JEE MAIN 2021

PAPER-1 (B.E. / B.TECH)

Reliable
INSTITUTE
Unleashing Potential

QUESTIONS & SOLUTIONS

Reproduced from Memory Retention

SHIFT-1

📅 **18 March, 2021**

🕐 **09:00 am to 12 Noon**

TRIGONOMETRY

Duration : 3 Hours

Max. Marks : 300

SUBJECT - MATHEMATICS

MATHEMATICS

1. Let circle
C$_1$: $x^2 + y^2 = 1$
C$_2$: $x^2 + y^2 - 2y = 1$
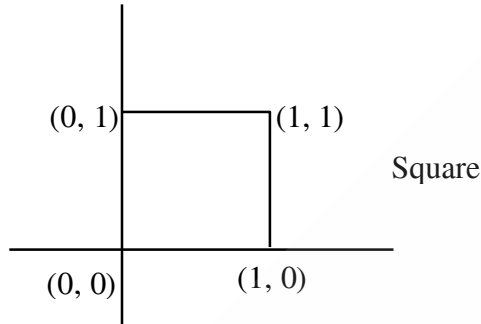C$_3$: $x^2 + y^2 - 2x = 1$
C$_4$: $x^2 + y^2 - 2x - 2y + 1 = 0$
figure formed by joining centres of C$_1$, C$_2$, C$_3$, C$_4$ is
(1) square   (2) rectangle   (3) rhombus   (4) parallelogram

Ans. (1)

Sol. Centres (0, 0), (1, 0), (0, 1), (1, 1)

2. The values of a and b for which the function $f(x) = \begin{cases} \dfrac{1}{|x|} & , |x| \geq 1 \\ ax^2 + b & , |x| < 1 \end{cases}$ is differentiable, are

respectively

(1) $\dfrac{1}{2}, \dfrac{3}{2}$ (2) $-\dfrac{1}{2}, \dfrac{3}{2}$ (3) $\dfrac{1}{2}, -\dfrac{3}{2}$ (4) $1, -2$

Ans. (2)

Sol. f(x) is continuous at x = 1 $\Rightarrow$ 1 = a + b
f(x) is differentiable at x = 1 $\Rightarrow$ -1 = 2a
$\Rightarrow a = -\dfrac{1}{2} \therefore b = \dfrac{3}{2}$

3. Find the sum of all the four digit numbers formed by using the digits 1,2,2,3.
(1) 36664   (2) 26664   (3) 24   (4) 46632

Ans. (2)

Sol. ....1 = $\dfrac{3!}{2!}$
...........2 = 3!
...........3 = $\dfrac{3!}{2!}$

sum of digits at unit's place = 3 + 12 + 9 = 24
sum of all four digits numbers = 24(1111) = 26664

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4. Find the number of points of intersection of the curves :
$S_1 : x^2 + y^2 - 22x + 10y + 137 = 0$ &
$S_2 : x^2 + y^2 - 10x - 10y + 41 = 0$

(1) 1   (2) 2   (3) 0   (4) 3

Ans. (1)

Sol. $S_1 : (x - 11)^2 + (y - 5)^2 = 9 \rightarrow C_1 = (11, 5)$

$S_2 : (x - 5)^2 + (y - 5)^2 = 9 \rightarrow C_2 = (5, 5)$

$r_1 = 3$ & $r_2 = 3$

$d(C_1C_2) = \sqrt{(11-5)^2} = 6$

$r_1 + r_2 = 6$

$\therefore$ Circles touch externally

Hence, 1 point of intersection.

5. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots\dots + a_{40}x^{40}$ then the value of $a_1 + a_2 + \dots + a_{37}$ is :

(1) $2^{18}(2^{19} - 19)$ (2) $2^{19}(2^{20} - 21)$ (3) $2^{18}(2^{20} - 38)$ (4) $2^{20}(2^{20} - 21)$

Ans. (2)

Sol. Put x = 1, – 1 and subtract

$4^{20} - 2^{20} = (a_0 + a_1 + \dots + a_{40}) - (a_0 - a_1 + \dots)$

$\Rightarrow 4^{20} - 2^{20} = 2(a_1 + a_3 + \dots + a_{39})$

$\Rightarrow a_1 + a_3 + \dots + a_{37} = 2^{39} - 2^{19} - a_{39}$

$a_{39} = $ coeff of x^{39} in $(1 + x + 2x^2)^{20} = {}^{20}C_1 2^{19}$

$\Rightarrow a_1 + a_3 + \dots + a_{37} = 2^{39} - 2^{19} - 20(2^{19})$

$= 2^{39} - 21(2^{19}) = 2^{19}(2^{20} - 21)$

6. Find the sum of the series :

$S = \dfrac{1}{3^2 - 1} + \dfrac{1}{5^2 - 1} + \dots\dots + \dfrac{1}{201^2 - 1}$

