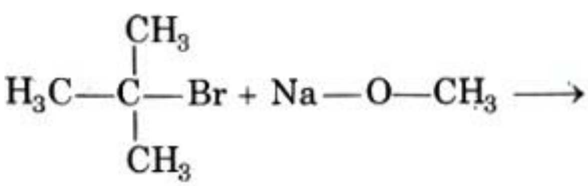
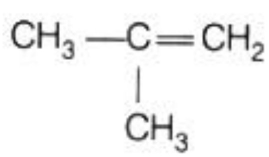
Chemistry

Single correct answer type:

1. What will be the product of the reaction?



(A)



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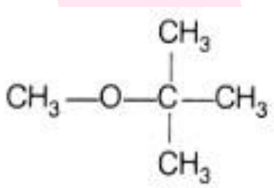
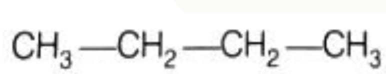
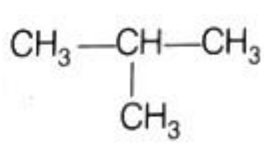


(C)



(D)



Solution: (A)

When tertiary alkyl halide is treated with sodium alkoxide than elimination reaction competes over substitution reaction because alkoxides are not only nucleophiles but strong base as well. Therefore, alkenes are formed instead of ethers.

2. Which of the following reaction is incorrect regarding Bohr's theory?

(A) Velocity of electron $\propto \frac{1}{n}$

(B) Frequency of revolution $\propto \frac{1}{n^2}$

(C) Radius of orbit $\propto n^2 z$

(D) Force on electron $\propto \frac{1}{n^4}$

Solution: (C)

Radius of orbit is directly proportional to ration of square of principal quantum number and atomic number

i.e., Radius of orbit $\propto \frac{n^2}{Z}$

For H-atom $r_n = \frac{n^2 \times 0.529 \times 10^{-8}}{z}\ cm$

3. Which of the following pair have identical shape?

(A) $CH_4 . SF_4$ (B) BCl_3, ClF_3

(C) $XeF_2, ZnCl_2$ (D) SO_2, CO_2

Solution: (C)

Molecules/compounds and their shape can be arranged as

Compound	Shape
XeF_2	Linear
$ZnCl_2$	Linear
BCl_3	Triangular planar
CiF_3	T-Shaped
CH_4	Tetrahedral
SF_4	See-saw

SO_2	Bent
CO_2	Linear

4. $10g$ of sample of mixture of $CaCl_2$ and $NaCl$ is treated to precipitate all the calcium as $CaCO_3$. This $CaCO_3$ is heated to convert all the Ca to CaO and the final mass of CaO us $1.62g$. The percent by mass of $CaCl_2$ in the original mixture is

(A) 32.1% (B) 16.2% (C) 21.8% (D) 12.0%

Solution: (A)

$CaCl_2 + NaCl = 10g$

Let weight of $CaCl_2 = xg$

Particle velocity

$$\mathrm{v}_P = \frac{dy}{dt} = \frac{d}{dt}\left[3\sin\left(25\pi t - \frac{\pi}{2}x\right)\right]$$

