JEE Main Exam 2023 - Session 1

25 Jan 2023 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. A car, moving along a straight line, covers the first half of the distance with uniform velocity of magnitude v_1 and the other half of the distance with uniform velocity of magnitude v_2 without turning. The magnitude of a verage velocity of the car is equal to

A) $\frac{2v_1v_2}{(v_1+v_2)}$

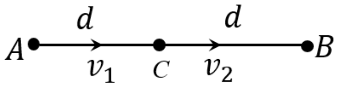
B) $\frac{(v_1+v_2)}{2}$

C) $v_1 + v_2$

D) $\sqrt{v_1 + v_2}$

Answer: $\frac{2v_1v_2}{(v_1+v_2)}$

Solution:



The magnitude of average velocity of car is:

$$v_{avg} = \frac{\text{magnitude of total displacement}}{\text{total time}}$$

Let the total distance covered by car be $2d$. Since the car is travelling along a straight line without turning, the magnitude of total displacement is same as the distance covered.

The time taken to cover the first half is $\frac{d}{v_1}$ and the time taken to cover the second half is $\frac{d}{v_2}$

$$\therefore \quad v_{avg} = \frac{2d}{\frac{d}{v_1}+\frac{d}{v_2}}$$

$$\Rightarrow \quad v_{avg} = \frac{2d(v_1v_2)}{v_1d+v_2d} = \frac{2v_1v_2}{v_1+v_2}$$

Hence, option A is correct.

Q.2. A car is moving with a constant speed of $2\ \text{m s}^{-1}$ along a circle of radius R. A pendulum is suspended from the ceiling of the car. Find the angle made by the pendulum with the vertical. Take $R = \frac{8}{15}\ \text{m}\ \&\ g = 10\ \text{m s}^{-2}$.

A) 30°

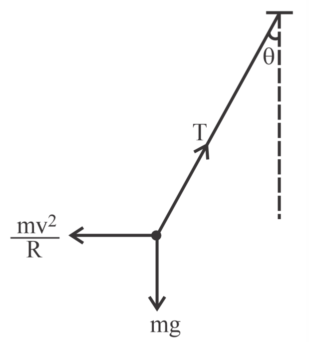
B) 53°

C) 37°

D) 60°

Answer: 37°

Solution:



Let the frame of reference to be attached to the car with x-axis along horizontal direction and y-axis along vertical direction. Since the frame of reference is a rotating frame, a centrifugal force term should be added while applying Newton's second law. Let θ be the angle made by the pendulum with the vertical.

Applying Newton's second law for horizontal direction:

$T\sin\theta - \frac{mv^2}{R} = 0$ (since the acceleration of the pendulum in the rotating frame is 0).

$\therefore T\sin\theta = \frac{mv^2}{R}$ ....(1)

Applying Newton's second law for vertical direction:

$T\cos\theta - mg = 0$

$\Rightarrow T\cos\theta = mg$ ....(2)

Dividing equation 1 by equation 2:

$$\tan\theta = \frac{v^2}{Rg} = \frac{(2)(2)}{\left(\frac{8}{15}\right)(10)} = \frac{3}{4}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right) = 37^\circ$$

Hence, option C is correct.

Q.3. A particle is dropped inside tunnel about a diameter of Earth. The particle starts oscillating with time period T. If R is radius of earth and g is the magnitude of acceleration due to gravity at Earth's surface, find the time period T.

A) $T = \pi\sqrt{\frac{R}{g}}$

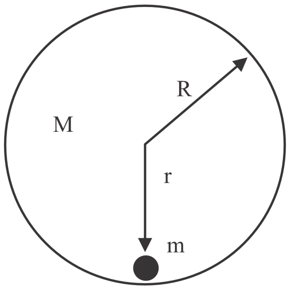
B) $T = 2\pi\sqrt{\frac{R}{g}}$

C) $T = 2\pi\sqrt{\frac{2R}{g}}$

D) $T = 2\pi\sqrt{\frac{3R}{g}}$

Answer: $T = 2\pi\sqrt{\frac{R}{g}}$

Solution:



The gravitational force acting on a particle of mass m at a distance $r\ (< R)$ from the centre of Earth is

$F = -G\dfrac{M'm}{r^2}$ ....(1)

where M' is the mass of portion of Earth enclosed by a sphere of radius r concentric with Earth. Here $-$ sign indicates radially inward direction of force.

Taking the density of Earth to be uniform

$$\frac{M'}{4\pi r^3} = \frac{M}{4\pi R^3}$$

$\Rightarrow M' = \dfrac{Mr}{R^3}$ ....(2)

Substituting M' from equation 2 into equation 1:

$$F = -\left(\frac{GMm}{R^3}\right) r$$

Which is similar to the equation of motion $F = -kx$ with $k = G\dfrac{Mm}{R^3} = \dfrac{mg}{R}\ \left(\because g = G\dfrac{M}{R^2}\right)$

For such motion, the time period is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Substituting k from above,

$$T = 2\pi\sqrt{\frac{R}{g}}$$

Hence, option B is correct.

Q.4. If T is the temperature of a gas then RMS velocity of the gas molecules is proportional to

A) $T^{\frac{1}{2}}$ B) $T^{-\frac{1}{2}}$ C) T D) T^2

Answer: $T^{\frac{1}{2}}$

Solution: The relation between RMS velocity, v_{rms} and absolute temperature, T is given by:

$$v_{rms} = \sqrt{\frac{3RT}{M_0}}$$

So, $v_{rms} \propto \sqrt{T}$

Hence, option A is correct.

Q.5. The time period of a simple pendulum at Earth's surface is T. Find the time period of the pendulum at a distance (from centre) which is twice the radius of earth.

A) $\frac{T}{4}$ B) $4T$ C) $\frac{T}{2}$ D) $2T$

Answer: $2T$

Solution: Time period of a simple pendulum at Earth's surface is given by:

$$T = 2\pi\sqrt{\frac{l}{g}} \;\ldots\left(1\right)$$

When distance from centre of Earth is $2R$, the acceleration due to gravity will be

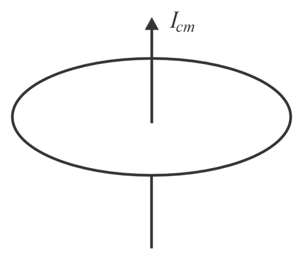
$$g' = \frac{GM}{(2R)^2} = \frac{GM}{4R^2} = \frac{g}{4}$$

Therefore, the time period will be

$$T' = 2\pi\sqrt{\frac{l}{\frac{g}{4}}} = 2T$$

Hence, option D is correct.

Q.6. Let I_{cm} be the moment of Inertia of a disc about an axis passing through its centre and perpendicular to its plane, I_{AB} be its moment of inertia about an axis AB that is along its plane and at a distance of $\frac{2r}{3}$ from its centre. Find $\frac{Icm}{I_{AB}}$



A) $\frac{1}{4}$

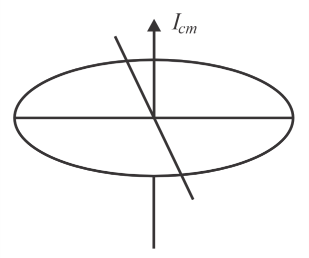
B) $\frac{18}{25}$

C) $\frac{9}{7}$

D) $\frac{1}{2}$

Answer: $\frac{18}{25}$

Solution:



$I_{cm}=\frac{1}{2}Mr^2 \;.....(1)$

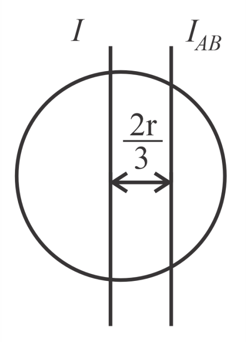
Let I_1 and I_2 be moment of inertia of the disc about two diameters perpendicular to each others. Using perpendicular axes theorem:

$I_1+I_2=I_{cm}\;....(2)$

Since I_1 and I_2 are similar, let $I_1=I_2=I$. Therefore, from equation 2

$2I=I_{cm}=\frac{1}{2}Mr^2$

$\Rightarrow I=\frac{1}{4}Mr^2$



Since AB is parallel to diameter and at a distance of $\frac{2}{3}r$ from it, using parallel axis theorem,

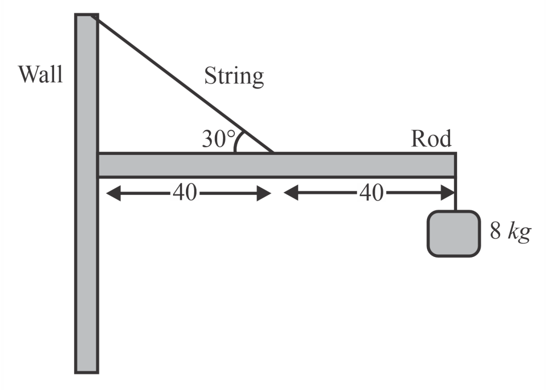
$I_{AB}=I+M\left(\frac{2r}{3}\right)^2=\frac{Mr^2}{4}+M\left(\frac{2r}{3}\right)^2$

$=Mr^2\left[\frac{1}{4}+\frac{4}{9}\right]=\frac{25}{36}Mr^2$

$\therefore \frac{Icm}{I_{AB}}=\frac{\frac{1}{2}Mr^2}{\frac{25}{36}Mr^2}=\frac{18}{25}$

Hence, option B is correct.

