}{32}-\frac{1}{8}[\ln 2]+\frac{1}{48}=\frac{1}{16}\left(\frac{11}{6}\right)-\frac{1}{8}[\ln 2]$

$\Rightarrow 16I=\frac{11}{6}-2\ln 2$

Q.15. In a city, 25% of the population is smoker and a smoker has 27 times more chance of being diagnosed with lung cancer. A person is selected at random and found to be diagnosed with lung cancer. If the probability of him being smoker is $\frac{k}{10}$, then the value of k is

Answer: 9

Solution: Given,

In a city, 25% of the population is smoker

So, $P(\text{smoker})=0.25$ so $P(\text{non-smoking})=0.75$

And smoker has 27 times more chance of being diagnosed with lung cancer,

So, if $P\left(\frac{\text{Lung cancer}}{\text{non-smoker}}\right)=p$ then $P\left(\frac{\text{Lung cancer}}{\text{smoker}}\right)=27p$,

Now A person is selected at random and found to be diagnosed with lung cancer,

So, total probability of having a lung cancer will be $P(\text{Lung cancer})=0.25\times 27p+0.75\times p$

So the probability of him being smoker (using bayes' theorem) will be $P\left(\frac{\text{smoker}}{\text{Lung cancer}}\right)=\frac{0.25\times 27p}{0.25\times 27p+0.75\times p}=\frac{27}{30}=\frac{9}{10}$

Now comparing with $\frac{k}{10}=\frac{9}{10}$,

Then the value of k will be 9