JEE Main Exam 2023 - Session 1

24 Jan 2023 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. Let γ_1 be the ratio of molar heat at constant pressure and constant volume for a monatomic gas. Let γ_2 be the ratio of molar heat at constant pressure and constant volume for a diatomic gas. Find $\frac{\gamma_1}{\gamma_2}$

A) $\frac{21}{25}$ B) $\frac{7}{3}$ C) $\frac{25}{21}$ D) $\frac{3}{7}$

Answer: $\frac{25}{21}$

Solution: As we know, $C_v = f\frac{R}{2}$

and $C_p = C_v + R = f\frac{R}{2} + R$

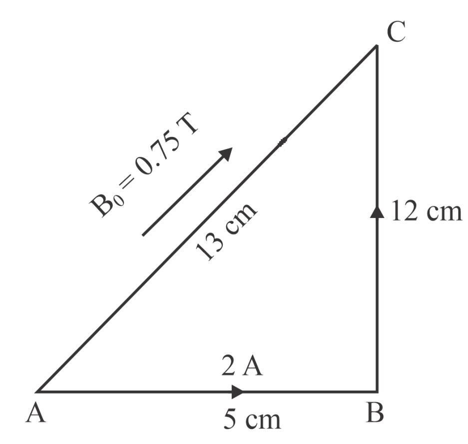
Therefore, $\gamma = \frac{Cp}{Cv} = 1 + \frac{2}{f}$

For monatomic gas, $f = 3$ and hence $\gamma_1 = \frac{5}{3}$

For diatomic gas, $f = 5$ and hence $\gamma_2 = \frac{7}{5}$

Required ratio, $\frac{\gamma_1}{\gamma_2} = \frac{25}{21}$

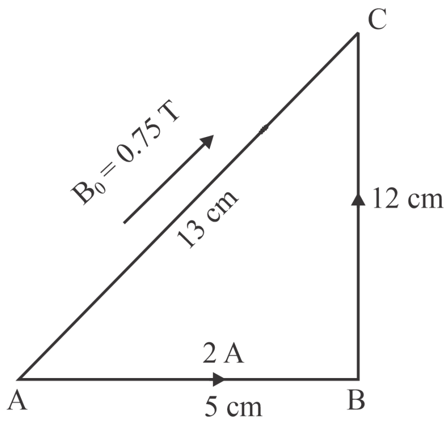
Q.2. A right angled triangle has current of $2\ \mathrm{A}$. The edge length are shown in the diagram. Magnetic field is acting in the plane of the triangle. The magnetic force acting on wire AB is



A) $\frac{5}{130}\ \mathrm{N}$ B) $\frac{15}{2}\ \mathrm{N}$ C) $\frac{3}{40}\ \mathrm{N}$ D) $\frac{9}{130}\ \mathrm{N}$

Answer: $\frac{9}{130}\ \mathrm{N}$

Solution:



$B = 0.75$ T

$F_{AB} = i \times (l_{AB}) B \sin\theta$ (angle between AC & AB)

$= 2 \times \left(\frac{5}{100}\right) \times 0.75 \times \frac{12}{13}$

$= \frac{9}{130}$ N

Q.3. Assertion (A) : Steel is used to build big structures
Reason (R) : Steel has more elastic modulus as compared to other materials.

A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true but R is false

D) Both A and R are false

Answer: Both A and R are true and R is the correct explanation of A

Solution: The relationship between Young's modulus, stress and strain is:

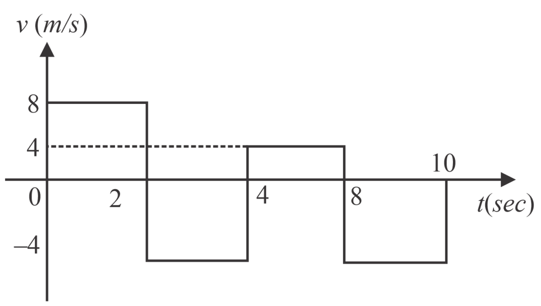
$\frac{stress}{strain} = Y$

Since the elastic modulus of steel is more, for the same amount of stress, the strain will be less resulting into less deformation of a building. Therefore, steel is used for building large structures.

Hence, both assertion and reason are correct and the reason correctly explains the assertion.

Q.4. The velocity-time graph of a body moving along straight line is given as shown.

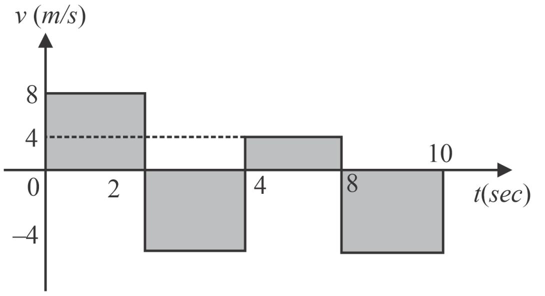
The ratio of displacement and distance is:



A) 1 : 1 B) 1 : 2 C) 1 : 3 D) 1 : 4

Answer: 1 : 3

Solution:



Area of upper rectangles, which will give us magnitude of the area under the velocity-time graph,

$A_1 = 8 \times 2 + 4 \times 4 = 32$

Area of lower rectangles, which will give us magnitude of the area under the velocity-time graph,

$A_2 = 4 \times 2 + 4 \times 2 = 16$

Therefore,

$displacement = A_1 - A_2 = 32 - 16 = 16$
$\& \; dis \tan ce = A_1 + A_2 = 32 + 16 = 48$

Hence, required ratio would be

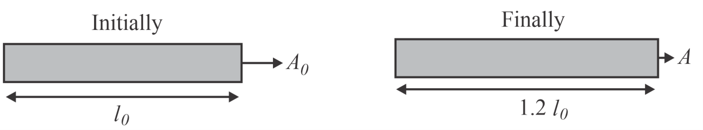
$= \frac{16}{48} = \frac{1}{3}$

Q.5. A copper wire is elongated such that its length is increased by 20%. Then the percentage increase in the resistance is

A) 20% B) 30% C) 44% D) 50%

Answer: 44%

Solution:



Before heating

$R_0 = \left(\frac{\rho l_0}{A_0}\right)$

After heating

Volume = Constant

Therefore,

$A_0 l_0 = 1.2 l_0 A$

$\Rightarrow A = \left(\frac{A_0}{1.2}\right)$

So,

$$R = \frac{\rho l}{A}$$
$$= \frac{\rho \times (1.2 l_0)}{\left(\frac{A_0}{1.2}\right)}$$
$$= \frac{\rho l_0}{A_0} \times 1.44$$
$$= 1.44\ R_0$$

Percentage increase in $R = \frac{R - R_0}{R_0} \times 100 = 44\%$

Q.6. Assertion (A) : acceleration due to gravity decreases with both height and depth from earth.
Reason (R) : if height and depth are equal for two points from surface of the earth, then acceleration due to gravity will be same at those points.

A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true and R is not the correct explanation of A

C) A is true but R is false

D) Both A and R are false

Answer: A is true but R is false

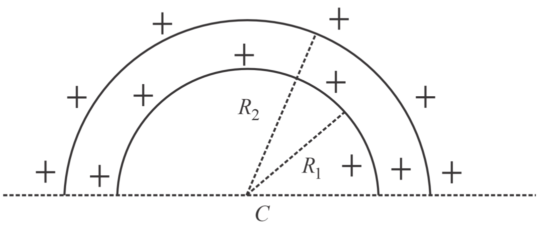
Solution: At any height, $g' = g\left(1 - \frac{2h}{R}\right)$

and at any depth,

$g' = g\left(1 - \frac{d}{R}\right)$

Clearly, g' decreases in both cases. But if $h = d$, then value of g' will not be the same.