Q.7. A massless rod is arranged as shown:

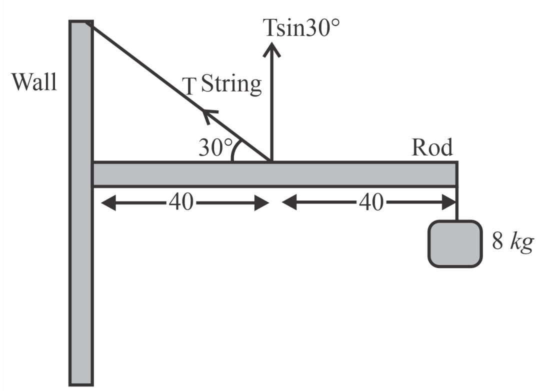


Find the tension in the string.

A) 320 N B) 640 N C) 160 N D) 480 N

Answer: 320 N

Solution:



Since the rod is at equilibrium, the net torque, about an axis perpendicular to the plane and passing through the point of contact of the rod and the wall, is zero. Therefore,

$T \sin 30^\circ \times 40 = 8g \times 80$

$\Rightarrow T = 320 \text{ N}$ (taking $g = 10 \text{ m s}^{-2}$)

Hence, option A is correct.

Q.8. Find the ratio of nuclear density of He & Ca.

A) $1:1$ B) $2:1$ C) $4:1$ D) $1:32$

Answer: $1:1$

Solution: The radius of a nucleus, R is related to its atomic number A as:

$R = R_\circ A^{1/3}$

where $R_\circ$ is the nuclear radius of hydrogen. Therefore, the density of a nucleus is given by:

$$d = \frac{\mu A}{\frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3\mu}{4\pi R_\circ^3} = \text{Constant}$$

(μ is atomic mass unit)

Hence, the ratio of nuclear density of any two nuclei will be $1:1$ and option A is correct.

Q.9.

Physical Quantity	SI Units
a. Pressure	i. $\mathrm{kg\ m\,s^{-1}}$
b. Impulse	ii. $\mathrm{kg\ m^{-1}\,s^{-1}}$
c. Coefficient of Viscosity	iii. $\mathrm{kg\ m^{2}\,s^{-1}}$
d. Angular momentum	iv. $\mathrm{kg\ m^{-1}\,s^{-2}}$

A) a-ii, b-ii, c-iv, d-i

B) a-iv, b-i, c-ii, d-iii

C) a-i, b-iv, c-iii, d-ii

D) a-iii, b-ii, c-i, d-i

Answer: a-iv, b-i, c-ii, d-iii

Solution: Pressure is related to force and area as:

$$P=\frac{F}{A}$$

$$\therefore unit\ of\ P=\frac{\mathrm{kg\ m\,s^{-2}}}{\mathrm{m^2}}=\mathrm{kg\ m^{-1}\,s^{-2}}$$

Impulse is related to force and time of application as:

$$J\ =\ F\cdot t$$

$$\therefore unit\ of\ J=\mathrm{kg\ m\ s^{-1}}$$

From stroke's law:

$$F=6\pi\eta \mathrm{r\ v}$$

$$\therefore unit\ of\ \eta\ =\ \frac{\mathrm{kg\ m\,s^{-2}}}{(\mathrm{m})\times(\mathrm{m\,s^{-1}})}=\mathrm{kg\ m^{-1}\,s^{-1}}$$

Angular momentum is related to radius of circle and speed as:

$$L=m\,r\,v$$

$$\therefore unit\ of\ L=\mathrm{kg\ m\ \frac{m}{s}}=\mathrm{kg\ m^{2}\,s^{-1}}$$

Hence, option B is the correct match.

Q.10. An EM wave is propagating along $+z$ direction. At an instant, its electric field is along $+y$ direction. Find direction of magnetic field at that instant.

A) $\hat{\imath}$

B) $-\hat{\imath}$

C) $\hat{\jmath}$

D) $-\hat{\jmath}$

Answer: $-\hat{\imath}$

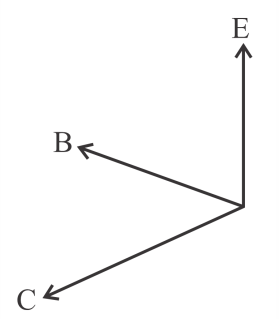
Solution: For an EM wave, unit vector along the direction of propagation of wave $\hat{c}$ is related to unit vector along the direction of electric field $\hat{E}$ and unit vector along the direction of magnetic field $\hat{B}$ as:

$\hat{c} = \hat{E} \times \hat{B}$

For the given wave, the direction of propagation is along $+z\ axis$ and the direction of electric field at the given instant is along $+y\ axis$,

$\therefore \hat{k} = \hat{j} \times \hat{B}$

The equation is satisfied for $\hat{B} = -\hat{i}$



Hence, option B is correct.

Q.11. An LC oscillator has an angular frequency of ω. If the inductance is increased to 2 times and capacitance 8 times, find the new value of ω.

A) $\frac{\omega}{2}$ B) $\frac{\omega}{4}$ C) 2ω D) $\frac{\omega}{8}$

Answer: $\frac{\omega}{4}$

Solution: Angular frequency expression in case of resonance is given by,

$\omega = \frac{1}{\sqrt{LC}}$.

Now for the second case,

$\omega' = \frac{1}{\sqrt{2L \times 8C}} = \frac{1}{4\sqrt{LC}}$

Therefore,

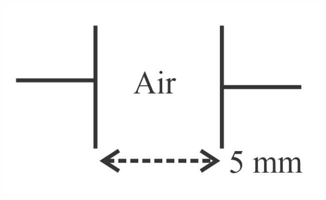
$\omega' = \frac{\omega}{4}$

Q.12. A parallel plate capacitor of capacitance C has plate area $40\ \text{cm}^2$ and plate separation $5\ \text{mm}$.If a dielectric slab of $K = 4$ and thickness $4\ \text{mm}$ is introduced between the plates, then the new capacitance will be

A) $2.5C$ B) $5C$ C) $15C$ D) $20C$

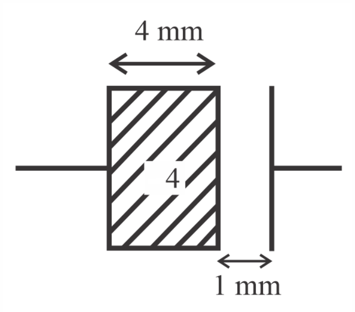
Answer: $2.5C$

Solution:



Initially,

$$C=\frac{\xi_0 \cdot A}{d}=\frac{\xi_0 \cdot A}{5\times10^{-3}}$$



After inserting dielectric slab,

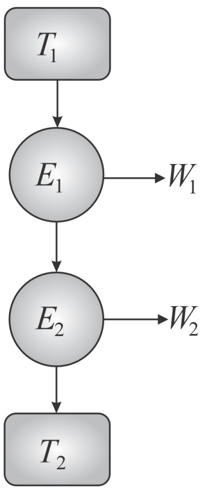
$$C_{new}=\frac{\xi_0 \cdot A}{\left(\frac{1}{1}+\frac{4}{4}\right)\times10^{-3}}$$

Therefore, ratio

$$\frac{C_{new}}{C}=\frac{\xi_0\frac{A}{2\times10^{-3}}}{\xi_0\frac{A}{5\times10^{-3}}}$$

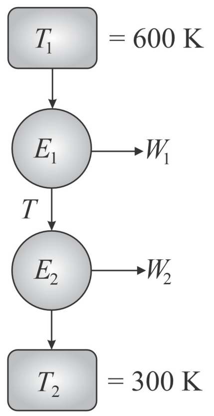
$$=\frac{5}{2}=2.5$$

Q.13. In the series sequence of two engines $E_1$and E_2 as shown. $T_1 = 600\ \mathrm{K}$ and $T_2=300\ \mathrm{K}$. It is given that both the engines work on Carnot principle and have same efficiency, then temperature T at which exhaust of E_1 is fed into E_2 is equal to $300\sqrt{n}\ K$. Value of n is equal to ______.



