

AREA AND PERIMETER

13

13.0 Introduction

Ira wants to find the area of her agricultural land, which is irregular in shape (Figure 1). So she divided her land into some regular shapes- triangles, rectangle, parallelogram, rhombus and square (Figure 2). She thought, ‘if I know the area of all these parts, I will know the area of my land.’



Figure 1

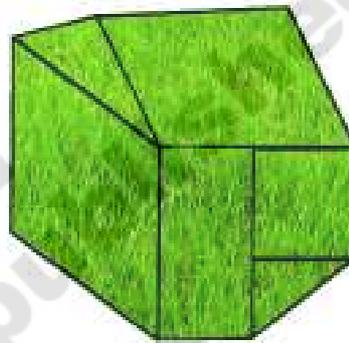


Figure 2

We have learnt how to find the area of a rectangle and square in earlier classes. In this chapter we will learn how to find the area of a parallelogram, triangle, rhombus. First let us review what we have learnt about the area and perimeter of a square and rectangle in earlier classes.



Exercise - 1

- Complete the table given below.

Diagram	Shape	Area	Perimeter
	Rectangle	$l \times b = lb$	_____
	Square	_____	4a

2. The measurements of some squares are given in the table below. However, they are incomplete. Find the missing information.

Side of a square	Area	Perimeter
15 cm	225 cm ²	
		88 cm

3. The measurements of some rectangles are given in the table below. However, they are incomplete. Find the missing information.

Length	Breadth	Area	Perimeter
20 cm	14 cm		
	12 cm		60 cm
15 cm		150 cm	

13.1 Area of a parallelogram

Look at the shape in Figure 1. It is a parallelogram. Now let us learn how to find its area-

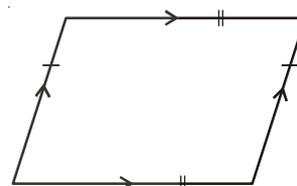


Figure 1

Activity 1

- Draw a parallelogram on a sheet of paper.
- Cut out the parallelogram.
- Now cut the parallelogram along the dotted line as shown in Figure 2 and separate the triangular shaped piece of paper.
- Keep the triangle on the other side as shown in Figure 3 and see if both the pieces together form a rectangle.

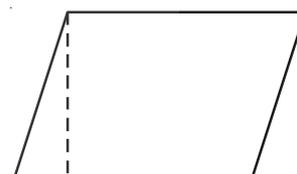


Figure 2

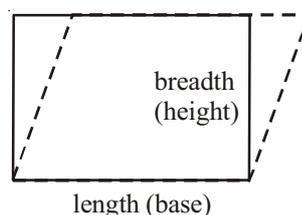


Figure 3

Can we say that the area of the parallelogram in Figure 2 equal to the area of the rectangle in Figure 3? You will find this to be true.

As you can see from the above activity the area of the parallelogram is equal to the area of the rectangle.

We know that the area of the rectangle is equal to length \times breadth.
We also know that the length of the rectangle is equal to the base of the parallelogram and the breadth of the rectangle is equal to its height.

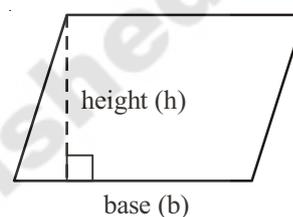


Therefore, Area of parallelogram = Area of rectangle

$$= \text{length} \times \text{breadth}$$

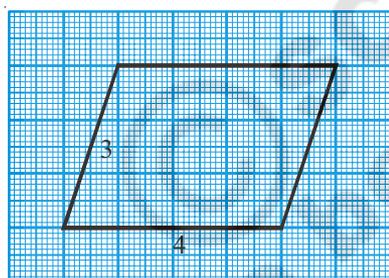
$$= \text{base} \times \text{height} \quad (\text{length} = \text{base} ; \text{breadth} = \text{height})$$

Thus, the area of the parallelogram is equal to the product of its base (b) and corresponding height (h) i.e., $A = bh$



Example 1 : Find the area of each parallelogram.

(i)



Solution :

Base (b) of a parallelogram = 4 units

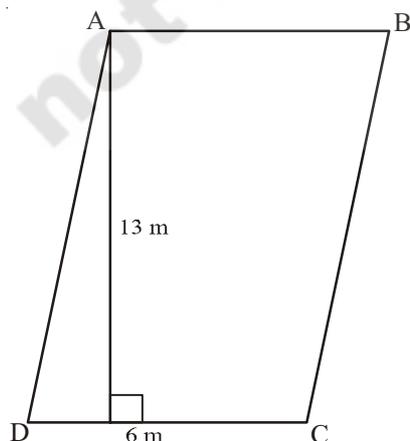
Height (h) of a parallelogram = 3 units

Area (A) of a parallelogram = bh

Therefore, $A = 4 \times 3 = 12$ sq. units

Thus, area of the parallelogram is 12 sq. units.

(ii)



Solution :

Base of a parallelogram (b) = 6 m.

Height of a parallelogram (h) = 13 m.

Area of a parallelogram (A) = bh

Therefore, $A = 6 \times 13 = 78$ m²

Thus, area of parallelogram ABCD is 78 m²



Try This

ABCD is a parallelogram with sides 8 cm and 6 cm. In Figure 1, what is the base of the parallelogram? What is the height? What is the area of the parallelogram?

In Figure 2, what is the base of the parallelogram? What is the height? What is the area of the parallelogram? Is the area of Figure 1 and Figure 2 the same?

