JEE Main Exam 2023 - Session 1

24 Jan 2023 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. The equation of wave is given as: $y = 0.05\ \sin(2x - 4t)$, where x is in metre and t is in second. Find the speed of wave.

A) $2\ \mathrm{m\ s^{-1}}$ B) $4\ \mathrm{m\ s^{-1}}$ C) $0.5\ \mathrm{m\ s^{-1}}$ D) $1\ \mathrm{m\ s^{-1}}$

Answer: $2\ \mathrm{m\ s^{-1}}$

Solution: The given wave equation is of the form:

$y = A\sin(kx - \omega t)$

with $k = 2$ and $\omega = 4$

Since, $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\mathrm{f}$

Then, $\frac{\omega}{k} = f\lambda = v$

$\Rightarrow v = \frac{4}{2} = 2\ \mathrm{m\ s^{-1}}$

Q.2. Two charges q_1 and q_2, separated by a distance d, are placed in a medium of dielectric constant k. If they are placed in the air, then what should be the separation between them such that they experience same force as before?

A) $d\sqrt{k}$ B) $k\sqrt{d}$ C) $2d\sqrt{k}$ D) $1.5d\sqrt{k}$

Answer: $d\sqrt{k}$

Solution: The force between the two charges when they are placed in dielectric is

$F_1 = \frac{1}{4\pi\varepsilon_o k}\frac{q_1q_2}{d^2}$

The force between the two charges when they are placed at separation d' is

$F_2 = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{d'^2}$

Since, the force between the two charges is same in both case

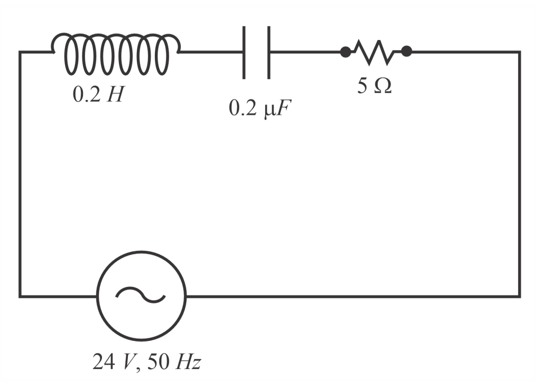
$F_1 = F_2$

$\Rightarrow \frac{1}{4\pi\varepsilon_o k}\frac{q_1q_2}{d^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{d'^2}$

$\Rightarrow d' = d\sqrt{k}$

Hence, the correct option is (a)

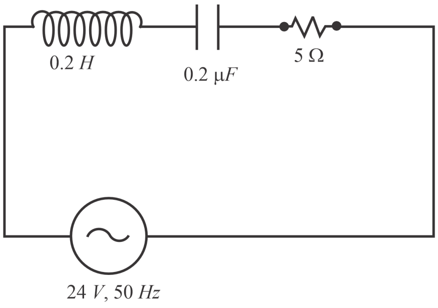
Q.3. Calculate the ratio between bandwidth and quality factor for the following circuit.



A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{1}{16}$ D) 4

Answer: $\frac{1}{8}$

Solution:



For an LCR circuit,

Band width $= \frac{R}{L}$

$= \frac{5}{0.2}\ \mathrm{Hz} = 25$

For an LCR circuit quality factor $= \frac{\sqrt{L}}{R\sqrt{C}}$

$= \frac{\sqrt{0.2}}{5\times\sqrt{0.2\times10^{-6}})} = 200$

Therefore, required ratio

$= \frac{25}{200} = \frac{1}{8}$

Q.4. Which statement is correct about photoelectric effect?

A) Maximum kinetic energy depends upon intensity of light.

B) Stopping potential is dependant only on work function of metal.

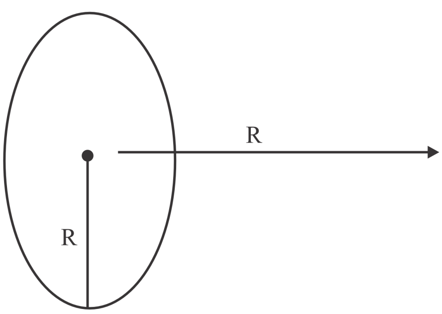
C) Photoelectric effect can be explained by wave nature of light.

D) Photoelectric effect can be explained by particle nature of light

Answer: Photoelectric effect can be explained by particle nature of light

Solution: In this phenomenon, when electromagnetic radiation (such as light) hits the material, the emission of electrons takes place. In the photoelectric effect, if the frequency is too low, no electron is seen getting freed. But, if the frequency is high enough, some electrons can be observed. These observations prove that the Light is made of particles. The energy of the particle increases with the frequency.

Q.5. Find the ratio of the magnitude of magnetic field B_1 at the centre of a loop of radius R to the magnitude of magnetic field B_2 along its axis at a distance R from the centre.



A) $3\sqrt{2}$

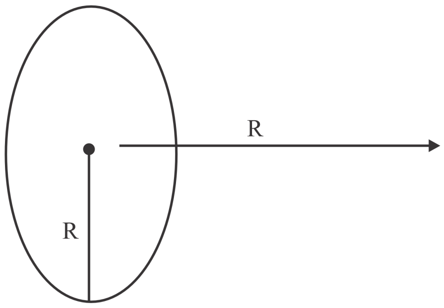
B) 2

C) $2\sqrt{2}$

D) $\frac{1}{2}$

Answer: $2\sqrt{2}$

Solution:



The magnitude of magnetic field at the centre of loop is

$B_1 = B_{\text{centre}} = \frac{\mu_0 i}{2R} \qquad \ldots(1)$

The magnitude of magnetic field at a point at a distance x from the centre along axis of loop is

$B_{\text{axis}} = \frac{\mu_0 i R^2}{2\left(R^2+x^2\right)^{\frac{3}{2}}}$

$B_2 = \frac{\mu_0 i R^2}{2\left(2^{\frac{3}{2}}\right)R^3} \qquad \ldots(2) \qquad (\text{since } x = R)$

Dividing equation 1 by equation 2, we get

$\frac{B_1}{B_2} = 2\sqrt{2}$

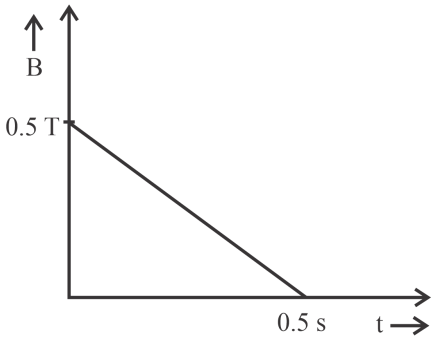
Q.6. A circular loop of radius $\frac{10}{\sqrt{\pi}}\text{cm}$ is placed in a linearly varying perpendicular magnetic field which has magnitude $0.5\ \text{T}$ at time $t = 0$. The magnetic field reduces to zero at $t = 0.5\ \text{s}$. Find the emf induced in the loop at $t = 0.25\ \text{s}$.

A) $0.01\ \text{V}$
B) $0.02\ \text{V}$
C) $0.005\ \text{V}$
D) $0.03\ \text{V}$

Answer: $0.01\ \text{V}$

Solution: Given: $B = 0.5\ \text{T}$ at $t = 0$ & $B = 0$, at $t = 0.5\ \text{s}$.

Magnetic field will decrease linearly as given below. We can say, value of magnetic field at $t = 0.25$ will be $B = 0.25$.



Therefore,

$\varepsilon_{\text{ind}} = \left|\frac{\Delta\phi}{\Delta t}\right| = \left|\frac{\Delta(BA)}{\Delta t}\right| = A\left|\frac{\Delta(B)}{\Delta t}\right|$

$= \pi \times \left(\frac{10}{\sqrt{\pi}}\right)^2 \times 10^{-4} \times \left(\frac{0.25}{0.25}\right)$

$= 10^{-2} \times 1$

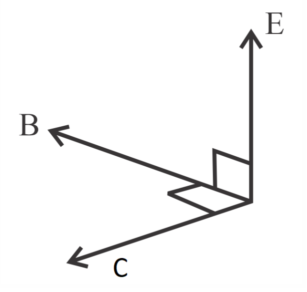
$= 0.01\ \text{V}$

Q.7. Find the correct relation, all the variables have their usual meaning.

A) $B = \omega(k \times E)$ B) $B = \left(\frac{\omega}{k} \times B\right)$ C) $E = \left(\frac{\omega}{k} B\right)$ D) $E = \omega(k \times B)$

Answer: $E = \left(\frac{\omega}{k} B\right)$

Solution: As we know, direction of electromagnetic wave will be in the direction of the vector $\vec{E} \times \vec{B}$.



Also, $c = \frac{\omega}{k}$

and $E = Bc$

Therefore,

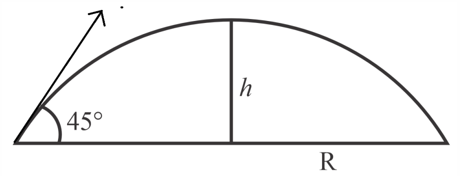
$E = B\frac{\omega}{k}$

Q.8. For a body projected obliquely, to attain maximum Range, the maximum height is 163 m. The maximum Horizontal Range is approximately _____

A) 450 m B) 650 m C) 850 m D) 1000 m

Answer: 650 m

Solution: For maximum range case, angle of projection would be 45°.



