JEE Main 2021 August 26, Shift 2 (Mathematics)

1. If matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, then the value of $A^{2025} - A^{2020}$ is:

(1) $A^6 - A$

(2) A^4

(3) A^5

(4) $A^5 - A$

Ans. (1)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

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$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

From the obtained general matrix A^n, we can get

$$A^{2025} - A^{2020} = \begin{bmatrix} 1 & 0 & 0 \\ 2024 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 2019 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^6 - A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, $A^{2025} - A^{2020} = A^6 - A$

2. The value of $2sin\left(\frac{\pi}{8}\right) sin\left(\frac{2\pi}{8}\right) sin\left(\frac{3\pi}{8}\right) \ldots \ldots sin\left(\frac{7\pi}{8}\right)$ is

(1) $\frac{1}{16}$

(2) $\frac{1}{8}$

(3) $\frac{1}{4}$

(4) $\frac{1}{2}$

Ans. (2)

Given

$2sin\left(\frac{\pi}{8}\right)sin\left(\frac{2\pi}{8}\right)sin\left(\frac{3\pi}{8}\right)\ldots\ldots sin\left(\frac{7\pi}{8}\right)$

$= 2sin\left(\frac{\pi}{8}\right)sin\left(\frac{2\pi}{8}\right)sin\left(\frac{3\pi}{8}\right)\ldots\ldots sin\left(\pi-\frac{2\pi}{8}\right)sin\left(\pi-\frac{\pi}{8}\right)$ (As $sin(\pi-\theta)$=$sin\theta$)

$= 2sin^2\frac{\pi}{8}sin^2\left(\frac{2\pi}{8}\right)sin^2\left(\frac{3\pi}{8}\right)\cdot sin\left(\frac{\pi}{2}\right)$

$= sin^2\frac{\pi}{8}sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$= sin^2\frac{\pi}{8}cos^2\frac{\pi}{8} = \frac{1}{4}\times 4sin^2\frac{\pi}{8}cos^2\frac{\pi}{8} = \frac{1}{4}sin^2\frac{\pi}{4} = \frac{1}{8}$

3. Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{1+sin^2x}{1+\pi^{sinx}}dx$

(1) $\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

(3) $\frac{\pi}{2}$

(4) $\frac{5\pi}{2}$

Ans. (2)

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{1+\sin^2x}{1+\pi^{\sin x}}dx \quad \ldots(i)$

Applying property of definite integral $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx$

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{1+\sin^2x}{1+\pi^{-\sin x}}dx \quad \ldots(ii)$ (As $sin(-\theta) = -sin(\theta)$)

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{\pi^{sinx}(1+sin^2x)}{1+\pi^{sinx}}dx \quad \ldots(iii)$

Adding equation (i) and (iii)

$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}(1+sin^2x)dx$

$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(1+\frac{1-cos2x}{2}\right)dx$

$2I = \frac{1}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}(3 - cos2x)dx$

$2I = \frac{1}{2}\left[3x - \frac{sin2x}{2}\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$

$2I = \frac{3\pi}{2} - 0 = \frac{3\pi}{2}$

$\Rightarrow I = \frac{3\pi}{4}$

4. The locus of mid-point of chord of the hyperbola $x^2 - y^2 = 4$ which touches $y^2 = 8x$ is:

(1) $x^3 + 2y^2 - y^2x = 0$

(2) $x^3 + 2y^2 + y^2x = 0$

(3) $x^3 - 2y^2 - y^2x = 0$

(4) $x^3 + 3y^2 - y^2x = 0$

Ans. (1)

Tangent to $y^2 = 8x$ is $y = mx + \frac{2}{m}$.... (i)

The equation of chord with mid-point (h, k) is $T = S_1$

$hx - ky - 4 = h^2 - k^2 - 4$

$\Rightarrow ky = hx \ + k^2 - h^2 \ldots.(ii)$ is the equation of chord with mid-point (h, k)

