Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/5/1

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

XII MATHEMATICS

QUESTION PAPER CODE 65/5/1

EXPECTED ANSWER/VALUE POINTS

Q. No.	VALUE POINTS	Marks
	SECTION - A	
	Question Numbers 1 to 20 carry 1 mark each.	
	Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:	
1	If A is a square matrix of order 3, such that $A(\text{adj}A)=10I$, then $\lvert\text{adj}A\rvert$ is equal to (a) 1 (b) 10 (c) 100 (d) 101 **Answer: (c)** 100	1
2	If A is a 3 x 3 matrix such that $\lvert A\rvert=8$, then $\lvert 3A\rvert$ equals. (a) 8 (b) 24 (c) 72 (d) 216 Answer: **(d)** 216	1
3	If $y=Ae^{5x}+Be^{-5x},$ then $\frac{d^2y}{dx^2}$ is equal to (a) $25y$ (b) $5y$ (c) $-25y$ (d) $15y$ **Answer: (a)** $25y$	1
4	$\int x^2 e^{x^3} dx$ equals (a) $\frac{1}{3}e^{x^3}+C$ (b) $\frac{1}{3}e^{x^4}+C$ (c) $\frac{1}{2}e^{x^3}+C$ (d) $\frac{1}{2}e^{x^2}+C$ **Answer: (a)** $\frac{1}{3}e^{x^3}+C$	1
5	If $\hat{i},\hat{j},\hat{k}$ are unit vectors along three mutually perpendicular directions, then (a) $\hat{i}.\hat{j}=1$ (b) $\hat{i}\times\hat{j}=1$ (c) $\hat{i}.\hat{k}=0$ (d) $\hat{i}\times\hat{k}=0$ **Answer: (c)** $\hat{i}.\hat{k}=0$	1
6	ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals (a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{AD}$ **Answer: (a)** $\vec{0}$	1
7	The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{-2}$ are mutually perpendicular if the value of k is (a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) -2 (d) 2 **Answer: (a)** $-\frac{2}{3}$	1

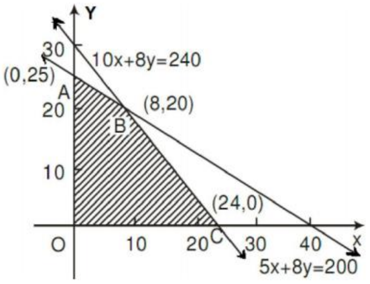
8	The graph of the inequality $2x+3y>6$ is (a) half plane that contains the origin (b) half plane that neither contains the origin nor the points of the line $2x+3y=6$ (c) whole XOY-plane excluding the points on the line $2x+3y=6$ (d) entire XOY-plane **Answer: (b)** half plane that neither contains the origin nor the points of the line $2x+3y=6$	1
9	A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is (a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ **Answer: (c)** $\frac{1}{4}$	1
10	A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P\left(A\cup B\right)$ is (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1 **Answer: (d)** 1	1
In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence:		
11	A relation in a set A is called ________ relation, if each element of A is related to itself. **Answer:** reflexive	1
12	If A + B = $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and A – 2B = $\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$, then A = _______________ **Answer:** $\begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{2}{3} & \frac{1}{3}\end{bmatrix}$	1
13	The least value of the function $f\left(x\right)=ax+\frac{b}{x}$ $\left(a>0,b>0,x>0\right)$ is____________. **Answer:** $2\sqrt{ab}$	1
14	The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is ____________. **Answer:** x^2 **OR** The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2=x$ is _______. **Answer:** 2	1 1

15	The vector equation of a line which passes through the points $(3,4,-7)$ and $(1,-1,6)$ is_________. **Answer:** $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$ Or, $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(2\hat{i}+5\hat{j}-13\hat{k})$	1
	OR The line of shortest distance between two skew lines is _______________ to both the lines. **Answer:** perpendicular	1
Q. 16 to 20 are very short answer questions.		
16	Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$. **Answer:** $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]=-\sin^{-1}\left[\sin\left(2\pi+\frac{\pi}{8}\right)\right]$ $=-\frac{\pi}{8}$	$\frac{1}{2}$ $\frac{1}{2}$
17	For $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, write A^{-1}. **Answer:** $\|A\|=1$ $A^{-1}=\begin{bmatrix}-1 & 4\\ -1 & 3\end{bmatrix}$	$1/2$ $1/2$
18	If the function f defined as $f(x)=\begin{cases}\frac{x^2-9}{x-3} & ,x\neq 3\\ k & ,x=3\end{cases}$ is continuous at $x=3$, find the value of k. **Answer:** $\lim\limits_{x\to 3}\frac{x^2-9}{x-3}=6,\ \therefore\ \ k=6$	$\frac{1}{2}+\frac{1}{2}$
19	If $f(x)=x^4-10$, then find the approximate value of $f(2.1)$. **Answer:** $f(2.1)\simeq f(2)+(0.1)f'(2)$ $=9.2$	$1/2$ $1/2$
	OR Find the slope of the tangent to the curve $y=2\sin^2(3x)$ at $x=\frac{\pi}{6}$. **Answer:** $\frac{dy}{dx}=6\sin 6x$ $\therefore$ slope of tangent $=0$	$1/2$ $1/2$
20	Find the value of $\int_1^4 \|x-5\|\,dx$. **Answer:** $\int_1^4 \|x-5\|\,dx=\int_1^4 (5-x)\,dx=\frac{15}{2}$	$1/2+1/2$

	SECTION – B	
Q. Nos. 21 to 26 carry 2 marks each.		
21	If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$, then show that $(fof)(x)=x$, for all $x\neq\frac{2}{3}$. Also, write inverse of f. **Answer:** $(fof)(x)=f\left(\frac{4x+3}{6x-4}\right)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{34x}{34}=x$ Now, $(fof)(x)=x \Rightarrow f^{-1}=f$ or $f^{-1}(x)=\frac{4x+3}{6x-4}$ **OR** Check if the relation R in the set **R** of real numbers defined as $R=\{(a,b):a<b\}$ is (i) symmetric, (ii) transitive. **Answer:** $(i)\, 1,2\in\mathbb{R}$ such that $1<2\Rightarrow(1,2)\in R$, but since 2 is not less than 1 $\Rightarrow(2,1)\notin R$. Hence R is not symmetric. (ii) Let $(a,b)\in R$ and $(b,c)\in R, \therefore a<b$ and $b<c$ $\Rightarrow a<c\Rightarrow(a,c)\in R \therefore R$ is transitive.	$1\frac{1}{2}$ $\frac{1}{2}$ 1 1
22	Find $\int\frac{x}{x^2+3x+2}dx$ **Answer:** $\int\frac{x}{x^2+3x+2}dx=\int\frac{x}{(x+1)(x+2)}dx=\int\left(\frac{-1}{x+1}+\frac{2}{x+2}\right)dx$ $=-\log\|x+1\|+2\log\|x+2\|+C$	1 1
23	If $x=a\cos\theta; y=b\sin\theta$, then find $\frac{d^2y}{dx^2}$. **Answer:** $\frac{dx}{d\theta}=-a\sin\theta, \frac{dy}{d\theta}=b\cos\theta\Rightarrow\frac{dy}{dx}=-\frac{b}{a}\cot\theta$ $\frac{d^2y}{dx^2}=\frac{b}{a}\operatorname{cosec}^2\theta\left(\frac{-1}{a\sin\theta}\right)=-\frac{b}{a^2}\operatorname{cosec}^3\theta$ **OR** Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$. **Answer:** Let $y=\sin^2 x$ and $z=e^{\cos x} \therefore \frac{dy}{dx}=2\sin x\cos x$ and $\frac{dz}{dx}=-\sin x.e^{\cos x}$ $\therefore\frac{dy}{dz}=\frac{2\sin x\cos x}{-\sin x\, e^{\cos x}}=\frac{-2\cos x}{e^{\cos x}}$ or $-2\cos x\, e^{-\cos x}$	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$