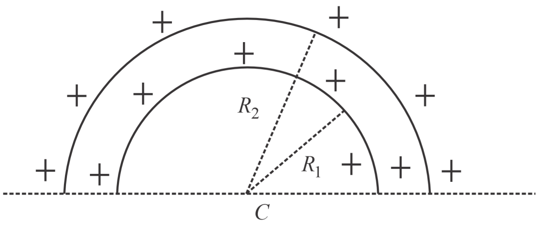
Q.7. Two concentric semi-circular rings (radii R_1 and R_2) have equal linear charge density (λ each) as shown:
Find the potential at centre C.



A) $\frac{\lambda}{\epsilon_0}$ B) $\frac{2\lambda}{\epsilon_0}$ C) $\frac{\lambda}{4\epsilon_0}$ D) $\frac{\lambda}{2\epsilon_0}$

Answer: $\frac{\lambda}{2\epsilon_0}$

Solution:



$V_C = V_1 + V_2$

$\Rightarrow V_C = \frac{1}{4\pi\epsilon_0} \times \frac{\lambda_1(\pi R_1)}{R_1} + \frac{1}{4\pi\epsilon_0} \times \frac{\lambda_1(\pi R_1)}{R_1}$

$= \frac{\lambda}{2\epsilon_0}$

Q.8. Electric field vector and magnetic field vector of an electromagnetic wave is given as $\vec{E} = E_0 \sin(\omega t - kx)\,\hat{\jmath}$ and $\vec{B} = B_0 \sin(\omega t - kx)\,\hat{k}$, then choose the correct option.

A) $\omega E_0 = kB_0$ B) $kE_0 = \omega B_0$ C) $\frac{E_0}{k} = \frac{B_0}{\omega}$ D) None of these

Answer: $kE_0 = \omega B_0$

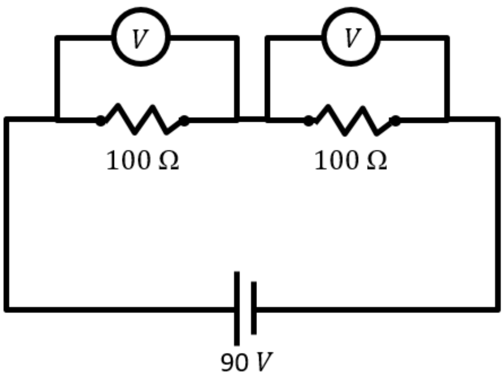
Solution: As we know,

$E_0 = cB_0 \left(\text{also } c = \frac{w}{k}\right)$

$\Rightarrow E_0 = \frac{\omega}{k} B_0$

$\Rightarrow \quad E_0 k = \omega B_0$

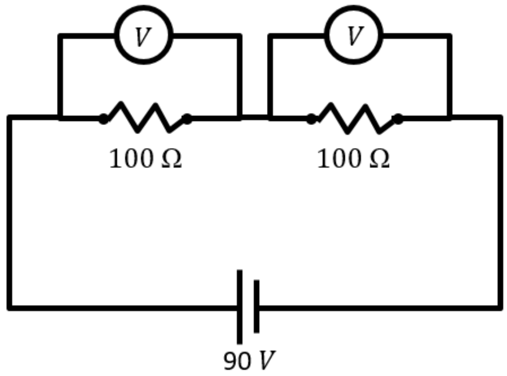
Q.9. Consider the circuit shown:
Resistance of each voltmeter is $400\,\Omega$. Find the reading of any one voltmeter.



A) 60 V B) 45 V C) 80 V D) 30 V

Answer: 45 V

Solution:



Due to symmetry, the potential drop across both the resistors will be $45\ V$. Since voltmeters is connected in parallel to the resistors, the reading of both voltmeters will be $45\ V$.

Q.10. Assertion (A) : The weight of an object at Mount Everest is lesser than its weight at sea level.
Reason (R) : The value of g decreases as height increases.

A) Assertion and Reason both are correct and Reason is the correct explanation of Assertion

B) Assertion and Reason both are correct but Reason is not the correct explanation

C) Assertion is correct reason in incorrect

D) Assertion is incorrect reason in correct

Answer: Assertion and Reason both are correct and Reason is the correct explanation of Assertion

Solution: The acceleration due to gravity decreases with height h $(h \ll R)$ above the surface as:

$$g' = g\left(1 - \frac{h}{R}\right)$$

Where g is the acceleration due to gravity at sea level. This results into decrease of weight mg' of an object at any height. Hence, both assertion and reason are correct and the reason correctly explains the assertion.

Q.11. A long solenoid has 70 turns per cm and carries current $2\ \mathrm{A}$. The magnetic field inside the solenoid is ($\mu_o = 4\pi \times 10^{-7}$ in SI units)

A) $125.2 \times 10^{-4}\ \mathrm{T}$

B) $835.2 \times 10^{-4}\ \mathrm{T}$

C) $176.0 \times 10^{-4}\ \mathrm{T}$

D) $880 \times 10^{-4}\ \mathrm{T}$

Answer: $176.0 \times 10^{-4}\ \mathrm{T}$

Solution: Magnetic field inside solenoid $= \mu_o n i$

where $n =$ Number of turns per unit length

$n = 7000$ turns/m

$$B_{\text{solenoid}} = \mu_o n i = 4\pi \times 10^{-7} \times 7000 \times 2$$

$$= 8 \times \frac{22}{7} \times 10^{-6} \times 7 \times 100$$

$$= 176 \times 10^{-4}\ \mathrm{T}$$

Q.12. When a beam of white light having parallel rays is incident on a convex lens, rays of different colours get separated. This phenomenon is called

A) Chromatic aberration

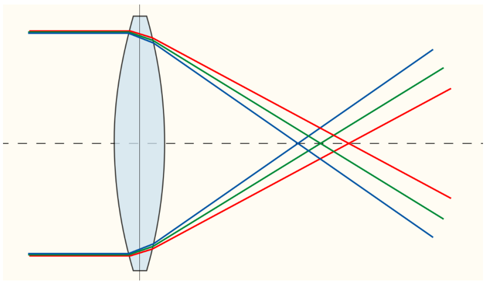
B) Polarization

C) Scattering

D) Spherical aberration

Answer: Chromatic aberration

Solution:



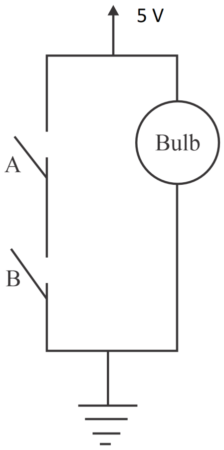
Due to difference in refractive index of a material for different wavelengths, the focal length will be different for different colours. This will cause a separation of parallel beam of white light. This phenomenon is known as chromatic aberration.

Polarisation is a property applying to transverse waves that specifies the geometrical orientation of the oscillations.

When energy waves (such as light, sound, and various electromagnetic waves) are caused to depart from a straight path due to imperfections in the medium it is called Scattering.

Spherical aberration is present when the outer parts of a lens do not bring light rays into the same focus as the central part.