As we know, $h = \frac{u^2 \sin^2 45^\circ}{2g}$

$= \frac{u^2}{7g} \times \frac{1}{2}$

$\Rightarrow h = \frac{u^2}{4g}$

Now, $R = \frac{u^2 \sin(2 \times 45^\circ)}{g}$

$\Rightarrow R = \frac{u^2}{g} \times 1$

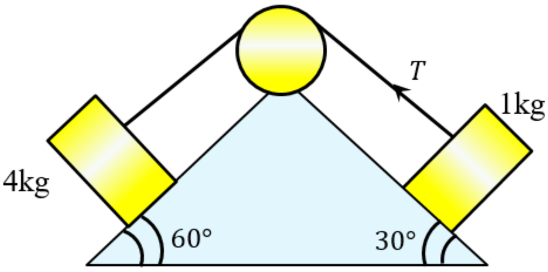
$= \frac{u^2}{4g} \times 4$

$= h \times 4$

$= 163 \times 4$

$\approx 650 \text{ m}$

Q.9. Find the tension in string.



A) $4\sqrt{3}$ N

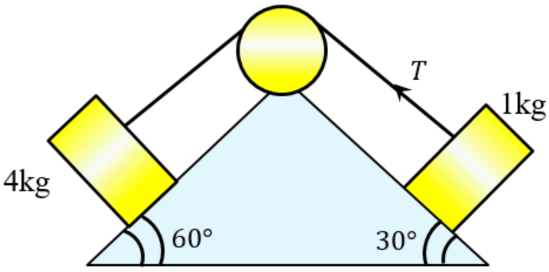
B) $4\left(\sqrt{3}-1\right)$ N

C) 4 N

D) $4\left(\sqrt{3}+1\right)$ N

Answer: $4\left(\sqrt{3}-1\right)$ N

Solution:



The magnitude of acceleration of both the blocks will be same and is given by

$a=\frac{4\,g\,\sin\,60^{\circ}-g\,\sin\,30^{\circ}}{4+1}$

$=\frac{20\sqrt{3}-5}{5}$

$a=\left(4\sqrt{3}-1\right)\text{ m s}^{-1}$

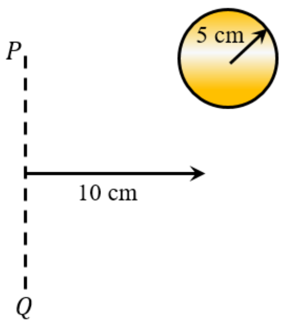
Applying Newton's second law for second block (along the direction parallel to the incline on which it is placed)

$T-5=1\times a$

$\Rightarrow T=5+4\sqrt{3}-1$

$=4\left(\sqrt{3}-1\right)$ N

Q.10. Find the radius of gyration for the uniform solid sphere of radius 5 cm about the axis PQ, as shown in the figure.



A) 5 cm

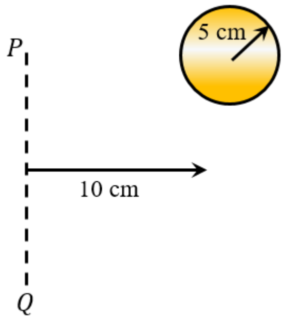
B) 10 cm

C) $\sqrt{110}$ cm

D) $\sqrt{90}$ cm

Answer: $\sqrt{110}$ cm

Solution:



Using parallel axis theorem,

$I_{PQ} = I_{\text{COM}} + Md^2$

$= \frac{2}{5}MR^2 + Md^2$

$= M\left[\frac{2}{5} \times 5^2 + 10^2\right]$

$= M\left[110\right] = Mk^2$

$\Rightarrow k = \sqrt{110}\ \text{cm}$

Q.11. Match the Column:

Physical Quantity	Dimension
I. h	a. ML^2T^{-1}
II. p	b. MLT^{-1}
III. V_0	c. $ML^2A^{-1}T^{-3}$
IV. ϕ	d. ML^2T^{-2}

A) i-a,ii-b,iii-c,iv-d

B) i-b,ii-a,iii-c,iv-d

C) i-d,ii-a,iii-c,iv-b

D) i-d,ii-a,iii-b,iv-c

Answer: i-a,ii-b,iii-c,iv-d

Solution: The dimension of momentum is

$p = \left[MLT^{-1}\right]$

From de Broglie equation

$\frac{h}{p} = \lambda$

$\left[h\right] = \left[p\right][\lambda] = \left[ML^2T^{-1}\right]$

The stopping potential is related to maximum kinetic energy of emitted electron as

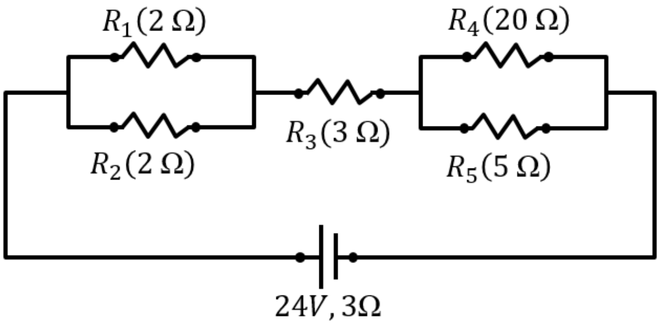
$V_0 = \frac{Km}{e}$

$= \frac{\left[ML^2T^{-2}\right]}{[AT]} = \left[ML^2A^{-1}T^{-3}\right]$

From Einstein's photoelectric equation, the work function ϕ has dimensions of energy

$[\varphi] = \left[ML^2T^{-2}\right]$

Q.12. In the circuit shown, Find current I_4 through R_4 and current I_5 through R_5



A) $I_4 = \frac{24}{55}A, I_4 = \frac{96}{55}$ A

B) $I_4 = \frac{96}{55}A, I_4 = \frac{24}{55}A$

C) $I_4 = \frac{24}{37}A, I_4 = \frac{96}{37}A$

D) $I_4 = \frac{96}{37}$ A, $I_4 = \frac{24}{37}$ A

Answer: $I_4 = \frac{24}{55}A, I_4 = \frac{96}{55}$ A

Solution: Since R_4 and R_5 are parallel, their equivalent resistance

$$R_{45} = \frac{R_4 R_5}{R_4 + R_5} = 4\ \Omega$$

Similarly,

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 1\ \Omega$$

Therefore, net resistance in circuit is

$$R_{net} = R_{12} + R_3 + R_{45} + r_{int}$$
$$= 1 + 3 + 4 + 3 = 11\Omega$$

The net current is

$$I_{net} = \frac{24}{11}\ A$$

Since,

$$I_4 + I_5 = I_{net} = \frac{24}{11}\ A \quad \ldots\ldots(1)$$

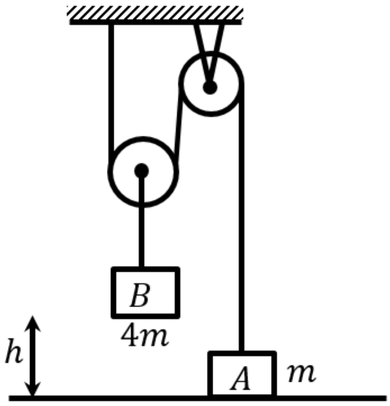
$$\frac{I_4}{I_5} = \frac{R_5}{R_4} = \frac{5}{20} = \frac{1}{4} \quad \ldots\ldots(2)$$

Solving equation 1 and 2

$$I_4 = \frac{24}{55}\ \text{A}$$

$$I_5 = \frac{96}{55}\ \text{A}$$

Q.13. In the given figure, masses of blocks A and B are equal to m and $4\ \mathrm{m}$ respectively. All strings and pulleys are ideal. The system is released from rest. Block B hits the ground after some time, while travelling a height 'h' vertically. The maximum height attained by block A is:



A) $3\mathrm{h}$

B) $\frac{11\mathrm{h}}{8}$

C) $\frac{24\mathrm{h}}{13}$

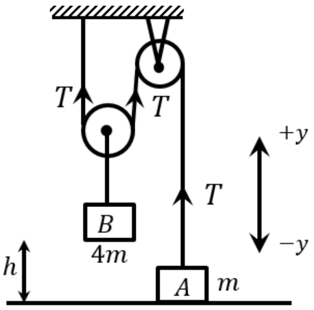
D) $\frac{13\mathrm{h}}{8}$

Answer: $3\mathrm{h}$

Solution: The magnitude of acceleration of two blocks are related as:

$a_A = 2a_B$

$\therefore$ If $a_A = a,\ a_B = \frac{a}{2}$



Applying Newton's second law for block A along y-axis

$T - mg = ma \qquad \ldots(1)$

Applying Newton's second law for block B along y-axis

$2T - 4mg = -4ma_B = -2ma$

$\Rightarrow T - 2mg = -ma \quad \ldots(2)$

Subtracting equation (2) from (1)

$2mg - mg = 2ma \ \Rightarrow a = \frac{g}{2}$

$\Rightarrow a_A = a = \frac{g}{2}\ \&\ a_B = \frac{a}{2} = \frac{g}{4}$

While block B moves downwards by height h, block A moves upwards by height $2h$. During this interval $a_A = \frac{g}{2}$. The velocity, v at end this interval is:

$v = \sqrt{2\left(\frac{g}{2}\right)2h} = \sqrt{2gh} \qquad \ldots..\left(3\right)$

Once block B hits the ground, block A is in free fall motion

$\therefore\ 0^2 - v^2 = 2gh'$ (where h' is further height attained by A). Substituting v from equation (3),

$\Rightarrow\ 0^2 - 2gh = 2gh'$

$\Rightarrow\ \ h = h'$

$\therefore$ Total height attained by block A is $h' + 2h = 3h$.