24	Evaluate $\int_1^2 \left[\frac{1}{x}-\frac{1}{2x^2}\right] e^{2x}dx$ **Answer:** Put $2x=t, \therefore dx=\frac{1}{2}dt$ $\therefore I=\int_1^2\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx=\int_2^4\left[\frac{1}{t}-\frac{1}{t^2}\right]e^t dt$ $=\left[\frac{1}{t}e^t\right]_2^4=\frac{e^4}{4}-\frac{e^2}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$
25	Find the value of $\int_0^1 x(1-x)^n\,dx$. **Answer:** $\int_0^1 x(1-x)^n\,dx=\int_0^1(1-x)(1-1+x)^n\,dx=\int_0^1\left(x^n-x^{n+1}\right)dx$ $=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1=\frac{1}{n+1}-\frac{1}{n+2}$ or $\frac{1}{(n+1)(n+2)}$	1 $\frac{1}{2}+\frac{1}{2}$
26	Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, find $P\left(A'\cap B'\right)$. **Answer:** $P\left(A'\cap B'\right)=P\left(A'\right)P\left(B'\right)$ $=(0.7)(0.4)=0.28$	1 1
SECTION – C		
Q. Nos. 27 to 32 carry 4 marks each.		
27	Solve for $x: \sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$. **Answer:** $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}\Rightarrow(1-x)=\sin\left(\frac{\pi}{2}+2\sin^{-1}x\right)$ $\Rightarrow(1-x)=\cos\left(2\sin^{-1}x\right)\Rightarrow 1-x=1-2x^2$ $\therefore 2x^2-x=0\Rightarrow x=0, x=\frac{1}{2}$ since $x=\frac{1}{2}$ does not satisty the given equation $\therefore x=0$ is the required solution.	$1\frac{1}{2}$ 1 1 $\frac{1}{2}$
28	If $y=(\log x)^x+x^{\log x}$, then find $\frac{dy}{dx}$	

	Answer: $y=(\log x)^x+x^{\log x}=u+v \Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	1
	$\therefore \log u = x\log(\log x)$ and $\log v=(\log x)^2$	$\frac{1}{2}$
	$\frac{du}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]$ and $\frac{dv}{dx}=x^{\log x}.\frac{2\log x}{x}$	1+1
	$\Rightarrow \frac{dy}{dx}=(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]+x^{\log x}.\frac{2\log x}{x}$	$\frac{1}{2}$
29	Solve the differential equation: $x\sin\left(\frac{y}{x}\right)\frac{dy}{dx}+x-y\sin\left(\frac{y}{x}\right)=0$, given that $x=1$ when $y=\frac{\pi}{2}$.	
	Answer:	
	Given differential equation gives $\frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-x}{x\sin\left(\frac{y}{x}\right)}$	$\frac{1}{2}$
	$\frac{y}{x}=v \Rightarrow y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$	$\frac{1}{2}$
	$\therefore v+x\frac{dv}{dx}=\frac{v\sin v-1}{\sin v} \Rightarrow x\frac{dv}{dx}=-\frac{1}{\sin v}$	1
	$\Rightarrow \int \sin v\, dv=-\int \frac{1}{x}dx$	$\frac{1}{2}$
	$\therefore \cos v=\log\lvert x\rvert+C$ or $\cos\left(\frac{y}{x}\right)=\log\lvert x\rvert+C$	1
	Given $x=1$ when $y=\frac{\pi}{2} \Rightarrow C=0$ $\therefore \cos\left(\frac{y}{x}\right)=\log\lvert x\rvert$ is the required solution.	$\frac{1}{2}$
30	If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.	
	Answer:	
	Diagonal vectors are: $\vec{a}+\vec{b}=3\hat{i}+6\hat{j}-2\hat{k}$ and $\vec{a}-\vec{b}=-\hat{i}-2\hat{j}+8\hat{k}$ (or, $\vec{b}-\vec{a}=\hat{i}+2\hat{j}-8\hat{k}$)	1+1
	$\therefore$ unit vectors are $\frac{(\vec{a}+\vec{b})}{\lvert\vec{a}+\vec{b}\rvert}=\frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$ and $\frac{(\vec{a}-\vec{b})}{\lvert\vec{a}-\vec{b}\rvert}=-\frac{1}{\sqrt{69}}\hat{i}-\frac{2}{\sqrt{69}}\hat{j}+\frac{8}{\sqrt{69}}\hat{k}$	1+1
	OR	
	Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B(2, –1,4) and C(4, 5, –1).	

	Answer: $\text{area of } \Delta ABC = \frac{1}{2}\lvert\text{cross product of any two side vectors}\rvert$ $\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k}$ and $\overrightarrow{BC} = 2\hat{i} + 6\hat{j} - 5\hat{k}$ $\overrightarrow{AB} \times \overrightarrow{BC} = 9\hat{i} + 7\hat{j} + 12\hat{k}$ $\therefore \text{area of } \Delta ABC = \frac{1}{2}\sqrt{81+49+144} = \frac{1}{2}\sqrt{274}$	1 $\frac{1}{2}+\frac{1}{2}$ 1 1
31	A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹100 each and for type B souvenir, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically. **Answer:** Let the company manufacture 'x' number of souvenirs of Type A And, 'y' number of souvenirs of Type B  $\therefore$ LPP is: Maximise $P = 100x + 120y$ subject to $5x + 8y \le 200$ $10x + 8y \le 240$ $x \ge 0, y \ge 0$ Correct Graph P(A) = ₹ 3,000 P(B) = ₹ 3,200 (Max.) P(C) = ₹2,400 $\therefore$ For Maximum profit, No. of souvenirs of Type A = 8 No. of souvenirs of Type B = 20	$\frac{1}{2}$ 1 $1\frac{1}{2}$ 1
32	Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples. **Answer:** Let X represents the number of rotten apples drawn. X : 0, 1, 2, 3 P (X) : $\frac{7}{10}.\frac{7}{10}.\frac{7}{10}$, $3.\frac{7}{10}.\frac{7}{10}.\frac{3}{10}$, $3.\frac{7}{10}.\frac{3}{10}.\frac{3}{10}$, $\frac{3}{10}.\frac{3}{10}.\frac{3}{10}$ $= \frac{343}{1000}$, $= \frac{441}{1000}$, $= \frac{189}{1000}$, $= \frac{27}{1000}$ X.P(X): 0, $\frac{441}{1000}$, $\frac{378}{1000}$, $\frac{81}{1000}$ Mean = $\sum X\,P(X) = \frac{900}{1000} = \frac{9}{10}$	$\frac{1}{2}$ 2 1 $\frac{1}{2}$

OR

In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in a shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

Answer:

E_1 :selecting shop X
E_2 :selecting shop Y
A :purchased tin is of type B — $\frac{1}{2}$

$$P(E_1) = P(E_2) = \frac{1}{2}$$
$$P(A|E_1) = \frac{4}{7}, P(A|E_2) = \frac{6}{11}$$ — 1

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$
$$= \frac{\frac{1}{2}.\frac{6}{11}}{\frac{1}{2}.\frac{4}{7} + \frac{1}{2}.\frac{6}{11}}$$ — 2

$$= \frac{21}{43}$$ — $\frac{1}{2}$

SECTION – D

Q. Nos. 33 to 36 carry 6 marks each.

33 Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines.

