CBSE NCERT Solutions for Class 11 Mathematics Chapter 12

Back of Chapter Questions

Exercise 12.1

1. A point is on the x-axis. What are its y-coordinates and z-coordinates?

Hint: If a point is on the x-axis, then its y-coordinates and z-coordinates are zero.

Solution:

Solutions step 1: If a point is on the x-axis, then its y-coordinates and z-coordinates are zero.

2. A point is in the XZ-plane. What can you say about its y-coordinate?

Hint: If a point is in the XZ plane, then its y-coordinate is zero.

Solution:

Solutions step1: If a point is in the XZ plane, then its y-coordinate is zero.

3. Name the octants in which the following points lie:$(1,2,3)$, $(4,-2,3)$, $(4,-2,-5)$, $(4,2,-5)$, $(-4,2,-5)$, $(-4,2,5)$, $(-3,-1,6)$, $(2,-4,-7)$

Hint: Check the positive and negative of number.

Solution:

Solution step 1: The x-coordinate, y-coordinate, and z-coordinate of point $(1,2,3)$ are all positive.

Therefore, this point lies in octant I.

The x-coordinate, y-coordinate, and z-coordinate of point $(4,-2,3)$ are positive, negative, and positive respectively. Therefore, this point lies in octant IV.

The x-coordinate, y-coordinate, and z-coordinate of point $(4,-2,-5)$ are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.

The x-coordinate, y-coordinate, and z-coordinate of point $(4,2,-5)$ are positive, positive, and negative respectively. Therefore, this point lies in octant V.

The x-coordinate, y-coordinate, and z-coordinate of point $(-4,2,-5)$ are negative, positive, and negative respectively. Therefore, this point lies in octant VI.

The x-coordinate, y-coordinate, and z-coordinate of point $(-4,2,5)$ are negative, positive, and positive respectively. Therefore, this point lies in octant II.

The x-coordinate, y-coordinate, and z-coordinate of point $(-3,-1,6)$ are negative, negative, and positive respectively. Therefore, this point lies in octant III.

The x-coordinate, y-coordinate, and z-coordinate of point $(2,-4,-7)$ are positive, negative, and negative respectively. Therefore, this point lies in octant VIII.

4. Fill in the blanks:

(i) The x-axis and y-axis taken together determine a plane known as_______.

Hint: xy- plane.

Solution:

Solution step 1: (i) The x-axis and y-axis taken together determine a plane known as xy- plane.

(ii) The coordinates of points in the XY-plane are of the form _______.

Hint: XY-plane are of the form $(x, y, 0)$.

Solution:

Solution step 1: (ii) The coordinates of points in the XY-plane are of the form $(x, y, 0)$.

(iii) Coordinate planes divide the space into ______ octants.

Hint: Coordinate planes divide the space into eight octants.

Solution:

Solution step 1: Coordinate planes divide the space into eight octants.

Exercise 12.2

1. Find the distance between the following pairs of points:

(i). _(2,3,5) and (4,3,1)

Hint: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step1

(i): The distance between points $P(x_1, y_1, z_1)$ and $P(x_2, y_2, z_2)$ is given by

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Distance between points (2,3,5) and (4,3,1)

$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$

1: 2

(ii)(– 3,7,2) and (2,4, – 1)

Hint: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step 1: Distance between points (– 3,7,2) and (2,4, – 1)

$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$

$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$

$= \sqrt{25 + 9 + 9}$

$= \sqrt{46}$

1: 2

(iii)(– 1,3, – 4) and (1, – 3,4)

Hint: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step 1: Distance between points $(-1,3,-4)$ and $(1,-3,4)$

$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (8)^2}$

$= \sqrt{4+36+64}$

$= \sqrt{104}$

$= 2\sqrt{26}$

(iv)$(2,-1,3)$ and $(-2,1,3)$

Hint: $PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Solution:

Solution step:(iv) Distance between points $(2,-1,3)$ and $(-2,1,3)$

$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$

$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

$= 2\sqrt{5}$

2. Show that the points $(-2,3,5)$, $(1,2,3)$ and $(7,0,-1)$ are collinear.

Hint: $PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Solution:

Solution step 1: Let points $(-2,3,5)$, $(1,2,3)$ and $(7,0,-1)$ be denoted by P, Q and R respectively.

Points P, Q and R are collinear if they lie on a line.

$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$

$= \sqrt{(3)^2 + (-1)^2 + (-2)^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$

$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{36 + 4 + 16}$

$= \sqrt{56}$

$= 2\sqrt{14}$

$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$

$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$

$= \sqrt{81 + 9 + 36}$

$= \sqrt{126}$

$= 3\sqrt{14}$

Here, $PQ + QR = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = PR$

Hence, points $P(-2,3,5), Q(1,2,3)$ and $R(7,0,-1)$ are collinear.