$$\mathrm{v}_P = 75\pi\cos\left(25\pi t - \frac{\pi}{2}x\right)$$

Maximum particle velocity, $(\mathrm{v}_P)_{max} = 75\pi\ m/s$

$$\Rightarrow \quad \frac{(\mathrm{v}_P)_{max}}{\mathrm{v}} = \frac{75\pi}{50}$$

$$= \frac{3}{2}\pi$$

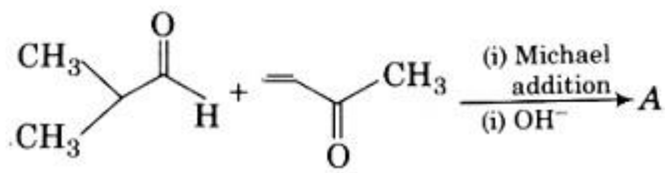
Mole of $CaO = \frac{1.62}{56}$

$$\therefore \quad \frac{x}{111} = \frac{1.62}{56}$$

$x = 3.21g$

$$\%\text{ of } CaCl_2 = \frac{3.21}{10} \times 100 = 32.1\%$$

5. How many chiral centre are possible for the product of following reaction?

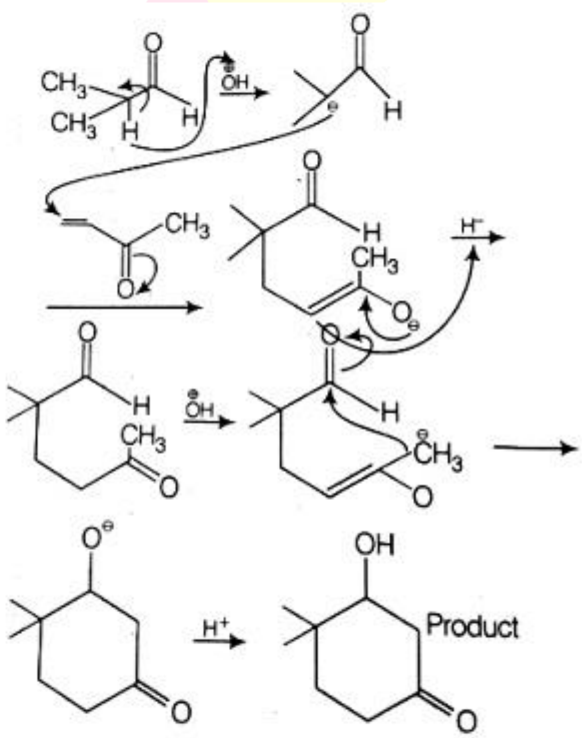


(A) 1 (B) 0 (C) 3 (D) 2

Solution: (A)

This problems includes conceptual mixing of Michael addition and number of chiral

Michael addition Addition of nucleophile to enone system is done in such a way that the addition looks like addition at 1^{st} and 4^{th} position of enone is known as Michael addition.



The number of chiral centre in product is 1 represented by star (*).

6. Elements/ions having same number of electrons are known as isoelectronic species. Arrange the following elements in correct order of atomic/ionic radii and choose the correct choice from the four choices given below

$O^{2-}, Na^{+}, Mg^{2+}, F^{-}, Al^{3+}$

(A) $Al^{3+} < Mg^{2+} < Na^{+} < F^{-} < O^{2-}$

(B) $Al^{3+} < Na^{+} < Mg^{2+} < F^{-} < O^{2-}$

(C) $Al^{3+} > Mg^{2+} > Na^{+} > F^{-} > O^{2-}$

(D) None of the above

Solution: (A)

Elements/ions having equal number of electrons are known as isoelectronic species. Among isoelectronic species, cations having highest charge are smallest while anion having highest charge are largest.

Cation < Neutral atom < Anion

Hence, correct choice is $Al^{3+} < Mg^{2+} < Na^{+} < F^{-} < CO^{2-}$

7. The ratio of oxidation states of Cl in potassium chloride to that in potassium chlorate is

(A) $+\frac{1}{5}$ (B) $-\frac{1}{5}$ (C) $-\frac{2}{5}$ (D) $+\frac{3}{5}$

Solution: (B)

Oxidation state of Cl in $KCl = -1$

Oxidation state of Cl in $KClO_3 = +5$

$\therefore$ Ratio of oxidation state of $Cl = \frac{-1}{5}$

8. A reaction, $Cu^{2+} + 2e^{-} \rightarrow Cu$ is given. For this reaction, graph between $E_{red}\ versus\ \ln[Cu^{2+}]$ is a straight line of intercept 0.34V, then the electrode oxidation potential of the half cell Cu/Cu^{2+} (0.1M) will be

(A) 0.35

(B) $0.34 + \frac{0.0591}{2}$

(C) $-0.34 - \frac{0.0591}{2}$

(D) $-0.34 + \frac{0.0591}{2}$

Solution: (D)

$Cu^{2+} + 2e^{-} \rightarrow Cu$

$$E_{Cu^{2+}/Cu} = E^o_{Cu^{2+}/Cu} - \frac{0.059}{2}\log\frac{1}{[Cu^{2+}]}$$

$$= E_{Cu^{2+}/Cu} - \frac{RT}{2F}\ln[Cu^{2+}]$$

Intercept $= 0.34 \Rightarrow E_{Cu^{2+}/Cu} = 0.34$

$$E_{Cu^{2+}/Cu} = 0.34 + \frac{0.059}{2}\log 0.1 = 0.31V$$

$$E_{Cu/Cu^{2+}=-}E_{Cu^{2+}/Cu} = -0.34 + \frac{0.059}{2}V$$

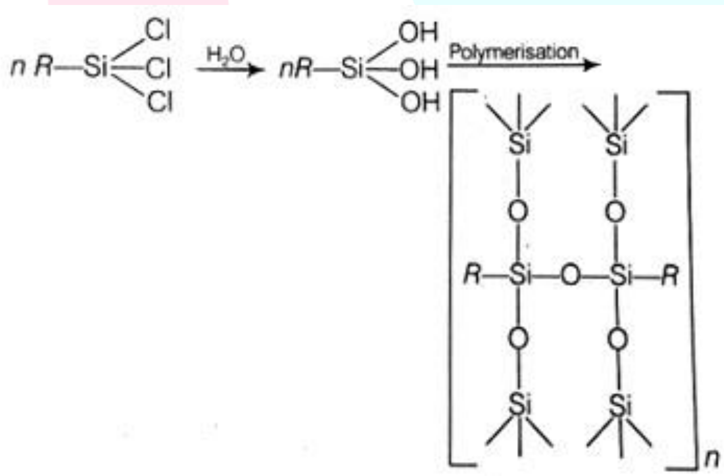
9. Which one of the following silanes on hydrolysis produces cross linked polymers?