Answer:

Let equation of required line is $\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c}$ ………….(i) — $\frac{1}{2}$

Since this line is perpendicular to $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$,

$a + 2b + 4c = 0$ ………………(ii)
$2a + 3b + 4c = 0$ ………………(iii) — 1

Solving (ii) and (iii) , $\frac{a}{-4} = \frac{b}{4} = \frac{c}{-1}$

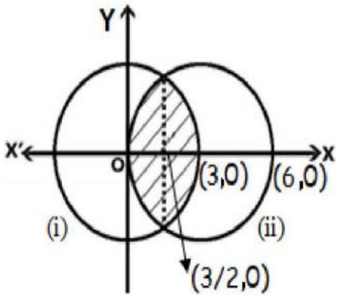
$\therefore$ DR′s of line in cartesian form is : $-4, 4, -1$ — 1

Equation of line in Cartesian form is: $\frac{x-1}{-4} = \frac{y-1}{4} = \frac{z-1}{-1}$ — 1

Vector form of line is $\vec{r} = (\hat{i} + j + k) + \lambda(-4\hat{i} + 4j - k)$ — 1

Let θ be the angle between given lines.

$$\cos\theta = \frac{1(2)+2(3)+4(4)}{\sqrt{1+4+16}\sqrt{4+9+16}} = \frac{24}{\sqrt{21}\sqrt{29}} \qquad \therefore \theta = \cos^{-1}\left(\frac{24}{\sqrt{21}\sqrt{29}}\right)$$ — $1+\frac{1}{2}$

34	Using integration find the area of the region bounded between the two circles $x^2+y^2=9$ and $(x-3)^2+y^2=9$.	
	Answer: Correct Figure	1
	Point of intersection of, $x^2+y^2=9;\left(x-3\right)^2+y^2=9 \Rightarrow \left(x-3\right)^2-x^2=0 \Rightarrow x=3/2$	1
	 Required area $=2\left[\int_0^{\frac{3}{2}}\sqrt{9-(x-3)^2}\,dx+\int_{\frac{3}{2}}^{3}\sqrt{9-x^2}\,dx\right]$ $=4\left[\int_{\frac{3}{2}}^{3}\sqrt{9-x^2}\,dx\right]$	$1\frac{1}{2}$
	$=4\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{\frac{3}{2}}^{3} = \left(6\pi-\frac{9\sqrt{3}}{2}\right)$	$1\frac{1}{2}+1$
	OR	
	Evaluate the following integral as the limit of sums $\int_1^4\left(x^2-x\right)dx$.	
	Answer:	
	$\int_1^4(x^2-x)\,dx=\lim_{h\to 0} h\left[f\left(1\right)+f\left(1+h\right)+f\left(1+2h\right)+.....f\left(1+\overline{n-1}\,h\right)\right]$	1
	where $f\left(x\right)=\left(x^2-x\right)$ and $nh=3$	1
	$\therefore \int_1^4(x^2-x)\,dx$	
	$=\lim_{h\to 0} h\left[\left(1-1\right)+\left(1+h^2+2h-h-1\right)+\left(1+4h^2+4h-2h-1\right)+...+\left(1+\left(n-1\right)^2h^2+2\left(n-1\right)h-\left(n-1\right)h-1\right)\right]$	2
	$=\lim_{h\to 0} h\left[h^2\left(1^2+2^2+3^2+.........+\left(n-1\right)^2\right)+h\left(1+2+3+.....+\left(n-1\right)\right)\right]$	1
	$=\lim_{h\to 0}\left[\frac{nh\left(nh-h\right)\left(2nh-h\right)}{6}+\frac{\left(nh\left(nh-h\right)\right)}{2}\right]=9+\frac{9}{2}=\frac{27}{2}$	1
35	Find the minimum value of $\left(ax+by\right)$, where $xy=c^2$.	
	Answer:	
	Let $S=ax+by$, where $y=\frac{c^2}{x}$ $\therefore$ $S=ax+\frac{bc^2}{x}$	1
	$\frac{dS}{dx}=a-\frac{bc^2}{x^2}$	1
	$\frac{dS}{dx}=0 \Rightarrow x^2=\frac{bc^2}{a}$ or $x=\sqrt{\frac{b}{a}}.c$	$1\frac{1}{2}$
	$\left.\frac{d^2S}{dx^2}\right\|_{x=\sqrt{\frac{b}{a}}.c}=\frac{2bc^2}{x^3}=2bc^2\left[\sqrt{\frac{a}{b}}\frac{1}{c}\right]^3>0$ for $a,b,c>0$ and $x=\sqrt{\frac{b}{a}}.c$	$1\frac{1}{2}$

	$\therefore \text{minimum value } = a\sqrt{\frac{b}{a}}\ .c+b.\frac{c^2}{c}\sqrt{\frac{a}{b}} = 2\sqrt{ab}.c$	1
36	If a, b, c are p^{th}, q^{th}, and r^{th} terms respectively of a G.P, then prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$ **Answer:** $a = AR^{p-1}, \quad b = AR^{q-1}, \quad c = AR^{r-1}$	$1\frac{1}{2}$
	$\therefore \Delta = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix} = \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix} - \log R \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix}$	1+1+1+1
	$= 0 + 0 + 0 = 0$	$\frac{1}{2}$
	OR	
	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the following system of equations: $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$ **Answer:** $\lvert A \rvert = 2(0) + 3(-2) + 5(1) = -1$	1
	$\Rightarrow A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ **(1 mark for any 4 correct co-factors)**	2
	Given equations can be written as $AX = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$	1
	$\therefore X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	1
	$\Rightarrow x = 1, \ y = 2, \ z = 3$	1

Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/5/2

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

XII MATHEMATICS

QUESTION PAPER CODE 65/5/2

EXPECTED ANSWER/VALUE POINTS

Q. No.	VALUE POINTS	Marks
	SECTION – A	
Question Numbers 1 to 20 carry 1 mark each.		
Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:		
1	If $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$, then x equals (a) 0 (b) –2 (c) –1 (d) 2 **Answer: (d)** 2	1
2	$\int 4^x 3^x \, dx$ equals (a) $\frac{12^x}{\log 12}+C$ (b) $\frac{4^x}{\log 4}+C$ (c) $\left(\frac{4^x.3^x}{\log 4.\log 3}\right)+C$ (d) $\frac{3^x}{\log 3}+C$ **Answer: (a)** $\frac{12^x}{\log 12}+C$	1
3	A number is chosen randomly from numbers 1 to 60. The probability that the chosen number is a multiple of 2 or 5 is (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{7}{10}$ (d) $\frac{9}{10}$ **Answer: (b)** $\frac{3}{5}$	1
4	ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals (a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{AD}$ **Answer: (a)** $\vec{0}$	1
5	If A is a square matrix of order 3, such that $A(\text{adj}A)=10I$, then $\lvert \text{adj}A \rvert$ is equal to (a) 1 (b) 10 (c) 100 (d) 101 **Answer: (c)** 100	1
6	A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is (a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ **Answer: (c)** $\frac{1}{4}$	1

7	If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then (a) $\hat{i}.\hat{j}=1$ (b) $\hat{i}\times\hat{j}=1$ (c) $\hat{i}.\hat{k}=0$ (d) $\hat{i}\times\hat{k}=0$ **Answer: (c)** $\hat{i}.\hat{k}=0$	1
8	The graph of the inequality $2x+3y>6$ is (a) half plane that contains the origin (b) half plane that neither contains the origin nor the points of the line $2x+3y=6$ (c) whole XOY-plane excluding the points on the line $2x+3y=6$ (d) entire XOY-plane **Answer: (b)** half plane that neither contains the origin nor the points of the line $2x+3y=6$	1
9	The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{-2}$ are mutually perpendicular if the value of k is (a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) -2 (d) 2 **Answer: (a)** $-\frac{2}{3}$	1
10	If $y=Ae^{5x}+Be^{-5x},$ then $\frac{d^2y}{dx^2}$ is equal to (a) $25y$ (b) $5y$ (c) $-25y$ (d) $15y$ **Answer: (a)** $25y$	1
In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence:		
11	A relation R on a set A is called ______________ , if $(a_1,a_2)\in R$ and $(a_2,a_3)\in R$ implies that $(a_1,a_3)\in R$, for $a_1,a_2,a_3\in A$. **Answer:** transitive	1
12	The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is ____________. **Answer:** x^2 **OR** The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2=x$ is _______. **Answer:** 2	1 1
13	The vector equation of a line which passes through the points $(3,4,-7)$ and $(1,-1,6)$ is **Answer:** $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$ Or, $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(2\hat{i}+5\hat{j}-13\hat{k})$ **OR** The line of shortest distance between two skew lines is _______________ to both the lines. **Answer:** perpendicular	1 1

14	If A + B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and A – 2B = $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A = ________________ **Answer:** $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$	1
15	The least value of the function $f(x) = ax + \frac{b}{x}$ $(a > 0, b > 0, x > 0)$ is____________. **Answer:** $2\sqrt{ab}$	1
Q. 16 to 20 are very short answer questions.		
16	Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$. **Answer:** $\sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = 1$	$\frac{1}{2} + \frac{1}{2}$
17	Using differential, find the approximate value of $\sqrt{36.6}$ up to 2 decimal places. **Answer:** $\sqrt{36.6} \simeq \sqrt{36} + \frac{1}{2\sqrt{36}}(0.6) = 6.05$ **OR** Find the slope of tangent to the curve $y = 2\cos^2(3x)$ at $x = \frac{\pi}{6}$. **Answer:** $\frac{dy}{dx} = -6\sin 6x$ $\therefore$ slope of tangent at $x = \frac{\pi}{6}$ is 0.	$\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
18	Find the value of $\int_1^4 \lvert x-5 \rvert dx$. **Answer:** $\int_1^4 \lvert x-5 \rvert dx = \int_1^4 (5-x)dx$ $= \frac{15}{2}$	$1/2$ $1/2$
19	If the function f defined as $f(x) = \begin{cases} \frac{x^2-9}{x-3} & , x \neq 3 \\ k & , x = 3 \end{cases}$ is continuous at $x = 3$, find the value of k.	