Q.14. A carrier wave of amplitude A_c carries a message signal of amplitude A_m. Modulation index of this signal is:

A) $\mu = \frac{Ac - Am}{Ac}$

B) $\mu = \frac{Ac}{Ac + Am}$

C) $\mu = \frac{Am}{Ac}$

D) $\mu = \frac{Ac}{Am}$

Answer: $\mu = \frac{Am}{Ac}$

Solution: Modulation index 'μ' is related to amplitude of modulation signal 'A_m' and amplitude of carrier wave 'A_c' as:

$\mu = \frac{Am}{Ac}$

Q.15. Weight of an object at earth's surface is $18\ \mathrm{N}$. If the object is taken $3200\ \mathrm{km}$ above the surface, then the weight of the object (in N) is
Given; radius of Earth $= 6400\ \mathrm{km}$

Answer: 8

Solution:

As height of the object is considerable compared to the radius of the earth, therefore approximation formula can not be used.

Force acting,

$$F = mg' = \frac{GMm}{\left(R+\frac{R}{2}\right)^2} = \frac{4}{9}\frac{GMm}{R^2}$$

$$\Rightarrow mg' = \frac{4}{9}(mg)$$

$$= \frac{4}{9} \times 18$$

$$= 8\ \mathrm{N}$$

Section B: Chemistry

Q.1. The phosphodiester bond of RNA is most stable at:

A) $\mathrm{pH}\ 2 — 3$ at $60^\circ\mathrm{C}$ B) $\mathrm{pH}\ 4 — 5$ at $90^\circ\mathrm{C}$ C) $\mathrm{pH}\ 9 - 10$ at $120^\circ\mathrm{C}$ D) $\mathrm{pH}\ 7 — 8$ at $90^\circ\mathrm{C}$

Answer: $\mathrm{pH}\ 4 — 5$ at $90^\circ\mathrm{C}$

Solution: Nucleotides are, in turn, joined to each other in polynucleotide chains through the 3' hydroxyl of 2' deoxyribose of one nucleotide and the phosphate attached to the 5' hydroxyl of another nucleotide. This is a phosphodiester linkage in which the phosphoryl group between the two nucleotides has one sugar esterified to it through a 5' hydroxyl and a second sugar esterified to it through a 5'-hydroxyl.

The RNA phosphodiester bond is most stable at $\mathrm{pH}\ 4 - 5$ at $90^\circ\mathrm{C}$.

Q.2. A $25\ \mathrm{mL}$ buffer solution is prepared by mixing CH_3COOH of concentration $0.1\ \mathrm{M}$ and CH_3COONa of concentration $0.01\ \mathrm{M}$. If the pH of the solution is 5, then calculate the pKa of CH_3COOH.

A) 4 B) 5 C) 6 D) 7

Answer: 6

Solution: The Henderson equation for the given acidic buffer can be written as

$$\mathrm{pH} = \mathrm{pKa} + \log \frac{[\mathrm{salt}]}{[\mathrm{weak\ \ acid}]}$$

$$5 = \mathrm{pKa} + \log \frac{10^{-2}}{0.1}$$

$$\Rightarrow \mathrm{pKa} = 6$$

Q.3. Statement-1: Noradrenaline is a neurotransmitter in human beings
Statement-2: Its Low concentration is not a cause of depression.

A) Both statement 1 and 2 are correct..

B) Statement 1 is correct but statement 2 is incorrect

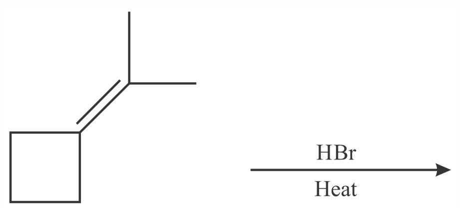
C) Statement 1 is incorrect but statement 2 is correct

D) Both statement 1 and 2 are correct

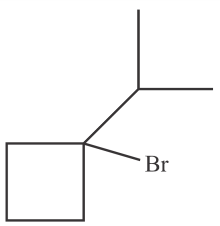
Answer: Statement 1 is correct but statement 2 is incorrect

Solution: Noradrenaline is one of the neurotransmitters that plays a role in mood changes. If the level of noradrenaline is low for some reason, then the signal-sending activity becomes low, and the person suffers from depression

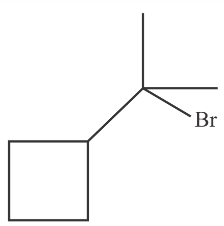
Q.4. The correct product of the following reaction is :



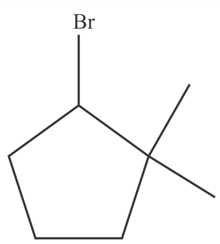
A)



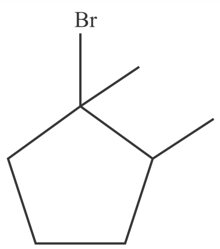
B)



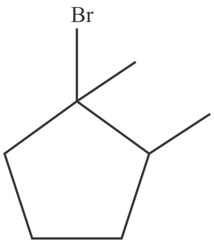
C)



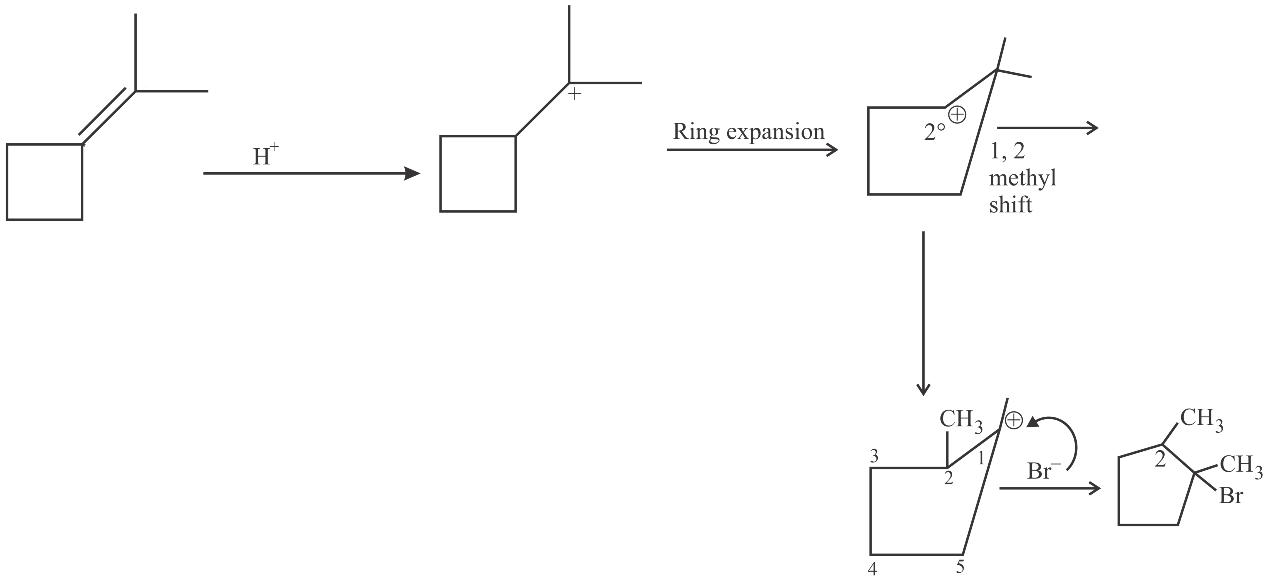
D)



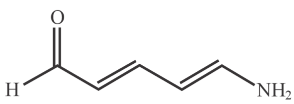
Answer:

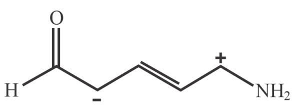


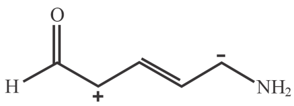
Solution:

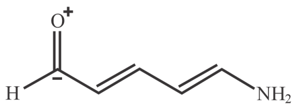


Q.5. The correct stability order of the resonating structure of



A 

B 

C 

A) $A > B > C$ B) $B > A > C$ C) $C > B > A$ D) $A > C > B$

Answer: $A > B > C$

Solution:

- All the atoms in the structure must have completed valence shell to increase the stability of the molecule
- A carbon atom that contains a positive charge has an incomplete octet therefore it is not stable
- The least stable structure is less contributor to the condition of hybrid resonance
- The structure with a lower number of atoms with formal charges is more stable than the greater number of molecular atom
- The structure must have a negative change on the strong electronegative atom to increase the stability

Q.6. The primary and secondary valency of cobalt in $\left[\mathrm{Co}\left(\mathrm{NH}_3\right)_5 \mathrm{Cl}\right] \mathrm{Cl}_2$ is

A) 3, 6 B) 2, 4 C) 3, 5 D) 2,5

Answer: 3, 6

Solution: The primary valency is the oxidation number of the metal ion. It can be calculated as:

$$x+(0)(5)+(-1)+(-2)\ =0$$
$$x=+3$$

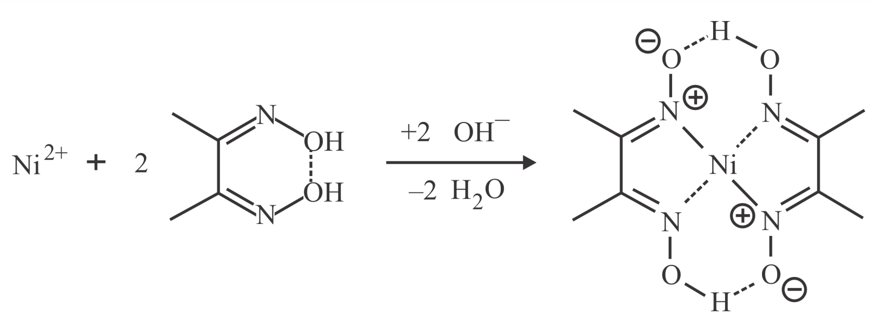
The secondary valency represents the coordination number of the central metal ion which is six in the present case

Q.7. The cation gives bright red colour with dimethylglyoxime. Which is that cation?