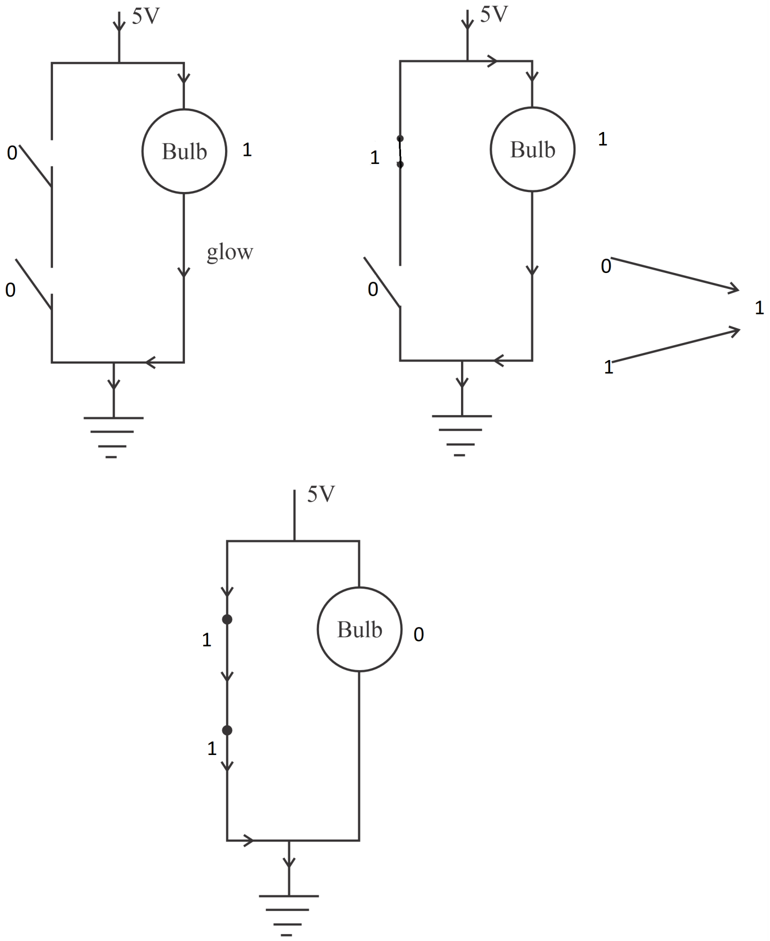
Q.13. Identify the logic gate from the given circuit



A) OR B) AND C) NAND D) XOR

Answer: NAND

Solution:



When both switches are open, the current will flow through the bulb and it will glow.

When switch A is closed and switch B is open, the current will flow through the bulb and it will glow.

When switch A is open and switch B is closed, the current will flow through bulb and it will glow.

When both the switches are closed, since the resistance of left arm of circuit is negligible, the current will flow through the left arm and hence the bulb will not glow.

The truth table is given as:

A	B	Bulb
0	0	1
1	0	1
0	1	1
1	1	0

Which shows that the given circuit is a NAND gate.

Section B: Chemistry

Q.1.

What is the hybridisation state and magnetic behaviour of metal atom ion in $\left[Co\left(NH_3\right)_6\right]^{3+}$

A) d^2sp^3 and diamagnetic

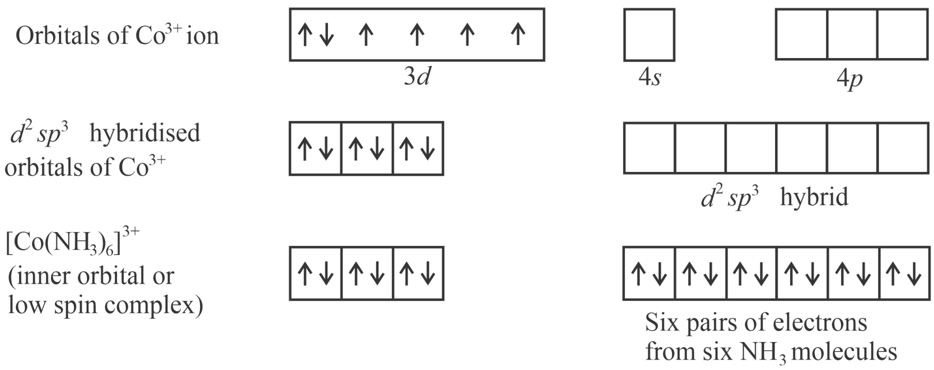
B) sp^3d^2 and diamagnetic

C) d^2sp^3 and paramagnetic

D) sp^3d^2 and paramagnetic

Answer: d^2sp^3 and diamagnetic

Solution: In $\left[Co\left(NH_3\right)_6\right]^{3+}$, Co is in $+3$ oxidation state with the configuration $3d^6$. In the presence of NH_3 a strong ligand, the 3d electrons pair up leaving two d-orbitals empty. Hence, the hybridisation is d^2sp^3 forming an inner orbital diamagnetic octahedral complex.



Q.2. In which of the following metal extraction, oxidation and reduction process both are involved?

A) Au B) Cu C) Fe D) Al

Answer: Au

Solution: $4M\,(s) + 8\,CN\,(aq) + 2H_2O\,(aq) + O_2\,(g) \rightarrow 4\left[M(CN)_2\right]^{-}(aq) +$
$4\,OH^{-}(aq)\left(M = Ag \text{ or } Au\right)$

In the above process metal undergo oxidation.

$2\left[M(CN)_2\right](aq) + Zn\,(s) \rightarrow \left[Zn(CN)_4\right]^{2-}(aq) + 2M\,(s)$

In the above process metal ion undergo reduction.

The overall process is known Mac Arthur Forrest cyanide process.

Q.3. α-particle, proton and electron have same kinetic energy. Select correct order of de-Broglie wavelength.

A) $\lambda_p = \lambda_\alpha = \lambda_e$ B) $\lambda_e > \lambda_p > \lambda_\alpha$ C) $\lambda_\alpha > \lambda_e > \lambda_p$ D) $\lambda_p > \lambda_e > \lambda_\alpha$

Answer: $\lambda_e > \lambda_p > \lambda_\alpha$

Solution: $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m\,KE}}$

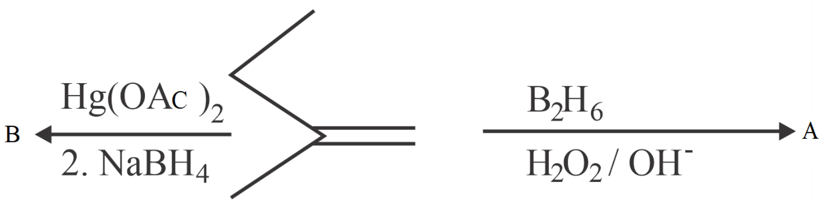
mass of $e^{-} = 9.1 \times 10^{-31}\,Kg$

mass of proton $= 1.67 \times 10^{-27}\,Kg$

mass of $\alpha-$particle $= 6.68 \times 10^{-27}\,Kg$

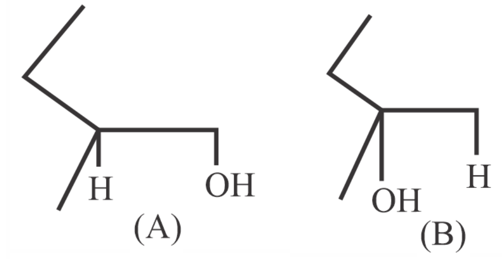
$\lambda_e > \lambda_p > \lambda_\alpha$

Q.4.