(1) $\dfrac{1}{4}$ (2) $\dfrac{25}{101}$ (3) $\dfrac{26}{99}$ (4) $\dfrac{26}{102}$

Ans. (2)

Sol. $S = \displaystyle\sum_{r=1}^{100} \dfrac{1}{(2r+1)^2 - 1} = \sum_{r=1}^{100} \dfrac{1}{(2r+1)\cdot 2(r)}$

$\therefore$ $S = \dfrac{1}{4} \displaystyle\sum_{r=1}^{100} \left[\dfrac{1}{r} - \dfrac{1}{r+1} \right]$
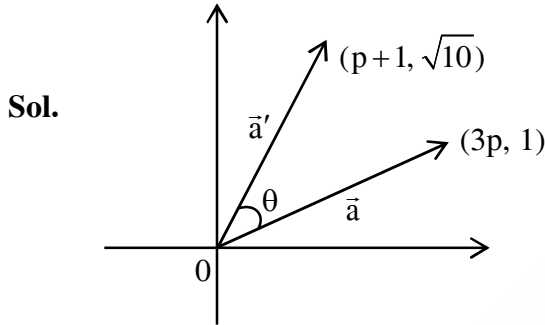
$S = \dfrac{1}{4}\left(\left(1 - \dfrac{1}{2}\right) + \left(\dfrac{1}{2} - \dfrac{1}{3}\right) + \left(\dfrac{1}{3} - \dfrac{1}{4}\right) + \dots\dots + \left(\dfrac{1}{100} - \dfrac{1}{101}\right) \right)$

$\therefore$ $S = \dfrac{1}{4}\left[\dfrac{100}{101} \right] = \dfrac{25}{101}$

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7. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesion system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, $\vec{a}$ has components (p + 1) and $\sqrt{10}$, then find the value of 'p'

(1) $-\dfrac{5}{4}$ (2) $\dfrac{4}{5}$ (3) 1   (4) −1

Ans. (4)

Sol.

given $\vec{a} = 3p\hat{i} + \hat{j}$, $\vec{a} = (p+1)\hat{i} + \sqrt{10}\hat{j}$

$|\vec{a}| = |\vec{a}'|$, (No Change in magnitude)

$\Rightarrow \sqrt{9p^2 + 1} = \sqrt{(p+1)^2 + 10}$

$9p^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0$

$p = -1$, $p = \dfrac{5}{4}$

8. The value of $\displaystyle\int \frac{(2x-1)\cos\sqrt{(2x-1)^2 + 4}}{\sqrt{4x^2 - 4x + 5}}dx$ is

(1) $\dfrac{1}{2}\sin\left(\sqrt{4x^2 - 4x + 5}\right) + C$ (2) $\sin\left(\sqrt{4x^2 - 4x + 5}\right) + C$

(3) $\tan\left(\sqrt{4x^2 - 4x + 5}\right) + C$ (4) None of these

Ans. (1)

Sol. Put $\tan^{-1}\left(\dfrac{2x-1}{2}\right) = \theta$

$\Rightarrow 2x - 1 = 2\tan\theta$

$\Rightarrow I = \displaystyle\int \frac{2\tan\theta\cos(2\sec\theta)}{2\sec\theta}\cdot\sec^2\theta\,d\theta$

$\Rightarrow I = \displaystyle\int \tan\theta\sec\theta\cos(2\sec\theta)\,d\theta$

Put $2\sec\theta = t \Rightarrow 2\sec\theta\tan\theta\, d\theta = dt$

$$\Rightarrow I = \frac{1}{2}\int \cos t\, dt = \frac{\sin t}{2} + c = \frac{\sin(2\sec\theta)}{2} + C$$

$$= \frac{1}{2}\sin\left(\sqrt{4x^2 - 4x + 5}\right) + C$$

9. Let $a|z|^2 + \overline{\alpha}z + \overline{z}\alpha + d = 0$ represents the equation of the circle when

(1) $\alpha^2 - ad > 0$ such that $a \in R$

(2) $\alpha^2 - ad \geq 0$ such that $a \in R - \{0\}$

(3) $\alpha^2 - ad = 0$ for $\forall\, a \in R$

(4) $\alpha^2 - ad \leq 0$ for $\forall\, a \in R$

Ans. (2)