Comparing the above equations we get,

$\frac{h}{m} = \frac{k}{1} = \frac{k^2-h^2}{\frac{2}{m}}$

$\Rightarrow m = \frac{h}{k}$ and $\frac{2}{m} = \frac{k^2-h^2}{k}$

$\Rightarrow \frac{2k}{h} = \frac{k^2-h^2}{k}$

$\Rightarrow h^3 + 2k^2 - k^2\,h = 0$

Replacing $h \rightarrow x$ & $k \rightarrow y$

$\therefore$ Equation of locus is $x^3 + 2y^2 - y^2x = 0$

5. $\sum_{r=1}^{50} \tan^{-1}\left(\frac{1}{2r^2}\right) = P$, then $\tan P$ is

(1) $\frac{50}{51}$

(2) $\frac{100}{101}$

(3) $\frac{101}{100}$

(4) $\frac{51}{50}$

Ans. (1)

$tan^{-1}\left(\frac{1}{2r^2}\right) = tan^{-1}\left(\frac{2}{4r^2}\right)$

$= tan^{-1}\left(\frac{2}{1+4r^2-1}\right)$

$= tan^{-1}\left(\frac{2}{1+(2r-1)(2r+1)}\right)$

$= tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$ $\left(\because tan^{-1}A - tan^{-1}B = tan^{-1}\left(\frac{A-B}{1+AB}\right)\right)$

$= tan^{-1}(2r+1) - tan^{-1}(2r-1)$

So $\sum_{r=1}^{50} \tan^{-1}\left(\frac{1}{2r^2}\right) = \tan^{-1}(3) - \tan^{-1}(1)$

$+tan^{-1}(5) - tan^{-1}(3)$

$+tan^{-1}(7) - tan^{-1}(5)$

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$+tan^{-1}(101) - tan^{-1}(99)$

$tan^{-1}(101) - tan^{-1}(1)$

$= tan^{-1}\left(\frac{101-1}{1+101}\right)$ $\left(\because tan^{-1}A - tan^{-1}B = tan^{-1}\left(\frac{A-B}{1+AB}\right)\right)$

$= tan^{-1}\left(\frac{100}{102}\right) = P$

So, $tanP = \frac{100}{102} = \frac{50}{51}$

6. If α, β are the roots of $x^2 - x + 2\lambda = 0$ and α, γ are the roots of $3x^2 - 4x + 27\lambda = 0$ then the value of $\frac{\beta\gamma}{\lambda}$ is

(A) $\frac{18}{17}$

(B) $\frac{18}{19}$

(C) 1

(D) $\frac{17}{16}$

Ans. (B)

Given

α, β are the roots of $x^2 - x + 2\lambda = 0$

$\alpha + \beta = 1$ ...(i)

$\alpha \cdot \beta = 2\lambda$ ...(ii)

α, γ are the roots of $3x^2 - 4x + 27\lambda = 0$

$\alpha + \gamma = \frac{4}{3}$ ...(iii)

$\alpha \cdot \gamma = \frac{27\lambda}{3} = 9\lambda$ ... (iv)

$(ii)/(iv)$

$\Rightarrow \frac{\beta}{\gamma} = \frac{2}{9}$ ... (v)

From (v) put γ in (iii)

$\alpha + \frac{9}{2}\beta = \frac{4}{3} \ldots$ (vi)

From $(vi) - (i)$

$\beta = \frac{2}{21}$ and $\gamma = \frac{3}{7}$

Put $\beta = \frac{2}{21}$ in (i)

$\alpha = \frac{19}{21}$

$\alpha \cdot \gamma = 9\lambda$ From (iv)

$\lambda = \frac{19}{441}$

$\frac{\beta\gamma}{\lambda} = \frac{2}{21} \times \frac{3}{7} \times \frac{441}{19} = \frac{18}{19}$

7. If $2x^2dy + (e^y - 2x)dx = 0$ & $y(e) = 1$, then the value of $y(1)$ is:

(A) $ln2$

(B) 2

(C) 0

(D) $ln3$

Answer (A)