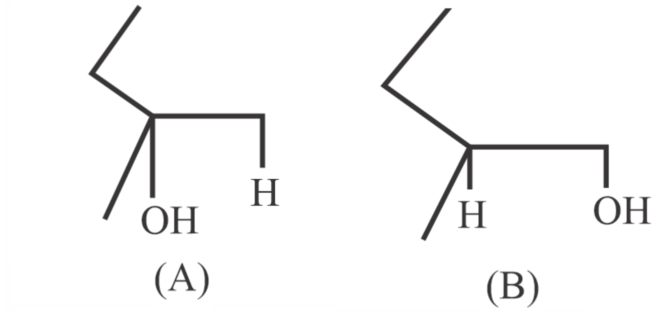


A and B are:

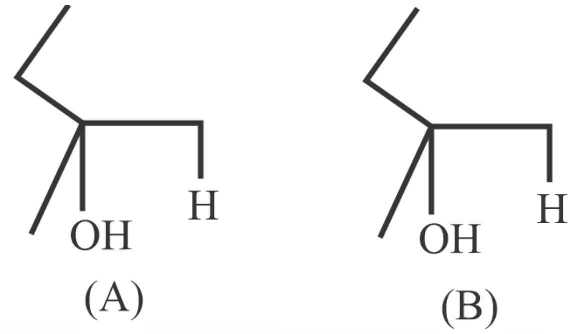
A)



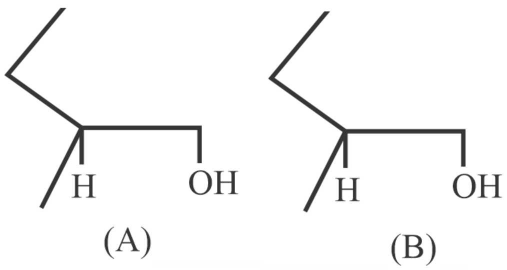
B)



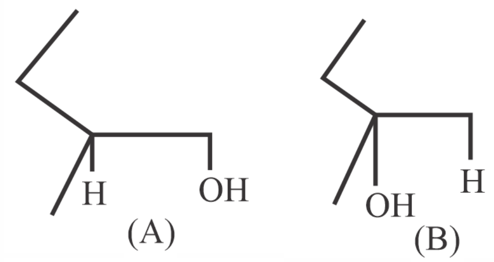
C)



D)

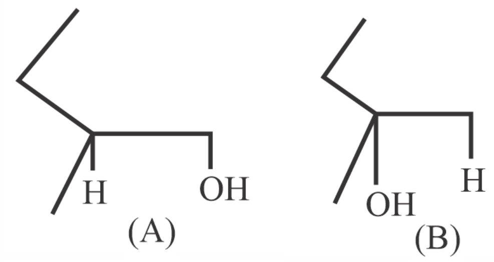


Answer:



Solution: The product A is formed by hydroboration oxidation reaction. Hence, the product formed according Anti Markonikov's rule.

The product B is formed by Mercuration and demercuration reaction. In this reaction product formed according to markonikov's rule without rearrangement.



Q.5. Which of the following is best of oxidising agent

A) Yu^{+2} B) Ce^{+2} C) Ce^{+4} D) Sm^{+2}

Answer: Ce^{+4}

Solution: The common oxidation state of lanthanides is +3. Hence, In their +2 oxidation states they acts as reducing agent and in their +4 oxidation states they acts as oxidising agents.

Q.6. Physisorption is

A) Highly specific in nature B) Having zero activation energy

C) Always monolayer D) High enthalpy of adsorption

Answer: Having zero activation energy

Solution: Physisorption is due to the formation of van der Waals forces. It is reversible in nature.Physisorption is not specific in nature. It has low adsorption enthalpy nearly 20 to 40 kJ/mol. It results in a multimolecular layer. Activation energy is less or zero in physisorption.

Q.7. The oxidation state of the most electronegative element in the products of the reaction:

$\mathrm{BaO_2}$+(diil)$\mathrm{H_2SO_4}$

A) -1 and -2 B) 0 and -2 C) -1 and -3 D) -3 and -2

Answer: -1 and -2

Solution: $\mathrm{BaO_2 + H_2SO_4 \rightarrow H_2O_2 + BaSO_4}$

So, the products are $\mathrm{H_2O_2}$ and $\mathrm{BaSO_4}$. The most Recriminative element in these two products is Oxygen (in both cases).

Thus, the oxidation states are -1in $\mathrm{H_2O_2}$ and -2 in $\mathrm{BaSO_4}$ respectively in each product.

Q.8. Which statement is correct?

A) Humans require more food than air B) Humans require more air than food

C) Humans need air 100 times more than food

D) Humans need air 15 times more than food

Answer: Humans need air 15 times more than food

Solution: Both air and food are essential for survival of humans. Humans need air 15 times more than food

Q.9. The number of peptide bond present in tripeptide VAL-PRO-GLY is

A) 1

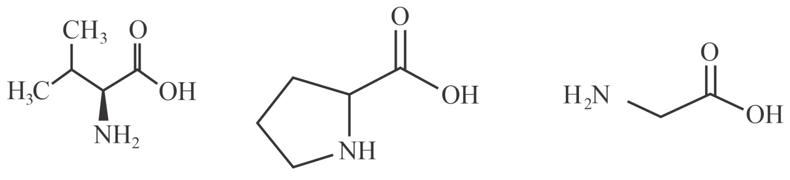
B) 2

C) 3

D) 4

Answer: 2

Solution: A peptide bond, also referred to as an amide bond, is formed between the α-nitrogen atom of one amino acid and the carbonyl carbon of a second.



Valine Proline Glycine

Q.10. Find out order of reaction of decomposition of AB3 using given information

Initial pressure of $AB_3(g)\,(mmHg)$	50	100	200	400
$t_{1/2}(sec)$	4	2	1	0.5

A) 0

B) 1

C) 2

D) 3

Answer: 2

Solution: $t_{1/2} \propto (P_o)^{1-n}$

$$\frac{\left(t_{1/2}\right)_1}{\left(t_{1/2}\right)_2} \propto \left[\frac{(Po)_1}{(Po)_2}\right]^{1-n}$$

$$\left(\frac{1}{2}\right) \propto (2)^{1-n}$$

$$(2)^{-1} \propto (2)^{1-n}$$

$$1-n=\frac{1}{2} \Rightarrow n=2$$

Q.11. Which of the following is the correct decreasing order of magnitude of standard reduction potential of Rb, Na and Li,in aqueous medium?

A) $Rb > Na > Li$

B) $Li > Rb > Na$

C) $Na > Rb > Li$

D) $Li > Na > Rb$

Answer: $Li > Rb > Na$

Solution: Electrode potential of given elements are:

$Li = -3.04$

$Na = -2.714$

$Rb = -2.930$

But, since only magnitude has to be considered and not their reactive tendencies, so the order is

$Li > Rb > Na$

Q.12. How many s-electrons are there in a Br-atom (Atomic No : of $Br = 35$)

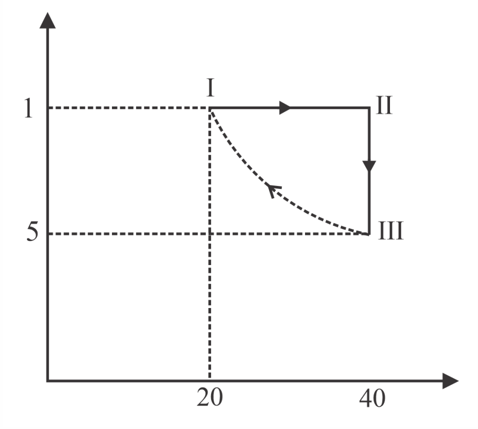
Answer: 8

Solution: The Electronic configuration of Br-atom is

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^5$

The number of s- electrons are eight.

Q.13.