	Answer: $\lim\limits_{x\to 3}\dfrac{x^2-9}{x-3}=6,\ \therefore\quad k=6$	$\frac{1}{2}+\frac{1}{2}$
20	For $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, write A^{-1}. **Answer:** $\lvert A\rvert=1$ $A^{-1}=\begin{bmatrix}-1 & 4\\ -1 & 3\end{bmatrix}$	 $1/2$ $1/2$
	SECTION – B	
Q. Nos. 21 to 26 carry 2 marks each.		
21	Find $\int\frac{x+1}{x(1-2x)}dx$. **Answer:** $\int\frac{x+1}{x(1-2x)}dx=\int\left(\frac{1}{x}+\frac{3}{1-2x}\right)dx$ $=\log\lvert x\rvert-\frac{3}{2}\log\lvert 1-2x\rvert+C$	 1 1
22	Evaluate $\int\frac{x\sin^{-1}\left(x^2\right)}{\sqrt{1-x^4}}dx$ **Answer:** $\int\frac{x\sin^{-1}\left(x^2\right)}{\sqrt{1-x^4}}dx=\frac{1}{2}\int t\ dt$, where $\sin^{-1}\left(x^2\right)=t$ $=\frac{t^2}{4}+C=\frac{1}{4}\left(\sin^{-1}x^2\right)^2+C$	 1 $\frac{1}{2}+\frac{1}{2}$
23	Find the value of $\int_0^1 x(1-x)^n\,dx$. **Answer:** $\int_0^1 x(1-x)^n\,dx=\int_0^1(1-x)(1-1+x)^n\,dx=\int_0^1\left(x^n-x^{n+1}\right)dx$ $=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1=\frac{1}{n+1}-\frac{1}{n+2}$ or $\frac{1}{(n+1)(n+2)}$	 1 $\frac{1}{2}+\frac{1}{2}$
24	If $x=a\cos\theta;\ y=b\sin\theta$, then find $\frac{d^2y}{dx^2}$. **Answer:** $\frac{dx}{d\theta}=-a\sin\theta,\ \frac{dy}{d\theta}=b\cos\theta\Rightarrow\frac{dy}{dx}=-\frac{b}{a}\cot\theta$ $\frac{d^2y}{dx^2}=\frac{b}{a}\operatorname{cosec}^2\theta\left(\frac{-1}{a\sin\theta}\right)=-\frac{b}{a^2}\operatorname{cosec}^3\theta$ **OR** Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.	 $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$

	Answer: Let $y=\sin^2 x$ and $z=e^{\cos x}$ $\therefore \frac{dy}{dx}=2\sin x\cos x$ and $\frac{dz}{dx}=-\sin x.e^{\cos x}$ $\therefore \frac{dy}{dz}=\frac{2\sin x\cos x}{-\sin x\, e^{\cos x}}=\frac{-2\cos x}{e^{\cos x}}$ or $-2\cos x\,\mathrm{e}^{-\cos x}$	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$
25	Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, find $P\left(A^{'}\cap B^{'}\right)$. **Answer:** $P\left(A^{'}\cap B^{'}\right)=P\left(A^{'}\right)P\left(B^{'}\right)$ $=(0.7)(0.4)=0.28$	1 1
26	If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$, then show that $(fof)(x)=x$, for all $x\neq\frac{2}{3}$. Also, write inverse of f. **Answer:** $(fof)(x)=f\left(\frac{4x+3}{6x-4}\right)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{34x}{34}=x$ Now, $(fof)(x)=x\Rightarrow f^{-1}=f$ or $f^{-1}(x)=\frac{4x+3}{6x-4}$ **OR** Check if the relation R in the set **R** of real numbers defined as defined as $R=\{(a,b):\lvert a\rvert<b\}$ is (i) symmetric, (ii) transitive. **Answer:** $(i)\ -1, 2\in\mathbb{R}$ such that $\lvert -1\rvert<2\Rightarrow(-1,2)\in R$, but since $\lvert 2\rvert$ is not less than $-1\Rightarrow(2,-1)\notin R$. Hence R is not symmetric. (ii) Let $(a,b)\in R$ and $(b,c)\in R, \therefore \lvert a\rvert<b$ and $\lvert b\rvert<c$ $\Rightarrow \lvert a\rvert<c\Rightarrow(a,c)\in R \therefore R$ is transitive. **Remark:** Since in Hindi version of this question, $R=\{(a,b):a<b\}$ is given, so full marks may be awarded to the student who solved it using $a<b$.	$1\frac{1}{2}$ $\frac{1}{2}$ 1 1
SECTION – C		
Q. Nos. 27 to 32 carry 4 marks each.		
27	Prove that $\tan\left[2\tan^{-1}\left(\frac{1}{2}\right)-\cot^{-1}3\right]=\frac{9}{13}$.	

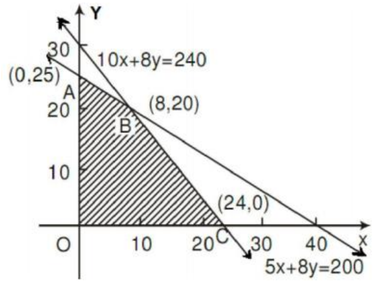
	Answer: $\text{LHS} = \tan\left[\tan^{-1}\left(\dfrac{2.\frac{1}{2}}{1-\frac{1}{4}}\right) - \tan^{-1}\dfrac{1}{3}\right]$ $= \tan\left[\tan^{-1}\left(\dfrac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}.\frac{1}{3}}\right)\right]$ $= \tan\left(\tan^{-1}\dfrac{9}{13}\right) = \dfrac{9}{13} = \text{RHS}$	$1\frac{1}{2}$ $1\frac{1}{2}$ 1
28	If $y = (\cos x)^x + \tan^{-1}\sqrt{x}$, find $\dfrac{dy}{dx}$. **Answer:** $y = (\cos x)^x + \tan^{-1}\sqrt{x} \quad = u + v, \therefore \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ $\log u = x\log(\cos x) \Rightarrow \dfrac{du}{dx} = (\cos x)^x\left[-x\tan x + \log(\cos x)\right]$ and $\dfrac{dv}{dx} = \dfrac{1}{1+x}.\dfrac{1}{2\sqrt{x}}$ $\therefore \dfrac{dy}{dx} = (\cos x)^x\left[-x\tan x + \log(\cos x)\right] + \dfrac{1}{2\sqrt{x}\ (1+x)}$	1 $1\frac{1}{2}$ 1 $\frac{1}{2}$
29	If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. **Answer:** Diagonal vectors are $\vec{a}+\vec{b} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{a}-\vec{b} = -\hat{i} - 2\hat{j} + 8\hat{k}$ (or $\vec{b}-\vec{a} = \hat{i} + 2\hat{j} - 8\hat{k}$). $\therefore$ unit vectors are $\dfrac{(\vec{a}+\vec{b})}{\lvert\vec{a}+\vec{b}\rvert} = \dfrac{3}{7}\hat{i} + \dfrac{6}{7}\hat{j} - \dfrac{2}{7}\hat{k}$ and $\dfrac{(\vec{a}-\vec{b})}{\lvert\vec{a}-\vec{b}\rvert} = -\dfrac{1}{\sqrt{69}}\hat{i} - \dfrac{2}{\sqrt{69}}\hat{j} + \dfrac{8}{\sqrt{69}}\hat{k}$ **OR** Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B (2, –1,4) and C(4, 5, –1). **Answer:** area of $\Delta ABC = \dfrac{1}{2}\lvert\text{cross product of any two side vectors}\rvert$ $\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k}$ and $\overrightarrow{BC} = 2\hat{i} + 6\hat{j} - 5\hat{k}$ $\overrightarrow{AB} \times \overrightarrow{BC} = 9\hat{i} + 7\hat{j} + 12\hat{k}$ $\therefore$ area of $\Delta ABC = \dfrac{1}{2}\sqrt{81+49+144} = \dfrac{1}{2}\sqrt{274}$	1+1 1+1 1 $\frac{1}{2}+\frac{1}{2}$ 1 1

30 A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹100 each and for type B souvenir, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically.