Answer: 2

Solution:



Equating efficiency for both the engines, we get

$1-\frac{T}{600}=1-\frac{300}{T}$
$\Rightarrow \frac{T}{600}=\frac{300}{T}$
$\Rightarrow T^2=180000$
$\Rightarrow T=\sqrt{180000}$
$=300\sqrt{2}$

Therefore,

$n=2.$

Q.14. A solenoid of length $2\ \mathrm{m}$, has 1200 turns. The magnetic field inside the solenoid, when $2\ \mathrm{A}$ current is passed through it is $\mathrm{N}\pi\times10^{-5}\ \mathrm{T}$. Find the value of N. (Diameter of solenoid is $0.5\ \mathrm{m}$)

Answer: 48

Solution: Magnetic field inside the solenoid is given by,

$B=\mu_0 ni$

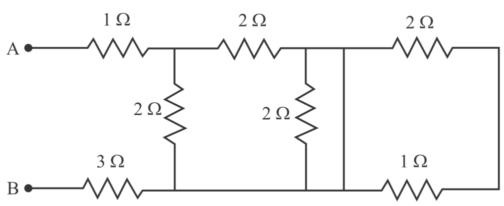
$=\left(4\pi\times10^{-7}\right)\times\left(\frac{1200}{2}\right)\times2$

$=48\pi\times10^{-5}$

Therefore,

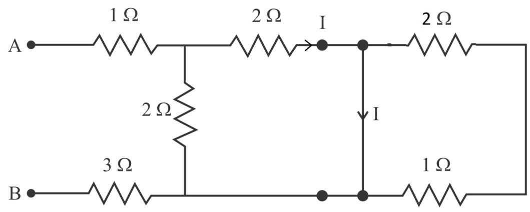
$N=48$

Q.15. Consider a network of resistors as shown. Find the effective resistance (in Ω) across A and B.

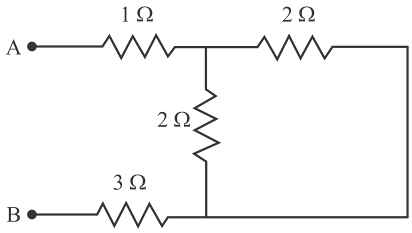


Answer: 5

Solution:



Due to plane wire connection in the right side of the $2\ \Omega$ resistor, the circuit will be a short circuit. Therefore, the middle $2\ \Omega$ will be rendered useless. Apart from this, the right side of the circuit will not have any current. So simplified circuit will be as given below.



Now, as we can see $2\ \Omega$ and $2\ \Omega$ are in parallel and hence will have equivalent resistance of $1\ \Omega$. Then, $1\ \Omega$, $1\ \Omega$ & $3\ \Omega$ will be in series across A and B, so the net resistance of the entire circuit will be

$= 1\ \Omega + 1\ \Omega + 3\ \Omega$

$= 5\ \Omega$

Section B: Chemistry

Q.1. Radius of 2^{nd} orbit of Li^{2+} ion is x, radius of 3^{rd} orbit of Be^{3+} will be

A) $\frac{27x}{16}$ B) $\frac{16x}{27}$ C) $\frac{4x}{3}$ D) $\frac{3x}{4}$

Answer: $\frac{27x}{16}$

Solution: $\text{n} = 3$

$\text{z} = 3$

$\text{r}_\text{n} = \frac{\text{r}_0 \text{n}^2}{\text{z}}$

for Li^{2+} $\text{x} = \frac{\text{r}_0 \times 2^2}{3}$

$\text{r}_\text{n} = \frac{3 \times \left(3\right)^2 \times \text{x}}{4 \times 4} = \frac{27\text{x}}{16}$

Q.2. If X-atoms are present at alternate corners and at body centre of a cube and Y-atoms are present at $1/3^{\text{rd}}$ of face centres then what will be the empirical formula?

A) X_2Y_5 B) X_5Y_2 C) X_1Y_5 D) X_3Y_2

Answer: X_3Y_2

Solution: Total number of corners = 8

4 corners are filled

$(X) = \frac{1}{8} \times 4 + 1 = \frac{3}{2}$

$(Y) = 6 \times \frac{1}{2} \times \frac{1}{3} = 1$

$X_{\frac{3}{2}} Y_1$

X_3Y_2

Q.3. Thionyl chloride on reaction with white phosphorus gives compound A. A on hydrolysis give compound B which is dibasic. Identify A and B

A) $A - PCl_5; B^{-}H_3PO_4$ B) $A - P_4O_{10}; B - H_3P^{-}O_4$ C) $A - POCl_3; B - H_3PO_4$ D) $A - PCl_3; B - H_3PO_3$

Answer: $A - PCl_3; B - H_3PO_3$

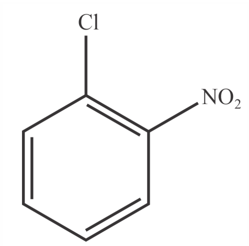
Solution:

$8\,SOCl_2 + P_4 \rightarrow 4\,PCl_3 + 4\,SO_2 + 2S_2Cl_2$
$PCl_3 + 3H_2O \rightarrow H_3PO_3 + 3\,HCl$

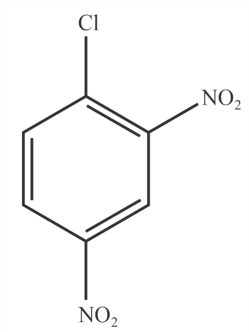
Phosphorus acid is a dibasic acid.

Q.4. Which of the following shows least reactivity towards nucleophilic substitution reaction?

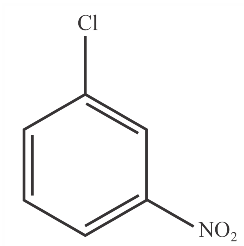
A)



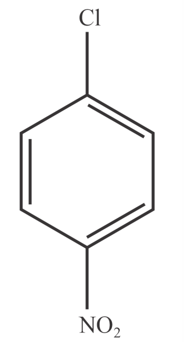
B)



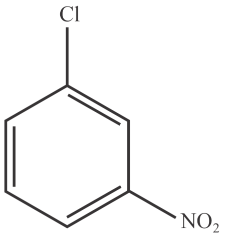
C)



D)



Answer:



Solution: Aromatic nucleophilic substitution reaction takes place via formation of carbanion intermediate. Hence, rate of reaction depends on the stability of carbanion. Electron withdrawing effect can stabilise the carbanion. Electron withdrawing nature of nitro group will be minimum at meta position.

Q.5. The correct decreasing order of positive electron gain enthalpy for the following inert gases

$\mathrm{He, Ne, Kr, Xe}$

A) $\mathrm{He > Ne > Kr > Xe}$ B) $\mathrm{He > Ne > Xe > Kr}$ C) $\mathrm{He > Xe > Ne > Kr}$ D) $\mathrm{Ne > Kr > Xe > He}$

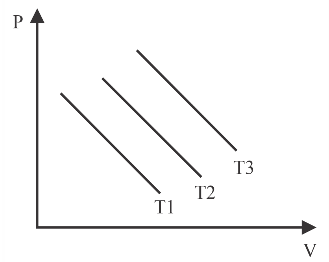
Answer: $\mathrm{Ne > Kr > Xe > He}$

Solution: Atomic and Physical Properties of Group 18 Elements

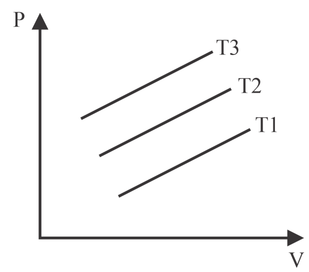
Propery	He	Ne	Ar	Kr	Xe	Rn
Atomic number	2	10	18	36	54	86
Electron gain enthalpy/$\mathrm{kJmol^{-1}}$	48	116	96	77	68	

Q.6. Which graph is correct for isothermal process at T_1, T_2 and T_3 if $(T_3 > T_2 > T_1)$

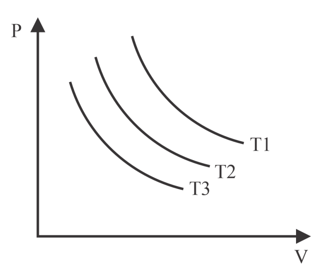
A)



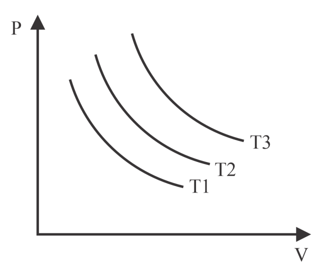
B)



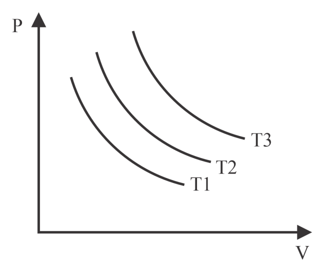
C)



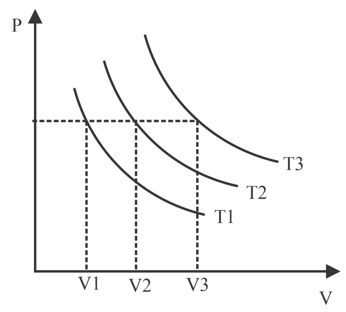
D)



Answer:



Solution:



For fixed pressure, we know that: $V_3 > V_2 > V_1$

Hence, $T_3 > T_2 > T_1$.

