## IOQM 2022-23 <br> Explanations for queries received

There were queries on Q13, Q17, Q19 and Q22. There were no significant queries on other questions.
13. A triangle with the maximum angle for $\angle A B C=59^{\circ}$ is shown below.

17. We need the smallest positive integer $N$ such that $f(f(f(N)))=97$. Note that 389 is a prime and

$$
f(f(f(389)))=f(f(388))=f(194)=97
$$

Thus the answer is $\lfloor\sqrt{389}\rfloor=19$.
19. The question asks for the number of values $x+2 y+3 z$ can take when $x+11 y+111 z=1000$. Note that when $x+11 y+111 z=1000$, we will have $x+2 y+3 z=n$ 1's in the string.

It is not difficult to see that under the condition $x+11 y+111 z=1000, x+2 y+3 z$ takes 108 values. Hence the answer is $1+0+8=09$.
22. It is not difficult to show that for $n \geq 5, F_{n}$ satisfies the recurrence

$$
F_{n}=F_{n-1}+F_{n-3}+F_{n-4}
$$

It is easy to see that $F_{1}=0, F_{2}=1, F_{3}=3$ and $F_{4}=4$. Using the recurrence, we compute the values of $F_{n}$ to obtain

$$
0,1,3,4,5,9,16,25,39,64,105
$$

and hence the smallest value of $n$ for which $F_{n}>100$ is 11 .

## IOQM 2022-23 Final Answers

| QNo | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Answer | 10 | 67 | 60 | 10 | 95 | 43 | 35 | 32 | $*$ | 88 | 08 | $*$ |
| QNo | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| Answer | 59 | 16 | 25 | 18 | 19 | 70 | 09 | 03 | 74 | 11 | 29 | 81 |

*     - The questions 9 and 12 had errors in their statements and all students will be awarded marks for them.