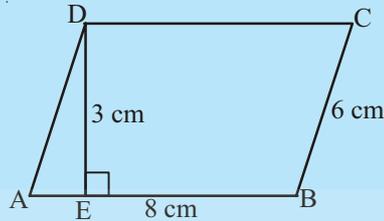


Figure 1

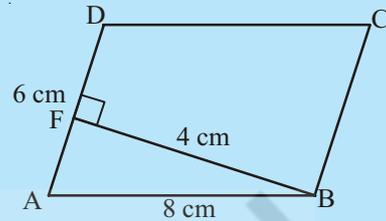
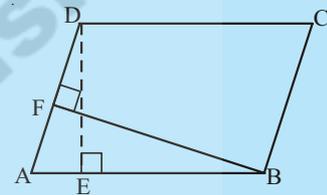


Figure 2

Any side of a parallelogram can be chosen as base of the parallelogram. In Figure 1, DE is the perpendicular falling on AB. Hence AB is the base and DE is the height of the parallelogram. In Figure 2, BF is the perpendicular falling on side AD. Hence, AD is the base and BF is the height.

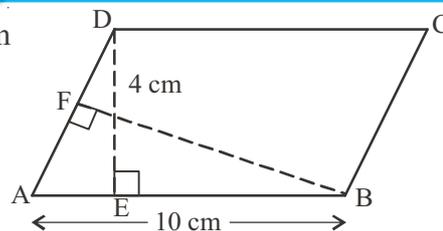


Do This

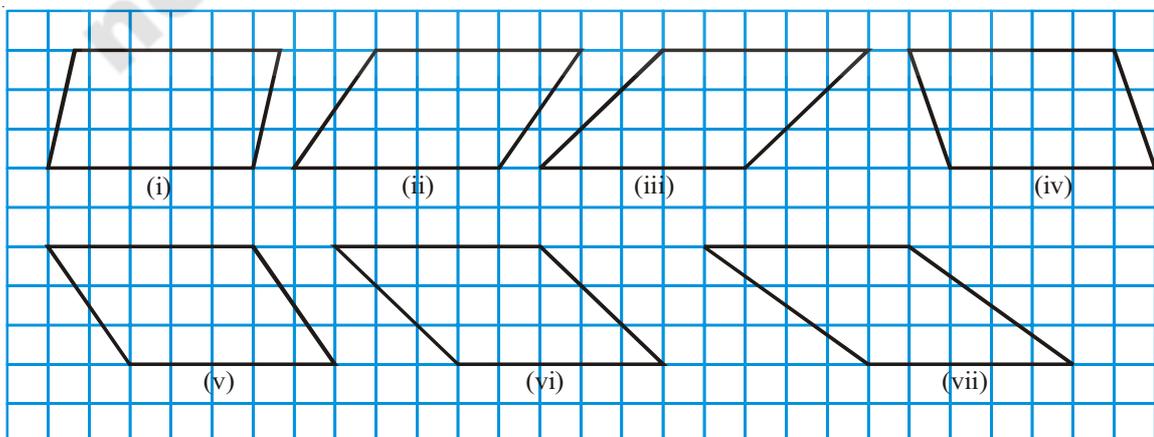
- In parallelogram ABCD, $AB = 10$ cm and $DE = 4$ cm

Find (i) The area of ABCD.

(ii) The length of BF, if $AD = 6$ cm



- Carefully study the following parallelograms.



- (i) Find the area of each parallelogram by counting the squares enclosed in it. For counting incomplete squares check whether two incomplete squares make a complete square in each parallelogram.

Complete the following table accordingly-

Parallelogram	Base	Height	Area	Area by counting squares		
				No. of full squares	No. of incomplete squares	Total squares (full)
(i)	5 units	3 units	$5 \times 3 = 15$ sq. units	12	6	15
(ii)						
(iii)						
(iv)						
(v)						
(vi)						
(vii)						

- (ii) Do parallelograms with equal bases and equal heights have the same area?



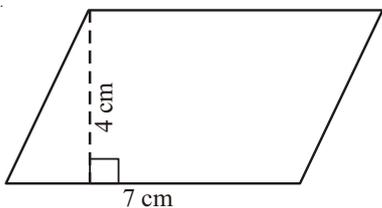
Try This

- Why is the formula for finding the area of a rectangle related to the formula for finding the area of a parallelogram?
- Explain why a rectangle is a parallelogram but a parallelogram may not be a rectangle.

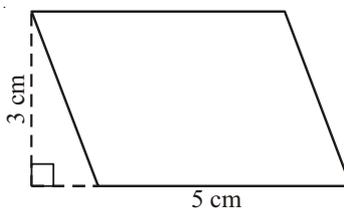


Exercise - 2

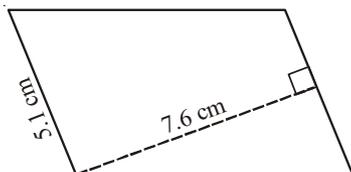
1. Find the area of the each of the following parallelograms.