A) Cu^{2+} B) Ni^{2+} C) Zn^{2+} D) Co^{2+}

Answer: Ni^{2+}

Solution: Nickel cation reacts with dimethylglyoxime forms an insoluble red precipitate of nickel dimethylglyoxime.



Q.8. Statement 1 : The freezing point of a solution decreases with decrease in amount of non-volatile solute

Statement 2 : Freezing point of the solution is less than that of the solvent.

A) Both statement 1 and 2 are correct

B) Statement 1 is correct but statement 2 is incorrect

C) Statement 1 is incorrect but statement 2 is correct

D) Both statement 1 and 2 are incorrect

Answer: Statement 1 is incorrect but statement 2 is correct

Solution: Vapour pressure of liquid and solid are equal at freezing point. Reduction in vapour pressure occurs when solute is added. Hence, it can be concluded that addition of non-volatile solute decreases the freezing point of a solvent and the depression in freezing point is directly proportional to the amount of non-volatile solute.

Q.9. Which of the following transition metal ion has magnetic moment 3.87 BM ?

A) Sc^{2+} B) Ti^{2+} C) V^{2+} D) Mn^{2+}

Answer: V^{2+}

Solution: The formula to calculate magnetic moment is:

$$\mu = \sqrt{n(n+2)}\ ,\ \text{where n is number of unpaired electrons}$$
$$\Rightarrow\ 3.87 = \sqrt{n(n+2)}$$
$$\Rightarrow\ n = 3$$

Since V^{2+} has 3 unpaired electrons. So it will have a magnetic moment of 3.87 BM

Q.10. The correct statement about Freons is

A) They are used as cancer medicines

B) They are chlorofluorocarbon compounds

C) These are toxic organic compounds

D) These are flammable compounds

Answer: They are chlorofluorocarbon compounds

Solution: Freons are the chlorofluoro derivatives of hydrocarbons.

(i) Freons are used as refrigerants in refrigerators and air conditioners.
(ii) It is used as propellents for aerosols and foams.
(iii) It is used as propellant for foams to spray out deodorants, shaving creams and insecticides.

Q.11.

Column I		Column II	
(a)	Aluminium	(1)	electrolysis
(b)	iron	(2)	Reverberatory furnace
(c)	silicon	(3)	Blast furnace
(d)	copper	(4)	Zone refining

A) A-1;B-3; C-4;D-2 B) A-2;B-3; C-1;D-4 C) A-3; B-2; C-1;D-4 D) A-2;B-4; C-1;D-3

Answer: A-1;B-3; C-4;D-2

Solution: The process by which refining of aluminium is done is called Hoop's electrolytic process.

The blast furnace is filled with iron ore, limestone and Coke. Once the impure iron is process, it forms a substance known as pig iron. Direct iron deduction uses methane gas instead of Coke to refine iron. The iron is heated hot enough to burn off impurities but not enough to melt the iron.

Zone refining is the technique for refining to acquire silicon of high immaculateness. This strategy is utilised for the most part to get semiconductors of high immaculateness like silicon.

The extraction of copper can be done in reverberatory furnace.

Q.12. Consider the following reaction given below

$BeO + HF + NH_3 \rightarrow A \xrightarrow{Heat} BeF_2 + NH_4F$, Identify the missing product A.

A) $Be(OH)_2$ B) $(NH_4)_2BeF_4$ C) $(NH_4)_2(BeF_3)$ D) $(NH_4)_2(BeF_6)$

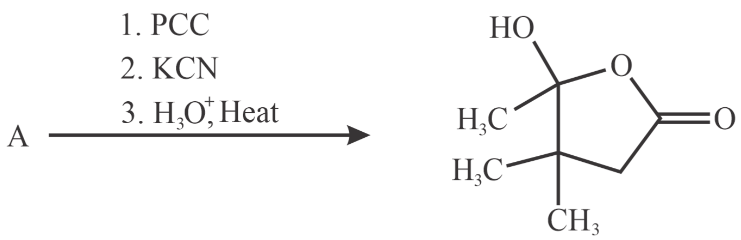
Answer: $(NH_4)_2BeF_4$

Solution:

$$BeO + HF + NH_3 \longrightarrow \underset{[A]}{(NH_4)_2[BeF_4]} \xrightarrow{\Delta} NH_4F + \underset{[B]}{BeF_2}$$

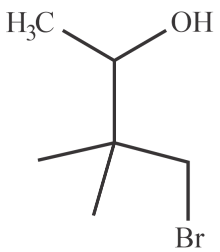
Oxidation state of Be in A is (+2)

Q.13.

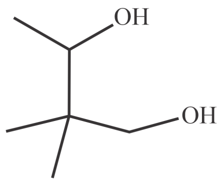


The correct structure of 'A' ?

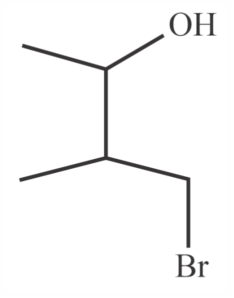
A)



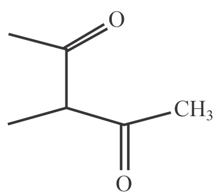
B)



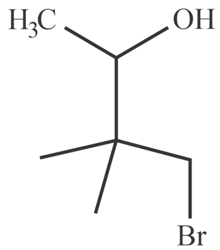
C)



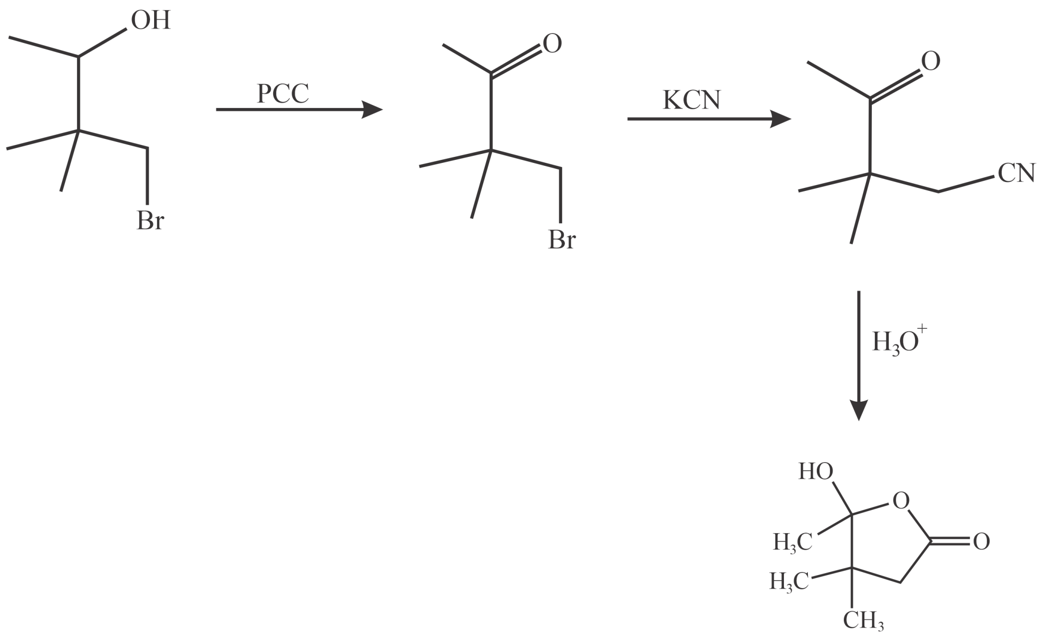
D)



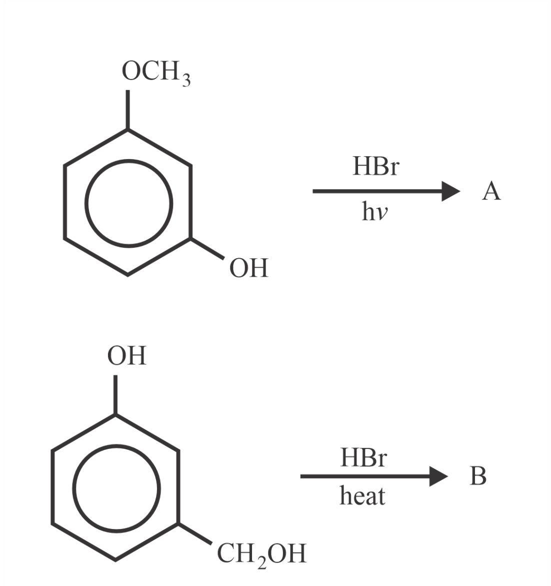
Answer:



Solution: PCC is the mild oxidising agent, converts alcohols to carbonyl compounds. In the second step nucleophilic substitution reaction takes place. The cyanide formed undergo hydrolysis to give caboxylic aicd, which undergo esterification.