Q.7. Identify the correct sequence of reactants for the following conversion

$\text{n} - \text{Heptane} \longrightarrow \longrightarrow \longrightarrow \text{PhCOOH} + \text{PhCH}_2\text{OH}$

A) $Al_2O_3/Cr_2O_3, CrO_2Cl_2/H_3O^+$, Conc. $NaOH, H_3O^+$

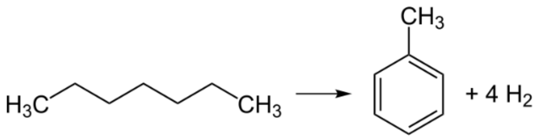
B) $Al_2O_3/Cr_2O_3, CrO_2Cl_2/H_3O^+, H_3O^+$, Conc. $NaOH$

C) CrO_2Cl_2, Al_2O_3, Conc. $NaOH, H_3O^+$

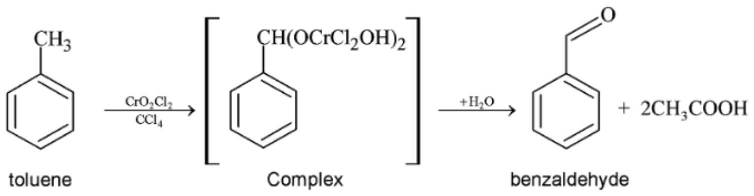
D) Sn/HCl, Conc. $NaOH, CrO_2Cl_2HNO_3$

Answer: $Al_2O_3/Cr_2O_3, CrO_2Cl_2/H_3O^+$, Conc. $NaOH, H_3O^+$

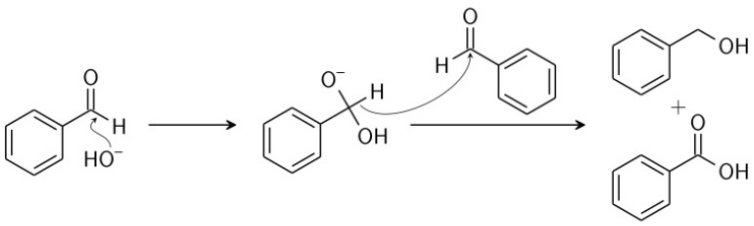
Solution: n-heptane undergo aromatisation reaction with $Al_2O_3/Cr_2O_3/\Delta$.



Toluene undergo Etard's reaction to give benzaldehyde.



Benzaldehyde undergo Cannizzaro's reaction as follows to get desired products.



Q.8. The correct order of basic strength in aqueous solution for

1. $CH_3 - NH_2$;
2. $CH_3 - NH - CH_3$;
3. $CH_3 - \overset{\displaystyle N}{\underset{\displaystyle CH_3}{|}} - CH_3$;
4. NH_3

A) $2 > 1 > 3 > 4$ B) $3 > 2 > 1 > 4$ C) $4 > 2 > 1 > 3$ D) $2 > 4 > 3 > 1$

Answer: $2 > 1 > 3 > 4$

Solution: The basicity of amines can be compared with respect to ammonia, by comparing the availability of pairs of electrons on nitrogen. In aqueous solution, electron donating inductive effect, solvation effect (H-bonding) and steric hindrance all together affect basic strength of substituted amines.

The overall order is 2>1>3>4

Q.9. Match the following

Column I		Column II	
(A)	K	(P)	Violet
(B)	Ca	(Q)	Brick red
(C)	Sr	(R)	Apple green
(D)	Ba	(S)	Crimson red

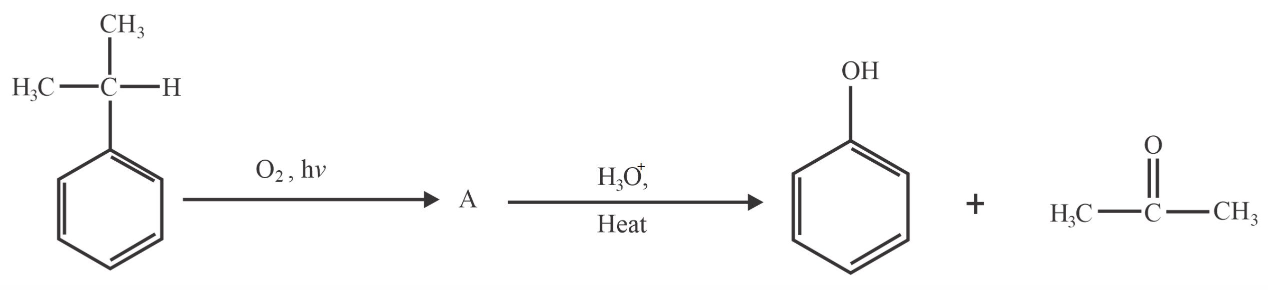
A) A-P; B-Q ; C-S; D-R B) A-Q; B-P ; C-S; D-R

C) A-R; B-S ; C-P; D-Q D) A-S; B-R ; C-Q; D-P

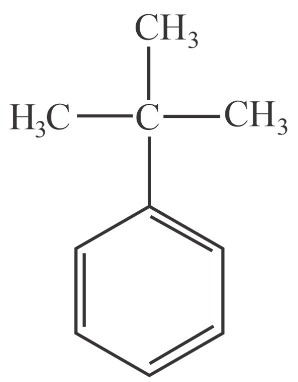
Answer: A-P; B-Q ; C-S; D-R

Solution: In alkaline earth metals, calcium gives brick-red flame, strontium gives crimson red flame, barium gives apple green flame. Potassium gives violet colour in flame test.

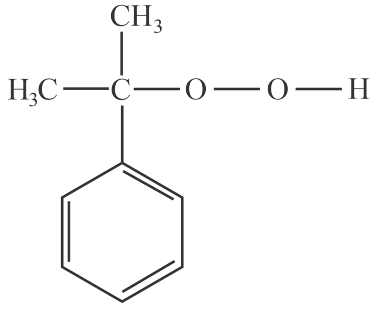
Q.10. Consider the following conversions.



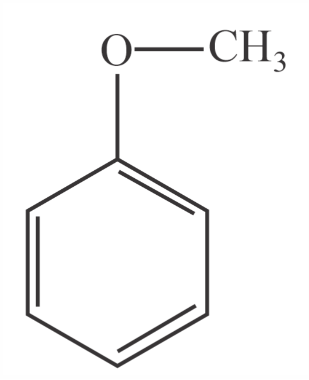
A)



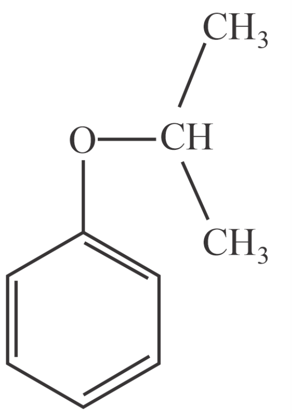
B)



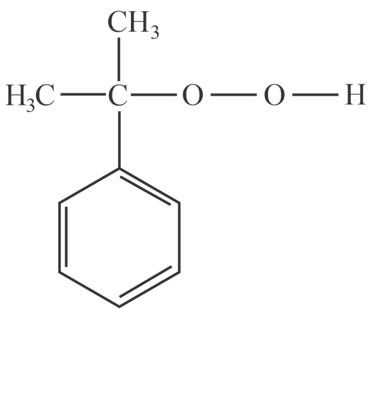
C)



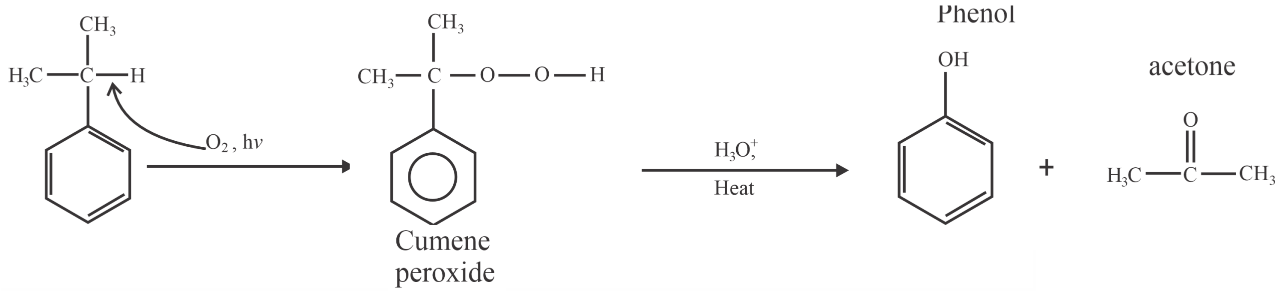
D)



Answer:



Solution:



Oxygen will attack the C-H bond and cumene peroxide will be formed which contains a peroxide bond

Q.11. Assertion: Acetal \& Ketal are stable in basic medium.
Reason: Leaving group tendency increasing as a result stability increases

A) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A)

B) Both Assertion (A) and Reason (R) are the true but Reason (R) is not a correct explanation of Assertion (A)

C) Assertion (A) is true and Reason (R) is false

D) Assertion (A) is false and Reason (R) is true

Answer: Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A)

Solution: The acetals are unstable in the presence of aqueous solutions of acid because they undergo hydrolysis in the presence of acidic conditions to produce the parent aldehyde or ketone or alcohols. But are stable and do not undergo hydrolysis in the presence of a base or under neutral conditions.