(A) $RSiCl_3$ (B) R_2SiCl_2

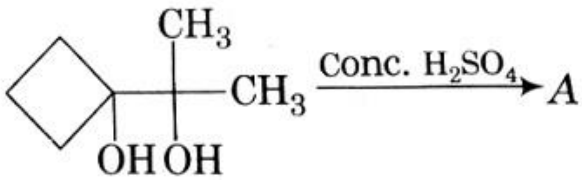
(C) R_3SiCl (D) R_4Si

Solution: (A)

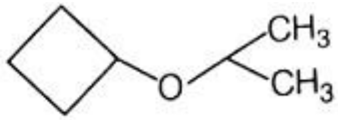
$RSiCl_3$ on hydrolysis produces cross linked polymer.



10. Identify the correct product formed during the following reaction.



(A)



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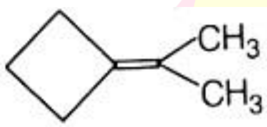

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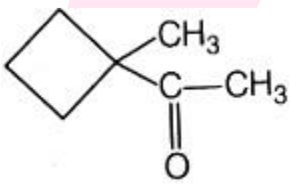
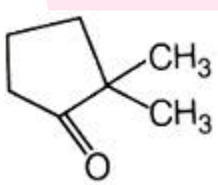
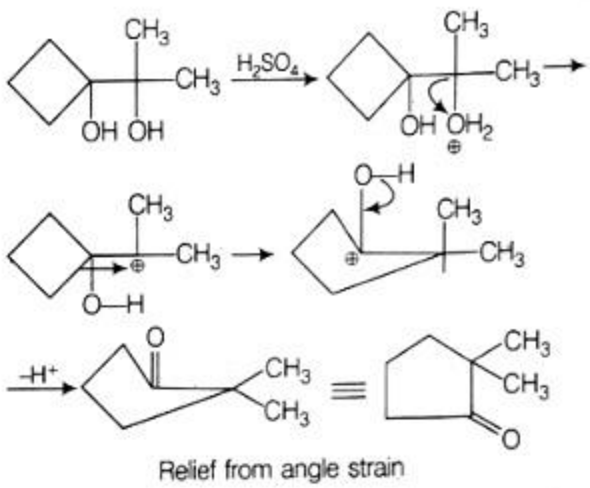


(D)



Solution: (D)

Pinacol-pinacolone rearrangement The diol is converted into α-hydroxy ketone when reacted in presence of acid is believe to proceeds through rearrangement of carbocation as shown.



Basic of carbocationic rearrangement is due to relief from angle strain.

11. Usually, $CaCl_2$ is preferred over $NaCl$ for cleaning snow on roads particulary in very cold countries. This is because

(A) $NaCl$ makes the road slippery but $CaCl_2$ does not

(B) $CaCl_2$ is hygroscopic but $NaCl$ is not

(C) $CaCl_2$ is less soluble in H_2O than $NaCl$

(D) Eutectic mixture of $CaCl_2/H_2O$ freezes at - 55^oC while that of $NaCl/H_2O$ freeze at - 18^oC.

Solution: (D)

A mixture of chemical compounds having a single chemical composition, solidifies at a lower temperature than any other composition made up of the same ingredients. This mixture is called eutectic mixture.

Freezing point eutectic mixture of $NaCl/H_2O$ is only - 180^oC but the ambient temperature of very cold countries is much lower than - 180^oC. In such situations, NaCl will be ineffective. Thus, for such situations eutectic mixture $CaCl_2/H_2O$ is used because it has freezing point of - 55^oC which is much lower than $NaCl$.

This mixture lowers the freezing point of ice that allows street snow or ice to melt at lower temperature.

12. The gold numbers of a few protective colloids are given

x 0.005
y 3.5
z 40

The protective nature of these colloidal solutions follow the order

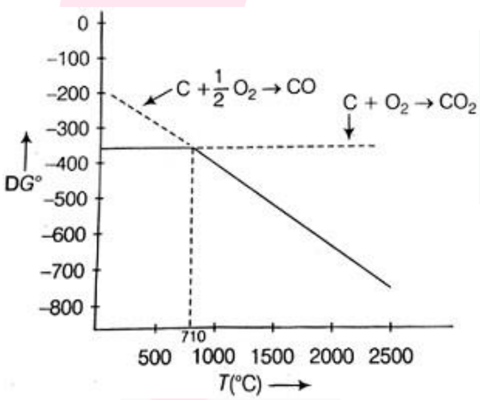
(A) $z > x > y$ (B) $x < y > z$ (C) $z > y > x$ (D) $x > y > z$

Solution: (D)

Smaller the gold number, greater is its protective power. Hence, the order would be $x > y > z$

Caution Point Gold number is used for calculating the protective powers of lyophilic colloids.