Answer:

Let the company manufacture ‘ x ’ number of souvenirs of Type A

And, ‘ y ’ number of souvenirs of Type B



$\therefore$ LPP is: Maximise $P = 100x + 120y$ — $\frac{1}{2}$

subject to $5x + 8y \le 200$

$10x + 8y \le 240$

$x \ge 0, y \ge 0$ — 1

Correct Graph — $1\frac{1}{2}$

P(A) = ₹ 3,000

P(B) = ₹ 3,200 (Max.)

P(C) = ₹2,400

$\therefore$ For Maximum profit, No. of souvenirs of Type A = 8

No. of souvenirs of Type B = 20 — 1

31 Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

Answer:

Let X represents the number of rotten apples drawn. — $\frac{1}{2}$

X :	0	1	2	3
P (X) :	$\frac{7}{10}.\frac{7}{10}.\frac{7}{10}$	$3.\frac{7}{10}.\frac{7}{10}.\frac{3}{10}$	$3.\frac{7}{10}.\frac{3}{10}.\frac{3}{10}$	$\frac{3}{10}.\frac{3}{10}.\frac{3}{10}$
	$= \frac{343}{1000}$	$= \frac{441}{1000}$	$= \frac{189}{1000}$	$= \frac{27}{1000}$
X.P(X):	0	$\frac{441}{1000}$	$\frac{378}{1000}$	$\frac{81}{1000}$

(P(X) — 2; X.P(X) — 1)

Mean = $\sum X\,P(X) = \frac{900}{1000} = \frac{9}{10}$ — $\frac{1}{2}$

OR

In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in a shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

	Answer: E_1 :selecting shop X E_2 :selecting shop Y A :purchased tin is of type B	$\frac{1}{2}$
	$P(E_1)=P(E_2)=\frac{1}{2}$ $P(A\|E_1)=\frac{4}{7}, P(A\|E_2)=\frac{6}{11}$	1
	$P(E_2\|A)=\frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)}$ $=\frac{\frac{1}{2}.\frac{6}{11}}{\frac{1}{2}.\frac{4}{7}+\frac{1}{2}.\frac{6}{11}}$	2
	$=\frac{21}{43}$	$\frac{1}{2}$
32	Solve the differential equation: $x\sin\left(\frac{y}{x}\right)\frac{dy}{dx}+x-y\sin\left(\frac{y}{x}\right)=0$, given that $x=1$ when $y=\frac{\pi}{2}$. **Answer:**	
	Given differential equation gives $\frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-x}{x\sin\left(\frac{y}{x}\right)}$	$\frac{1}{2}$
	$\frac{y}{x}=v \Rightarrow y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$	$\frac{1}{2}$
	$\therefore v+x\frac{dv}{dx}=\frac{v\sin v-1}{\sin v} \Rightarrow x\frac{dv}{dx}=-\frac{1}{\sin v}$	1
	$\Rightarrow \int \sin v\, dv=-\int \frac{1}{x}dx$	$\frac{1}{2}$
	$\therefore \cos v=\log\|x\|+C$ or $\cos\left(\frac{y}{x}\right)=\log\|x\|+C$	1
	Given $x=1$ when $y=\frac{\pi}{2} \Rightarrow C=0$ $\therefore \cos\left(\frac{y}{x}\right)=\log\|x\|$ is the required solution.	$\frac{1}{2}$

SECTION – D

Q. Nos. 33 to 36 carry 6 marks each.

33	If a, b, c are pth, q th, and r th terms respectively of a G.P, then prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$	
	Answer:	
	$a = AR^{p-1}, \quad b = AR^{q-1}, \quad c = AR^{r-1}$	$1\frac{1}{2}$
	$\therefore \Delta = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix} = \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix} - \log R \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix}$	1+1+1+1
	$= 0 + 0 + 0 = 0$	$\frac{1}{2}$
	OR	
	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}.	
	Using A^{-1}, solve the following system of equations: $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$	
	Answer:	
	$\lvert A \rvert = 2(0) + 3(-2) + 5(1) = -1$	1
	$\Rightarrow A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ **(1 Mark for any 4 correct co-factors)**	2
	Given equations can be written as $AX = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$	1
	$\therefore X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	1
	$\Rightarrow x = 1, \; y = 2, \; z = 3$	1

34 Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines.

Answer:

Let equation of required line is $\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c}$ ………….(i) — $\frac{1}{2}$

Since this line is perpendicular to $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$,

$a+2b+4c=0$ ………………(ii)

$2a+3b+4c=0$ ………………(iii) — 1

Solving (ii) and (iii) , $\frac{a}{-4}=\frac{b}{4}=\frac{c}{-1}$ — 1

$\therefore$ DR′s of line in cartesian form is : $-4,4,-1$ — 1

Equation of line in Cartesian form is: $\frac{x-1}{-4}=\frac{y-1}{4}=\frac{z-1}{-1}$ — 1

Vector form of line is $\vec{r}=(\hat{i}+j+k)+\lambda(-4\hat{i}+4j-k)$

Let θ be the angle between given lines.

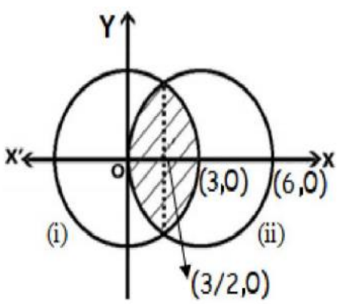
$\cos\theta=\frac{1(2)+2(3)+4(4)}{\sqrt{1+4+16}\ \sqrt{4+9+16}}=\frac{24}{\sqrt{21}\ \sqrt{29}}$ $\quad\therefore\theta=\cos^{-1}\left(\frac{24}{\sqrt{21}\ \sqrt{29}}\right)$ — $1+\frac{1}{2}$

35 Using integration find the area of the region bounded between the two circles $x^2+y^2=9$ and $(x-3)^2+y^2=9$.

Answer:

Correct Figure — 1

Point of intersection of, $x^2+y^2=9;(x-3)^2+y^2=9\Rightarrow(x-3)^2-x^2=0\Rightarrow x=3/2$ — 1



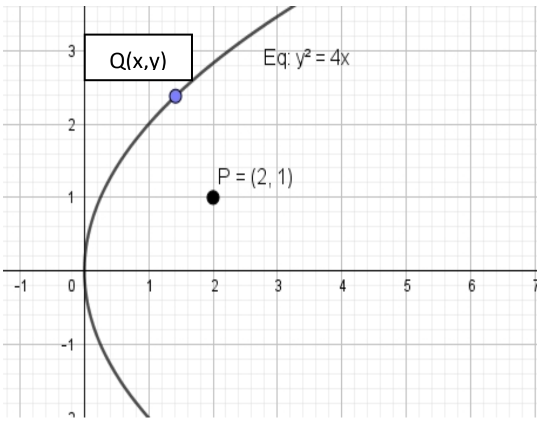
Required area $=2\left[\int_0^{\frac{3}{2}}\sqrt{9-(x-3)^2}\,dx+\int_{\frac{3}{2}}^{3}\sqrt{9-x^2}\,dx\right]$

$=4\left[\int_{\frac{3}{2}}^{3}\sqrt{9-x^2}\,dx\right]$ — $1\frac{1}{2}$

$=4\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{\frac{3}{2}}^{3}=\left(6\pi-\frac{9\sqrt{3}}{2}\right)$ — $1\frac{1}{2}+1$

OR

Evaluate the following integral as the limit of sums $\int_1^4\left(x^2-x\right)dx$.