3. (i)Verify the following:

(i) $(0,7,-10), (1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle

Hint: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step 1: Let points $(0,7,-10), (1,6,-6)$ and $(4,9,-6)$ be denoted by A, B and C respectively.

$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2}$

$= \sqrt{(1)^2 + (-1)^2 + (4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\text{CA} = \sqrt{(0-4)^2 + (7-9)^2 + (-10-6)^2}$

$= \sqrt{(-4)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{16 + 4 + 16}$

$= \sqrt{36}$

$= 6$

Here, $\text{AB} = \text{BC} \neq \text{CA}$

Thus, the given points are the vertices of an isosceles triangle.

(ii)$(0,7,10)$, $(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

Hint: $\text{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step 1: Let $(0,7,10)$, $(-1,6,6)$ and $(-4,9,6)$ be denoted by A, B and C respectively.

$\text{AB} = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + (-4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\text{BC} = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$

$= \sqrt{(-3)^2 + (3)^2 + (0)^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\text{CA} = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$

$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$

$= \sqrt{16 + 4 + 16}$

$= \sqrt{36}$

$= 6$

Now, $\text{AB}^2 + \text{BC}^2 = \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 = 18 + 18 = 36 = \text{AC}^2$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii)$(-1,2,1), (1,-2,5), (4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Hint: $\text{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step 1: Let $(-1,2,1), (1,-2,5), (4,-7,8)$ and $(2,-3,4)$ be denoted by A, B, C and D respectively.

$\text{AB} = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$

$= \sqrt{4 + 16 + 16}$

$= 6$

$\text{BC} = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$

$\text{CD} = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$

$= \sqrt{4 + 16 + 16}$

$= \sqrt{36}$

$= 6$

$\text{DA} = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$

Here, $\mathrm{AB} = \mathrm{CD} = 6, \mathrm{BC} = \mathrm{AD} = \sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

4. Find the equation of the set of points which are equidistant from the points $(1,2,3)$ and $(3,2,-1)$.

Hint: $\mathrm{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step 1: Let $\mathrm{P}(x, y, z)$ be the point that is equidistant from points $\mathrm{A}(1,2,3)$ and $\mathrm{B}(3,2,-1)$. Accordingly, $\mathrm{PA} = \mathrm{PB}$

$\Rightarrow \mathrm{PA}^2 = \mathrm{PB}^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

5. Find the equation of the set of points P, the sum of whose distances from $\mathrm{A}(4,0,0)$ and $\mathrm{B}(-4,0,0)$ is equal to10.

Hint: $\mathrm{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Solution:

Solution step 1: Let the coordinates of P be $(\mathrm{x}, \mathrm{y}, \mathrm{z})$.

The coordinates of points A and B are $(4,0,0)$ and $(-4,0,0)$ respectively.

It is given that $\mathrm{PA} + \mathrm{PB} = 10$.

$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$

$\Rightarrow \sqrt{(x-4)^2+y^2+z^2} = 10 - \sqrt{(x+4)^2+y^2+z^2}$

On squaring both sides, we obtain

$\Rightarrow (x-4)^2+y^2+z^2 = 100 - 20\sqrt{(x+4)^2+y^2+z^2} + (x+4)^2+y^2+z^2$

$\Rightarrow x^2-8x+16+y^2+z^2 = 100 - 20\sqrt{x^2+8x+16+y^2+z^2} + x^2+8x+16+y^2+z^2$

$\Rightarrow 20\sqrt{x^2+8x+16+y^2+z^2} = 100+16x$

$\Rightarrow 5\sqrt{x^2+8x+16+y^2+z^2} = (25+4x)$

On squaring both sides again, we obtain

$25\,(x^2+8x+16+y^2+z^2) = 625+16x^2+200x$

$\Rightarrow 25x^2+200x+400+25y^2+25z^2 = 625+16x^2+200x$

$\Rightarrow 9x^2+25y^2+25z^2-225 = 0$

Thus, the required equation is $9x^2+25y^2+25z^2-225=0$.