Q.12. Volume of $1.2 \mathrm{~g/L}$ solution of monobasic acid $(M = 24.2 \mathrm{~g/mol})$ needed to neutralise $25 \mathrm{ml}$ of $0.24 \mathrm{MNaOH}$.

Answer: 121

Solution: HA (Mono Basic Acid)

1L of soln $\rightarrow 1.2 \mathrm{~g}$ of HA

$1 \mathrm{L}$ of soln $\rightarrow \frac{1.2}{24.2}$ mol of HA

milli equivalents of Acid $=$ milli equivalents of Base

$\mathrm{N}_1 \times \mathrm{V}_1(\mathrm{ml}) = \mathrm{N}_2 \times \mathrm{V}_2(\mathrm{ml})$

$\frac{1.2}{24.2} \times \mathrm{V}_1\left(\mathrm{ml}\right) = 0.24 \times 25$

$\Rightarrow \mathrm{V}_1 = 121 \mathrm{~ml}$

Q.13. Which of the following are paramagnetic?

$\mathrm{V}^{2+}, \mathrm{Ti}^{2+}, \mathrm{Cr}^{3+}, \mathrm{Ni}^{2+}$

Answer: 4

Solution: The species with unpaired electrons are paramagnetic.

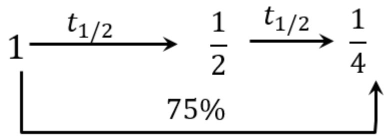
	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
	21	22	23	24	25	26	27	28	29	30
M^{2+}	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	$3d^8$	$3d^9$	$3d^{10}$
M^{3+}	[Ar]	$3d^1$	$3d^2$	$3d^3$	$3d^4$	$3d^5$	$3d^6$	$3d^7$	-	-

Hence, the given four elements are having unpaired electrons.

Q.14. For a first order reaction $A \rightarrow B, t_{1/2}$ is 30 minutes. Then find the time (in minutes) required for 75% completion of reaction?

Answer: 60

Solution: $t_{75\%} = 2 \times t_{1/2} = 2 \times 30 = 60$ mins



Q.15. How many of the following ions/elements has/have the same value of magnetic moment?

$V^{3+}, Cr^{3+}, Fe^{2+}, Ni^{2+}$

Answer: 2

Solution: The ions with same number of unpaired electrons have same value of magnetic moment.

V^{3+} – 2 unpaired electron
Cr^{3+} – 3 unpaired electron
Fe^{2+} – 4 unpaired electron
Ni^{2+} – 2 unpaired electron

Q.16. How many of the following complexes is(are) paramagnetic:

$[Fe(CN)_6]^{3-}$, $[Fe(CN)_6]^{4-}$, $[NiCl_4]^{2-}$, $[Ni(CN)_4]^{2-}$, $[CuCl_4]^{2-}$, $[Cu(CN_4)^{3-}$, $[Cu(H_2O)_4]^{2+}$

Answer: 4

Solution: The species with unpaired electrons are paramagnetic.

$Fe^{+3}(26)$	$3d^5$	CN^-strong field ligand pairing occurs and one unpaired electron is present hence, it is para magnetic.
$Fe^{+2}(26)$	$3d^6$	CN^-strong field ligand pairing occurs and zero unpaired electron is present hence, it is diamagnetic
$Ni^{+2}(28)$	$3d^8$	Cl^- is weak field ligand. two unpaired electrons are present. Hence paramagnetic.
$Ni^{+2}(28)$	$3d^8$	CN^- strong field ligand pairing occurs and, zero unpaired electron is present hence, it is diamagnetic
$Cu^{+2}(29)$	$3d^9$	paramagnetic
$Cu^{+}(29)$	$3d^{10}$	diamagnetic
$Cu^{+2}(29)$		paramgnetic

$[Fe(CN)_6]^{3-}$ $[NiCl_4]^{2-}$ $[CuCl_4]^{2-}$ $[Cu(H_2O)_4]^{2+}$ are paramgnetic complexes.

Q.17. Consider the following cell representation:

$Pt|H_2 \ |H^+ \ || \ Fe^{3+}|Fe^{2+}$
(1 atm) (1 M)

The value of $\frac{[Fe^{2+}]}{[Fe^{3+}]}$ is

Given that $E_{cell} = 0.712$ V & $E^0_{cell} = 0.771$ V

Answer: 10

Solution: Oxidation half reaction: $H_{2(g)} \longrightarrow 2H^{+}_{(aq)} + 2e^{-}$

Reduction half reaction: $2\,Fe^{3+}_{(aq)} + 2e^{-} \rightarrow 2\,Fe^{2+}_{(aq)}$

$$E_{cell} = E_{cell}{}^{\circ} - \frac{0.059}{2}\log\left[\frac{\left[Fe^{2+}\right]^{2}\left[H^{+}\right]^{2}}{\left[Fe^{2+}\right]^{2}}\right]$$

$$E_{cell} = E_{cell}{}^{\circ} - \frac{0.059}{2}\log\left[\frac{\left[Fe^{2+}\right]^{2}\left[H^{+}\right]}{\left[Fe^{2+}\right]^{2}}\right]^{2}$$

$0.712 \quad 0.771$

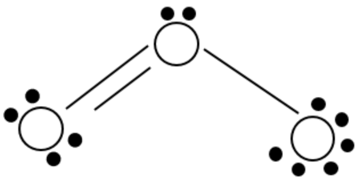
$$-0.059 = -0.059\ \log\frac{\left[Fe^{2+}\right]}{\left[Fe^{3+}\right]}$$

$$\log 10 = \log\frac{\left[Fe^{2+}\right]}{\left[Fe^{3+}\right]}$$

Q.18. Total number of lone pairs in Ozone

Answer: 6

Solution:



Total no of lone pairs $= 6$.

Section C: Mathematics

Q.1. The logical statement $(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$ is a

A) Tautology B) Fallacy C) Equivalent to $p \vee \sim q$ D) Equivalent to $p \wedge \sim q$

Answer: Tautology

Solution: We have,

p	q	$\sim q$	$(p \wedge \sim q)$	$(p \vee \sim q)$	$p \rightarrow \sim q$	$(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$
T	T	F	F	T	F	T
T	F	T	T	T	T	T
F	T	F	F	F	T	T
F	F	T	F	T	T	T

So, $(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$ is a tautology.

Q.2. If a_r is the coefficient of x^{10-r} in the expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is

A) 390 B) 1210 C) 485 D) 220

Answer: 1210

Solution: Since, a_r is the coefficient of x^{10-r} in the expansion of $(1+x)^{10}$, therefore

$a_r = {}^{10}C_{10-r} = {}^{10}C_r$

$a_{r-1} = {}^{10}C_{r-1}$

So,

$$\frac{a_r}{a_{r-1}} = \frac{{}^{10}C_r}{{}^{10}C_{r-1}}$$

$$\Rightarrow \frac{a_r}{a_{r-1}} = \frac{10!}{r! \times (10-r)!} \times \frac{(r-1)! \times (11-r)!}{10!}$$

$$\Rightarrow \frac{a_r}{a_{r-1}} = \frac{11-r}{r}$$

Now,

$$\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = \sum_{r=1}^{10} r^3 \left(\frac{11-r}{r}\right)^2$$

$$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = \sum_{r=1}^{10} r(11-r)^2$$

$$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = \sum_{r=1}^{10} r\left(121 + r^2 - 22r\right)$$

$$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = \sum_{r=1}^{10} \left(121r + r^3 - 22r^2\right)$$

$$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = \frac{121 \times 10 \times 11}{2} + \left(\frac{10 \times 11}{2}\right)^2 - \frac{22 \times 10 \times 11 \times 21}{6}$$

$$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = 6655 + 3025 - 8470$$

$$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2 = 1210$$

Q.3. Find the value of limit $\lim_{n \to \infty} \frac{1+2-3+4+5-6+.....(3n-2)+(3n-1)-3n}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$