13. Consider the following Ellingham diagram for carbon



Which of the statement is incorrect for the above Ellingham diagram?

(A) Upto 710^oC, the reaction of formation of CO_2 is energetically more favourable but above 710^oC, the formation of CO is preferred

(B) Carbon can be used to reduce any metal oxide at a sufficiently high temperature

(C) Carbon reduces many oxides at elevated temperature because ΔG^o vs temperature line has a negative slope

(D) $\Delta S^o\left[C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g)\right] < \Delta S^o[C(s) + O_2(g) \rightarrow CO(g)]$

Solution: (D)

Since, $\Delta G^o = \Delta H^o - T\Delta S^o$

Where, $\Delta G^o =$ standard Gibb's free energy of the reaction

$\Delta S^o =$ standard entropy of the reaction

$\Delta H^o =$ standard enthalpy of the reaction

$T =$ temperature

From, the above equation, it is clear that, ΔG^o will be more negative when ΔS^o is less negative (or ΔS^o is high).

In the diagram, ΔG^o value for $C + \frac{1}{2}O_2 \rightarrow CO$ is less negative, (lower) than that for $C + O_2 \rightarrow CO_2$

Therefore, ΔS^o would be higher for

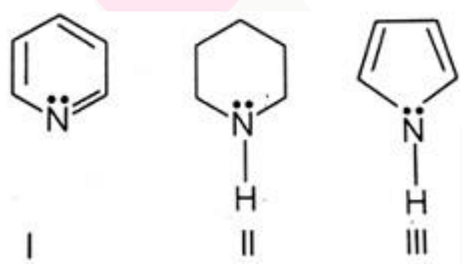
$$C + \frac{1}{2}O_2 \rightarrow CO$$

Than that for $C + O_2 \rightarrow O_2$

Hence,

$$\Delta S^o \left[C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g)\right] > \Delta S^o[C(s) + O_2(g) \rightarrow CO(g)]$$

14. Arrange the following in correct order of basicity



(A) $I > II > III$

(B) $III > II > I$

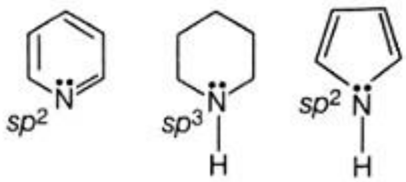
(C) $I > III > II$

Solution: (C)

This problem includes conceptual mixing of basic strength, hybridization of nitrogen atom and extent of conjugation.

While solving such problem students are advised to draw the structure and mark the type of hybridization on N-atom, then answer the question by using combined concept of hybridization and conjugation.

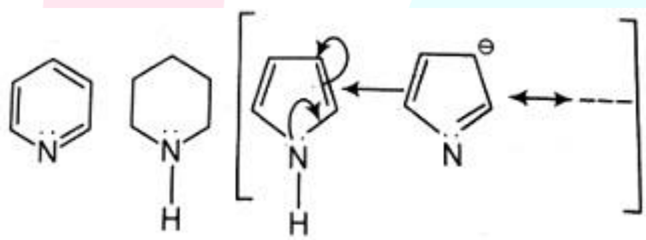
Hybridization of N-atom in below compounds are sp^2, sp^3 and sp^2 respectively.



I II III

Greater the s-character more will be electronegativity of N-atom and lesser will be its basicity on this basic I is less basic than II.

Conjugation If lone pairs of electron of N is involved in conjugation causes decrease in basicity of compound due to lesser availability of lone pair for donation to show basic nature.



Lone pair involved in formation of aromatic sextet of 6π-electron (least basic).

15. What is the density of Na_2O having antifluorite type crystal structure, if the edge length of the cube is 100pm and what is the effect on density by 0.05% Frenkel defect?

(A) $823.5\ g\ cm^{-3}$, density increases

(B) $414.16\ g\ cm^{-3}$, density decreases

(C) $823.5\ g\ cm^{-3}$, density remains same

(D) $414.16\ g\ cm^{-3}$, density remains same

Solution: (D)

As, density $(\rho) = \frac{Z_{eff} \times \text{Molecular weight}}{N_A \times a^3}$

(For antifluorite, $Z_{eff} = \frac{4}{\text{unit cell}}$

$$\rho = \frac{4 \times (23 \times 2 + 16)}{6 \times 10^{23} \times (100pm \times 10^{-10})^3}$$

$= 414.16\ g\ cm^{-3}$

$[1 \text{ picometer} = 10^{-12} m = 10^{-10} cm]$

Caution point Frenkel defect is the type of stoichiometric defect in which density of the crystal does not change.