	Answer: $\int_1^4 (x^2 - x)\,dx = \lim_{h\to 0} h\left[f(1) + f(1+h) + f(1+2h) + \ldots.. f\left(1+\overline{n-1}\,h\right)\right]$ where $f(x) = \left(x^2 - x\right)$ and $nh = 3$ $\therefore \int_1^4 (x^2 - x)\,dx =$ $\lim_{h\to 0} h\left[(1-1) + \left(1 + h^2 + 2h - h - 1\right) + \left(1 + 4h^2 + 4h - 2h - 1\right) + \ldots + \left(1 + (n-1)^2 h^2 + 2(n-1)h - (n-1)h - 1\right)\right]$ $= \lim_{h\to 0} h\left[h^2\left(1^2 + 2^2 + 3^2 + \ldots\ldots.. + (n-1)^2\right) + h\left(1 + 2 + 3 + \ldots.. + (n-1)\right)\right]$ $= \lim_{h\to 0}\left[\frac{nh(nh-h)(2nh-h)}{6} + \frac{(nh(nh-h))}{2}\right] = 9 + \frac{9}{2} = \frac{27}{2}$	1 1 2 1 1
36	Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2,1)$. **Answer:** Let $Q(x, y)$ be the point on curve $y^2 = 4x$, which is nearest to the point P (2, 1).  $\Rightarrow (PQ)^2 = (x-2)^2 + (y-1)^2$ Let $(PQ)^2 = S$, then $S = \left(\frac{y^2}{4} - 2\right)^2 + (y-1)^2$ $\frac{dS}{dy} = 2\left(\frac{y^2}{4} - 2\right)\frac{2y}{4} + 2(y-1) = \frac{y^3 - 8}{4}$ $\frac{dS}{dy} = 0 \Rightarrow y = 2 \;\therefore\; x = 1$ $\frac{d^2S}{dy^2} = \frac{3y^2}{4} > 0$ $\therefore$ (1, 2) will be at minimum distance from (2, 1)	1 1 $1\frac{1}{2}$ 1 1 $\frac{1}{2}$

Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/5/3

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**

2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**

3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.

5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).

11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
- Leaving answer or part thereof unassessed in an answer book.
- Giving more marks for an answer than assigned to it.
- Wrong totaling of marks awarded on a reply
- Wrong transfer of marks from the inside pages of the answer book to the title page.
- Wrong question wise totaling on the title page.
- Wrong totaling of marks of the two columns on the title page.
- Wrong grand total.
- Marks in words and figures not tallying.
- Wrong transfer of marks from the answer book to online award list.
- Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
- Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

XII MATHEMATICS

QUESTION PAPER CODE 65/5/3

EXPECTED ANSWER/VALUE POINTS

Q. No.	VALUE POINTS	Marks
	SECTION – A	
	Question Numbers 1 to 20 carry 1 mark each.	
	Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:	
1	If A is a skew symmetric matrix of order 3, then the value of $\|A\|$ is (a) 3 (b) 0 (c) 9 (d) 27 **Answer: (b)** 0	1
2	If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then (a) $\hat{i}.\hat{j}=1$ (b) $\hat{i}\times\hat{j}=1$ (c) $\hat{i}.\hat{k}=0$ (d) $\hat{i}\times\hat{k}=0$ **Answer: (c)** $\hat{i}.\hat{k}=0$	1
3	A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is (a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ **Answer: (c)** $\frac{1}{4}$	1
4	If A is a 3 x 3 matrix such that $\|A\|=8$, then $\|3A\|$ equals. (a) 8 (b) 24 (c) 72 (d) 216 Answer: **(d)** 216	1
5	$\int x^2 e^{x^3} dx$ equals (a) $\frac{1}{3}e^{x^3}+C$ (b) $\frac{1}{3}e^{x^4}+C$ (c) $\frac{1}{2}e^{x^3}+C$ (d) $\frac{1}{2}e^{x^2}+C$ **Answer: (a)** $\frac{1}{3}e^{x^3}+C$	1
6	If $y=\log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ equals: (a) $-\frac{1}{x}$ (b) $-\frac{1}{x^2}$ (c) $\frac{2}{x^2}$ (d) $-\frac{2}{x^2}$ **Answer: (d)** $-\frac{2}{x^2}$	1

7	A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B the event that the number obtained is less than 5. Then $P(A \cup B)$ is (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1 **Answer: (d)** 1	1
8	ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals (a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{AD}$ **Answer: (a)** $\vec{0}$	1
9	The distance of the origin (0, 0, 0) from the plane $-2x+6y-3z=-7$ is (a) 1 unit (b) $\sqrt{2}$ units (c) $2\sqrt{2}$ units (d) 3 units **Answer: (a)** 1 unit	1
10	The graph of the inequality $2x+3y>6$ is (a) half plane that contains the origin (b) half plane that neither contains the origin nor the points of the line $2x+3y=6$ (c) whole XOY-plane excluding the points on the line $2x+3y=6$ (d) entire XOY-plane **Answer: (b)** half plane that neither contains the origin nor the points of the line $2x+3y=6$	1
In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence:		
11	If A and B are square matrices each of order 3 and $\|A\|=5$, $\|B\|=3$, then the value of $\|3AB\|$ is __________. **Answer:** 405	1
12	The least value of the function $f(x)=ax+\frac{b}{x}$ $(a>0, b>0, x>0)$ is____________. **Answer:** $2\sqrt{ab}$	1
13	The vector equation of a line which passes through the points $(3,4,-7)$ and $(1,-1,6)$ is **Answer:** $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$ Or, $\vec{r}=(3\hat{i}+4\hat{j}-7\hat{k})+\lambda(2\hat{i}+5\hat{j}-13\hat{k})$ **OR** The line of shortest distance between two skew lines is _______________ to both the lines. **Answer:** perpendicular	1 1

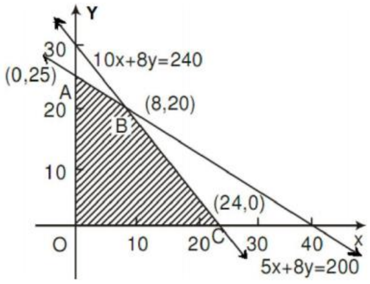
14	The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is ____________. **Answer:** x^2 **OR** The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2=x$ is _______. **Answer:** 2	1 1
15	A relation in a set A is called ________ relation, if each element of A is related to itself. **Answer:** reflexive	1
Q. 16 to 20 are very short answer questions.		
16	Find the cofactors of all the elements of $\begin{bmatrix}1 & -2\\ 4 & 3\end{bmatrix}$. **Answer:** $A_{11}=3,\ A_{12}=-4,\ A_{21}=2,\ A_{22}=1$	½ for any two correct 1
17	Let $f(x)=x\lvert x\rvert$, for all $x\in R$ check its differentiability at $x=0$. **Answer:** $f(x)=\begin{cases}-x^2, & x<0\\ x^2, & x\geq 0,\end{cases}$,differentiable at $x=0$.	$\frac{1}{2}+\frac{1}{2}$
18	Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$. **Answer:** $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]=-\sin^{-1}\left[\sin\left(2\pi+\frac{\pi}{8}\right)\right]$ $=-\frac{\pi}{8}$	$\frac{1}{2}$ $\frac{1}{2}$
19	Find the value of $\int_1^4 \lvert x-5\rvert\,dx$. **Answer:** $\int_1^4 \lvert x-5\rvert\,dx=\int_1^4 (5-x)\,dx$ $=\frac{15}{2}$	$\frac{1}{2}$ $\frac{1}{2}$
20	If $f(x)=x^4-10$, then find the approximate value of $f(2.1)$. **Answer:** $f(2.1)\simeq f(2)+(0.1)f'(2)$ $=9.2$	$1/2$ $1/2$