Given

$2x^2dy + (e^y - 2x)dx = 0$ & $y(e) = 1$

$\frac{dy}{dx} = \frac{-e^y}{2x^2} + \frac{1}{x}$

Dividing by e^y

$e^{-y}\frac{dy}{dx} = \frac{e^{-y}}{x} - \frac{1}{2x^2}$

Let $e^{-y} = t$ ... (i)

$e^{-y}(-1)\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{2x^2}$

This is a first order linear differential equation

I.F. $= e^{\int \frac{1}{x}dx} = e^{lnx} = x$

$tx = \int \frac{1}{2x^2} \cdot x dx + C$

$tx = \int \frac{1}{2x} dx + C$

Using equation (i)

$e^{-y}x = \frac{1}{2} lnx + C$

Applying $y(e) = 1$

$e^{-1}e = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

$e^{-y}x = \frac{1}{2}(1 + lnx)$

Put $x = 1$ then $e^{-y}x = \frac{1}{2}(1 + lnx)$ is

$e^{-y} = \frac{1}{2}(1 + 0)$ is $e^{-y} = \frac{1}{2} \Rightarrow y = ln2$

8. The circle (C) touches the line $2y = x$ at point $(2, 1)$ and it intersect the circle $C_1(x^2 + y^2 + 2y - 5 = 0)$ at P & Q. If PQ is diameter of C_1, then calculate square of diameter of circle (C)

Ans. 245

Family of circle touching line $2y = x$ at point $(2, 1)$

$(x - 2)^2 + (y - 1)^2 + \lambda(x - 2y) = 0 \;\ldots(i)$

Common chord PQ is

$(x - 2)^2 + (y - 1)^2 + \lambda(x - 2y) - x^2 - y^2 - 2y + 5 = 0$

(PQ is diameter of C_1 which passes through $(0, -1)$)

$4 + 4 + 2\lambda - 1 + 2 + 5 = 0$

$2\lambda + 14 = 0$

$\lambda = -7$ put in (i)

$(x - 2)^2 + (y - 1)^2 - 7(x - 2y) = 0$

$x^2 - 4x + 4 + y^2 - 2y + 1 - 7x + 14y = 0$

$x^2 + y^2 - 11x + 12y + 5 = 0$

$r = \sqrt{\frac{121}{4} + 36 - 5} = \sqrt{\frac{121+124}{4}} = \sqrt{\frac{245}{4}}$

$4r^2 = 245$

Hence, $d^2 = 245$

9. If a die is rolled till 6 comes and x represents number of throws, then the probability of $P\left(\frac{x \geq 5}{x > 2}\right)$ is

(1) $\frac{1}{6}$

(2) $\frac{24}{36}$

(3) $\frac{25}{36}$

(4) $\frac{18}{36}$

Ans. (3)

Applying Conditional Probability

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$P\left(\frac{x \geq 5}{x > 2}\right) = \frac{P(x \geq 5 \cap x > 2)}{P(x > 2)}$

$= \frac{P(x=5)+P(x=6)+P(x=7)+\cdots.}{P(x=3)+P(x=4)+P(x=5)+\cdots}$

$= \frac{\left(\frac{5}{6}\right)^4 \frac{1}{6} + \left(\frac{5}{6}\right)^5 \frac{1}{6} + \cdots \ldots}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \frac{1}{6} + \cdots \ldots \ldots}$

Applying Sum of infinite GP $= \frac{\mathrm{a}}{1-\mathrm{r}}$

$= \frac{\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1-\frac{5}{6}}}{\frac{\left(\frac{5}{6}\right)^2 \frac{1}{6}}{1-\frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

10. The mean and variance of the 4 observations $3, 7, x, y$ (where $x > y$) is 5 and 10 respectively. The mean of the observations $4 + x + y, 7 + x, x + y, x - y$ will be:

(A) 10

(B) 12

(C) 8

(D) 14

Ans. (B)