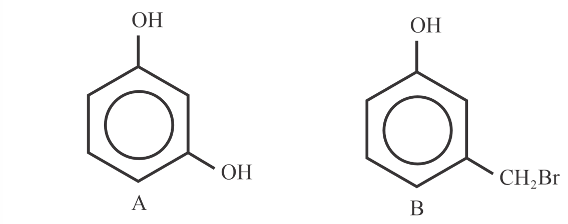


Q.14.

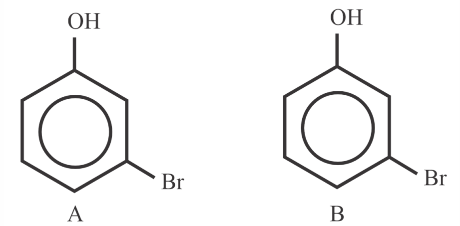


Identify the products A and B

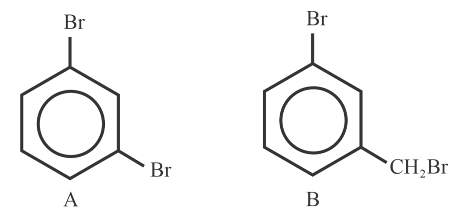
A)



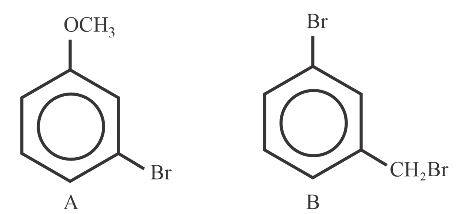
B)



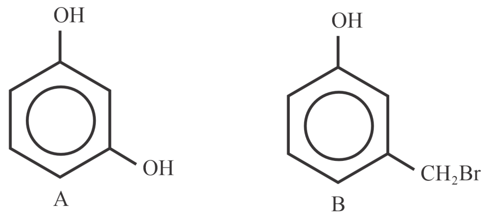
C)



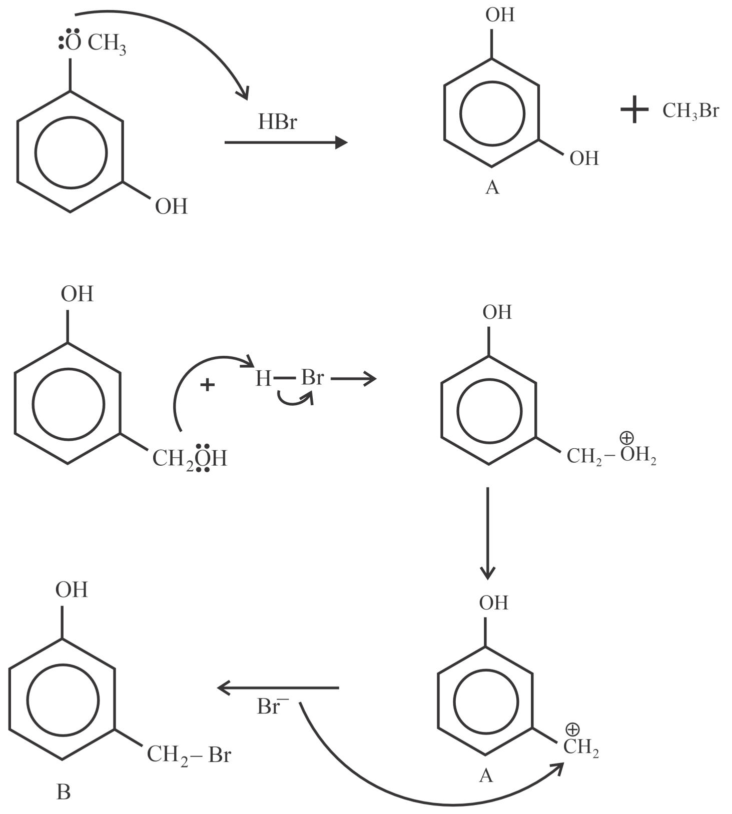
D)



Answer:



Solution:



Q.15. Match Column I with Column II

	Column I		Column II
A	Soda ash	1	NaF
B	Chlorophyll	2	$Ca(OH)_2$
C	Whitewashing	3	Na_2CO_3
D	Toothpaste	4	Mg^{2+} ions

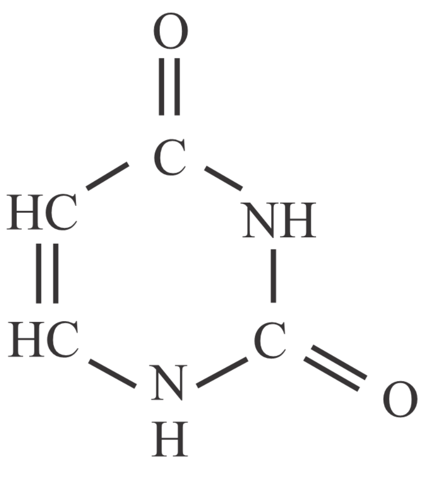
A) A-3,B-4,C-2,D-1 B) A-1,B-4,C-2,D-3 C) A-3,B-2,C-4,D-1 D) A-4,B-3,C-2,D-1

Answer: A-3,B-4,C-2,D-1

Solution:

Soda ash	Na_2CO_3
chlorophyll	Mg^{2+} ions
Whitewashing	$Ca\,(OH_2)$
toothpaste	Na_2CO_3

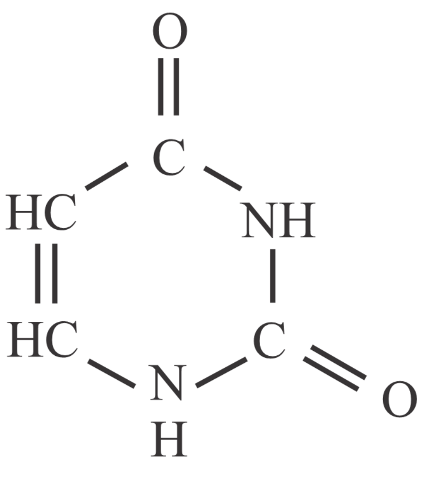
Q.16. Find mass $\%$ of N in uracil.



Uracil (U)

Answer: 25

Solution:



Uracil (U)

The molecular formula of uracil is $C_4H_4\,N_2O_2$

The molar mass of uracil = 112 g/mol

Mass $\%$ of $\mathrm{N} = \frac{28}{112} \times 100$ = 25 %

Q.17. In the complex $[\mathrm{CoCl}_4]^{2-}$ with CFT configuration of $\mathrm{e}^{\mathrm{n}}\mathrm{t}_2^{\mathrm{m}}$, find the value of (n + number of unpaired electrons)

Answer: 7

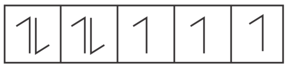
Solution: The oxidation state of Co in complex $[\mathrm{CoCl_4}]^{2-}$ is

$x - 4 = -2$

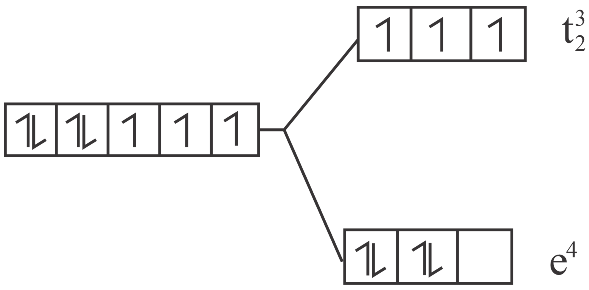
$x = +2$

Electronic configuration of Co is ${}_{27}Co = [Ar]4s^2 3d^7$

So, $Co^{+2} = \left[\mathrm{Ar}\right]3d^7$



Since CI^- is a weak field ligand, so the given complex will have a tetrahedral geometry with following Crystal-field splitting



Hence, number of e-electrons $= 4$ and the total number of unpaired electrons $= 3$

So, value of $(n)+$(unpaired electrons) is $(4) + (3) = 7$.

Q.18. 5.0 g of NaOH is dissolved in water to get 450 mL solution. What volume of the solution in mL is required to prepare 500 mL of $0.1\,\mathrm{M NaOH}$ solution?

Answer: 180

Solution: $M_1V_1 = M_2V_2$

$\left(\frac{5 \times 1000 \times V_1}{40 \times 450}\right) = 0.1 \times \frac{500}{1000}$

$V_1 = \frac{180}{1000} = 0.18\ \mathrm{L}$

Q.19. If v(frequency) $\propto Z^n$ where Z is the atomic number. What is the value of n?

Answer: 2

Solution: $\sqrt{v}$ is directly proportional to Z according to Moseley.