Sol. $az\overline{z} + \alpha\overline{z} + \overline{\alpha}z + d = 0$ ..... (i)

$z\overline{z} + \dfrac{\alpha}{a}\overline{z} + \dfrac{\overline{\alpha}}{a}z + \dfrac{d}{a} = 0 \Rightarrow$ circle

centre $= -\dfrac{\alpha}{a}$, $r = \sqrt{|\alpha|^2 - c}$

$\Rightarrow \left|\dfrac{\alpha}{a}\right|^2 - \dfrac{d}{a} \geq 0$ for equation (i) to represents a circle

$\Rightarrow |\alpha|^2 - ad \geq 0$

10. Let the system of equation $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$, $\gamma u + \alpha v + \beta w = 0$ has non trial solution and α, β, γ are distinct roots of $x^3 + ax^2 + bx + c = 0$ find value of $\dfrac{a^2}{b}$

(1) 0   (2) 1   (3) 2   (4) 3

Ans. (1)

Sol. $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow \alpha^2 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$\therefore \quad \alpha + \beta + \gamma = 0$ or $\alpha = \beta = \gamma$

Here $\alpha + \beta + \gamma = 0$

$\therefore \quad (\alpha, \beta, \gamma$ distinct)

$\therefore \quad a = 0$

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11. Let $f(x) = \sqrt{x}$ and $g(x) \sqrt{1-x}$ and fucntion

$f(x) + g(x), f(x) - g(x)$, $\dfrac{f(x)}{g(x)}, \dfrac{g(x)}{f(x)}$, $g(x) - f(x)$

Find the set of value of 'x' which is common to the domain of all the five functions.

(1) $x \in (0,1)$ (2) $x \in [0,1]$ (3) $x \in [0,1)$ (4) $x \in (0,1]$

Ans. (1)

Sol. $D_f = [0, \infty)$

$D_g = (-\infty, 1]$

$D_{f+g} = D_{f-g} = D_{g-f} = [0, \infty) \cap (-\infty, 1] = [0,1]$

for $\dfrac{f}{g}$ and $\dfrac{g}{f}$ to be defined , $x \ne 0,1 \Rightarrow$ common domain $= (0,1)$

12. If $(100)^{\alpha} - (199)^{\beta} = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$. Then find the slope of line joining (α, β) and origin.

(1) 540   (2) 550   (3) 530   (4) 545

Ans. (2)

Sol. RHS $= \displaystyle\sum_{r=0}^{99}(100-r)(100+r)$

$= (100)^3 - \dfrac{99 \times 100 \times 199}{6} = (100)^3 - (1650) \, 199$

LHS $= (100)^{\alpha} - (199) \, \beta$

So, $\alpha = 3, \beta = 1650$

Slope $= \tan\theta = \dfrac{\beta}{\alpha}$

$\tan\theta = 550$

13. Let the straight lines are $y = (mx + 1)$, $3x + 4y = 9$. Find number of integral values of m for which abscissa of point of intersection is integer

(1) 1   (2) 2   (3) 3   (4) 0

Ans. (2)

Sol. $3x + 4(mx + 1) = 9$

$x(3 + 4m) = 5$

$x = \dfrac{5}{(3 + 4m)}$

$(3 + 4m) = \pm 1, \pm 5$

$4m = -3 \pm 1, -3 \pm 5$

$4m = -4, -2, -8, 2$

$m = -1, -\dfrac{1}{2}, -2, \dfrac{1}{2}$

Two integral value of m

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14. Find the differential equation of the curve $y^2 = 4a(x + a)$ where a is any arbitrary constant.