Mean$= \frac{3+7+x+y}{4} = 5$

$\Rightarrow 10 + x + y = 20$

$\Rightarrow x + y = 10 \ldots (i)$

Variance $= 10 = \frac{9+49+x^2+y^2}{4} - 25$

$\Rightarrow 35 = \frac{9+49+x^2+y^2}{4}$

$\Rightarrow x^2 + y^2 = 82 \ldots (ii)$

From (i) and (ii)

$(x + y)^2 = x^2 + y^2 + 2xy$

$100 = 82 + 2xy$

$xy = 9$

$(x-y)^2 = x^2 + y^2 - 2xy$

$(x-y)^2 = 82 - 18$

$x - y = 8$ ... (iii) as $x > y$

From (i) and (iii)

$\Rightarrow x = 9, y = 1$

$\therefore$ Mean $= \frac{4+x+y+7+x+x+y+x-y}{4}$

$= \frac{11+4x+y}{4} = \frac{11+36+1}{4} = 12$

11. Let the plane P passes through the point $(1, 2, 3)$ and it contains the line of intersection of $\vec{r} \cdot (\hat{i} + \hat{j} + 4\hat{k}) = 16$ and $\vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 6$. Then which of the following point does not lie on P?

(1) $(-8,8,6)$

(2) $(-4,3,5)$

(3) $(8,-5,1)$

(4) $(-8,8,5)$

Ans. (4)

$\vec{r} \cdot (\hat{i} + \hat{j} + 4\hat{k}) = 16$ and $\vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 6$ are two equations of planes in vector form.

$x + y + 4z - 16 = 0$ and $-x + y + z = 6$ are two equations of plane in cartesian form

Equation of the plane using family of plane

$P_1 + \lambda P_2 = 0$

$(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0$

it passes through the point $(1, 2, 3)$

$(1 + 2 + 12 - 16) + \lambda(-1 + 2 + 3 - 6) = 0$

$\Rightarrow -1 - 2\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$

Plane P ; $(1-\lambda)x + (1+\lambda)y + (4+\lambda)z = 16 + 6\lambda$

$\Rightarrow \frac{3}{2}x + \frac{1}{2}y + \frac{7}{2}z = 13$

$\Rightarrow 3x + y + 7z = 26$

On substituting options in equation of plane P we can say that $(-8,8,5)$ does not lie on the plane.

12. If $f(x) = \left(\frac{2}{x}\right)^{x^2}$ then the maximum value of $f(x)$ is

(1) e^2

(2) $e^{2e^{-1}}$

(3) $e^{e^{-1}}$

(4) e

Ans. (2)

$f'(x) = 0$ for maximum value

Let $y = \left(\frac{2}{x}\right)^{x^2}$

$lny = x^2 ln\frac{2}{x}$

$\frac{1}{y}y' = 2xln\frac{2}{x} + x^2\frac{1}{\frac{2}{x}} \times \frac{-2}{x^2}$

$y' = (y)x\left(2ln\frac{2}{x} - 1\right)$

$y' = \left(\frac{2}{x}\right)^{x^2} x\left(2ln\frac{2}{x} - 1\right)$

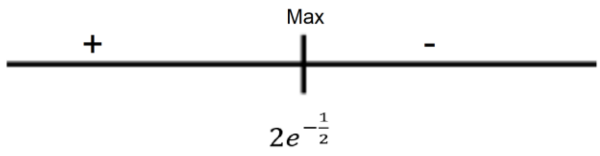
$y' = 0$ as $x \neq 0$

$2ln\frac{2}{x} = 1$

$\left(\frac{2}{x}\right) = e^{\frac{1}{2}}$

$x = 2e^{-\frac{1}{2}}$

Sign scheme of y' is



Then maximum value will be

$f\left(2e^{-\frac{1}{2}}\right) = \left(\frac{2}{2e^{-\frac{1}{2}}}\right)^{4e^{-1}}$