$\Rightarrow \sqrt{v}\ \alpha\ Z$

$\Rightarrow\ v\ \alpha\ Z^2$

So, the value of n = 2

v (frequency) □ Z^n

Q.20. First line of the Paschen series has wavelength of 720 nm. Obtain the wavelength of its second line

Answer: 492

Solution: Paschen series are spectral lines coming to $n=3$

1^{st} line of Paschen series makes a transition from, $4 \rightarrow 3$

So, 2^{nd} line of Paschen series makes a transition from, $5 \rightarrow 3$

We know that,

$$\frac{1}{\lambda}=R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] Z^2$$

For 1^{st} line, $\frac{1}{\lambda}=R\left[\frac{1}{(3)^2}-\frac{1}{(4)^2}\right] Z^2$

and for 2^{nd} line,

$$\frac{1}{\lambda'}=R\left[\frac{1}{(3)^2}-\frac{1}{(5)^2}\right] Z^2$$

or $\frac{\left(\frac{1}{\lambda}\right)}{\left(\frac{1}{\lambda'}\right)}=\frac{R\left[\frac{7}{9\times16}\right] Z^2}{R\left[\frac{16}{9\times25}\right] Z^2}$

Given, $\lambda=720 \text{ nm}$

or $\frac{\left(\frac{1}{720}\right)}{\left(\frac{1}{\lambda'}\right)}=\frac{175}{256}$

or $\frac{\lambda'}{720}=\frac{175}{256}$

or $\lambda' \approx 492 \text{ nm}$

Section C: Mathematics

Q.1. $\int_0^{\frac{\pi}{2}} \frac{\sin^{2023} x}{\sin^{2023} x+\cos^{2023} x} dx=$

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{8}$ D) $\frac{\pi}{3}$

Answer: $\frac{\pi}{4}$

Solution: $I=\int_0^{\frac{\pi}{2}} \frac{\sin^{2023} x}{\sin^{2023} x+\cos^{2023} x} dx \quad \ldots(i)$

Let $x \rightarrow \frac{\pi}{2}-x$

Applying $\int_0^a f(x) dx=\int_0^a f(a-x) dx$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sin^{2023}\left(\frac{\pi}{2}-x\right)}{\sin^{2023}\left(\frac{\pi}{2}-x\right)+\cos^{2023}\left(\frac{\pi}{2}-x\right)} dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\cos^{2023} x}{\cos^{2023} x+\sin^{2023} x} dx \quad \ldots(ii)$$

Adding (i) and (ii), we get

$$2I=\int_0^{\frac{\pi}{2}} \frac{\sin^{2023} x+\cos^{2023} x}{\cos^{2023} x+\sin^{2023} x} dx=\int_0^{\frac{\pi}{2}} dx$$

$$I=\frac{1}{2}\times\frac{\pi}{2}=\frac{\pi}{4}$$

Q.2. $\int_0^3\left|x^2-3x+2\right| dx=$

A) 0 B) 2 C) $\frac{11}{6}$ D) $\frac{13}{6}$

Answer: $\frac{11}{6}$

Solution: $I=\int_0^3\left|x^2-3x+2\right|dx$

$=\int_0^3|(x-1)(x-2)|dx$

$=\int_0^1\left(x^2-3x+2\right)dx-\int_1^2\left(x^2-3x+2\right)dx+\int_2^3\left(x^2-3x+2\right)dx$

$=\left[\frac{x^3}{3}-3\frac{x^2}{2}+2x\right]_0^1-\left[\frac{x^3}{3}-3\frac{x^2}{2}+2x\right]_1^2+\left[\frac{x^3}{3}-3\frac{x^2}{2}+2x\right]_2^3$

$=\left(\frac{1}{3}-\frac{3}{2}+2\right)-\left[\left(\frac{8}{3}-6+4\right)-\left(\frac{1}{3}-\frac{3}{2}+2\right)\right]+\left[\left(9-\frac{27}{2}+6\right)-\left(\frac{8}{3}-6+4\right)\right]$

$=\left(\frac{2}{3}-3+4\right)-\left(\frac{16}{3}-12+8\right)+\left(9-\frac{27}{2}+6\right)$

$=20-\frac{109}{6}=\frac{11}{6}$

Q.3. Shortest distance between the lines $\frac{x-2}{3}=\frac{y-1}{3}=\frac{z-0}{2}$ and $\frac{x-1}{3}=\frac{y-2}{2}=\frac{z-1}{3}$ is

A) $\frac{6}{\sqrt{43}}$ B) $\frac{11}{\sqrt{43}}$ C) $\frac{3}{\sqrt{43}}$ D) $\frac{5}{\sqrt{43}}$

Answer: $\frac{11}{\sqrt{43}}$

Solution: Given equations are

$\frac{x-2}{3}=\frac{y-1}{3}=\frac{z-0}{2}$ is passing through a point $(2,1,0)$ and it's direction ratios are $3,3,2$.

So,

$\vec{a}_1=2\hat{i}+\hat{j}+0\hat{k}$

$\vec{b}_1=3\hat{i}+3\hat{j}+2\hat{k}$

And,

$\frac{x-1}{3}=\frac{y-2}{2}=\frac{z-1}{3}$ is passing through a point $(1,2,1)$ and it's direction ratios are $3,2,3$.

$\vec{a}_2=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}_2=3\hat{i}+2\hat{j}+3\hat{k}$

Now,

$\vec{a}_2-\vec{a}_1=\left(\hat{i}+2\hat{j}+\hat{k}\right)-\left(2\hat{i}+\hat{j}+0\hat{k}\right)=-\hat{i}+\hat{j}+\hat{k}$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 3 & 2\\ 3 & 2 & 3\end{vmatrix}$

$\Rightarrow\vec{b}_1\times\vec{b}_2=5\hat{i}-3\hat{j}-3\hat{k}$

So,

$\Rightarrow\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{25+9+9}=\sqrt{43}$

We know that the shortest distance between two skew lines is $\frac{\left|\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$=\left|\frac{\left(-\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(5\hat{i}-3\hat{j}-3\hat{k}\right)}{\sqrt{43}}\right|$

$=\frac{|-5-3-3|}{\sqrt{43}}=\frac{11}{\sqrt{43}}$ units

Q.4. If $y^2+\log_e\left(\cos^2 x\right)=y$ and $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then

A) $y''(0)=0$

B) $|y'(0)|=|y''(0)|$

C) $|y''(0)|=2$

D) $|y'(0)|+|y'(1)|=3$

Answer: $|y''(0)|=2$

Solution: Given,

$y^2 + \log_e\left(\cos^2 x\right) = y$

Now differentiating both side with respect to x we get,

$2 \times y \times y'(x) + \frac{2\cos x(-\sin x)}{\cos^2 x} = y'(x)$

$\Rightarrow y'(x) = \frac{2\tan x}{2y-1}$

$\Rightarrow y'(0) = 0$

Now differentiating $y'(x)$ we get,

$\Rightarrow y''(x) = \frac{(2y-1)2\sec^2 x - 2\tan x\left(2y'(x)\right)}{(2y-1)^2}$

$\Rightarrow y''(0) = \frac{(2y-1)2\sec^2 0 - 2\tan 0\left(2y'(0)\right)}{(2y-1)^2}$

$\Rightarrow y''(0) = \frac{(2y-1)\times 2\times 1}{(2y-1)^2} = \frac{2}{2y-1}$

Now in equation $y^2 + \log_e\left(\cos^2 x\right) = y$ at $x = 0$, the value of $y = 0 \text{ or } 1$,

So putting the value of y in $y''(0) = \frac{2}{2y-1}$ we get,

$y''(0) = \frac{2}{2\times 0-1} = -2 \text{ at } y = 0$

And $y''(0) = \frac{2}{2\times 1-1} = 2 \text{ at } y = 1$

Hence, $|y''(0)| = 2$

Q.5. Tangent drawn at a point on the parabola $y^2 = 24x$ intersects the hyperbola $xy = 2$ at points A and B. The locus of mid-point of AB is

A) $y^2 = 3x$

B) $y^2 = -3x$

C) $y^2 = 6x$

D) $y^2 = -6x$

Answer: $y^2 = -3x$

Solution: Given:

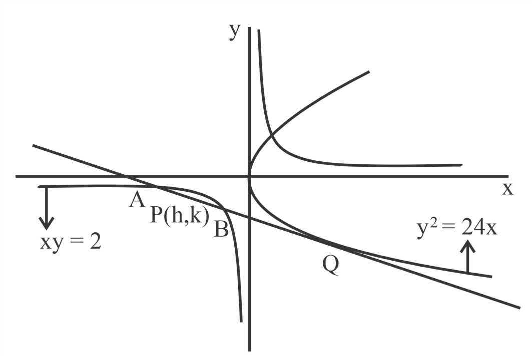
$y^2 = 24x$

Comparing with $y^2 = 4ax$, we get

$4a = 24 \Rightarrow a = 6$

Also given $xy = 2$

Let any point $Q \equiv \left(at^2, 2at\right) \equiv \left(6t^2, 12t\right)$ on the parabola.



Equation of tangent at Q is

$12yt = 12\left(x + 6t^2\right)$

$\Rightarrow yt = x + 6t^2$

$\Rightarrow x - yt + 6t^2 = 0 \quad \ldots.(\text{i})$

Let mid-point of AB be $P(h, k)$.

Equation of chord of hyperbola is

$\frac{xk+yh}{2} = hk$

$\Rightarrow xk + hy - 2hk = 0 \quad \ldots(\text{ii})$

Since, (i) & (ii) are same, so on comparing, we get

$\frac{k}{1} = \frac{h}{-t} = \frac{-2hk}{6t^2}$

$\Rightarrow t = -\frac{h}{k}$ & $\frac{h}{-t} = \frac{-2hk}{6t^2} \Rightarrow t = \frac{k}{3}$

So,

$\frac{k}{3} = -\frac{h}{k}$

$\Rightarrow k^2 = -3h$

Hence, locus is $y^2 = -3x$.