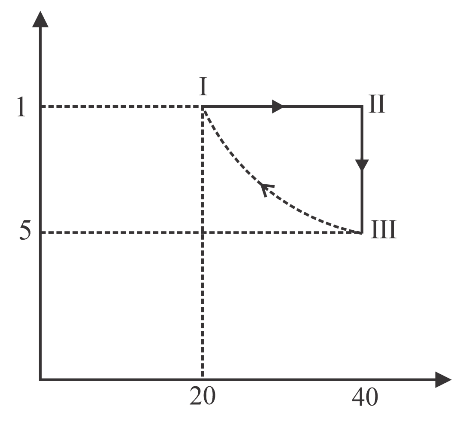


I $\longrightarrow$ II Isobaric
II $\rightarrow$ III Isochoric
III $\longrightarrow$ I Isothermal

All process are reversible. Find out work done by gas for complete cyclic process (in atm.lit)

Answer: 6

Solution:



$W_{I \to II} = -1 \times (40 - 20) = -20$ atm. lit

$W_{II \to III} = 0$

$W_{III \to I} = 2.303(1 \times 20) \times \log 2$

$= +13.818$

$W_{I \to II \to II \to IV} = -20 + 13.818$

$= -6.182$ atm. lit

Work done by the gas $= +6.182$ atm. lit

Q.14. Find the sum of number of unpaired electrons in the following diatomic molecules:
N_2, N_2^+, O_2, O_2^+?

Answer: 4

Solution: No. of unpaired electrons in $N_2 = 0$

No. of unpaired electrons in $N_2^+ = 1$

No. of unpaired electrons in $O_2 = 2$

No. of unpaired electrons in $O_2^+ = 1$

Sum $= 0 + 1 + 2 + 1 = 4$

Q.15. pK_a of lactic acid is 4. Find the pH of $0.005M$ calcium lactate at $27^{\circ}C$ is :

Answer: 8

Solution: The salt undergo anionic hydrolysis.

$$pH = \frac{1}{2}\left[pK_w + pK_a + \log_{10} C\right]$$

$$pH = \frac{1}{2}\left[k_w + 4 + \log_{10}(0.005)\right]$$

$$pH = 7 + \frac{1}{2}[4 - 3 + 0.7]$$

$$pH \simeq 8$$

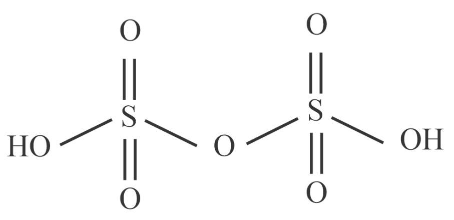
Q.16. Find the sum of number of π bonds in peroxdisulphuric acid and pyrosulphuric acid?

Answer: 8

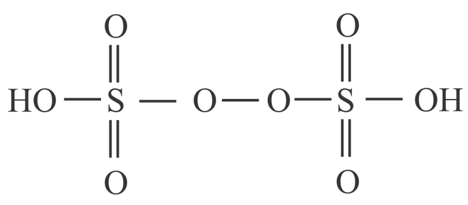
Solution: Peroxydisulphuric acid $(H_2\,S_2O_8)$ has 4π bonds

Pyrodisulphuric $(H_2\,S_2O_7)$ acid has4π bonds

Total number of π bonds $= 8$



$H_2S_2O_7$ 4π



4π

$H_2S_2O_8$

Q.17. How many of the following concentration terms are temperature independent? Mole fraction, mass percent (96w/w), Molarity (M), Molality (m), ppm, volume percent (%V/V)

Answer: 4

Solution: There is no volume term involved in the temperature independent concentration terms.

Temperature independent concentration terms are :

Mole fraction,

Molality (m),

Parts Per Million (ppm),

Mass percentage (%w/w)

Q.18. Total no. of s-electron in an unipositive ion containing 55 protons

Answer: 10

Solution: The atom with 55 protons is Caesium. The electronic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^1$

The electronic configuration of unipositive ion is $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^0$

Hence, the number of s-electrons are 10.

Q.19. One atom of
X has
$25\ \mathrm{MeV}$ energy. The energy in 102 g of
X is
$P \times 10^{25}\ \mathrm{MeV}$. Then find the value of
P?

Given :
X has molar mass :
$61\ g$ and
$N_A = 6 \times 10^{23}$

Answer: 3

Solution: Total energy $= 25 \times 6 \times 10^{23} = 3 \times 10^{25} MeV$

$n_X = \frac{102}{61}$

$N_A = 6 \times 10^{23}$

$n \times N_A =$No. of atoms

Total energy $= 25 \times \frac{102}{61} \times 6 \times 10^{23} = 3 \times 10^{25}$

Section C: Mathematics

Q.1. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx =$

A) π

B) 2π

C) 3π

D) 4π

Answer: 2π

Solution: Let

$$I=\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}\left(\frac{48}{\sqrt{9-4x^2}}\right)dx$$

$$\Rightarrow I=\frac{48}{2}\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}\left(\frac{1}{\sqrt{\frac{9}{4}-x^2}}\right)dx$$

$$\Rightarrow I=24\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}\left(\frac{1}{\sqrt{\left(\frac{3}{2}\right)^2-x^2}}\right)dx$$

$$\Rightarrow I=24\left[\sin^{-1}\left(\frac{2x}{3}\right)\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$$

$$\Rightarrow I=24\left[\sin^{-1}\left(\frac{2}{3}\times\frac{3\sqrt{3}}{4}\right)-\sin^{-1}\left(\frac{2}{3}\times\frac{3\sqrt{2}}{4}\right)\right]$$

$$\Rightarrow I=24\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$\Rightarrow I=24\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$\Rightarrow I=2\pi$$

Q.2.

Find the value of $\left(\frac{1+\cos\frac{2\pi}{9}+i\sin\frac{2\pi}{9}}{1+\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}}\right)^3$

A) $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$ B) $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ C) $\frac{1}{2}-i\frac{\sqrt{3}}{2}$ D) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$

Answer: $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$

Solution: Given,

$$\left(\frac{1+\cos\frac{2\pi}{9}+i\sin\frac{2\pi}{9}}{1+\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}}\right)^3$$

$$=\left(\frac{2\cos^2\frac{\pi}{9}+i2\sin\frac{\pi}{9}\cos\frac{\pi}{9}}{2\cos^2\frac{\pi}{9}-i2\sin\frac{\pi}{9}\cos\frac{\pi}{9}}\right)^3$$

$$=\left(\frac{2\cos\frac{\pi}{9}\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)}{2\cos\frac{\pi}{9}\left(\cos\frac{\pi}{9}-i\sin\frac{\pi}{9}\right)}\right)^3$$

$$=\left(\frac{\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)}{\left(\cos\frac{\pi}{9}-i\sin\frac{\pi}{9}\right)}\right)^3$$

$$=\left(\frac{e^{i\frac{\pi}{9}}}{e^{-i\frac{\pi}{9}}}\right)^3 \left\{\text{as } e^{i\theta}=\cos\theta+i\sin\theta\right\}$$

$$=\left(e^{i\frac{2\pi}{9}}\right)^3$$

$$=e^{i\frac{2\pi}{3}}$$

$$=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$$

$$=\frac{-1}{2}+i\frac{\sqrt{3}}{2}$$

Q.3. If $\vec{a}=\hat{i}+2\hat{j}+m\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+m\hat{k}$. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $m=$

A) $\pm\sqrt{2}$ B) $\pm\sqrt{3}$ C) ±2 D) $\pm\sqrt{5}$

Answer: $\pm\sqrt{3}$

Solution: Given:

$\vec{a}=\hat{i}+2\hat{j}+m\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+m\hat{k}$

So,

$\vec{a}\cdot\vec{b}=1-4+m^2$

$\Rightarrow \vec{a}\cdot\vec{b}=m^2-3$

Since, $\vec{a}$ and $\vec{b}$ are perpendicular, therefore

$\vec{a}\cdot\vec{b}=0$

$\Rightarrow m^2=3$

$\Rightarrow m=\pm\sqrt{3}$

Q.4. The sum of the coefficient of first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n$ is 376. The coefficient of x^4 is equal to:

A) 695 B) 410 C) 405 D) 395

Answer: 405

Solution: We know that general term in the expansion of $\left(x-\frac{3}{x^2}\right)^n$ is given by,

$T_{r+1}={}^nC_r x^{n-r}\left(\frac{-3}{x^2}\right)^r$

$=(-1)^r\times{}^nC_r 3^r\, x^{n-r-2r}$

$T_{r+1}=(-1)^r\times{}^nC_r 3^r\, x^{n-3r}$..............(1)

So, $T_1=T_{0+1}={}^nC_0 3^0\, x^n=x^n$

$T_2=(-1)\times{}^nC_1 3^1\, x^{n-3}$

$T_3={}^nC_2 3^2\, x^{n-6}$

Now given sum $1-{}^nC_1 3+{}^nC_1 3^2=376$

$\Rightarrow 1-3n+\frac{n(n-1)}{2}\cdot 9=376$

$\Rightarrow 1-3n+\frac{n^2-n}{2}\cdot 9=376$

$\Rightarrow 2-6n+9n^2-9n=752$

$\Rightarrow 9n^2-15n-750=0$

$\Rightarrow 3n^2-5n-250=0$

$\Rightarrow n=\frac{5\pm\sqrt{25+3000}}{6}$

$\Rightarrow n=\frac{5\pm 55}{6}$

$\Rightarrow n=10\ \{\text{ignoring negative sign}\}$

Now $T_{r+1}=(-1)^r\,{}^{10}C_r\,3^r\,x^{10-3r}$

So for coefficient of x^4 we take $10-3r=4$

$\Rightarrow 3r=6$

$\Rightarrow r=2$

So, coefficient of x^4 is given by,

$T_{2+1}=(-1)^2\cdot{}^{10}C_2\,3^2=45\times 9=405.$

Q.5. Let $A=\{a,b,c,d\}$ and a relation $A\to A$ be $R=\{(a,b),(b,d),(b,c),(b,a)\}$ then minimum number of elements required to make R equivalent is

A) 7 B) 10 C) 12 D) 14

Answer: 12

Solution: Given that,

$A=\{a,\ b,\ c,d\},\ R\ :\ A\to A$

Given by $R=\{(a,b),(b,d),(b,c),(b,a)\}$

Let the new set of relation is R' :

$R'=\{(a,a),(b,b),(c,c),(d,d),(c,b),(d,b),(a,c),(c,a),(a,d),(d,a),(c,d),(d,c)\}$

R' is the set containing minimum number of elements which on adding in R gives an equivalence relation.

Thus, $R'\cup R=\{(a,a),(b,b),(c,c),(d,d),(a,b),(b,a),(b,c),(c,b),(b,d),(d,b),(a,c),(c,a),(a,d),(d,a),(c,d),(d,c)\}$

Which is a equivalence relation,

Hence, from $R'=\{(a,a),(b,b),(c,c),(d,d),(c,b),(d,b),(a,c),(c,a),(a,d),(d,a),(c,d),(d,c)\}$

we can say that minimum 12 elements are required to make a equivalence relation.

Q.6. Three urns $A,\ B$ and C contains 4 red, 6 black; 5 red, 5 black and λ red, 4 black balls respectively. A ball is drawn and found to be red. If probability that ball was drawn from urn C is 0.4, then the square of side of equilateral triangle in parabola $y^2=\lambda x$ with one vertex at vertex of parabola is

A) 144 B) 432 C) 368 D) 284

Answer: 432

Solution: Let R be the event of drawing red balls.

$P\left(\frac{R}{A}\right)=\frac{4}{10}$

$P\left(\frac{R}{B}\right)=\frac{5}{10}$

$P\left(\frac{R}{C}\right)=\frac{\lambda}{4+\lambda}$

Now,

$P\left(\frac{C}{R}\right)=0.4$

$\Rightarrow \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right)+P(B)P\left(\frac{R}{B}\right)+P(C)P\left(\frac{R}{C}\right)}=\frac{4}{10}$

$\Rightarrow \frac{\frac{1}{3}\times\left(\frac{\lambda}{\lambda+4}\right)}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\times\left(\frac{\lambda}{\lambda+4}\right)}=\frac{4}{10}$

$\Rightarrow \frac{\left(\frac{\lambda}{\lambda+4}\right)}{\frac{9}{10}+\left(\frac{\lambda}{\lambda+4}\right)}=\frac{4}{10}$

$\Rightarrow \frac{\lambda}{\lambda+4}=\frac{4}{10}\left[\frac{9}{10}+\left(\frac{\lambda}{\lambda+4}\right)\right]$

$\Rightarrow \frac{5\lambda}{\lambda+4}=2\left[\frac{9}{10}+\left(\frac{\lambda}{\lambda+4}\right)\right]$

$\Rightarrow \frac{5\lambda}{\lambda+4}=\frac{19\lambda+36}{5(\lambda+4)}$

$\Rightarrow 25\lambda=19\lambda+36$

$\Rightarrow 6\lambda=36$

$\Rightarrow \lambda=6$

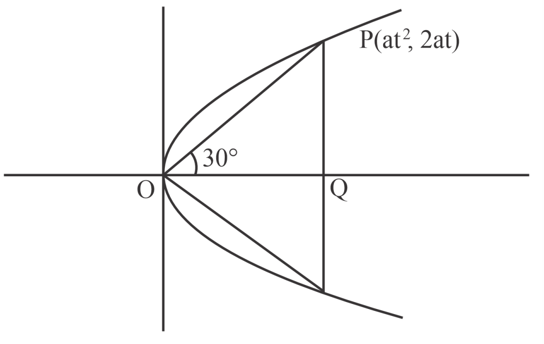
So, parabola is

$y^2=6x=4\times\frac{2}{3}\times x$

Comparing with $y^2=4ax$, we get

$a=\frac{2}{3}$

Hence, we have



Let $P \equiv \left(\frac{2}{3}t^2, \frac{4}{3}t\right)$

In ΔOPQ, we have

$\tan\left(30^\circ\right) = \frac{2at}{at^2} = \frac{2}{t}$

$\Rightarrow \frac{2}{t} = \frac{1}{\sqrt{3}}$

$\Rightarrow t = 2\sqrt{3}$

Side length of triangle$= 4at = 12\sqrt{3}$

Square of side of triangle

$= \left(12\sqrt{3}\right)^2 = 432$ units

Q.7. If the shortest distance between the lines $\frac{x-\sqrt{6}}{1} = \frac{y+\sqrt{6}}{2} = \frac{z-\sqrt{6}}{3}$ and $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z-3\sqrt{6}}{5}$ is 6, then sum of squares of all possible values(s) of λ is

A) 1024 B) 732 C) 416 D) 312

Answer: 732

Solution: Given equations are

$\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z-3\sqrt{6}}{5}$ is passing through a point $\left(\lambda, 2\sqrt{6}, 3\sqrt{6}\right)$ and it's direction ratios are $3, 4, 5$.

So,

$\vec{a}_1=\lambda\hat{i}+2\sqrt{6}\hat{j}+3\sqrt{6}\hat{k}$

$\vec{b}_1=3\hat{i}+4\hat{j}+5\hat{k}$

And,

$\frac{x-\sqrt{6}}{1}=\frac{y+\sqrt{6}}{2}=\frac{z-\sqrt{6}}{3}$ is passing through a point $\left(\sqrt{6}, -\sqrt{6}, \sqrt{6}\right)$ and it's direction ratios are $1, 2, 3$.