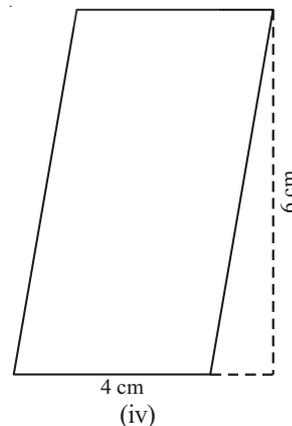
(i)



(ii)

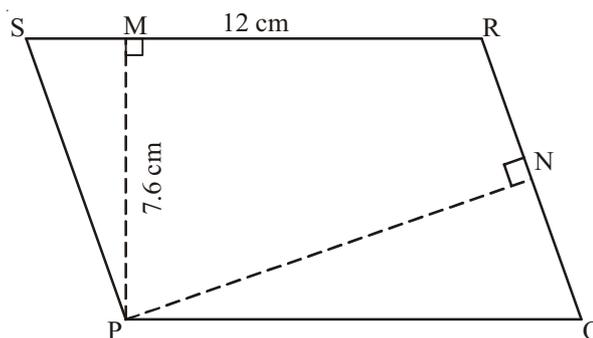


(iii)

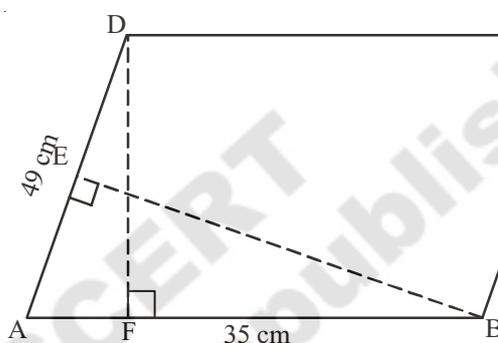


(iv)

2. PQRS is a parallelogram. PM is the height from P to \overline{SR} and PN is the height from P to \overline{QR} . If $SR = 12$ cm and $PM = 7.6$ cm.



- (i) Find the area of the parallelogram PQRS
- (ii) Find PN, if $QR = 8$ cm.
3. DF and BE are the height on sides AB and AD respectively in parallelogram ABCD. If the area of the parallelogram is 1470 cm^2 , $AB = 35$ cm and $AD = 49$ cm, find the length of BE and DF.



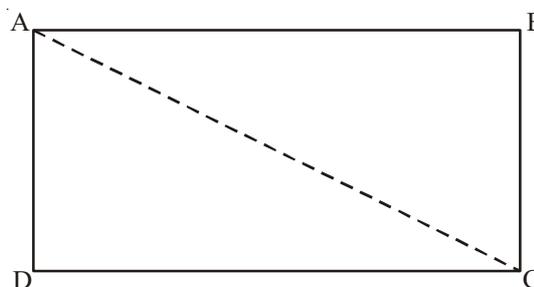
4. The height of a parallelogram is one third of its base. If the area of the parallelogram is 192 cm^2 , find its height and base.
5. In a parallelogram the base and height are in the ratio of 5:2. If the area of the parallelogram is 360 m^2 , find its base and height.
6. A square and a parallelogram have the same area. If a side of the square is 40m and the height of the parallelogram is 20m, find the base of the parallelogram.

13.2 Area of triangle

13.2.1 Triangles are parts of rectangles

Draw a rectangle Cut the rectangle along its diagonal to get two triangles.

Superimpose one triangle over the other. Are they exactly the same in area? Can we say that the triangles are congruent?



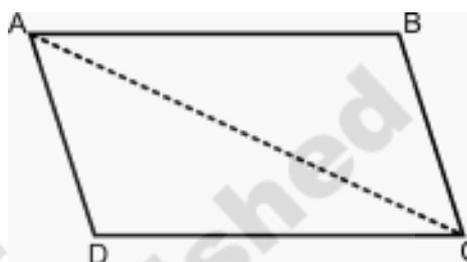
You will find that both the triangles are congruent. Thus, the area of the rectangle is equal to the sum of the area of the two triangles.

Therefore, the area of each triangle = $\frac{1}{2} \times (\text{area of rectangle})$

$$= \frac{1}{2} \times (l \times b) = \frac{1}{2} lb$$

13.2.2 Triangles are parts of parallelograms

Make a parallelogram as shown in the Figure. Cut the parallelogram along its diagonal. You will get two triangles. Place the triangles one on top of each other. Are they exactly the same size (area)?



You will find that the area of the parallelogram is equal to the area of both the triangles.

We know that area of parallelogram is equal to product of its base and height. Therefore,

$$\text{Area of each triangle} = \frac{1}{2} \times (\text{area of parallelogram})$$

$$\text{Area of triangle} = \frac{1}{2} \times (\text{base} \times \text{height})$$

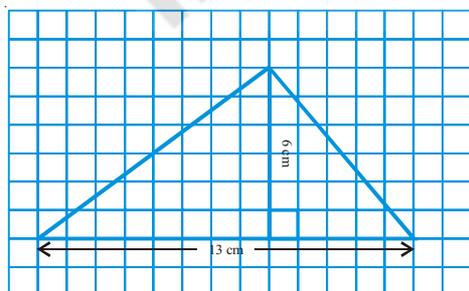
$$= \frac{1}{2} \times b \times h = \frac{1}{2} bh$$



Thus, the area of a triangle is equal to half the product of its base (b) and height (h) i.e.,

$$A = \frac{1}{2} bh$$

Example 2 : Find the area of the triangle.



Solution :

Base of triangle (b) = 13 cm

Height of triangle (h) = 6 cm

$$\text{Area of a triangle (A)} = \frac{1}{2} (\text{base} \times \text{height}) \text{ or } \frac{1}{2} bh$$

$$\text{Therefore, } A = \frac{1}{2} \times 13 \times 6$$

$$= 13 \times 3 = 39 \text{ cm}^2$$

Thus the area of the triangle is 39 cm^2

Example 3: Find the area of $\triangle ABC$.