	OR Find the slope of the tangent to the curve $y = 2\sin^2(3x)$ at $x = \frac{\pi}{6}$. **Answer:** $\frac{dy}{dx} = 6\sin 6x$ $\therefore$ slope of tangent $= 0$	 $1/2$ $1/2$
SECTION – B		
Q. Nos. 21 to 26 carry 2 marks each.		
21	Find $\int \frac{x+1}{(x+2)(x+3)}dx$ **Answer:** $\int \frac{x+1}{(x+2)(x+3)}dx = \int\left(-\frac{1}{x+2}+\frac{2}{x+3}\right)dx$ $= -\log\lvert x+2\rvert + 2\log\lvert x+3\rvert + C$	 1 1
22	If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, then show that $(fof)(x) = x$, for all $x \neq \frac{2}{3}$. Also, write inverse of f. **Answer:** $(fof)(x) = f\left(\frac{4x+3}{6x-4}\right) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{34x}{34} = x$ Now, $(fof)(x) = x \Rightarrow f^{-1} = f$ or $f^{-1}(x) = \frac{4x+3}{6x-4}$ **OR** Check if the relation R in the set **R** of real numbers defined as defined as $R = \{(a,b): a<b\}$ is (i) symmetric, (ii) transitive. **Answer:** (i) $1, 2 \in \mathbb{R}$ such that $1<2 \Rightarrow (1,2) \in R$, but since 2 is not less than 1 $\Rightarrow (2,1) \notin R$. Hence R is not symmetric. (ii) Let $(a,b) \in R$ and $(b,c) \in R, \therefore a<b$ and $b<c$ $\Rightarrow a<c \Rightarrow (a,c) \in R \therefore R$ is transitive.	 $1\frac{1}{2}$ $\frac{1}{2}$ 1 1
23	Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, find $P(A' \cap B')$. **Answer:** $P(A' \cap B') = P(A')P(B')$ $= (0.7)(0.4) = 0.28$	 1 1

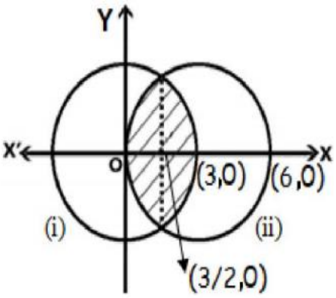
24	Evaluate $\int_1^2 \left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx$ **Answer:** Put $2x=t, \therefore dx=\frac{1}{2}dt$ $\therefore I=\int_1^2\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx=\int_2^4\left[\frac{1}{t}-\frac{1}{t^2}\right]e^t dt$ $=\left[\frac{1}{t}e^t\right]_2^4=\frac{e^4}{4}-\frac{e^2}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$
25	If $x=a\cos\theta; y=b\sin\theta$, then find $\frac{d^2y}{dx^2}$. **Answer:** $\frac{dx}{d\theta}=-a\sin\theta, \frac{dy}{d\theta}=b\cos\theta \Rightarrow \frac{dy}{dx}=-\frac{b}{a}\cot\theta$ $\frac{d^2y}{dx^2}=\frac{b}{a}\cos ec^2\theta\left(\frac{-1}{a\sin\theta}\right)=-\frac{b}{a^2}\cos ec^3\theta$ **OR** Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$. **Answer:** Let $y=\sin^2 x$ and $z=e^{\cos x}$ $\therefore \frac{dy}{dx}=2\sin x\cos x$ and $\frac{dz}{dx}=-\sin x.e^{\cos x}$ $\therefore \frac{dy}{dz}=\frac{2\sin x\cos x}{-\sin x\, e^{\cos x}}=\frac{-2\cos x}{e^{\cos x}}$ or $-2\cos x\, e^{-\cos x}$	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$
26	Find the value of $\int_0^1 \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)dx$. **Answer:** $\int_0^1 \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)dx=\int_0^1\tan^{-1}\left(\frac{(1-x)-x}{1+(1-x)x}\right)dx=\int_0^1\tan^{-1}(1-x)dx-\int_0^1\tan^{-1}x\,dx$ $=0$ as $\int_0^1\tan^{-1}x\,dx=\int_0^1\tan^{-1}(1-x)dx$	1 1
SECTION – C		
Q. Nos. 27 to 32 carry 4 marks each.		
27	Solve the equation for x: $\sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2} \quad (x\neq 0)$ **Answer:** Given equation can be written as $\sin^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2}-\sin^{-1}\left(\frac{5}{x}\right)\Rightarrow\sin^{-1}\left(\frac{12}{x}\right)=\cos^{-1}\left(\frac{5}{x}\right)$	1

	$\therefore \sin^{-1}\left(\frac{12}{x}\right) = \sin^{-1}\left(\frac{\sqrt{x^2-25}}{x}\right)$	1
	$\Rightarrow \frac{12}{x} = \frac{\sqrt{x^2-25}}{x}$ $\Rightarrow x^2 - 25 = 144 \quad \Rightarrow x = \pm 13,$	$1\frac{1}{2}$
	since $x = -13$ does not satisfy the given equation, $\therefore$ required solution is $x = 13$.	$\frac{1}{2}$
28	Find the general solution of the differential equation $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy \;, y \neq 0$. **Answer:** Given differential equation can be written as	
	$\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y^2}{y e^{\frac{x}{y}}}$	$\frac{1}{2}$
	Put $\frac{x}{y} = v \quad \Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$	1
	$\Rightarrow \quad v + y\frac{dv}{dy} = \frac{v e^v + y}{e^v} \qquad \Rightarrow y\frac{dv}{dy} = \frac{y}{e^v}$	1
	$\therefore \int e^v \, dv = \int dy \qquad \Rightarrow \quad e^v = y + C$	1
	$\Rightarrow \quad e^{\frac{x}{y}} = y + C$, which is the required solution.	$\frac{1}{2}$
29	If $y = (\log x)^x + x^{\log x}$, then find $\frac{dy}{dx}$. **Answer:**	
	$y = (\log x)^x + x^{\log x} = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	1
	$\therefore \log u = x \log(\log x)$ and $\log v = (\log x)^2$	$\frac{1}{2}$
	$\frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right]$ and $\frac{dv}{dx} = x^{\log x} . \frac{2\log x}{x}$	1+1
	$\Rightarrow \frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x} . \frac{2\log x}{x}$	$\frac{1}{2}$
30	Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples. **Answer:** Let X represents the number of rotten apples drawn. X : 0 1 2 3	$\frac{1}{2}$

	P (X) : $\frac{7}{10}.\frac{7}{10}.\frac{7}{10}$ $3.\frac{7}{10}.\frac{7}{10}.\frac{3}{10}$ $3.\frac{7}{10}.\frac{3}{10}.\frac{3}{10}$ $\frac{3}{10}.\frac{3}{10}.\frac{3}{10}$ $=\frac{343}{1000}$ $=\frac{441}{1000}$ $=\frac{189}{1000}$ $=\frac{27}{1000}$	2
	X.P(X): 0 $\frac{441}{1000}$ $\frac{378}{1000}$ $\frac{81}{1000}$	1
	Mean = $\sum X\,P(X) = \frac{900}{1000} = \frac{9}{10}$	$\frac{1}{2}$
	OR In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in a shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y. **Answer:** E_1 :selecting shop X E_2 :selecting shop Y A:purchased tin is of type B	$\frac{1}{2}$
	$P(E_1)=P(E_2)=\frac{1}{2}$ $P(A\|E_1)=\frac{4}{7}, P(A\|E_2)=\frac{6}{11}$	1
	$P(E_2\|A)=\frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)}$ $=\frac{\frac{1}{2}.\frac{6}{11}}{\frac{1}{2}.\frac{4}{7}+\frac{1}{2}.\frac{6}{11}}$	2
	$=\frac{21}{43}$	$\frac{1}{2}$
31	A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹100 each and for type B souvenir, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically.	