A) $\frac{3}{2}\left(\sqrt{2}+1\right)$

B) $\frac{2}{3}\left(\sqrt{2}+1\right)$

C) $\frac{2}{3\sqrt{2}}$

D) $2\sqrt{2}$

Answer: $\frac{3}{2}\left(\sqrt{2}+1\right)$

Solution: Given,

$$\lim_{n\to\infty}\frac{1+2-3+4+5-6+.....(3n-2)+(3n-1)-3n}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$$

Now simplifying the expression by using summation formula we get,

$$\lim_{n\to\infty}\frac{\sum_{n=1}^{n}[(3n-2)+(3n-1)-3n]}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$$

$$=\lim_{n\to\infty}\frac{\sum_{n=1}^{n}[(3n-3)]}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$$

$$=\lim_{n\to\infty}\frac{3\sum_{n=1}^{n}n-3\sum_{n=1}^{n}1}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$$

$$=\lim_{n\to\infty}\frac{3\times\frac{n(n+1)}{2}-3n}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$$

$$=\lim_{n\to\infty}\frac{3n(n+1)-6n}{2\left(\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}\right)}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n}{2\left(\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}\right)}$$

$$=\lim_{n\to\infty}\frac{n^2\left(3-\frac{3}{n}\right)}{2n^2\left(\sqrt{2+\frac{3}{n^3}+\frac{1}{n^4}}-\sqrt{1+\frac{1}{n^3}+\frac{3}{n^4}}\right)}$$

$$=\lim_{n\to\infty}\frac{\left(3-\frac{3}{n}\right)}{2\left(\sqrt{2+\frac{3}{n^3}+\frac{1}{n^4}}-\sqrt{1+\frac{1}{n^3}+\frac{3}{n^4}}\right)}$$

$$=\frac{(3-0)}{2(\sqrt{2+0+0}-\sqrt{1+0+0})}$$

$$=\frac{3}{2\left(\sqrt{2}-1\right)}=\frac{3(\sqrt{2}+1)}{2}.$$

Q.4. Mean of a data set is 10 and variance is 4. If one entry of data set changes from 8 to 12, then new mean becomes 10.2. Then, new variance of the data set is

A) 3.92 B) 3.96 C) 4.04 D) 4.08

Answer: 3.96

Solution: Let the number of observations of the data set is n.

So,

$\sum x_i = 10n \quad \ldots.(\mathrm{i})$

Also,

$\sum x_i - 8 + 12 = 10.2n$

$10n + 4 = 10.2n$

$\Rightarrow 0.2n = 4$

$\Rightarrow n = 20$

Also,

$\sigma^2 = 4 = \frac{\sum x_i^2}{20} - 100$

$\Rightarrow \sum x_i^2 = 104 \times 20 = 2080$

Now, new variance is

$= \frac{\sum x_i^2 - 8^2 + 12^2}{20} - (10.2)^2$

$= \frac{2080 + 4 \times 20}{20} - 104.04$

$= 108 - 104.04$

$= 3.96$

Q.5. If $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$ when $z_1 = 2 + 3i$ & $z_2 = 3 + 4i$, then the locus of z is

A) Straight line with slope $\frac{-1}{2}$

B) Circle with radius $\frac{1}{\sqrt{2}}$

C) Hyperbola with eccentricity $\sqrt{2}$

D) Hyperbola with eccentricity $\frac{5}{2}$

Answer: Circle with radius $\frac{1}{\sqrt{2}}$

Solution: We know that $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$ represents Pythagoras property of a right-angled triangle right angle at z,

So if z moves it will always move on locus which making $90°$ from two fixed point z_1 & z_2 which will be diametric endpoint of a circle,

So radius of the circle will be given by $\frac{1}{2}|z_1 - z_2| = \frac{1}{2}\sqrt{1^2 + 1^2} = \frac{1}{\sqrt{2}}$

Q.6. If $f(x) = x^b + 3$, $g(x) = ax + c$. If $(g(f(x)))^{-1} = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$, then $fog(ac) + gof(b)$ is

A) 189

B) 195

C) 194

D) 89

Answer: 189

Solution: Given:

$f(x) = x^b + 3$

$g(x) = ax + c$

Now,

$y = g(f(x))$

$\Rightarrow y = a\left(x^b + 3\right) + c$

$\Rightarrow y = ax^b + 3a + c$

$\Rightarrow x = \left(\frac{y-c-3a}{a}\right)^{\frac{1}{b}}$

So,

$(g(f(x)))^{-1} = \left(\frac{x-c-3a}{a}\right)^{\frac{1}{b}}$

Hence,

$\left(\frac{x-c-3a}{a}\right)^{\frac{1}{b}} = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

So,

$a = 2, \quad b = 3, \ 3a + c = 7 \Rightarrow c = 1$

So,

$f(x) = x^3 + 3$
$g(x) = 2x + 1$

Now,

$fog(ac) = fog(2) = f(5) = 128$

$gof(b) = g(f(3)) = g(30) = 61$

Hence,

$fog(ac) + gof(b) = 128 + 61 = 189$

Q.7. If $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$ then find the value of $\left|adj\left(adjA^2\right)\right|$

A) 6^4 B) 4^8 C) 4^5 D) 2^8

Answer: 2^8

Solution: We know that $|adj\,(adj A)| = |A|^{(n-1)^2}$ where n is order of matrix,

Now using the formula we will solve $\left|adj\left(adj A^2\right)\right| = \left|A^2\right|^{(n-1)^2} = \left|A^2\right|^4 = |A|^8$

Now finding $|A|$ by solving $|A| = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{vmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 2 & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 3 \end{vmatrix}$$

$$\Rightarrow |A| = \frac{1}{\log x \times \log y \times \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix}$$

$$\Rightarrow |A| = \frac{\log x \times \log y \times \log z}{\log x \times \log y \times \log z} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 5 - 2 - 1 = 2,$$

Hence, $\left|adj\left(adj A^2\right)\right| = |A|^8 = |2|^8$

Q.8. If $f(x) = \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)} dx$ and $f(3) = \frac{1}{2}(\ln 5 - \ln 6)$, then $f(4)$ is

A) $\frac{1}{2}(\ln 17 - \ln 19)$

B) $\frac{1}{2}(\ln 19 - \ln 17)$

C) $\ln 19 - \ln 17$

D) $\ln 17 - \ln 19$

Answer: $\frac{1}{2}(\ln 17 - \ln 19)$

Solution: Let

$I=\int\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx$

Put $x^2=t\Rightarrow 2xdx=dt$

$I=\int\frac{1}{(t+1)(t+3)}dt$

$\Rightarrow I=\frac{1}{2}\int\frac{2}{(t+1)(t+3)}dt$

$\Rightarrow I=\frac{1}{2}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)dt$

$\Rightarrow I=\frac{1}{2}[\ln(t+1)-\ln(t+3)]+C$

$\Rightarrow f(x)=\frac{1}{2}\left[\ln\left(x^2+1\right)-\ln\left(x^2+3\right)\right]+C$

Put $x=3$, then

$\frac{1}{2}[\ln 5-\ln 6]=\frac{1}{2}[\ln 10-\ln 12]+C$

$\frac{1}{2}[\ln 5-\ln 6]=\frac{1}{2}[\ln 2+\ln 5-\ln 2-\ln 6]+C$

$\Rightarrow C=0$

So,

$f(x)=\frac{1}{2}\left[\ln\left(x^2+1\right)-\ln\left(x^2+3\right)\right]$

$\Rightarrow f(4)=\frac{1}{2}(\ln 17-\ln 19)$

Q.9. For $a>1$ the value of $\lim_{x\to\infty}\frac{\cot^{-1}\left(x^{-a}\log_a x\right)}{\sec^{-1}\left(a^x\log_x a\right)}$ is equal to

A) -1 B) 1 C) 2 D) $\log_a 2$

Answer: 1

Solution: Let $f(x)=x^{-a}\log_a x$

Now $\lim_{x\to\infty}x^{-a}\log_a x=\lim_{x\to\infty}\frac{\log_e x}{x^a\log_e a}$

$=\lim_{x\to\infty}\frac{1/x}{x^{a-1}\log_e a}=0$

Let $g(x)=a^x\log_x a$

Now $\lim_{x\to\infty}a^x\log_x a=\lim_{x\to\infty}\frac{a^x\log_e a}{\log_e x}$

$=\lim_{x\to\infty}\frac{a^x(\log_e a)^2}{1/x}=\infty$

So, $\lim_{x\to\infty}\frac{\cot^{-1}\left(x^{-a}\log_a x\right)}{\sec^{-1}\left(a^x\log_x a\right)}=\frac{\cot^{-1}0}{\sec^{-1}\infty}$

$=\frac{\pi/2}{\pi/2}=1$

Q.10. If $f(x)=\int_0^2 e^{|x-t|}dt$, then the minimum value of $f(x)$ is

A) $2(e-1)$ B) $2(e+1)$ C) $2e-1$ D) $2e+1$

Answer: $2(e-1)$

Solution: Given:

$f(x)=\int_0^2 e^{|x-t|}dt$

Case 1: When $x<0$, then

$f(x)=\int_0^2 e^{t-x}dt=e^{-x}\int_0^2 e^{t}dt$

$\Rightarrow f(x)=e^{-x}\left(e^2-1\right)$

Case 2: When $x\geq 2$, then

$f(x)=\int_0^2 e^{x-t}dt=e^{x}\int_0^2 e^{-t}dt$

$\Rightarrow f(x)=e^{x}\left(1-e^{-2}\right)$

Case 3: When $0\leq x<2$, then

$f(x)=\int_0^x e^{x-t}dt+\int_x^2 e^{t-x}dt$

$\Rightarrow f(x)=e^{x}\left(1-e^{-x}\right)+e^{-x}\left(e^2-e^{x}\right)$

$\Rightarrow f(x)=e^{x}+e^{2-x}-2$

So,

$$f(x)=\begin{cases} e^{-x}\left(e^2-1\right); & x<0 \\ e^{x}+e^{2-x}-2; & 0\leq x<2 \\ e^{x}\left(1-e^{-2}\right); & x\geq 2 \end{cases}$$

$$f'(x)=\begin{cases} -e^{-x}\left(e^2-1\right); & x<0 \\ e^{x}-e^{2-x}; & 0<x<2 \\ e^{x}\left(1-e^{-2}\right); & x>2 \end{cases}$$

For $x>2$; $f'(x)>0$, hence function is increasing, so minimum value is

$f(2)=e^2\left(1-e^{-2}\right)=e^2-1$

For $0\leq x<2$, we have

$f'(x)=e^{x}-e^{2-x}$

$f''(x)=e^{x}+e^{2-x}$

For critical points

$f'(x)=0\Rightarrow e^{x}-e^{2-x}=0\Rightarrow x=1$

So,

$f''(1)=e+e=2e>0$

So, minima at $x=1$, which is

$f(1)=e+e-2=2(e-1)$

For $x<0$, we have

$f'(x)=-e^{-x}\left(e^2-1\right)<0$

So,

$f_{\min}=\left(e^2-1\right)$

Q.11. The term independent of x in the expansion of $\left(2x+\frac{1}{x^7}-7x^2\right)^5$ is

A) 6860

B) 13720

C) -13720

D) -6860

Answer: -13720

Solution: Any term in the expansion of $\left(2x+\frac{1}{x^7}-7x^2\right)^5$ is given by

$\frac{5!}{a!b!c!}(2x)^a\left(x^{-7}\right)^b\left(-7x^2\right)^c$ where $a+b+c=5 \quad \ldots(i)$

$=2^a(-7)^c\frac{5!}{a!b!c!}x^a x^{-7b}x^{2c}=2^a(-7)^c\frac{5!}{a!b!c!}x^{a-7b+2c}$

Now for term to be independent of x, we know

$a-7b+2c=0 \quad \ldots(ii)$

Solving (i) & (ii), we get

$b=\frac{c+5}{8}$

Also $a,\ b,\ c\in\{1,2,3,4,5\}$

So, $a=1,\ b=1,\ c=3$

Therefore, the term independent of x will be $2^1(-7)^3\frac{5!}{1!1!3!}=-13720$

Q.12. If sum of two integers is 66 and μ is the maximum value of their product, $S=\left\{x\in Z,\ x(66-x)\geq\frac{5\mu}{9}\right\},\ x\neq 0$, then probability of A when $A=\{x\in S,\ x=3k,\ x\in N\}$ is

A) $\frac{1}{4}$ B) $\frac{2}{3}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$

Answer: $\frac{1}{3}$

Solution: Given,

Sum of two integer is 66, so one number will be x and other will be $66-x$,

And given μ is maximum value of their product,

So let $y=x(66-x)$

$\Rightarrow y=66x-x^2$

Now differentiating to find maxima and minima we get,

$\frac{dy}{dx}=66-2x$

Now equating with zero to find point of maxima as $y=66x-x^2$ represents a downward parabola so it will give maxima,

So $\frac{dy}{dx}=0\Rightarrow 66-2x=0\Rightarrow x=33,$

Hence, the value of $\mu=33\times 33=1089$

Now solving $x(66-x)\geq\frac{5\mu}{9}$

$\Rightarrow x(66-x)\geq\frac{5\times 1089}{9}$

$\Rightarrow x(66-x)\geq 605$

$\Rightarrow x^2-66x+605\leq 0$

$\Rightarrow (x-11)(x-55)\leq 0$

So $x\in[11,55]$ $\rightarrow$total 45 numbers,

Now for probability of A, favourable outcomes will be $x=3k\Rightarrow x=\{12,15,18,\ldots..54\}\rightarrow$ total 15 numbers,

So probability will be $\frac{15}{45}=\frac{1}{3}$

Q.13. Let $L_1:\frac{x-3}{1}=\frac{y-2}{2}=\frac{z-1}{3}$ and $L_2:\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and direction ratios of line L_3 are $<1,-1,3>$. P and Q are points of intersection of L_1 & L_3 and L_2 & L_3 respectively, then distance between P and Q is

A) $\frac{10}{3}\sqrt{6}$ B) $\frac{8}{3}\sqrt{11}$ C) $\frac{4}{3}\sqrt{11}$ D) $\frac{11}{3}\sqrt{6}$

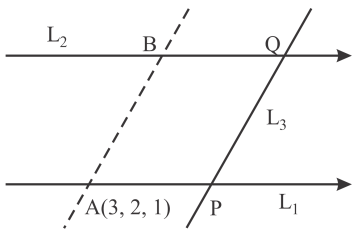
Answer: $\frac{4}{3}\sqrt{11}$

Solution: Let $A(3,2,1)$

Equation of line AB is

$\frac{x-3}{1}=\frac{y-2}{-1}=\frac{z-1}{3}=k$ (Let)

$\Rightarrow x=k+3, y=-k+2, z=3k+1$



Here, $PQ=AB$.

$L_2: \frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}=\lambda$

Since, B lies on L_2, so

$B(\lambda+1, 2\lambda+2, 3\lambda+3)$

For point of intersection

$k+3=\lambda+1 \Rightarrow \lambda-k=2$

$2-k=2\lambda+2 \Rightarrow 2\lambda+k=0 \Rightarrow k=-2\lambda$

$\Rightarrow 3\lambda=2 \Rightarrow \lambda=\frac{2}{3}$

Hence, $B\left(\frac{5}{3}, \frac{10}{3}, 5\right)$

Therefore,

$AB=PQ=\sqrt{\left(\frac{4}{3}\right)^2+\left(\frac{4}{3}\right)^2+16}=\frac{4}{3}\sqrt{11}$ units

Q.14. If $\vec{a}=-\hat{i}+2\hat{j}+\hat{k}$ is rotated by 90° about origin passing through $y-$axis which gives a new vector $\vec{b}$, then projection of $\vec{b}$ on $\vec{c}=5\hat{i}+4\hat{j}+3\hat{k}$ is given by

A) $\frac{6}{5}$ B) $\frac{3}{5}$ C) $\frac{6}{5\sqrt{3}}$ D) $\frac{6}{5}\sqrt{3}$

Answer: $\frac{6}{5}$

Solution: Given,

$\vec{a} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ is rotated by 90° about origin passing through $y-$axis which gives a new vector $\vec{b}$,

So, let $\vec{b} = t\vec{a} + s\hat{\jmath}$ {note here given vector is passing through y-axis so $s\hat{\jmath}$ has been introduced.}

$\Rightarrow \vec{b} = t\left(-\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) + s\hat{\jmath}$

$\Rightarrow \vec{b} = -t\hat{\imath} + (2t + s)\hat{\jmath} + t\hat{k}$

Also $\vec{a} \cdot \vec{b} = 0$ as they both are perpendicular,

So, $\left(-\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) \cdot \left(-t\hat{\imath} + (2t + s)\hat{\jmath} + t\hat{k}\right) = 0$

$\Rightarrow t + 4t + 2s + t = 0$

$\Rightarrow 3t = -s$

Hence, $\vec{b} = -t\hat{\imath} + (2t - 3t)\hat{\jmath} + t\hat{k}$

$\Rightarrow \vec{b} = t\left(-\hat{\imath} - \hat{\jmath} + \hat{k}\right)$

Now using $\left|\vec{a}\right| = \left|\vec{b}\right|$ as only direction is changing not length,

So, $\sqrt{1^2 + 2^2 + 1^2} = |t|\sqrt{1^2 + 1^2 + 1^2} \Rightarrow t = \pm\sqrt{2}$