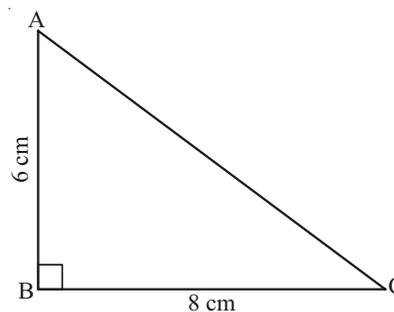
Solution: Base of the triangle (b) = 8 cm

Height of the triangle (h) = 6 cm

$$\text{Area of the triangle (A)} = \frac{1}{2} bh$$

$$\text{Therefore, } A = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Thus, the area of $\triangle ABC = 24 \text{ cm}^2$



Notice that in a right angle triangle two of its sides can be the height.

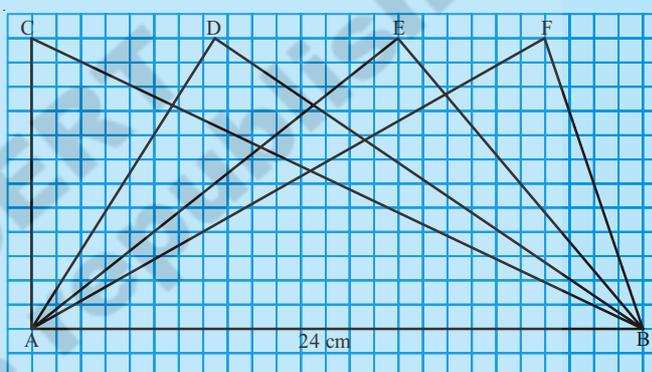


Try This

In Figure all the triangles are on the base AB = 24 cm. Is the height of each of the triangles drawn on base AB, the same?

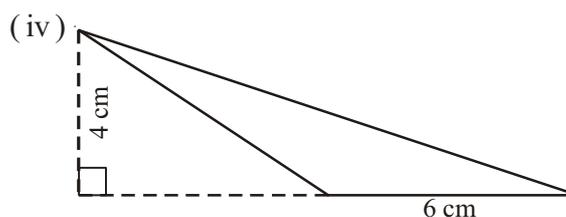
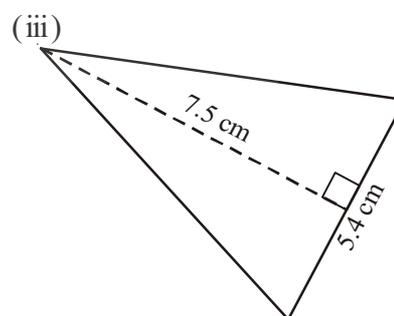
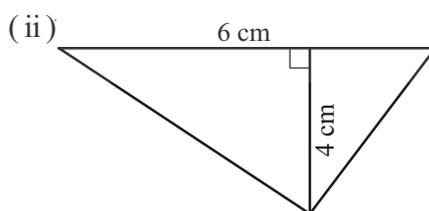
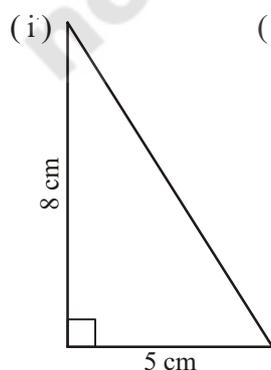
Will all the triangles have equal area? Give reasons to support your answer.

Are the triangles congruent also?

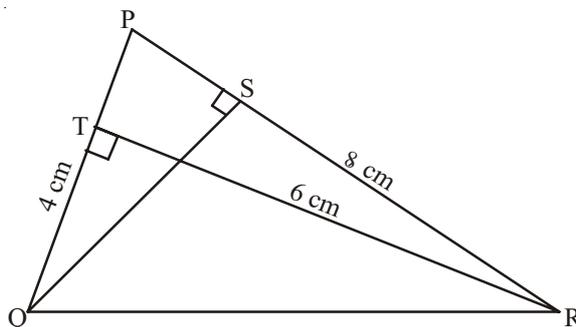


Exercise - 3

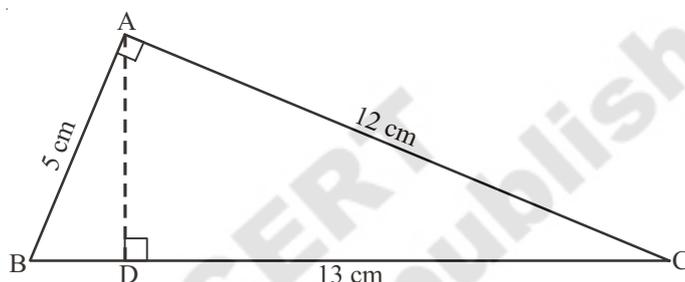
1. Find the area of each of the following triangles.



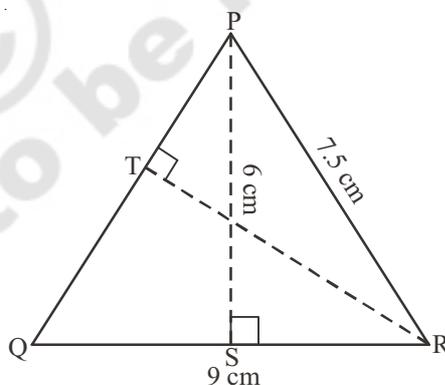
2. In ΔPQR , $PQ = 4$ cm, $PR = 8$ cm and $RT = 6$ cm. Find (i) the area of ΔPQR (ii) the length of QS .