Q.6. $\lim_{t \to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + 3^{\frac{1}{\sin^2 t}} \ldots\ldots n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$

A) 0 B) n C) $\frac{n^2-n}{2}$ D) n^2+n

Answer: n

Solution:

$$L = \lim_{t\to 0}\left(\frac{1}{1\,\sin^2 t}+\frac{1}{2\,\sin^2 t}+\frac{1}{3\,\sin^2 t}\ldots\ldots\frac{1}{n\,\sin^2 t}\right)^{\sin^2 t}$$

$$= \lim_{t\to 0}\left(1^{\operatorname{cosec}^2 t}+2^{\operatorname{cosec}^2 t}+3^{\operatorname{cosec}^2 t}\ldots\ldots n^{\operatorname{cosec}^2 t}\right)^{\sin^2 t}$$

$$= \lim_{t\to 0}\frac{\sin^2 t}{n\,\sin^2 t}\left(\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t}+\left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t}+\left(\frac{3}{n}\right)^{\operatorname{cosec}^2 t}\ldots\ldots 1\right)^{\sin^2 t}$$

$$= n\lim_{t\to 0}\ (0+0+0\ldots\ldots 1)^0 = n$$

Q.7. The area(in sq. units) enclosed between the curves $y^2=-4x+4$ and $y=2x+2$ is

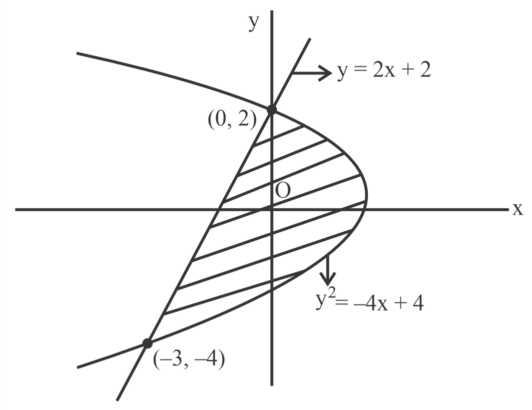
A) 3 B) 9 C) 4 D) 12

Answer: 9

Solution: Given:

$y^2=-4x+4$

$y=2x+2$



Required area

$$= \int_{-4}^{2}\left(\frac{4-y^2}{4}-\frac{y-2}{2}\right)dy$$

$$= \int_{-4}^{2}\left(2-\frac{y^2}{4}-\frac{y}{2}\right)dy$$

$$= \left[2y-\frac{y^3}{12}-\frac{y^2}{4}\right]_{-4}^{2}$$

$$= \left[\left(4-\frac{8}{12}-1\right)-\left(-8+\frac{64}{12}-\frac{16}{4}\right)\right]$$

$= 15-6$

$= 9$ sq. units

Q.8. $\sim(\sim p\wedge q)\Rightarrow(\sim p\vee q)$ is equivalent to

A) $\sim p\vee q$ B) $\sim p\wedge q$ C) $p\wedge q$ D) $p\vee q$

Answer: $\sim p\vee q$

Solution:

p	q	~p	~$p\wedge q$	~$p\vee q$	~ (~$p\wedge q$)	~ (~$p\wedge q$) $\Rightarrow$ (~$p\vee q$)
T	T	F	F	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	F	T	F	T	T	T

So, ~ (~$p\wedge q$) $\Rightarrow$ (~$p\vee q$) is equivalent to (~$p\vee q$).

Q.9. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}=$

A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{3}$

Answer: $\frac{\pi}{3}$

Solution:

$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}$

$=\tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}\left(1+\sqrt{3}\right)}\right)+\sec^{-1}\sqrt{\frac{4\left(2+\sqrt{3}\right)}{3\left(2+\sqrt{3}\right)}}$

$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{4}{3}}$

$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cos^{-1}\frac{\sqrt{3}}{2}=\frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$

Q.10. Which of the following is equivalent to $\sum_{r=0}^{22}{}^{22}C_r\cdot{}^{23}C_r$

A) ${}^{44}C_{22}$ B) ${}^{45}C_{22}$ C) ${}^{45}C_{23}$ D) ${}^{44}C_{23}$

Answer: ${}^{45}C_{23}$

Solution: Let, $S=\sum_{r=0}^{22}{}^{22}C_r\cdot{}^{23}C_r$

Now using ${}^{n}C_r={}^{n}C_{n-r}$ we get,

$S=\sum_{r=0}^{22}{}^{22}C_r\cdot{}^{23}C_{23-r}$

$\Rightarrow S={}^{22}C_0\cdot{}^{23}C_{23}+{}^{22}C_1\cdot{}^{23}C_{22}+\ldots\ldots\ldots\ldots{}^{22}C_{22}\cdot{}^{23}C_1$

Which is sum of coefficient of x^{23} in the expansion of $(1+x)^{22}\cdot(1+x)^{23}$ or $(1+x)^{23+22}=(1+x)^{45}$

Hence, $\sum_{r=0}^{22}{}^{22}C_r\cdot{}^{23}C_{23-r}={}^{45}C_{23}$

Q.11. General solution of the differential equation $\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$ is

A) $y=\left(1+\frac{1}{x}\right)+ce^{\frac{1}{x}}$ B) $y=\left(1-\frac{1}{x}\right)+ce^{\frac{1}{x}}$ C) $y=\left(x+\frac{1}{x}\right)+ce^{\frac{1}{x}}$ D) $y=\left(x-\frac{1}{x}\right)+ce^{\frac{1}{x}}$

Answer: $y=\left(1+\frac{1}{x}\right)+ce^{\frac{1}{x}}$

Solution: Given:

$\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$

This is a linear differential equation.

$\mathrm{I.F.}=e^{\int\frac{dx}{x^2}}=e^{-\frac{1}{x}}$

Solution of the given differential equation is

$ye^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}\left(\frac{1}{x^3}\right)dx$

Put $-\frac{1}{x}=t\Rightarrow\frac{dx}{x^2}=dt$

$ye^{-\frac{1}{x}}=-\int te^t dt$

$\Rightarrow ye^{-\frac{1}{x}}=-\left[te^t-e^t\right]+c$

$\Rightarrow ye^{-\frac{1}{x}}=e^{-\frac{1}{x}}+\frac{1}{x}e^{-\frac{1}{x}}+c$

$\Rightarrow y=\left(1+\frac{1}{x}\right)+ce^{\frac{1}{x}}$

Q.12. If $R=\{(a,b):\gcd(a,b)=1;\ a\ \&\ b \text{ are integers}\}$, then the relation is

A) Symmetric B) Reflexive C) Transitive D) None of these

Answer: Symmetric

Solution: Since, R is defined as $R=\{(a,b):\gcd(a,b)=1;\ a\ \&\ b \text{ are integers}\}$.

Reflexive : $aRa,\ \gcd\ (a,a)\neq 1$ as $\gcd\ (a,a)=a$

So, R is not reflexive.

Symmetric : aRb iff $\gcd\ (a,b)=\gcd\ (b,a)$

Which is true, hence, R is symmetric.

Transitive : aRb iff $\gcd\ (a,b)=1\ \&\ \gcd\ (b,c)=1\ \Rightarrow\gcd\ (c,a)=1$, which is not possible always, for example if $\gcd\ (2,3)=1\ \&\ \gcd\ (3,4)=1$ that does not imply that $\gcd\ (4,2)=1$, hence we can say that relation is not transitive.

Q.13. If $\left(1-\sqrt{3}i\right)^{100}=2^{99}(p+iq)$, then the equation with roots $p-q+q^2$ and $p+q+q^2$ will be

A) $x^2-4x+1=0$ B) $x^2-4x=0$ C) $x^2-2x+\sqrt{3}=0$ D) $x^2-4x+3=0$

Answer: $x^2-4x+1=0$

Solution: We know $\frac{1}{2}-\frac{\sqrt{3}}{2}i=-\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=-\omega$

Now, $\left(1-\sqrt{3}i\right)^{100}=(-2\omega)^{100}=2^{100}\omega$

$\Rightarrow 2^{100}\omega=2^{99}(p+iq)$

$\Rightarrow 2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=p+iq$

$\Rightarrow p=-1, q=\sqrt{3}$

So roots of the equation are $2-\sqrt{3}$ and $2+\sqrt{3}$

i.e. the required equation is $x^2-4x+1=0$

Q.14. The sum of all the values of x satisfying $\cos^{-1}x-2\sin^{-1}x=\cos^{-1}2x$ is

A) 0 B) 1 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

Answer: 0

Solution: Given:

$\cos^{-1}x - 2\sin^{-1}x = \cos^{-1}2x$

$\Rightarrow \frac{\pi}{2} - \sin^{-1}x - 2\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}2x$

$\Rightarrow 3\sin^{-1}x = \sin^{-1}2x$

$\Rightarrow \sin\left(3\sin^{-1}x\right) = \sin\left(\sin^{-1}2x\right)$

$\Rightarrow 3\sin\left(\sin^{-1}x\right) - 4\left[\sin\left(\sin^{-1}x\right)\right]^3 = \sin\left(\sin^{-1}2x\right)$

$\Rightarrow 3x - 4x^3 = 2x$

$\Rightarrow x - 4x^3 = 0$

$\Rightarrow x\left(1 - 4x^2\right) = 0$

$\Rightarrow x = 0,\ \frac{1}{2},\ -\frac{1}{2}$

Required sum is

$= 0 + \frac{1}{2} - \frac{1}{2} = 0$

Q.15. Let function $f(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right); & x \neq 0 \\ 0; & x = 0 \end{cases}$

A) $f(x)$ is continuous but not differentiable at $x = 0$

B) $f(x)$ is continuous at $x = 0$

C) $f'(x)$ is differentiable but not continuous

D) $f'(x)$ is continuous but not differentiable

Answer: $f(x)$ is continuous at $x = 0$

Solution: Given:

$$f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right); & x\neq 0 \\ 0; & x=0 \end{cases}$$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\left[x^2\sin\left(\frac{1}{x}\right)\right]$$

$$\Rightarrow \lim_{x\to 0} f(x)=\lim_{x\to 0} x^2\times[\text{number between } -1 \text{ to } 1]$$

$$\Rightarrow \lim_{x\to 0} f(x)=0=f(0)$$

Hence, $f(x)$ is continuous at $x=0$.