(1) $y(y')^2 + 2xy' - y = 0$

(2) $y(y')^2 - 2xy' + y = 0$

(3) $y(y')^2 + 2xy' + y = 0$

(4) $y(y')^2 - 2xy' - y = 0$

Ans. (1)

$y^2 = 4a(x + a)$ ...... (i)

$2yy' = 4a$

$\therefore yy' = 2a$

$\therefore$ by (i) $y^2 = 2yy'\left(x + \dfrac{yy'}{2}\right)$

$y^2 = 2yy'x + (yy')^2$

$\Rightarrow y(y')^2 + 2xy' - y = 0$

(as $y \neq 0$)

15. If $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$, $(x \in (0, \pi))$

Then the values of 'x' which satisfy this are :

(1) $\dfrac{\pi}{12}, \dfrac{5\pi}{12}$ (2) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$ (3) $\dfrac{2\pi}{3}, \dfrac{\pi}{3}$ (4) $\dfrac{\pi}{9}, \dfrac{2\pi}{9}$

Ans. (1)

Sol. $R_1 \rightarrow R_1 + R_2$

$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

$C_1 \rightarrow C_1 - C_2$

$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1+\cos^2 x & \cos^2 x \\ 0 & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

$\therefore 2 + 8\sin 2x - 4\sin 2x = 0$

$\Rightarrow \sin 2x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

16. If $2A + B = \begin{bmatrix} 0 & 0 & 3 \\ 10 & 1 & 9 \\ -1 & 4 & 0 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 0 & -5 & 9 \\ -5 & 3 & 2 \\ -3 & 2 & 0 \end{bmatrix}$

Find the value of $(t_r(A) - t_r(B))$.

(1) 1   (2) – 2   (3) – 1   (4) 2

Ans. (4)

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Sol. $t_r(2A + B) = 1 \Rightarrow 2t_r(A) + t_r(B) = 1$ .....(1)

$t_r(A - 2B) = 3 \Rightarrow t_r(A) - 2t_r(B) = 3$ .....(2)

$\Rightarrow t_r(A) - 2[1 - 2t_r(A)] = 3$

$\Rightarrow t_r(A) = 1, t_r(B) = -1$

$\therefore t_r(A) - t_r(B) = 2$

17. Let $y = \left(3\sqrt{2}\right)x - 1$ be given line then find equation of straight line passes through A(1, 3) and makes an angle $\tan^{-1}\sqrt{2}$ with given line

(1) $5y + 4\sqrt{2}x = 4\sqrt{2} + 15$ (2) $5y - 4\sqrt{2}x = 4\sqrt{2} + 15$

(3) $4\sqrt{2}x - 5y = 4\sqrt{2} - 15$ (4) None of these

Ans. (1)

Sol. $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\sqrt{2} = \left|\dfrac{m_1 - 3\sqrt{2}}{1 + 3\sqrt{2}m_1}\right|$

$\Rightarrow \pm\sqrt{2}(1 + 3\sqrt{2}m_1) = (m_1 - 3\sqrt{2})$

$\sqrt{2} + 6m_1 = m_1 - 3\sqrt{2}$ [for positive sign]

$m_1 = \dfrac{-4\sqrt{2}}{5}$

$-\sqrt{2} - 6m_1 = m_1 - 3\sqrt{2}$

$2\sqrt{2} = 7m_1 \Rightarrow m_1 = \dfrac{2\sqrt{2}}{7}$

18. Find number of times, the digit '3' appears while writing integers from 1 to 1000.

Ans. 300

Sol. $\underline{3}\,\_\,\_ = (9 \times 9) \times 1$

$\_\,\underline{3}\,\_ = (9 \times 9) \times 1$

$\_\,\_\,\underline{3} = (9 \times 9) \times 1$

$\underline{3}\,\underline{3}\,\_ = (9) \times 2$

$\underline{3}\,\_\,\underline{3} = (9) \times 2$

$\_\,\underline{3}\,\underline{3} = (9) \times 2$

$\underline{3}\,\underline{3}\,\underline{3} = (1) \times 3$

total way = 243 + 54 + 3 = 300

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19. Let P_1 be the plane $x - 2y + 2z - 3 = 0$ and $A(1, 2, 3)$ lie on it. Let there be another plane P_2 which is parallel to P_1 and at unit distance from A. If P_2 is $ax + by + cz + d =$ the find positive value of $\left(\dfrac{b-d}{c-a}\right)$

Ans. 4

Sol. Let $P_2 : x - 2y + 2z + \lambda = 0$

$$\left|\frac{1-4+6+\lambda}{\sqrt{1+4+4}}\right| = 1$$

$|\lambda + 3| = 3$

$(\lambda + 3) = \pm 3$

$\lambda = -3 \pm 3$

$\lambda = 0 \text{ or } -6$

$P_2 : x - 2y + 2z + 0 = 0$

$x - 2y + 2z - 6 = 0$

$\dfrac{b-d}{c-a} = \dfrac{(-2)-(0 \text{ or } -6)}{(2-1)} = -2 \text{ or } 4$

so positive is 4

20. If $f(x^2) + g(4 - x) = 4x^3$ and $g(x) + g(4 - x) = 0$, then the value of $\displaystyle\int_{-4}^{4} f(x^2) \, dx$

Ans. 512

Sol. $I = 2 \displaystyle\int_{0}^{4} f(x^2) \, dx$ ....(i)