16. A swimmer coming put from a pool is covered with a film of water weiging about 18g. Calculate the internal energy of vaporization at $100^o C$.

$\left[\Delta_{vap} H^{\ominus} \text{ for water at } 373\ K = 40.66\ kJ\ mol^{-1}\right]$

The correct option is

(A) $35.67\ kJ\ mol^{-1}$ (B) $37.56\ kJ\ mool^{-1}$

(C) $36.57\ kJ\ mol^{-1}$ (D) $38.75\ kJ\ mol^{-1}$

Solution: (B)

We can represent the process of evaporation as

$18\ g\ H_2O(l) \xrightarrow{\text{vporisation}} 18\ g\ H_2O(g)$

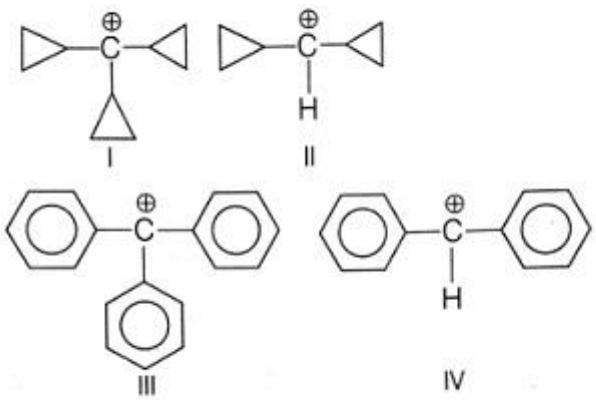
Number of oles in $18\ g\ H_2O(l)$ is $= \frac{18\ g}{18\ g\ mol^{-1}} = 1\ mole$

$\Delta_{\text{vap}} U = \Delta_{\text{vap}} H^{\ominus} - p\Delta V$

$= \Delta_{\text{vap}} H^{\ominus} - \Delta n_g RT$

Assume steam behave as an ideal gas.
$\Delta_{\text{vap}} U = (40.66) - (1)(8.314 \times 10^{-3})\ (373)$
$= 40.66 - 3.10$
$= 37.56\ kJ\ mol^{-1}$

17. Which of the following is correct order of stability of carbocation?



(A) $IV > III > II > I$ (B) $I > II > III > IV$

(C) $III > II > I > IV$ (D) $I > III > II > IV$

Solution: (D)

In case of cyclopropyl carbocation, stability of carbocation depends upon conjugation between bent orbitals of cyclopropyl ring and vacant p-orbital of cationic carbon. This type of bonding is known as banana bonding.

18. Mercury is a liquid metal because

(A) It has a completely filled d-orbital that causes d-d overlapping

(B) It has completely filled d-orbital that prevents d-d overlapping

(C) It has a completely filled s-orbital

(D) It has a small atomic size

Solution: (B)

The electronic configuration of mercury is $[Xe]4f^{10}, 5d^{10}, 6s^2$. Its d-subshell is completely filled, thus, it prevents the overlapping of d-orbitals (d-d overlapping). Hence, it is liquid metal at room temperature.

19. The volume of 10N and 4N HCl required to make 1L of 7N HCl are

(A) $0.75\ L$ of $10\ N\ HCl$ and $0.25\ L$ of $4\ N\ HCl$

(B) $0.50\ L$ of $10\ N\ HCl$ and $0.50\ L$ of $4\ N\ HCl$

(C) $0.65\ L$ of $10\ N\ HCl$ and $0.5\ L$ of $4\ N\ HCl$

(D) $0.85\ L$ of $10\ N\ HCl$ and $0.15\ L$ of $4\ N\ HCl$

Solution: (B)

Let V litre of 10 N HCl be mixed with (1 – V) litre of 4 N HCl to give (V + 1 – V) = 1 L of 7N HCl

As we know that,

$N_1V_1 + N_2V_2 = NV$

$10V + 4(1 - V) = 7 \times 1$

$10V + 4 - 4V = 7$

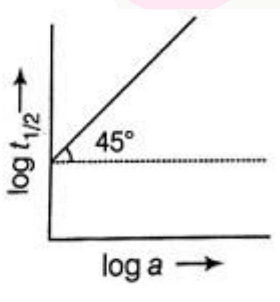
$6V = 7 - 4$

$V = \frac{3}{6} = 0.50L$

Volume of $10\ N\ HCl\ =\ 0.50\ L$

Volume of $4\ N\ HCl\ =\ 1 - 0.50\ =\ 0.50\ L$

20. Following is the graph between $\log T_{50}$ and $\log a(a = \text{initial concentration})$ for a given reaction at 27^oC. Hence, order is