Exercise 12.3

1. (i)Find the coordinates of the point which divides the line segment joining the points $(-2,3,5)$ and $(1,-4,6)$ in the ratio (i) $2:3$ internally,

Hint: $\mathrm{P}(x_1,y_1,z_1)$ and $\mathrm{Q}(x_2,y_2,z_2)$ internally in the ratio $m:n$ are $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+my_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$

Solution:

Solution step 1: The coordinates of point R that divides the line segment joining points $\mathrm{P}(x_1,y_1,z_1)$ and $\mathrm{Q}(x_2,y_2,z_2)$ internally in the ratio $m:n$ are

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+my_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$$

Let $\mathrm{R}(x,y,z)$ be the point that divides the line segment joining points $(-2,3,5)$ and $(1,-4,6)$ internally in the ratio $2:3$

$x=\frac{2(1)+3(-2)}{2+3}, y=\frac{2(-4)+3(3)}{2+3}$ and $z=\frac{2(6)+3(5)}{2+3}$

i.e., $x=\frac{-4}{5}, y=\frac{1}{5}$ and $z=\frac{27}{5}$

Thus, the coordinates of the required point are $\left(-\frac{4}{5},\frac{1}{5},\frac{27}{5}\right)$.

(ii) $2:3$ externally

Solution:

Solution step 1: externally in the ratio $m:n$ are $\left(\frac{mx_2+nx_1}{m-n}, \frac{my_2-my_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$

The coordinates of point R that divides the line segment joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ externally in the ratio $m:n$ are

$$\left(\frac{mx_2+nx_1}{m-n}, \frac{my_2-my_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$$

Let $R(x, y, z)$ be the point that divides the line segment joining points $(-2,3,5)$ and $(1,-4,6)$ externally in the ratio $2:3$

$x = \frac{2(1)-3(-2)}{2-3}, y = \frac{2(-4)-3(3)}{2-3}$ and $z = \frac{2(6)-3(5)}{2-3}$

i.e., $x = -8, y = 17$ and $z = 3$

Thus, the coordinates of the required point are $(-8,17,3)$.

2. Given that $P(3,2,-4), Q(5,4,-6)$ and $R(9,8,-10)$ are collinear. Find the ratio in which Q divides PR.

Hint: $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m:n$ are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+my_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

Solution:

Solution step: 1 Let point $Q(5,4,-6)$ divide the line segment joining points $P(3,2,-4)$ and $R(9,8,-10)$ in the ratio $k:1$.

Therefore, by section formula,

$$(5,4,-6) = \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1}\right)$$

$$\Rightarrow \frac{9k+3}{k+1} = 5$$

$$\Rightarrow 9k+3 = 5k+5$$

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

Thus, point Q divides PR in the ratio$1:2$.

3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2,4,7)$ and $(3,-5,8)$.

Hint: $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m : n$ are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+my_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

Solution:

Solution step:1 Let the YZ plane divide the line segment joining points $(-2,4,7)$ and $(3,-5,8)$ in the ratio $k : 1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the x-coordinate of any point is zero.

$$\frac{3k-2}{k+1} = 0$$

$$\Rightarrow 3k - 2 = 0$$

$$\Rightarrow k = \frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio $2 : 3$.

4. Using section formula, show that the points $A(2,-3,4)$, $B(-1,2,1)$ and $C\left(\frac{0,1}{3}, 2\right)$are collinear.

Hint: $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m : n$ are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+my_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

Solution:

Solution step:1 The given points are $A(2,-3,4)$, $B(-1,2,1)$ and $C\left(\frac{0,1}{3}, 2\right)$.

Let P be a point that divides AB in the ratio $k : 1$.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+2}$, we obtain $k = 2$.

For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$.

i.e., C $\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio $2 : 1$ and is the same as point P.

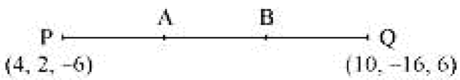
Hence, points A, B and C are collinear.

5. Find the coordinates of the points which trisect the line segment joining the points $P(4,2,-6)$ and $Q(10,-16,6)$.

Hint: $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m : n$ are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+my_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

Solution:

Solution step 1: Let A and B be the points that trisect the line segment joining points $P(4,2,-6)$ and $Q(10,-16,6)$



Point A divides PQ in the ratio $1 : 2$. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10) + 2(4)}{1 + 2}, \frac{1(-16) + 2(2)}{1 + 2}, \frac{1(6) + 2(-6)}{1 + 2}\right) = (6, -4, -2)$$

Point B divides PQ in the ratio $2 : 1$. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10) + 1(4)}{2 + 1}, \frac{2(-16) + 1(2)}{2 + 1}, \frac{2(6) - 1(6)}{2 + 1}\right) = (8, -10, 2)$$

Thus, $(6, -4, -2)$ and $(8, -10, 2)$ are the points that trisect the line segment joining points $P(4,2,-6)$ and $Q(10,-16,6)$.