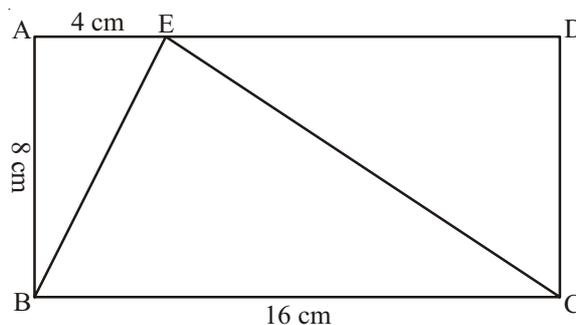
3. ΔABC is right-angled at A . AD is perpendicular to BC , $AB = 5$ cm, $BC = 13$ cm and $AC = 12$ cm. Find the area of ΔABC . Also, find the length of AD .



4. ΔPQR is isosceles with $PQ = PR = 7.5$ cm and $QR = 9$ cm. The height PS from P to QR , is 6 cm. Find the area of ΔPQR . What will be the height from R to PQ i.e. RT ?

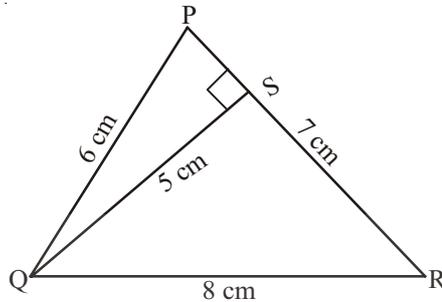


5. $ABCD$ rectangle with $AB = 8$ cm, $BC = 16$ cm and $AE = 4$ cm. Find the area of ΔBCE . Is the area of ΔBEC equal to the sum of the area of ΔBAE and ΔCDE . Why?

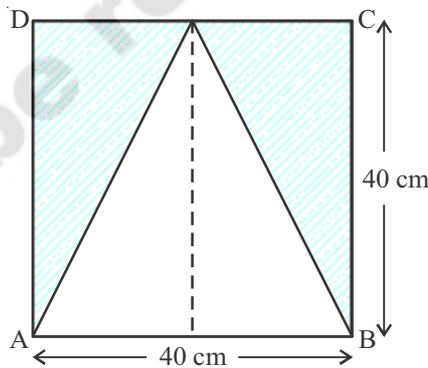


6. Ramu says that the area of ΔPQR is, $A = \frac{1}{2} \times 7 \times 5 \text{ cm}^2$

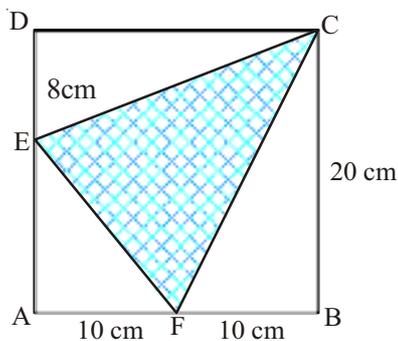
Gopi says that it is, $A = \frac{1}{2} \times 8 \times 5 \text{ cm}^2$. Who is correct? Why?



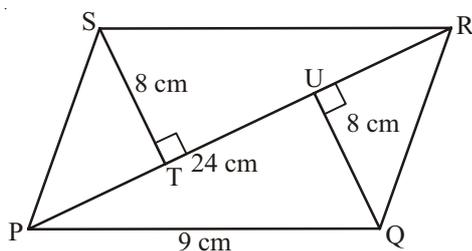
7. Find the base of a triangle whose area is 220 cm^2 and height is 11 cm .
8. In a triangle the height is double the base and the area is 400 cm^2 . Find the length of the base and height.
9. The area of triangle is equal to the area of a rectangle whose length and breadth are 20 cm and 15 cm respectively. Calculate the height of the triangle if its base measures 30 cm .
10. In Figure ABCD find the area of the shaded region.



11. In Figure ABCD, find the area of the shaded region.



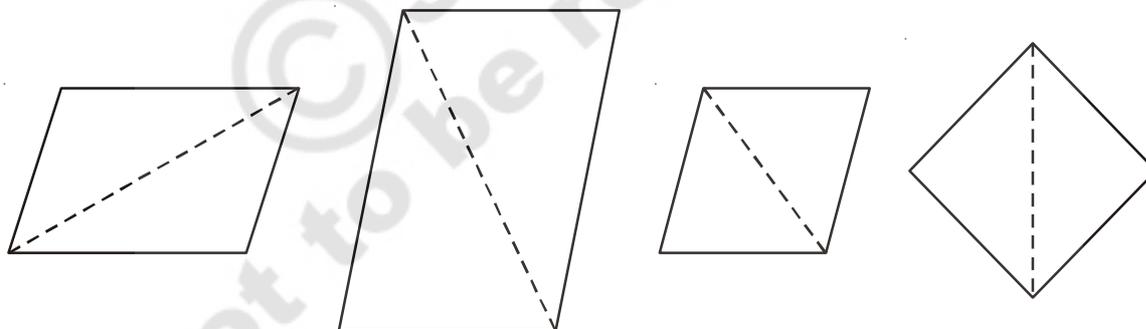
12. Find the area of a parallelogram PQRS, if $PR = 24$ cm and $QU = ST = 8$ cm.



13. The base and height of the triangle are in the ratio 3:2 and its area is 108 cm^2 . Find its base and height.

13.3 Area of a rhombus

Santosh and Akhila are good friends. They are fond of playing with paper cut-outs. One day, Santosh gave different triangle shapes to Akhila. From these she made different shapes of parallelograms. These parallelograms are given below-



Santosh asked Akhila, 'which parallelograms has 4 equal sides?'

Akhila said, 'the last two have equal sides.'