(A) 1 (B) 2 (C) 3 (D) 0

Solution: (D)

$$t_{\frac{1}{2}} \propto \left(\frac{1}{a}\right)^{n-1}\ or\ t_{\frac{1}{2}} = k(a)^{1-n}$$

$\log t_{\frac{1}{2}} = \log k + (1-n)\log a$

(It represents straight line equation; $y = c + mx$)

Slope $= (1-n) = \tan 45^o = 1$

$\therefore \quad (1-n) = 1$

$\Rightarrow \quad n = 0$

21. The catalyst used for olefin polymerization is

(A) Ziegler-Natta catalyst

(B) Raneynickel catalyst

(C) Wilkinson catalyst

(D) Merrified resin

Solution: (A)

Ziegler-Natta catalyst $[TiCl_4 + Al(C_2H_5)_3]$ is used as a catalyst in the polymerization of olefins.

22. Which one of the following is a covalent hydride?

(A) CaH_2 (B) NaH (C) BH_3 (D) BeH_2

Solution: (C)

Hydrides are binary compounds of hydrogen. These can be classified into four groups

(i) Ionic hydrides : NaH, CaH_2, LiH

(ii) Covalent hydrides: $B_2H_6, NH_3, NaBH_4$

(iii) Polynuclear hydrides : $LiAlH_4, NaBH_4$

(iv) Interstitial hydrides are those in which hydrogen is trapped in the interstitial spaces of transition metals.

Here, B_2H_6 is a dimeric form of $BH_3 . BH_3$ covalently combined with another BH_3 molecule to form $B_2H_6 . B_2H_6$ contain 3 centre $2e^-$ bonds.

23. Which one of the following is used for the separation of noble gas mixture from air?

(A) Charcoal

(B) $90\%\ CaC_2 + 10\% CaCl_2$

(C) Soda lime + potash solution

(D) $90\%\ CaCO_3 + 10\%$ urea

Solution: (B)

The method used to separate noble gas mixture from air is called Fischer-Ringe's method. When air free from moisture and CO_2 is passed over a heated mixture (800^oC) of 90% $CaC_2 + 10\% CaCl_2$ in an iron sealed tube, the following reactions take place

$$CaC_2 + N_2 \xrightarrow{800^oC} CaCN_2 + C$$

$$2C + O \rightarrow 2CO$$

$$C + O_2 \rightarrow CO_2$$

$$2CaC_2 + 3CO_2 \rightarrow 2CaCO_3 + 5C$$

$$CuO + CO \rightarrow Cu + CO_2$$

CO_2 gas is absorbed by KOH solution. Thus, a mixture of inert gases are obtained.

24. Consider the following statements.

I. NCl_5 does not exist while PCl_5 does.

II. Both O_2^+ and NO are paramagnetic.

III. The three $C-O$ bonds are not equal in carbonate ion.

IV. Head prefers to form tetravalent compound. Which of the above statements are incorrect?

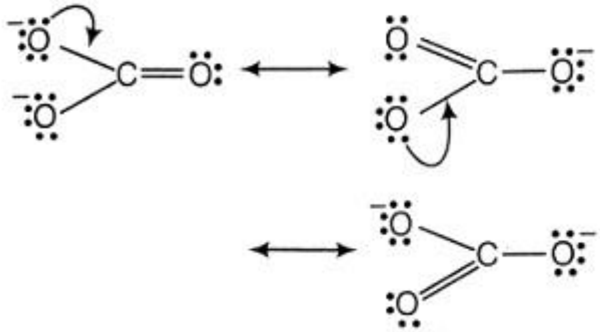
(A) I and III (B) I, III and IV (C) II and III (D) III and IV

Solution: (D)

I. In nitrogen, d-orbitals are absent, hence, it does not form NCl_5. Thus, NCl_5 does not exist but PCl_5 does.

II. O_2^+ and NO are isoelectronic and contains one unpaired electron each. Thus, both are paramagnetic.

III. In carbonate ion. CO_3^{2-} all three $C - O$ bonds are identical due to resonance



IV. Pb^{2+} is more stable than Pn^{4+} due to inert pair effect, hence, prefers to form divalent compounds.

Thus, the incorrect statements are III and IV.

25. The liquefied metal that expand on solidification is

(A) Al (B) Zn (C) Ga (D) Cu

Solution: (C)

Gallium, Ga is a soft silvery white metal and is liquid at room temperature. When it solidifies, expands by 3.1%. Thus, it should not be stored in glass or metal containers.