$= e^{2e^{-1}}$

13. Let $a_1, a_2, a_3, \dots \dots a_{10}$ are in A.P. with common difference -3 and $b_1, b_2, b_3 \dots \dots b_{10}$ are in G.P. with common ratio 2 and $C_k = a_k + b_k$ where $k = 1,2,3 \dots \dots 10$. If $C_2 = 12$ and $C_3 = 13$ then the value of $\sum_{k=1}^{10} C_k$

Ans. 2021

Given $C_2 = 12$ and $C_3 = 13$

Also $C_k = a_k + b_k$ where $k = 1,2,3 \dots \dots 10$

Now, $a_k = a_1 + (k-1)(-3)$ (as $T_n = a + (n-1)d$)

and $b_k = b_1 2^{k-1}$ (As $T_n = br^{n-1}$)

$C_k = a_1 + (k-1)(-3) + b_1 2^{k-1}$

$$\sum_{k=1}^{10} C_k = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$C_2 = a_1 - 3 + b_1(2) = 12$

$C_3 = a_1 - 6 + 4b_1 = 13$

By solving

$2b_1 - 3 = 1 \Rightarrow b_1 = 2$ and $a_1 = 11$

$\sum_{k=1}^{10} C_k = \frac{10}{2}(22 + 9(-3)) + 2(2^{10} - 1)$

$= -25 + 2046 = 2021$

14. If $(\sqrt{3} + i)^{100} = 2^{99}(p + iq)$ then p and q are the roots of the equation

(1) $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

(2) $x^2 + (\sqrt{3} + 1)x - \sqrt{3} = 0$

(3) $x^2 - (\sqrt{3} - 1)x + \sqrt{3} = 0$

(4) $x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$

Ans. (4)

Given

$(\sqrt{3} + i)^{100} = 2^{99}(p + iq)$

Converting it into polar form then

for z $= \sqrt{3} + i, r = 2$ & $\theta = \frac{\pi}{6}$

$(\sqrt{3} + i)^{100} = 2^{100}\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)^{100} = 2^{99}(p + iq)$

Now converting into Euler's form

$2^{100}e^{i\frac{50\pi}{3}} = 2^{99}(p + iq)$

$2\left(cos\frac{50\pi}{3} + isin\frac{50\pi}{3}\right) = p + iq$

$2\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right) = p + iq$

$p = -1$ and $q = \sqrt{3}$

p and q roots of the equation

$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$

15. The value of $\lim_{x\to 2} \sum_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4}$ is

(A) $\frac{13}{44}$

(B) $\frac{9}{44}$

(C) $\frac{9}{22}$

(D) $\frac{9}{41}$

Ans. (B)

$$\lim_{x \to 2} \sum_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4}$$

$$= \sum_{n=1}^{9} \frac{2}{4n(n+1) + 4(2n+1) + 4}$$

$$= \frac{1}{2} \sum_{n=1}^{9} \frac{1}{n^2 + 3n + 2} = \frac{1}{2} \sum_{n=1}^{9} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$= \frac{1}{2}\left[\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots \left(\frac{1}{10} - \frac{1}{11}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{2} - \frac{1}{11}\right] = \frac{9}{44}$$

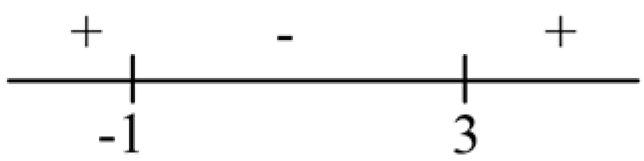
16. If the function $f(x) = 2x^3 - 6x^2 - 18x$ has local maxima at $x = a$ and local minima at $x = b$ and the area bounded by $y = f(x)$ from $x = a$ to $x = b$ with x-axis is A. Then the value of $4A$ is:

Ans. 404

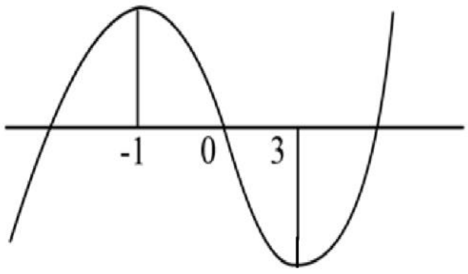
Sol. $f'(x) = 6x^2 - 12x - 18$

$= 6(x^2 - 2x - 3)$

$= 6(x - 3)(x + 1)$



$a = -1, b = 3$



$A = \int_{-1}^{0} (2x^3 - 6x^2 - 18x)\,dx + \left|\int_{0}^{3} (2x^3 - 6x^2 - 18x)\,dx\right|$

$= \left[\frac{x^4}{2} - 2x^3 - 9x^2\right]_{-1}^{0} + \left|\left[\frac{x^4}{2} - 2x^3 - 9x^2\right]_{0}^{3}\right|$

$= 0 - \left(\frac{1}{2} + 2 - 9\right) + \left|\frac{81}{2} - 54 - 81\right|$

$\Rightarrow A = 101 \Rightarrow 4\,A = 404\ sq.$ unit

17. Minimum value of 'n' for which $\frac{(2i)^n}{(1-i)^{n-2}}$ is positive integer

Ans. 6

Given $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(1-i)^{2\frac{(n-2)}{2}}}$

$= \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}} = \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$ is positive integer.

$\frac{n+2}{2} = 4\alpha$ and $\frac{n-2}{2} = 2\beta$

Clearly n must be even $n = 2, 4$ rejected.

So for $n = 6 \Rightarrow (2i)^4 = 16$ and $(-1)^2 = 1$

Hence, $n = 6$ is the correct answer.

18. Angle between two body diagonals PB and CQ of a cuboid is $cos^{-1}\left(\frac{1}{5}\right)$ and length and breath are equal in length, which is 10. Find height of cuboid:

(1) $5\sqrt{2}$

(2) 50

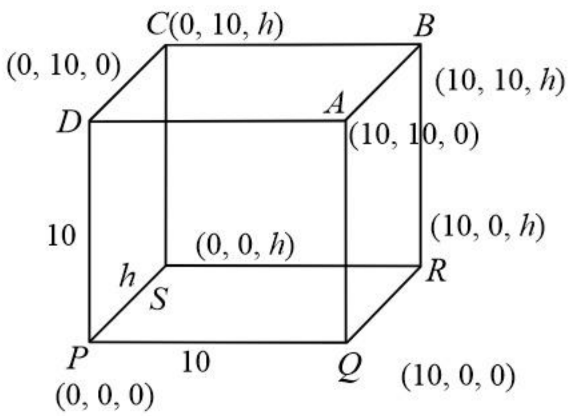
(3) 25

(4) $5\sqrt{5}$

Ans. (1)

Given that length and breath are equal in length, which is 10.

Assuming one of the vertex as (0,0,0) and edges of the cube along coordinate axes then we can find the coordinates of other vertices as shown in diagram-



To find the angle between two body diagonals PB and CQ

Position vectors of PB and CQ are

$\vec{CQ} = (10-0)\hat{i} + (0-10)\hat{j} + (0-h)\hat{k} = 10\hat{i} - 10\hat{j} - h\hat{k}$

$\vec{PB} = (10-0)\hat{i} + (10-0)\hat{j} + (h-0)\hat{k} = 10\hat{i} + 10\hat{j} + h\hat{k}$

Angle between two vectors is

$cos\theta = \left|\frac{\vec{CQ}\cdot\vec{PB}}{|\vec{CQ}|\cdot|\vec{PB}|}\right|$

$cos\theta = \left|\frac{100-100-h^2}{\left(\sqrt{200+h^2}\right)^2}\right|$

$\frac{1}{5} = \frac{h^2}{200+h^2}$

$200 = 4h^2$

$h^2 = 50$

$h = 5\sqrt{2}$ unit.