$\Rightarrow \ I = 2 \displaystyle\int_{0}^{4} f((4-x)^2) \, dx$ ....(ii)

Adding equation (i) & (ii)

$2I = 2\displaystyle\int_{0}^{4} \left[f(x^2) + f(4-x)^2 \right] dx$ ....(iii)

Now using $f(x^2) + g(4 - x) = 4x^3$ ....(iv)
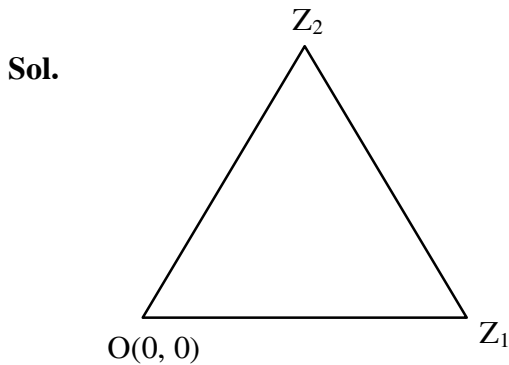
$x \to 4 - x$

$f((4-x)^2) + g(x) = 4(4-x)^3$ ....(v)

Adding equation (iv) & (v)

$f(x^2) + f(4-x^2) + g(x) + g(4-x) = 4(x^3 + (4-x)^3]$

$\Rightarrow \ f(x^2) + f(4-x^2) = 4[x^3 + (4-x)^3]$

Now, $I = 4 \displaystyle\int_{0}^{4} (x^3 + (4-x)^3) \, dx = 512$

21. If roots of $x^2 + ax + 12 = 0$ and origin form equilateral triangle in complex plane, find $|a|$ equal to

Ans. 6

Sol.

for equilateral triangle

$$Z_1^2 + Z_2^2 + O^2 = Z_1 Z_2 + 0 + 0$$

$$(Z_1 + Z_2)^2 = 3 Z_1 Z_2$$

$$\Rightarrow (-a^2) = 3(12)$$

$$\Rightarrow a^2 = 36$$

$$a = -6 \text{ or } 6$$

$$|a| = 6$$

22. Average age of 25 teachers of a school is 40. If a teacher of age 60 year leave the school and a new teacher of 'x' age joined and new mean of their ages is 39 then find the value of 'x'

Ans. 35

Sol. $\overline{x} = \dfrac{\sum\limits_{i=1}^{25} x_i}{25} = 40$

$$\sum_{i=1}^{25} x_i = 1000$$

when a teacher left school

$$\sum_{i=1}^{24} x_i = 1000 - 60 = 940$$

$$\overline{x}' = \dfrac{\sum\limits_{i=1}^{24} (x_i) + x}{25}$$

$$39 \times 25 = 940 + x$$

$$x = 35$$

23. If $\lim\limits_{x \to 0} \dfrac{\sin^{-1}x - \tan^{-1}x}{x^3} = \ell$ then

find $6\ell + 1$.

Ans. 4

Sol. $\ell = \lim\limits_{x \to 0} \dfrac{\left(x + \dfrac{x^3}{6} + ...\right) - \left(x - \dfrac{x^3}{3} ....\right)}{x^3}$

$\therefore \ell = \dfrac{1}{6} + \dfrac{1}{3} = \dfrac{1}{2}$

$\therefore 6\ell = 4$

24. Find number of solutions of equation

$|\cot x| = \cot x + \dfrac{1}{\sin x}$ in $x \in [0, 2\pi]$

Ans. 1

Sol. Case I : $x \in \left[0, \dfrac{\pi}{2}\right] \cup \left[\pi, \dfrac{3\pi}{2}\right]$

$\cot x = \cot x + \dfrac{1}{\sin x} \quad \Rightarrow$ not possible

Case : II $x \in \left[\dfrac{\pi}{2}, \pi\right] \cup \left[\dfrac{3\pi}{2}, 2\pi\right]$

$-\cot x = \cot x + \dfrac{1}{\sin x} \Rightarrow \dfrac{-2\cos x}{\sin x} = \dfrac{1}{\sin x}$

$\Rightarrow \cos x = \dfrac{-1}{2} \Rightarrow x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

$= 1$

25. If $\int \dfrac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = f(x) + c$ and if $f(0) = 0$ & $f(1) = \dfrac{1}{k}$, then find k.