Santosh said, '**If all the sides of a parallelogram are equal, it is called a Rhombus.**'

Now let us learn how to calculate the area of a Rhombus.

Like in the case of a parallelogram and triangle, we can use the method of splitting into congruent triangles to find the area of a rhombus.

ABCD is a rhombus.

Area of rhombus ABCD = (area of $\triangle ACD$) + (area of $\triangle ACB$)

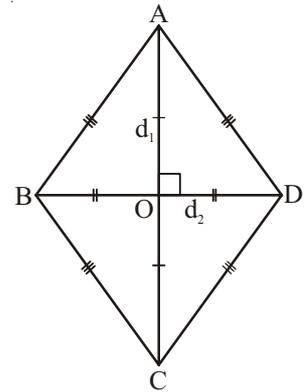
$$= \left(\frac{1}{2} \times AC \times OD \right) + \left(\frac{1}{2} \times AC \times OB \right)$$

diagonals bisect perpendicularly

$$= \frac{1}{2} AC \times (OD + OB)$$

$$= \frac{1}{2} AC \times BD$$

$$= \frac{1}{2} d_1 \times d_2 \text{ (as } AC = d_1 \text{ and } BD = d_2)$$



In other words, the area of a rhombus is equal to half the product of its diagonals i.e.,

$$A = \frac{1}{2} d_1 d_2$$

Example 4 : Find the area of rhombus ABCD

Solution : Length of the diagonal (d_1) = 7.5 cm

Length of the other diagonal (d_2) = 5.6 cm

$$\text{Area of the rhombus (A)} = \frac{1}{2} d_1 d_2$$

$$\text{Therefore, } A = \frac{1}{2} \times 7.5 \times 5.6 = 21 \text{ cm}^2$$

Thus, area of rhombus ABCD = 21 cm²

Example 5 : The area of a rhombus is 60 cm² and one of its diagonals is 8 cm. Find the other diagonal.

Solution : Length one diagonal (d_1) = 8 cm

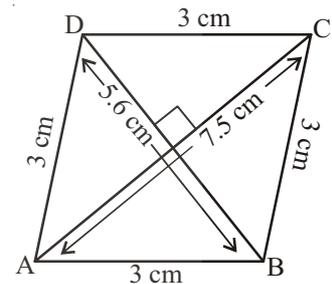
Length of the other diagonal = d_2

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$\text{Therefore, } 60 = \frac{1}{2} \times 8 \times d_2$$

$$d_2 = 15 \text{ cm.}$$

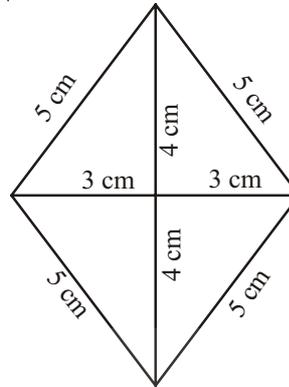
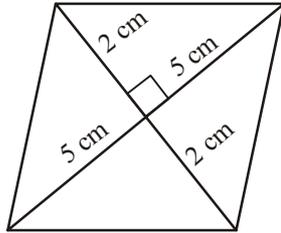
Thus, length of the other diagonal is 15 cm.





Exercise 4

1. Find the area of the following rhombuses.



2. Find the missing values.

Diagonal-1 (d_1)	Diagonal-2 (d_2)	Area of rhombus
12 cm	16 cm	
27 mm		2025 mm ²
24 m	57.6 m	

3. If length of diagonal of a rhombus whose area 216 sq. cm. is 24 cm. when find the length of second diagonal.
4. The floor of a building consists of 3000 tiles which are rhombus shaped. The diagonals of each of the tiles are 45 cm and 30 cm. Find the total cost of polishing the floor, if cost per m² is ₹.25.

13.4 Circumference of a circle

Nazia is playing with a cycle tyre. She is rotating the tyre with a stick and running along with it.

What is the distance covered by tyre in one rotation?

The distance covered by the tyre in one rotation is equal to the length around the wheel. The length around the tyre is also called the circumference of the tyre .

What is the relation between the total distance covered by the tyre and number of rotations?



Total distance covered by the tyre = number of rotations \times length around the tyre.



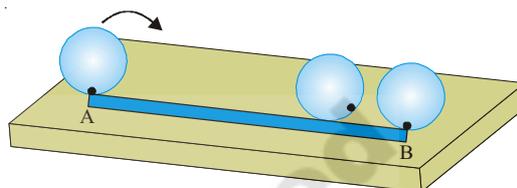
Activity 2

Jaya cut a circular shape from a cardboard. She wants to stick lace around the card to decorate it. Thus, the length of the lace required by her is equal to the circumference of the card. Can she measure the circumference of the card with the help of a ruler?



Let us see what Jaya did?

Jaya drew a line on the table and marked its starting point A. She then made a point on the edge of the card. She placed the circular card on the line, such that the point on the card coincided with point A. She then rolled the card along the line, till the point on the card touched the line again. She marked this point B. The length of line AB is the circumference of the circular card. The length of the lace required around the circular card is the distance AB.



Try This

Take a bottle cap, a bangle or any other circular object and find its circumference using a string.

It is not easy to find the circumference of every circular shape using the above method. So we need another way for doing this. Let us see if there is any relationship between the diameter and the circumference of circles.

A man made six circles of different radii with cardboard and found their circumference using a string. He also found the ratio between the circumference and diameter of each circle.

