Permutation & Combination Formulas

Permutation Formulas

★ When repetition is not allowed: P is a permutation or arrangement of r things from a set of n things without replacement. We define P as:

$$nP_r = \frac{n!}{(n-r)!}$$

★ When repetition is allowed: P is a permutation or arrangement of r things from a set of n things when repetition is allowed. We define P as:

$$nPr = n^r$$

Derivation of Permutation Formula:

Let us assume that there are r boxes and each of them can hold one thing. There will be as many permutations as there are ways of filling in *r* vacant boxes by *n* objects.

– No. of ways the first box can be filled: ***n***

– No. of ways the second box can be filled: **(*n* – 1)**

– No. of ways the third box can be filled: **(*n* – 2)**

– No. of ways the fourth box can be filled: **(*n* – 3)**

– No. of ways r^{th} box can be filled: **[*n* – (*r* – 1)]**

The number of permutations of *n* different objects taken *r* at a time, where $0 < r \le n$ and the objects do not repeat is: ***n*(*n* – 1)(*n* – 2)(*n* – 3) . . . (*n* – r + 1)**

⇒ $nP_r = n(n-1)(n-2)(n-3)...(n-r+1)$

Multiplying and dividing by $(n-r)(n-r-1)...3 \times 2 \times 1$, we get:

$$nP_r = \frac{[n(n-1)(n-2)(n-3)...(n-r+1)(n-r)(n-r-1)..3\times2\times1]}{(n-r)(n-r-1)..3\times2\times1} = \frac{n!}{(n-r)!}$$

$$nP_r = \frac{n!}{(n-r)!}$$

Combination Formulas

★ When repetition is not allowed: C is a combination of n distinct things taking r at a time (order is not important). We define C as:

$$nC_r = \frac{nP_r}{r!} = \frac{n!}{(n-r)!r!}$$

★ When repetition is allowed: C is a combination of n distinct things taking r at a time (order is not important) with repetition. We define C as:

$$nCr = (n + r - 1)!/[r!(n-1)!]$$

Derivation of Combination Formula:

Let us assume that there are r boxes and each of them can hold one thing.

– No. of ways to select the first object from *n* distinct objects: ***n***

– No. of ways to select the second object from (*n-1)* distinct objects: **(*n-1)***

– No. of ways to select the third object from (*n-2)* distinct objects: **(*n-2)***

– No. of ways to select r^{th} object from [*n-(r-1)*] distinct objects: **[*n-(r-1)*]**

Completing the selection of r things from the original set of n things creates an ordered subset of ***r*** elements.

∴ The number of ways to make a selection of r elements of the original set of *n* elements is: **$n(n-1)(n-2)(n-3)\ldots(n-(r-1))$ or $n(n-1)(n-2)\ldots(n-r+1)$**.

Let us consider the ordered subset of *r* elements and all its permutations. The total number of all permutations of this subset is equal to r! because *r* objects in every combination can be rearranged in *r!* ways.

Hence, the total number of permutations of *n* different things taken *r* at a time is ($nC_r \times r!$). It is nothing but nP_r.

$$nP_r = nC_r \times r!$$

$$nC_r = \frac{nP_r}{r!} = \frac{n!}{(n-r)!r!}$$