So, $\vec{b} = \pm 2\left(-\hat{\imath} - \hat{\jmath} + \hat{k}\right)$

Now projection of $\vec{b}$ on $\vec{c} = 5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ will be $\left|\frac{\vec{b} \cdot \vec{c}}{\left|\vec{c}\right|}\right| = \sqrt{2}\left|\frac{-5-4+3}{\sqrt{5^2+4^2+3^2}}\right| = \sqrt{2}\left|\frac{-6}{5\sqrt{2}}\right| = \frac{6}{5}$

Q.15. If $\frac{dy}{dx} = \frac{y}{x}\left(1 + xy^2(1 + \ln x)\right)$ and $y(1) = 3$, then the value of $\frac{y^2(3)}{9}$ is

A) $\frac{1}{-43+27\ln 3}$

B) $\frac{1}{43+27\ln 3}$

C) $\frac{9}{59-162(1+\ln 3)}$

D) $\frac{1}{27-43\ln 3}$

Answer: $\frac{9}{59-162(1+\ln 3)}$

Solution: Given:

$\frac{dy}{dx}=\frac{y}{x}\left(1+xy^2(1+\ln x)\right)$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}+y^3(1+\ln x)$

$\Rightarrow \frac{dy}{dx}-\frac{y}{x}=y^3(1+\ln x)$

$\Rightarrow \frac{1}{y^3}\frac{dy}{dx}-\frac{1}{xy^2}=1+\ln x$

Put $\frac{1}{y^2}=t\Rightarrow\frac{2}{y^3}\frac{dy}{dx}=-\frac{dt}{dx}$

$-\frac{1}{2}\frac{dt}{dx}-\frac{t}{x}=1+\ln x$

$\Rightarrow \frac{dt}{dx}+\left(\frac{2}{x}\right)t=-2(1+\ln x)$

This is a linear differential equation.

$\text{IF}=e^{\int\frac{2}{x}dx}=e^{2\ln|x|}=x^2$

So, solution is

$tx^2=-2\int x^2(1+\ln x)dx$

$\Rightarrow \frac{x^2}{y^2}=-2\left[(1+\ln x)\frac{x^3}{3}-\int\frac{x^2}{3}dx\right]$

$\Rightarrow \frac{x^2}{y^2}=-2\left[(1+\ln x)\frac{x^3}{3}-\frac{x^3}{9}\right]+C$

Now, $y(1)=3$, so

$\frac{1}{9}=-\frac{4}{9}+C$

$\Rightarrow C=\frac{5}{9}$

So,

$\frac{x^2}{y^2}=-2\left[(1+\ln x)\frac{x^3}{3}-\frac{x^3}{9}\right]+\frac{5}{9}$

So,

$\frac{9}{y^2(3)}=-2\left[(1+\ln 3)\frac{27}{3}-\frac{27}{9}\right]+\frac{5}{9}$

$\Rightarrow \frac{9}{y^2(3)}=-18(1+\ln 3)+\frac{59}{9}$

$\Rightarrow \frac{9}{y^2(3)}=\frac{59-162(1+\ln 3)}{9}$

$\Rightarrow \frac{y^2(3)}{9}=\frac{9}{59-162(1+\ln 3)}$

Q.16. If $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$ where $x\in[-1,1]$ and sum of all solutions is $\alpha-\frac{4}{\sqrt{3}}$ then value of α is

Answer: 2

Solution: Given,

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$$ where $x\in[-1,1]$

Now taking case 1 when $x<0$, then

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$$

$$\Rightarrow 2\tan^{-1}x+\pi+2\tan^{-1}x=\frac{\pi}{3}$$

$$\Rightarrow 4\tan^{-1}x=-\pi+\frac{\pi}{3}$$

$$\Rightarrow 4\tan^{-1}x=\frac{-2\pi}{3}$$

$$\Rightarrow \tan^{-1}x=\frac{-\pi}{6}$$

$$\Rightarrow x=\frac{-1}{\sqrt{3}}$$

Now taking case 2 when $x\geq 0$,

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$$

$$\Rightarrow 2\tan^{-1}x+2\tan^{-1}x=\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{12}$$

$$\Rightarrow x=2-\sqrt{3}$$

So, sum of solution will be $\frac{-1}{\sqrt{3}}+2-\sqrt{3}=2-\frac{4}{\sqrt{3}}$

Now on comparing with $\alpha-\frac{4}{\sqrt{3}}$ we get, $\alpha=2$

Q.17. If $y=(1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)$ then the value of $y'(-1)-y''(-1)$ will be

Answer: 496

Solution: Given,

$$y=(1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)$$

$$\Rightarrow y=\frac{(1-x)(1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)}{1-x}$$

$$\Rightarrow y=\frac{\left(1-x^2\right)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)}{1-x}$$

$$\Rightarrow y=\frac{\left(1-x^4\right)\left(1+x^4\right)\left(1+x^8\right)\left(1+x^{16}\right)}{1-x}$$

$$\Rightarrow y=\frac{\left(1-x^8\right)\left(1+x^8\right)\left(1+x^{16}\right)}{1-x}$$

$$\Rightarrow y=\frac{\left(1-x^{16}\right)\left(1+x^{16}\right)}{1-x}=\frac{\left(1-x^{32}\right)}{1-x}$$

Now differentiating with respect to x we get,

$$y'(x)=\frac{(1-x)\left(-32x^{31}\right)-\left(1-x^{32}\right)(-1)}{(1-x)^2}=\frac{\left(-32x^{31}\right)+32x^{32}+\left(1-x^{32}\right)}{(1-x)^2}$$

$$y'(-1)==\frac{32+32+(1-1)}{(2)^2}=16$$

Now finding $y''(x)$ we get,

$$y''(x)=\frac{(1-x)^2\left[\left(-32\times31x^{30}\right)+32\times32x^{31}-32x^{31}\right]+(2(1-x))\left(\left(-32x^{31}\right)+32x^{32}+\left(1-x^{32}\right)\right)}{(1-x)^4}$$

$$\Rightarrow y''(-1)=\frac{(2)^2[-32\times31-32\times32+32]+(2(2))(32+32+(1-1))}{(2)^4}$$

$$\Rightarrow y''(-1)=\frac{[-32\times31-32\times32+32]+64}{4}=\frac{[-32\times62]+64}{4}=-8\times62+16=-480$$

So, $y'(-1)-y''(-1)=16-(-480)=496$

Q.18. If $\log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(3^{2\alpha-4}\cdot\frac{5}{2}+1\right)=2$ then maximum integral value of β for which the equation, $x^2-\left((\Sigma\alpha)^2x\right)+\left(\Sigma(\alpha+1)^2\right)\beta=0$ has real root is,

Answer: 6

Solution: Given,

$\log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(3^{2\alpha-4}\cdot\frac{5}{2}+1\right)=2$

Now let $3^{2\alpha-4}=t$, so the equation becomes,

$\log_2\left(t^2+13\right)-\log_2\left(\frac{5t}{2}+1\right)=2$

$\Rightarrow \log_2\frac{\left(t^2+13\right)}{\left(\frac{5t}{2}+1\right)}=2$

$\Rightarrow \frac{\left(t^2+13\right)}{\left(\frac{5t}{2}+1\right)}=2^2$

$\Rightarrow t^2+13=10t+4$

$\Rightarrow t^2-10t+9=0$

$\Rightarrow t=1 \text{ or } 9$

So, $3^{2\alpha-4}=1 \text{ or } 9$

$\Rightarrow 3^{2\alpha-4}=3^0 \text{ or } 3^2$

$\Rightarrow \alpha=2,\ 3$

Now solving $x^2-\left((\Sigma\alpha)^2x\right)+\left(\Sigma(\alpha+1)^2\right)\beta=0$

$\Rightarrow x^2-\left((2+3)^2x\right)+\left(3^2+4^2\right)\beta=0$

$\Rightarrow x^2-25x+25\beta=0$

Now for real roots $D\geq 0\Rightarrow 25^2-4\times 25\beta\geq 0$

$\Rightarrow \beta\leq\frac{25}{4}$

So maximum integral value will be $\beta=6$

Q.19. If $a,\ b\in[1,25]$ & $a,\ b\in N$ such that $a+b$ is a multiple of 5, then the number of ordered pair (a,b) is

Answer: 125

Solution: Since $a,\ b\in[1,25]$ & $a,\ b\in N$, so the numbers can be in the form of

$5m\in\{5,10,15,20,25\}$

$5m+1\in\{1,6,11,16,21\}$

$5m+2\in\{2,7,12,17,22\}$

$5m+3\in\{3,8,13,18,23\}$ and

$5m+4\in\{4,9,14,19,24\}$

So, now for $a+b$ from $5m+1$ & $5m+4$ will have total $5\times 5+5\times 5=50$ ordered pairs

Similarly $a+b$ from $5m+2$ & $5m+3$ will have total 50 ordered pairs

And $a+b$ from $5m$ type will be total $5\times 5=25$ ordered pairs.

Hence, total number of ordered pairs will be 125