He recorded his observations in the following table-

Circle	Radius	Diameter	Circumference	Ratio of circumference and diameter
1.	3.5 cm	7.0 cm	22.0 cm	$\frac{22}{7} = 3.14$
2.	7.0 cm	14.0 cm	44.0 cm	$\frac{44}{14} = 3.14$
3.	10.5 cm	21.0 cm	66.0 cm	
4.	21.0 cm	42.0 cm	132.0 cm	
5.	5.0 cm	10.0 cm	32.0 cm	
6.	15.0cm	30.0 cm	94.0 cm	

What can you infer from the above table? Is the ratio between the circumference and the diameter of each circle approximately the same? Can we say that the circumference of a circle is always about three times its diameter?

The approximate value of the ratio of the circumference to the diameter of a circle is $\frac{22}{7}$ or 3.14.

Thus it is a constant and is denoted by π (pi).

Therefore, $\frac{c}{d} = \pi$ where 'c' is the circumference of the circle and 'd' its diameter.

Since, $\frac{c}{d} = \pi$

$$c = \pi d$$

Since, diameter of a circle is twice the radius i.e. $d = 2r$

$$c = \pi \times 2r \quad \text{or} \quad c = 2\pi r$$

Thus, circumference of a circle = πd or $2\pi r$

Example 6 : Find the circumference of a circle with diameter 10 cm. (Take $\pi = 3.14$)

Solution : Diameter of a circle (d) = 10 cm.

$$\begin{aligned} \text{Circumference of circle (c)} &= \pi d \\ &= 3.14 \times 10 \\ c &= 31.4 \text{ cm} \end{aligned}$$

Thus, the circumference of the circle is 31.4 cm.

Example 7 : Find the circumference of a circle with radius 14 cm. (Take $\pi = \frac{22}{7}$)

Radius of a circle (r) = 14 cm

Circumference of a circle (c) = $2\pi r$

$$\text{Therefore, } c = 2 \times \frac{22}{7} \times 14$$

$$c = 88 \text{ cm}$$

Thus, the circumference of the circle is 88 cm.



Exercise - 5

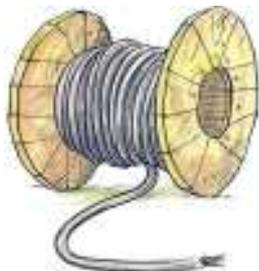
- Find the circumference of a circle whose radius is-
(i) 35 cm (ii) 4.2 cm (iii) 15.4 cm
- Find the circumference of circle whose diameter is-
(i) 17.5 cm (ii) 5.6 cm (iii) 4.9 cm

Note : take $\pi = \frac{22}{7}$ in the above two questions.

- Taking $\pi = 3.14$, find the circumference of a circle whose radius is
(a) 8 cm (b) 15 cm (c) 20 cm
 - Calculate the radius of a circle whose circumference is 44cm?
- If the circumference of a circle is 264 cm, find its radius. Take $\pi = \frac{22}{7}$.
- If the circumference of a circle is 33 cm, find its diameter.
- How many times will a wheel of radius 35cm be rotated to travel 660 m?
(Take $\pi = \frac{22}{7}$).
- The ratio of the diameters of two circles is 3 : 4. Find the ratio of their circumferences.
- A road roller makes 200 rotations in covering 2200 m.
Find the radius of the roller.
- The minute hand of a circular clock is 15 cm.
How far does the tip of the minute hand move in 1 hour?
(Take $\pi = 3.14$)



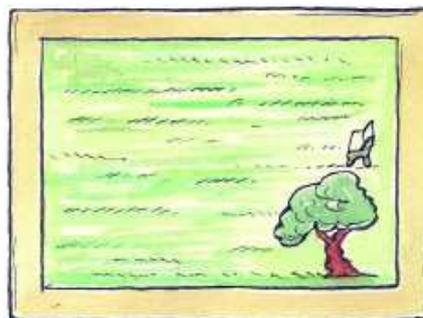
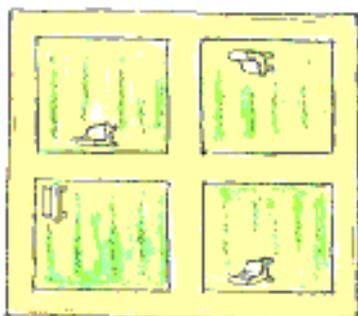
10.



A wire is bent in the form of a circle with radius 25 cm. It is straightened and made into a square. What is the length of the side of the square?

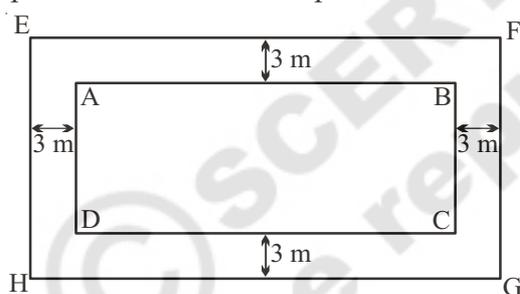


13.5 Rectangular Paths



We often come across such walking paths in garden areas. Now we shall learn how to measure the areas of such paths as this often useful in calculating their costs of construction.

Example 8 : A plot is 60m long and 40m wide. A path 3m wide is to be constructed around the plot. Find the area of the path.



Solution : Let ABCD be the given plot. A 3m wide path is running all around it. To find the area of this path we have to subtract the area of the smaller rectangle ABCD from the area of the bigger rectangle EFGH.