$\vec{a}_2=\sqrt{6}\hat{i}-\sqrt{6}\hat{j}+\sqrt{6}\hat{k}$

$\vec{b}_2=\hat{i}+2\hat{j}+3\hat{k}$

Now,

$\vec{a}_2-\vec{a}_1=\left(\sqrt{6}\hat{i}-\sqrt{6}\hat{j}+\sqrt{6}\hat{k}\right)-\left(\lambda\hat{i}+2\sqrt{6}\hat{j}+3\sqrt{6}\hat{k}\right)=\left(\sqrt{6}-\lambda\right)\hat{i}-3\sqrt{6}\hat{j}-2\sqrt{6}\hat{k}$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 4 & 5\\ 1 & 2 & 3\end{vmatrix}$

$\Rightarrow \vec{b}_1\times\vec{b}_2=2\hat{i}-4\hat{j}+2\hat{k}$

So,

$\Rightarrow \left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{4+16+4}=\sqrt{24}$

We know that the shortest distance between two skew lines is $\frac{\left|\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}=6$

$\Rightarrow \left|\frac{\left(\left(\sqrt{6}-\lambda\right)\hat{i}-3\sqrt{6}\hat{j}-2\sqrt{6}\hat{k}\right)\cdot\left(2\hat{i}-4\hat{j}+2\hat{k}\right)}{\sqrt{24}}\right|=6$

$\Rightarrow \left|\frac{5\sqrt{6}-\lambda}{\sqrt{6}}\right|=6$

$\Rightarrow \frac{5\sqrt{6}-\lambda}{\sqrt{6}}=\pm 6$

$\Rightarrow 5\sqrt{6}-\lambda=\pm 6\sqrt{6}$

$\Rightarrow \lambda=\ -\sqrt{6},\ 11\sqrt{6}$

So, required sum is

$=-\left(\sqrt{6}\right)^2+\left(11\sqrt{6}\right)^2=732$

Q.8. Total number of numbers formed using the digits $3, 5, 6, 7, 8$ (without repetition) which are greater than 7000 will be

A) 148 B) 168 C) 144 D) 124

Answer: 168

Solution: Case 1

forming 4 digit number,

When 7 is at 1000's place

$7\_\_\_,$

So total number of ways will be $4 \times 3 \times 2 = 24$ ways,

Case 2

forming 4 digit number when 8 is at 1000's place

$8\_\_\_,$

So total number of ways will be $4 \times 3 \times 2 = 24$ ways,

Case 3

forming 5 digit number,

So total number of ways will be $5! = 120$ ways,

Now adding all the cases we get, $120 + 24 + 24 = 168$ ways.

Q.9. If a 5×5 matrix whose each entry is either 0 or 1, is such that sum of entries of each column as well as each row is 1, then number of such matrices will be

A) 30 B) 60 C) 90 D) 120

Answer: 120

Solution: Given,

A 5×5 matrix whose each entry is either 0 or 1, is such that sum of entries of each column as well as each row is 1,

Taking a possible case we get,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now arranging the above case we get,

$$\left.\begin{matrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} & \begin{matrix} \rightarrow 5 \text{ ways} \\ \rightarrow 4 \text{ ways} \\ \rightarrow 3 \text{ ways} \\ \rightarrow 2 \text{ ways} \\ \rightarrow 1 \text{ way} \end{matrix}\end{matrix}\right\}$$ because if we fix 1 at first row and first column then in second row first column 1 cannot come and vice-versa.

So total number of such matrices will be $5 \times 4 \times 3 \times 2 \times 1 = 120$

Q.10. The proposition $\sim(p \wedge (p \rightarrow \sim q))$ is equivalent to

A) $p \wedge (p \vee q)$ B) $\sim p \vee q$ C) $p \vee q$ D) $\sim p \wedge q$

Answer: $\sim p \vee q$

Solution: We have,

$\sim(p \wedge (p \rightarrow \sim q))$

p	q	$\sim p$	$\sim q$	$p \rightarrow \sim q$	$p \wedge (p \rightarrow \sim q)$	$\sim(p \wedge (p \rightarrow \sim q))$	$p \wedge (p \vee q)$	$\sim p \vee q$	$p \vee q$	$\sim p \wedge q$
T	T	F	F	F	F	T	T	T	T	F
T	F	F	T	T	T	F	T	F	T	F
F	T	T	F	T	F	T	F	T	T	T
F	F	T	T	T	F	T	F	T	F	F

So, $\sim(p \wedge (p \rightarrow \sim q)) \equiv (\sim p \vee q)$

Q.11. For a 3×3 matrix, if $|adj\,(adj\,(adj A))| = 12^4$, then $\left|A^{-1}(adj A)\right| =$

A) 1 B) $2\sqrt{3}$ C) 6 D) 12

Answer: $2\sqrt{3}$

Solution: Given $|adj\,(adj\,(adj A))| = 12^4$

$\Rightarrow |A|^{(3-1)^3} = 12^4$

$|A|^8 = 12^4$

$|A| = \pm 2\sqrt{3}$

Now $\left|A^{-1}(adj A)\right| = \left|A^{-1}\right| |adj A|$

$= \frac{1}{|A|}|A|^2 = |A| = \pm 2\sqrt{3}$

Q.12. If $\lim_{x \to \alpha} |[x-5] - [2x+2]| = 0$, then (where, $[\cdot]$ is GIF)

A) $\alpha \in (-7.5, -6.5)$ B) $\alpha \in [-7.5, -6.5)$ C) $\alpha \in (-7.5, -6.5]$ D) $\alpha \in [-7.5, -6.5]$

Answer: $\alpha \in [-7.5, -6.5)$

Solution: We have,

$\lim_{x \to \alpha} |[x-5] - [2x+2]| = 0$

$\Rightarrow \lim_{x \to \alpha} |[x] - 5 - [2x] - 2| = 0$

$\Rightarrow \lim_{x \to \alpha} |[x] - [2x] - 7| = 0$

If $\alpha = -7.5$, then

$\lim_{x \to -7.5} |[x] - [2x] - 7| = |[-7.5] - [-15] - 7| = |-8 + 15 - 7| = 0$

So, $\alpha = -7.5$ is included.

If $\alpha = -6.5$, then

$\lim_{x \to -6.5} |[x] - [2x] - 7| = |[-6.5] - [-13] - 7| = |-7 + 13 - 7| = 1 \neq 0$

So, $\alpha = -6.5$ is excluded.

Hence, $\alpha \in [-7.5, -6.5)$

Q.13. Let $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$, then

A) $f(0) = f(1) + f(2) + f(3)$ B) $f(3) + 2f(0) = f(2) + f(1)$ C) $2f(0) = f(1) - f(2)$ D) $f(3) - f(1) = 2f(2)$

Answer: $f(0) = f(1) + f(2) + f(3)$

Solution: Let

$f(x) = x^3 - ax^2 + bx - c$

$f'(x) = 3x^2 - 2ax + b$

$f''(x) = 6x - 2a$

$f'''(x) = 6$

Also,

$f'(1) = a$

$\Rightarrow 3 - 2a + b = a$

$\Rightarrow 3a = b + 3$

Also,

$f''(2) = 12 - 2a = b$

$\Rightarrow 12 - 2a = 3a - 3$

$\Rightarrow 5a = 15 \Rightarrow a = 3$

$\Rightarrow b = 6$

And,

$f'''(3) = c \Rightarrow c = 6$

Hence,

$f(x) = x^3 - 3x^2 + 6x - 6$

Now,

$f(3) + 2f(0) = 27 - 27 + 18 - 6 - 12 = 0$

$f(2) + f(1) = 8 - 12 + 12 - 6 + 1 - 3 + 6 - 6 = 0$

So,

$f(3) + 2f(0) = f(2) + f(1)$

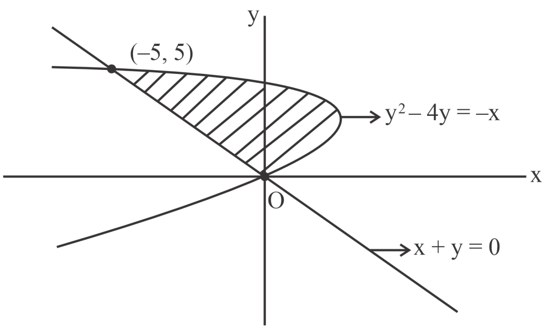
Q.14. If area bounded by $y^2 - 4y = -x$ and $x + y = 0$ is A, then $6A =$