26. Point out the correct statement for the set of characteristics of ZnS crystal.

(A) Coordination number (4 : 4); ccp; Zn^{2+} ion in the alternate tetrahedral voids

(B) Coordination number (6 : 6); hcp; Zn^{2+} ion in all tetrahedral voids

(C) Coordination number (6 : 4); hcp; Zn^{2+} ion in all octahedral voids

(D) Coordination number (4 : 4); ccp; Zn^{2+} ion in all tetrahedral voids

Solution: (A)

Zns has zinc blende type structure (i.e., ccp structure). The S^{2-} ions are present at the corners of the cube and at the centre of each face. Zinc ions occupy half of the tetrahedral sites. Each zinc ion is surrounded by four sulphide ions which are disposed towards the corner of regular tetrahedron. Similarly, S^{2-} ion is surrounded by four Zn^{2+} ions.

27. Arrange the following compounds in the increasing order of nucleophilic addition reaction.

I. $HCHO$

II. CH_3COCH_3

III. $C_6H_5COCH_3$

IV. $C_3H_5COC_6H_5$

(A) $I < II < III < IV$ (B) $IV < III < II < I$

(C) $IV < II < III < I$ (D) $III < IV < II < I$

Solution: (C)

Reactivity of nucleophilic addition reaction depends upon the electron deficiency of carbonyl group and steric hinderance. Steric hinderance decreases the rate of reaction. This steric hinderance is minimum in methanol and maximum in benzophenone.

28. The heat of combustion of sucrose, $C_{12}H_{12}O_{11}(s)$ at constant volume is 1348.9 $kcal\ mol^{-1}$ at 25^oC, then the heat of reaction at constant pressure when steam is produced

(A) $-1348.9\ kcal$ (B) $-1342.34\ kcal$

(C) $+1250\ kcal$ (D) None of the above

Solution: (B)

The combustion equation of sucrose is $C_{12}H_{22}O_{11}(s) + 12O_2(G) \rightarrow 12CO_2(g) + 11H_2O(g)$

Here, $\Delta n = 12 + 11 - 12 = 11$

As we know,

$\Delta H = \Delta E + \Delta nRT$

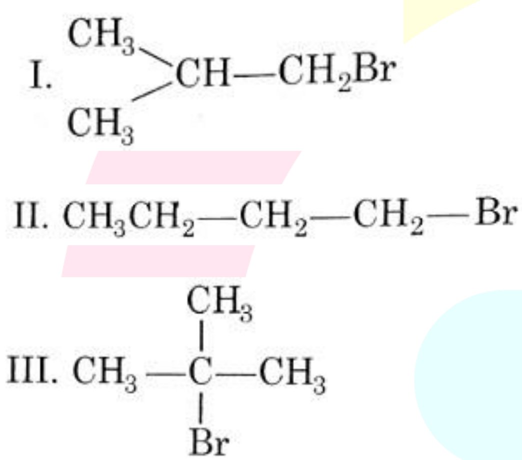
$\Delta H = (-1348.9 \times 10^3) + 11 \times 2 \times 298$

$= -1348900 + 6556$

$= -1342344\ cal$

$= -1342.344\ kcal$

29. Arrange the following compounds in increasing order of their boiling points.



(A) $II < I < III$ (B) $I < II < III$

(C) $III < I < II$ (D) $III < II < I$

Solution: (C)

Boiling point decreases with increase in branching. Compound (III) has two branches, compound (I) has one branch and compound (II) is a normal alkyl halide with no branch. So, the boiling point is minimum for compound (III) and maximum for compound (II).

30. Which of the following compounds will give positive iodoform test with I_2 and $NaOH$?

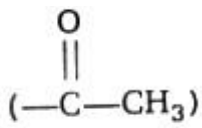
(A) $C_6H_5COC_6H_5$

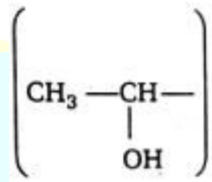
(B) CH_3CH_2CHO

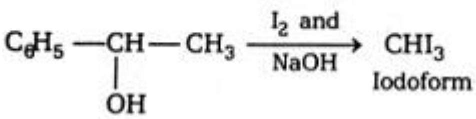
(C) $C_6H_4COCH_2CH_3$

(D)

Solution: (D)

Aldehydes and ketones posses  group will give positive iodoform test,

Apart from methylated carbonyl compounds, alcohols with  group also give positive iodoform test.

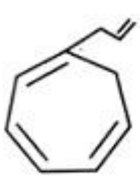


$+C_6H_5COONa$

31. What will be the product when most acidic species among following will react with 3-chloroprop-1-ene?

(A)

(B)



(C)

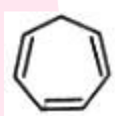
(D)

Solution: (A)

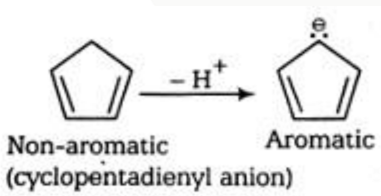
This problem includes conceptual mixing of acidic character, aromaticity and nucleophilic substitution reaction.