Length of inner rectangle	=	60m
Breadth of inner rectangle	=	40m
Area of the plot ABCD	=	$(60 \times 40) \text{ m}^2$
	=	2400 m^2
Width of the path	=	3m
Length of outer rectangle	=	$60 \text{ m} + (3+3) \text{ m}$
	=	66 m
Breadth of outer rectangle	=	$40 \text{ m} + (3+3) \text{ m}$
	=	46 m

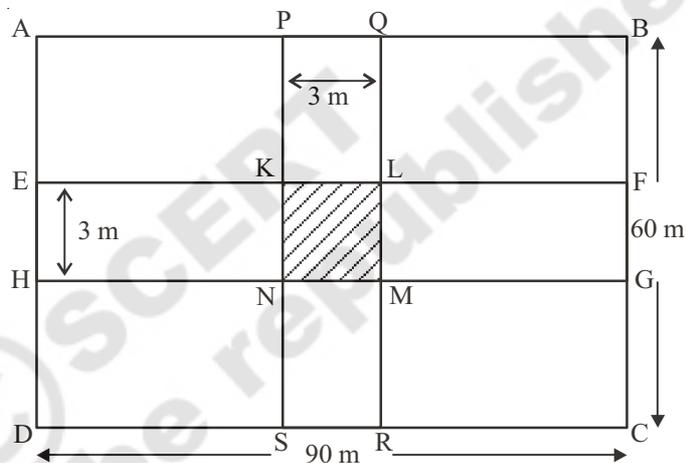
$$\begin{aligned} \text{Area of the outer rectangle} &= 66 \times 46 \text{ m}^2 \\ &= 3036 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Therefore, area of the path} &= (3036 - 2400) \text{ m}^2 \\ &= 636 \text{ m}^2 \end{aligned}$$

Example 9 : The dimensions of a rectangular field are 90 m and 60 m. Two roads are constructed such that they cut each other at the centre of the field and are parallel to its sides. If the width of each road is 3 m, find-

- (i) The area covered by the roads.
- (ii) The cost of constructing the roads at the rate of ₹110 per m^2 .

Solution : Let ABCD be the rectangular field. PQRS and EFGH are the 3m roads.



- (i) Area of the crossroads is the area of the rectangle PQRS and the area of the rectangle EFGH. As is clear from the picture, the area of the square KLMN will be taken twice in this calculation thus needs to be subtracted once.

From the question we know that,

$$PQ = 3 \text{ m, and } PS = 60 \text{ m}$$

$$EH = 3 \text{ m, and } EF = 90 \text{ m}$$

$$KL = 3 \text{ m } \quad KN = 3 \text{ m}$$

$$\begin{aligned} \text{Area of the roads} &= \text{Area of the rectangle PQRS} + \text{area of the rectangle EFGH} \\ &\quad - \text{Area of the square KLMN} \end{aligned}$$

$$= (PS \times PQ) + (EF \times EH) - (KL \times KN)$$

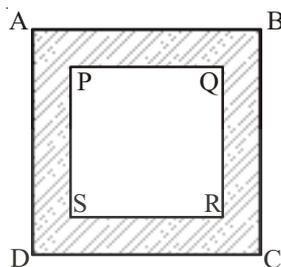
$$= (60 \times 3) + (90 \times 3) - (3 \times 3)$$

$$= (180 + 270 - 9) \text{ m}^2$$

$$= 441 \text{ m}^2$$

(ii) Cost of construction = ₹110 × m²
 Cost of constructing the roads = 110 × 441
 = ₹48,510

Example 10 : A path 5m wide runs around a square park of side 100m. Find the area of the path. Also find the cost of cementing it at the rate of ₹250 per 10m²



Solution : Let PQRS be the square park of the side 100 m. The shaded region represents the 5m wide path.

Length of AB = 100 + (5 + 5) = 110 m

Area of the square PQRS = (side)² = (100 m)² = 10000 m²

Area of the square ABCD = (side)² = (110 m)² = 12100 m²

Therefore, area of the path = (12100 - 10000) = 2100 m²

Cost of the cementing per 10 m² = ₹250

Therefore, cost of the cementing 1 m² = $\frac{250}{10}$

Thus, cost of cementing 2100 m² = $\frac{250}{10} \times 2100$

= ₹52,500



Exercise - 6

1. A path 2.5 m wide is running around a square field whose side is 45 m. Determine the area of the path.
2. The central hall of a school is 18m long and 12.5 m wide. A carpet is to be laid on the floor leaving a strip 50 cm wide near the walls, uncovered. Find the area of the carpet and also the uncovered portion?

3. The length of the side of a grassy square plot is 80 m. Two walking paths each 4 m wide are constructed parallel to the sides of the plot such that they cut each other at the centre of the plot. Determine the area of the paths.
4. A verandah 2 m wide is constructed all around a room of dimensions 8 m × 5 m. Find the area of the verandah
5. The length of a rectangular park is 700 m and its breadth is 300 m. Two crossroads, each of width 10 m, cut the centre of a rectangular park and are parallel to its sides. Find the area of the roads. Also, find the area of the park excluding the area of the crossroads.



Looking Back

- The area of the parallelogram is equal to the product of its base (b) and corresponding height (h) i.e., $A = bh$. Any side of the parallelogram can be taken as the base.
- The area of a triangle is equal to half the product of its base (b) and height (h) i.e., $A = \frac{1}{2} bh$.
- The area of a rhombus is equal to half the product of its diagonals i.e., $A = \frac{1}{2} d_1 d_2$.
- The circumference of a circle = $2 \pi r$ where r is the radius of the circle and $\pi = \frac{22}{7}$ or 3.14.

Archimedes (Greece)

287 - 212 BC

He calculated the value of π first time.

He also evolved the mathematical formulae for finding out the circumference and area of a circle.