Ans. 4

Sol. $\int \dfrac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = \int \dfrac{5x^8 + 7x^6}{x^{14}\left(2 + \dfrac{1}{x^5} + \dfrac{1}{x^7}\right)^2} dx$
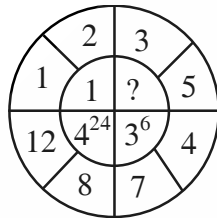
$$= \int \frac{\dfrac{5}{x^6} + \dfrac{7}{x^8}}{\left(2 + \dfrac{1}{x^5} + \dfrac{1}{x^7}\right)} dx \quad : \text{put } 2 + \frac{1}{x^5} + \frac{1}{x^7} = t$$

$$= \int \frac{-dt}{t^2} = \frac{1}{t} + c \Rightarrow -\left(\frac{5}{x^6} + \frac{7}{x^8}\right) dx = dt$$

$$= f(x) = \frac{1}{2 + \dfrac{1}{x^5} + \dfrac{1}{x^7}} = \frac{x^7}{2x^7 + 1 + x^2}$$

$$f(x) = \frac{1}{4} = \frac{1}{k} \Rightarrow k = 4$$

26. Find the missing term in

Ans. 16

Sol. 4^{24} has base 4 (= 12 − 8)

3^6 has base 3 (= 7 − 4)

(?) will have base 2 (= 5 − 3)

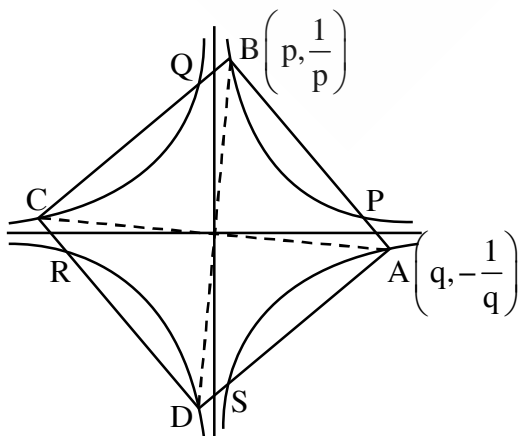Power 24 = 6 × 4 = (no. of divisor of 12) × (no. of divisor of 8)

Power 6 = 2 × 3 = (no. of divisor of 7) × (no. of divisor of 4)

(?) will have power = (no. of divisor of 3) × (no. of divisor of 5) = 2 × 2 = 4

27. Four points lying on the curve $x^2y^2 = 1$ form a square such that mid-points of sides also lie on the given curve. Find the square of area of the square

Ans. 80

Sol.

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OA ⊥ OB

$\Rightarrow \left(\dfrac{1}{p^2}\right)\left(-\dfrac{1}{q^2}\right) = -1$

$\Rightarrow p^2 q^2 = 1$

$P\left(\dfrac{p+q}{2}, \dfrac{\dfrac{1}{p} - \dfrac{1}{q}}{2}\right)$ lies

on $x^2 y^2 = 1$

$\Rightarrow (p+q)^2 \left(\dfrac{1}{p} - \dfrac{1}{q}\right)^2 = 16$

$\Rightarrow (p+q)^2 (p-q)^2 = 16$

$\Rightarrow (p^2 - q^2)^2 = 16$

$\Rightarrow p^2 - \dfrac{1}{p^2} = \pm 4$

$\Rightarrow p^4 \pm 4p^2 - 1 = 0$

$\Rightarrow p^2 = \dfrac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$

$\Rightarrow p^2 = 2 + \sqrt{5}$ or $-2 + \sqrt{5}$

$OB^2 = p^2 + \dfrac{1}{p^2} = 2 + \sqrt{5} + \dfrac{1}{2+\sqrt{5}}$ or $-2 + \sqrt{5} + \dfrac{1}{-2+\sqrt{5}} = 2\sqrt{5}$

$Area = 4\left(\dfrac{1}{2}\right)(OA)(OB) = 2(OB)^2 = 4\sqrt{5}$