Now,

$$\text{RHD}=f'\left(0^+\right)=\lim_{h\to 0}\left[\frac{f(0+h)-f(0)}{h}\right]$$

$$\Rightarrow \text{RHD}=f'\left(0^+\right)=\lim_{h\to 0}\left[\frac{h^2\sin\left(\frac{1}{h}\right)-0}{h}\right]$$

$$\Rightarrow \text{RHD}=f'\left(0^+\right)=\lim_{h\to 0}\left[h\sin\left(\frac{1}{h}\right)\right]=0$$

And,

$$\text{LHD}=f'\left(0^-\right)=\lim_{h\to 0}\left[\frac{f(0-h)-f(0)}{-h}\right]$$

$$\Rightarrow \text{LHD}=f'\left(0^-\right)=\lim_{h\to 0}\left[\frac{-h^2\sin\left(\frac{1}{h}\right)-0}{-h}\right]=0$$

Since, $\text{LHD}=\text{RHD}=\text{finite}$

So, $f(x)$ is differentiable at $x=0$.

Now,

$$f'(x)=\begin{cases} 2x\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right); & x\neq 0 \\ 0; & x=0 \end{cases}$$

$$\Rightarrow f''(x)=\begin{cases} 2\sin\left(\frac{1}{x}\right)-\frac{2}{x}\cos\left(\frac{1}{x}\right)-\frac{1}{x^2}\sin\left(\frac{1}{x}\right); & x\neq 0 \\ 0; & x=0 \end{cases}$$

Since, $\lim_{x\to 0} f''(x)$ does not exists finitely, therefore $f'(x)$ is not differentiable at $x=0$.

Now,

$$\lim_{x\to 0} f'(x)=\lim_{x\to 0}\left[2x\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)\right]$$

$$\Rightarrow \lim_{x\to 0} f'(x)=\lim_{x\to 0}\left[2x\left\{\frac{1}{x}-\frac{1}{3!}\frac{1}{x^3}+\frac{1}{5!}\frac{1}{x^5}-\ldots.\right\}-\left\{1-\frac{1}{2!}\frac{1}{x^2}+\frac{1}{4!}\frac{1}{x^4}-\ldots.\right\}\right]$$

$$\Rightarrow \lim_{x\to 0} f'(x)=\lim_{x\to 0}\left[\left\{1-\frac{2}{3!}\frac{1}{x^2}+\frac{2}{5!}\frac{1}{x^4}-\ldots.\right\}-\left\{-\frac{1}{2!}\frac{1}{x^2}+\frac{1}{4!}\frac{1}{x^4}-\ldots.\right\}\right]$$

This limit does not exists finitely, hence $f'(x)$ is discontinuous at $x=0$.

Q.16. The distance of the point $P(1,2,3)$ from the plane containing the points $A(1,4,5)$, $B(2,3,4)$ and $C(3,2,1)$ is equal to:

A) $5\sqrt{2}$ B) $3\sqrt{2}$ C) 1 D) $\sqrt{2}$

Answer: $\sqrt{2}$

Solution: Given,

The plane is passing through the points $A(1,4,5),\ B(2,3,4)$ and $C(3,2,1)$,

Now direction ratio of normal vector of plane is given by,

$$\vec{n}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3-1 & 2-4 & 1-5\\ 2-1 & 3-4 & 4-5\end{vmatrix}$$

$$\Rightarrow \vec{n}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & -2 & -4\\ 1 & -1 & -1\end{vmatrix}$$

$$\Rightarrow \vec{n}=-2\hat{\imath}-2\hat{\jmath}+0\hat{k}$$

Now equation of plane is given by,

$$-2(x-1)-2(y-4)+0(z-5)=0$$

$$\Rightarrow 2x+2y-10=0$$

$$\Rightarrow x+y-5=0$$

Now distance of point $P(1,2,3)$ from plane $x+y-5=0$ is given by

Distance $D=\left|\frac{1\times1+2\times1+3\times0-5}{\sqrt{1^2+1^2+0^2}}\right|=\left|\frac{-2}{\sqrt{2}}\right|=\sqrt{2}$

Q.17. If $\vec{v}\cdot\vec{w}=2$ and $\vec{u}\times\vec{w}=\vec{v}+\alpha\vec{u}$, $\vec{u}=2\hat{\imath}+3\hat{\jmath}-4\hat{k}$, $\vec{v}=\hat{\imath}+2\hat{\jmath}-4\hat{k}$, then $\vec{u}\cdot\vec{w}$ is

A) $\frac{28}{12}$ B) $\frac{12}{29}$ C) $-\frac{29}{12}$ D) $\frac{29}{12}$

Answer: $\frac{29}{12}$

Solution: Given:

$\vec{v}\cdot\vec{w}=2$ and

$$\vec{u}=2\hat{\imath}+3\hat{\jmath}-4\hat{k}$$

$$\Rightarrow |\vec{u}|=\sqrt{4+9+16}=\sqrt{29}$$

Now,

$$\vec{u}\times\vec{w}=\vec{v}+\alpha\vec{u}$$

$$\Rightarrow \vec{u}\cdot\left(\vec{u}\times\vec{w}\right)=\vec{u}\cdot\vec{v}+\alpha|\vec{u}|^2$$

$$\Rightarrow \vec{u}\cdot\vec{v}+\alpha|\vec{u}|^2=0$$

$$\Rightarrow (2+6+16)+29\alpha=0$$

$$\Rightarrow \alpha=-\frac{24}{29}\quad\ldots(\mathrm{i})$$

Also,

$$\vec{u}\times\vec{w}=\vec{v}+\alpha\vec{u}$$

$$\Rightarrow \vec{w}\cdot\left(\vec{u}\times\vec{w}\right)=\vec{v}\cdot\vec{w}+\alpha\left(\vec{u}\cdot\vec{w}\right)$$

$$\Rightarrow \vec{v}\cdot\vec{w}+\alpha\left(\vec{u}\cdot\vec{w}\right)=0$$

$$\Rightarrow 2-\frac{24}{29}\left(\vec{u}\cdot\vec{w}\right)=0$$

$$\Rightarrow \left(\vec{u}\cdot\vec{w}\right)=\frac{29}{12}$$

Q.18. There are 12 languages. One can atmost chose 2 out of 5 particular languages. The number of ways in which one can select 5 languages is

Answer: 546

Solution: Case I -

When 0 language is selected from 5 particular language.

Total number of ways$= {}^{7}C_{5} = 21$

Case II -

When 1 language is selected from 5 particular language.

Total number of ways$= {}^{7}C_{4}{}^{5}C_{1} = 175$

Case III -

When 2 language is selected from 5 particular language.

Total number of ways$= {}^{7}C_{3}{}^{5}C_{2} = 350$

So, total number of required ways$= 546$

Q.19. How many 9 digit number can be formed using the digits $1,1,1,2,2,3,3,4,4$ in which even digits are at even places

Answer: 60

Solution: According to the question,

$1,\ 1,\ 1,\ 2,\ 2,\ 3,\ 3,\ 4,\ 4$

there are four even digits as 2 and 4 are repeating two times each.
So, the even digits will occupy the even places in $\frac{4!}{(2!\,2!)} = 6$ ways

3 and 1 are repeating two and three times respectively.
So, the remaining places will be occupied by odd digits in $\frac{5!}{(3!\,2!)} = 10$ ways

Therefore, total 9 digits that can be formed $= 6 \times 10 = 60$.
Hence, the answer is 60.

Q.20. Find the value of α if $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$

Answer: 1012

Solution: Given,

$\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$

Now taking L.H.S we get,

$\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = \sum_{r=1}^{2023} r^2 \cdot \frac{2023}{r} \cdot {}^{2022}C_{r-1}$

$\Rightarrow \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = \sum_{r=1}^{2023} r \cdot 2023 \cdot {}^{2022}C_{r-1}$

$\Rightarrow \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \sum_{r=1}^{2023} (r-1+1) \cdot {}^{2022}C_{r-1}$

$\Rightarrow \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \left[\sum_{r=1}^{2023} (r-1) \cdot {}^{2022}C_{r-1} + \sum_{r=1}^{2023} {}^{2022}C_{r-1}\right]$

$\Rightarrow \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \left[2022 \sum_{r=2}^{2023} {}^{2021}C_{r-2} + \sum_{r=1}^{2023} {}^{2022}C_{r-1}\right]$

$\Rightarrow \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \left[2022 \times 2^{2021} + 2^{2022}\right]$

$\Rightarrow \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times 2^{2022} [1011 + 1]$

$\Rightarrow \sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times 1012 \times 2^{2022}$

Now on comparing with $\sum_{r=0}^{2023} r^2 \cdot {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$ we get,

$\alpha = 1012$