Answer: 125

Solution: We have,

$y^2 - 4y = -x$

$x + y = 0$



Required area

$= \int_0^5 \left(\left(4y - y^2\right) + y\right) dy$

$= \int_0^5 \left(5y - y^2\right) dy$

$= \left[\frac{5y^2}{2} - \frac{y^3}{3}\right]_0^5$

$= \left[\frac{125}{2} - \frac{125}{3}\right]$

$= \frac{125}{6}$ sq. units

So,

$A = \frac{125}{6} \Rightarrow 6A = 125$

Q.15. Let $a_1, a_2, \ldots, a_6$ be in $\mathrm{A.P.}$ such that $a_1 + a_3 = 10$ and mean of $a_1, a_2, \ldots, a_6$. Then $8\sigma^2$ is equal to, (where σ^2 is variance)

Answer: 210

Solution: Let $a_1=a$ and common difference be d.

Now,

$a_1+a_3=10$

$\Rightarrow 2a+2d=10$

$\Rightarrow a+d=5 \quad \ldots(\mathrm{i})$

Now,

Mean$=\frac{19}{2}$

$\Rightarrow \frac{a+(a+d)+(a+2d)+\ldots+(a+5d)}{6}=\frac{19}{2}$

$\Rightarrow \frac{6a+15d}{6}=\frac{19}{2}$

$\Rightarrow 2a+5d=19 \quad \ldots(\mathrm{ii})$

Solving (i) & (ii), we get

$a=2,\ d=3$

So, AP is $2,\ 5,\ 8,\ 11,\ 14,\ 17.$

$\sigma^2=\frac{4+25+64+121+196+289}{6}-\left(\frac{19}{2}\right)^2$

$\Rightarrow \sigma^2=\frac{699}{6}-\frac{361}{4}$

$\Rightarrow \sigma^2=\frac{105}{4}$

$\Rightarrow 8\sigma^2=210$

Q.16. If $f(x)=\frac{2^{2x}}{2^{2x}+2}$ then find the value of $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)\ldots\ldots\ldots f\left(\frac{2022}{2023}\right)$

Answer: 1011

Solution: Given,

$f(x)=\frac{2^{2x}}{2^{2x}+2}$,

so $f(1-x)=\frac{2^{2-2x}}{2^{2-2x}+2}=\frac{2^2}{2^2+2^{2x+1}}=\frac{2}{2+2^{2x}}$

Now $f(x)+f(1-x)=\frac{2^{2x}}{2^{2x}+2}+\frac{2}{2+2^{2x}}=1$

So by using the relation $f(x)+f(1-x)=1$ we will solve the given equation

$f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)\ldots\ldots\ldots f\left(\frac{2022}{2023}\right)$

$=f\left(\frac{1}{2023}\right)+f\left(\frac{2022}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{2021}{2023}\right)+f\left(\frac{3}{2023}\right)+f\left(\frac{2020}{2023}\right)\ldots\ldots\ldots f\left(\frac{1011}{2023}\right)+f\left(\frac{1012}{2023}\right)$

$=\underbrace{f\left(\frac{1}{2023}\right)+f\left(\frac{2022}{2023}\right)}_{1}+\underbrace{f\left(\frac{2}{2023}\right)+f\left(\frac{2021}{2023}\right)}_{1}+\underbrace{f\left(\frac{3}{2023}\right)+f\left(\frac{2020}{2023}\right)}_{1}\ldots\ldots\ldots+\underbrace{f\left(\frac{1011}{2023}\right)+f\left(\frac{1012}{2023}\right)}_{1}$

$=1+1+1\ldots\ldots\ldots 1011\text{ times}=1011$

Q.17. If $\frac{1^3+2^3+3^3\ldots\ldots n^3}{1\cdot3+2\cdot5+3\cdot7+\ldots.n(2n+1)}=\frac{9}{5}$ then the value of n is

Answer: 5

Solution: Given,

$$\frac{1^3+2^3+3^3.......n^3}{1\cdot3+2\cdot5+3\cdot7+....n(2n+1)}=\frac{9}{5}$$

Now we know that $1^3+2^3+3^3.......n^3=\left[\frac{n(n+1)}{2}\right]^2$

And $\Sigma n(2n+1)=2\Sigma n^2+\Sigma n$

$\Rightarrow \Sigma n(2n+1)=2\times\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

$\Rightarrow \Sigma n(2n+1)=\frac{n(n+1)(2n+1)}{3}+\frac{n(n+1)}{2}$

So putting the value in the given equation we get,

$$\frac{1^3+2^3+3^3.......n^3}{1\cdot3+2\cdot5+3\cdot7+....n(2n+1)}=\frac{9}{5}$$

$$\Rightarrow \frac{\left[\frac{n(n+1)}{2}\right]^2}{\frac{n(n+1)(2n+1)}{3}+\frac{n(n+1)}{2}}=\frac{9}{5}$$

$$\Rightarrow \frac{\frac{1}{4}[n(n+1)]}{\frac{(2n+1)}{3}+\frac{1}{2}}=\frac{9}{5}$$

$$\Rightarrow \frac{6[n(n+1)]}{4(2(2n+1)+3)}=\frac{9}{5}$$

$$\Rightarrow \frac{[n(n+1)]}{2(2(2n+1)+3)}=\frac{3}{5}$$

$\Rightarrow 5[n(n+1)]=6(2(2n+1)+3)$

$\Rightarrow 5n^2+5n=6(4n+5)$

$\Rightarrow 5n^2+5n=24n+30$

$\Rightarrow 5n^2-19n-30=0$

$\Rightarrow 5n^2-25n+6n-30=0$

$\Rightarrow (5n+6)(n-5)=0$

$\Rightarrow n=5$

Q.18. If $F(x+y)=F(x)\cdot F(y)$, $F(1)=3$ and $\sum_{r=1}^{n}F(r)=3279$ then find the value of n

Answer: 7

Solution: Given,

$F(x+y)=F(x)\cdot F(y)$ and $F(1)=3$

Now taking $x=1$ & $y=1$ we get,

$F(1+1)=F(1)\cdot F(1)\Rightarrow F(2)=3^2$

Similarly $F(2+1)=F(2)\cdot F(1)\Rightarrow F(3)=3^2\times 3=3^3$

And so on $F(n)=3^n$

So, $\sum_{r=1}^{n}F(r)=3279$

$\Rightarrow F(1)+F(2)+F(3)\ldots\ldots F(n)=3279$

$\Rightarrow 3+3^2+3^3\ldots\ldots+3^n=3279$

$\Rightarrow 3\times\frac{3^n-1}{3-1}=3279$

$\Rightarrow \frac{3^n-1}{2}=1093$

$\Rightarrow 3^n-1=2186$

$\Rightarrow 3^n=2187$

$\Rightarrow 3^n=3^7$

$\Rightarrow n=7$