	Answer: Let the company manufacture ‘ x ’ number of souvenirs of Type A And, ‘ y ’ number of souvenirs of Type B  $\therefore$ LPP is: Maximise $P = 100x + 120y$ subject to $5x + 8y \le 200$ $10x + 8y \le 240$ $x \ge 0, y \ge 0$ Correct Graph P(A) = ₹ 3,000 P(B) = ₹ 3,200 (Max.) P(C) = ₹2,400 ∴ For Maximum profit, No. of souvenirs of Type A = 8 No. of souvenirs of Type B = 20	$\frac{1}{2}$ 1 $1\frac{1}{2}$ 1
32	If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. **Answer:** Diagonal vectors are $\vec{a} + \vec{b} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{a} - \vec{b} = -\hat{i} - 2\hat{j} + 8\hat{k}$ (or $\vec{b} - \vec{a} = \hat{i} + 2\hat{j} - 8\hat{k}$). $\therefore$ unit vectors are $\frac{(\vec{a}+\vec{b})}{\lvert\vec{a}+\vec{b}\rvert} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ and $\frac{(\vec{a}-\vec{b})}{\lvert\vec{a}-\vec{b}\rvert} = -\frac{1}{\sqrt{69}}\hat{i} - \frac{2}{\sqrt{69}}\hat{j} + \frac{8}{\sqrt{69}}\hat{k}$ **OR** Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B(2, –1,4) and C(4, 5, –1). **Answer:** area of $\Delta ABC = \frac{1}{2}\lvert\text{cross product of any two side vectors}\rvert$ $\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k}$ and $\overrightarrow{BC} = 2\hat{i} + 6\hat{j} - 5\hat{k}$ $\overrightarrow{AB} \times \overrightarrow{BC} = 9\hat{i} + 7\hat{j} + 12\hat{k}$ $\therefore$ area of $\Delta ABC = \frac{1}{2}\sqrt{81 + 49 + 144} = \frac{1}{2}\sqrt{274}$	1+1 1+1 1 $\frac{1}{2} + \frac{1}{2}$ 1 1
	SECTION – D	
	Q. Nos. 33 to 36 carry 6 marks each.	
33	Find the distance of the point $P(3,4,4)$ from the point, where the line joining the points $A(3,-4,-5)$ and $B(2,-3,1)$ intersects the plane $2x + y + z = 7$	

	Answer: Equation of the line passing through $A(3,-4,-5)$ and $B(2,-3,1)$ is $\frac{x-3}{1}=\frac{y+4}{-1}=\frac{z+5}{-6}$ ...(i) Any point on the line (i) is $Q(\lambda+3,-\lambda-4,-6\lambda-5)$. Since the point Q lies on the given plane $2x+y+z=7$, $\therefore 2(\lambda+3)+(-\lambda-4)+(-6\lambda-5)=7 \Rightarrow \lambda=-2$ Then Q is $(1,-2,7)$. $PQ=\sqrt{(2)^2+(6)^2+(-3)^2}=7$ units	$1\frac{1}{2}$ 1 $1\frac{1}{2}$ 1 1
34	Find the minimum value of $(ax+by)$, where $xy=c^2$. **Answer:** Let $S=ax+by$, where $y=\frac{c^2}{x}$ $\therefore$ $S=ax+\frac{bc^2}{x}$ $\frac{dS}{dx}=a-\frac{bc^2}{x^2}$ $\frac{dS}{dx}=0 \Rightarrow x^2=\frac{bc^2}{a}$ or $x=\sqrt{\frac{b}{a}}.c$ $\left.\frac{d^2S}{dx^2}\right\|_{x=\sqrt{\frac{b}{a}}.c}=\frac{2bc^2}{x^3}=2bc^2\left[\sqrt{\frac{a}{b}}\ \frac{1}{c}\right]^3>0$ for $a,b,c>0$ and $x=\sqrt{\frac{b}{a}}.c$ $\therefore$ minimum value $=a\sqrt{\frac{b}{a}}.c+b.\frac{c^2}{c}\sqrt{\frac{a}{b}}=2\sqrt{ab}.c$	1 1 $1\frac{1}{2}$ $1\frac{1}{2}$ 1
35	If a, b, c are p[th], q[th], and r[th] terms respectively of a G.P, then prove that $\begin{vmatrix}\log a & p & 1\\ \log b & q & 1\\ \log c & r & 1\end{vmatrix}=0$ **Answer:** $a=AR^{p-1},\ \ b=AR^{q-1},\ \ c=AR^{r-1}$ $\therefore\ \Delta=\begin{vmatrix}\log A+(p-1)\log R & p & 1\\ \log A+(q-1)\log R & q & 1\\ \log A+(r-1)\log R & r & 1\end{vmatrix}=\log A\begin{vmatrix}1 & p & 1\\ 1 & q & 1\\ 1 & r & 1\end{vmatrix}+\log R\begin{vmatrix}p & p & 1\\ q & q & 1\\ r & r & 1\end{vmatrix}-\log R\begin{vmatrix}1 & p & 1\\ 1 & q & 1\\ 1 & r & 1\end{vmatrix}$ $=0+0+0=0$	$1\frac{1}{2}$ 1+1+1+1 $\frac{1}{2}$

	OR	
	If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, then find A^{-1}.	
	Using A^{-1}, solve the following system of equations: $2x-3y+5z=11$; $3x+2y-4z=-5$; $x+y-2z=-3$	
	Answer:	
	$\lvert A\rvert=2(0)+3(-2)+5(1)=-1$	1
	$\Rightarrow A^{-1}=-\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}$ or $\begin{bmatrix}0 & 1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix}$ **(1 mark for any 4 correct co-factors)**	2
	Given equations can be written as $AX=B$, where $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}$	1
	$\therefore X=A^{-1}B=\begin{bmatrix}0 & 1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix}\begin{bmatrix}11\\ -5\\ -3\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$	1
	$\Rightarrow x=1,\ y=2,\ z=3$	1
36	Using integration find the area of the region bounded between the two circles $x^2+y^2=9$ and $(x-3)^2+y^2=9$.	
	Answer: Correct Figure	1
	Point of intersection of, $x^2+y^2=9;(x-3)^2+y^2=9\Rightarrow(x-3)^2-x^2=0\Rightarrow x=3/2$	1
		
	Required area $=2\left[\int_0^{\frac{3}{2}}\sqrt{9-(x-3)^2}\,dx+\int_{\frac{3}{2}}^{3}\sqrt{9-x^2}\,dx\right]$ $=4\left[\int_{\frac{3}{2}}^{3}\sqrt{9-x^2}\,dx\right]$	$1\frac{1}{2}$
	$=4\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{\frac{3}{2}}^{3}=\left(6\pi-\frac{9\sqrt{3}}{2}\right)$	$1\frac{1}{2}+1$

	OR Evaluate the following integral as the limit of sums $\int_1^4 \left(x^2 - x\right) dx$. **Answer:** $\int_1^4 (x^2 - x)\, dx = \lim_{h \to 0} h\left[f\,(1) + f\,(1+h) + f\,(1+2h) + \ldots . f\left(1+\overline{n-1}\,h\right)\right]$ where $f\,(x) = \left(x^2 - x\right)$ and $nh = 3$ $\therefore \int_1^4 (x^2 - x)\, dx =$ $\lim_{h \to 0} h\left[(1-1) + \left(1 + h^2 + 2h - h - 1\right) + \left(1 + 4h^2 + 4h - 2h - 1\right) + \ldots + \left(1 + (n-1)^2 h^2 + 2(n-1)h - (n-1)h - 1\right)\right]$ $= \lim_{h \to 0} h\left[h^2\left(1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + (n-1)^2\right) + h\left(1 + 2 + 3 + \ldots . + (n-1)\right)\right]$ $= \lim_{h \to 0}\left[\frac{nh(nh-h)(2nh-h)}{6} + \frac{(nh(nh-h))}{2}\right] = 9 + \frac{9}{2} = \frac{27}{2}$	 1 1 2 1 1