Student are advised to identify the most stable intermediate obtained among all (after the removal of H^+) keeping in mind the concept of conjugation and aromaticity. Then complete the reaction further using concept of nucleophilic substitution reaction.

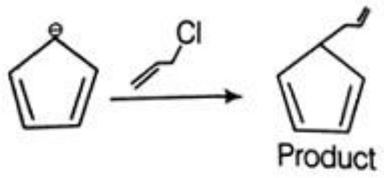
Acidic character The species which easily donate its hydrogen and produces stable conjugate base is acid. The species which produces more stable conjugate base is more stronger acid.

 and  do not looses H^+ hence are not acidic.

 looses the H easily and produces more stablearomatic cyclopentadienyl anion.



Now, cyclopentadienyl anion on reaction with 3-chloro prop-1-ene produces the product via nucleophilic substitution reaction.



32. $CaCO_3(s) \xrightarrow{\text{Heat}} X(s) + Z(g)$

$X(s) \xrightarrow{\text{Carbon.heat}} C(s) + D(g)$

$C(s) + H_2O \rightarrow E(g)$.

(A) C_2H_2, CaO (B) C_2H_2, CaC_2

(C) CH_4, CaC_2 (D) CH_4, CaO

Solution: (B)

33. Which of the following will not form optical isomers?

(A) $[Co(en)_3]^{3+}$ (B) $[Co(NH_3)_3\,(NO_2)_3]$

(C) $[Pt(en)_2Cl_2]^{2+}$ (D) $[CrCl_2(ox)_2]^{3-}$

Solution: (B)

Optical isomerism is shown by only those complexes which lack symmetry. Complex $[Co(NH_3)_3\,(NO_2)_3]$ shows facial and meridional isomerism. Both isomers of this complex contain place of symmetry. So, it will not form optical isomers.

34.

The true statement about the product is

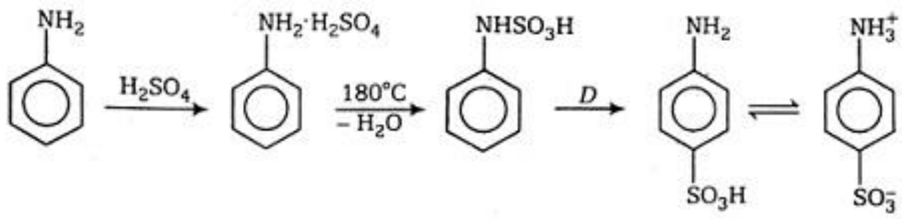
(A) It does not exist as Zwitter ion

(B) It does not act as inner salt

(C) $-SO_3$ diminishes the basic character of $-NH_2$

(D) $-NH_2$ displays a powerful basic character

Solution: (C)



Sulphanilic acid

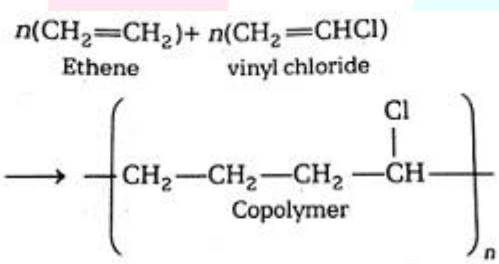
Sulphanilic acid exists as a dipolar ion which has acidic as well as basic groups in the same molecule. Such ions are called Zwitter ions or inner salts.

35. A copolymer of ethene and vinyl chloride contains alternate monomers of each type. What is the mass percentage of vinyl chloride in this copolymer?

(A) 38% (B) 69% (C) 72% (D) 82%

Solution: (B)

The structure of copolymer of ethane and vinyl chloride is shown below



Molecular weight of ethane $(CH_1CH_2) = 28$

Molecular weight of vinyl chloride $(CH_2CHCl) = 62.5$

Empirical formula weight of copolymer $= 28 + 62.5 = 90.5$

Mass % of vinyl chloride in the copolymer $= \frac{62.5\times100}{90.5} = 69.06 \approx 69\%$

36. The number of disulphide linkages present in insulin are

(A) 1 (B) 2 (C) 3 (D) 4

Solution: (B)

Insulin is composed of two peptide chains referred to chain A and B. Chain A of 21 residues and chain B of 30 residues are cross linked by two disulphide bridges.

37. Which of the following statement is not true about the drug barbital?

(A) It is used in sleeping pills

(B) It is a non-hypnotic drug

(C) It is transquilizer

(D) It causes addiction

Solution: (B)

Barbital is a sleep-producing drug, hypnotic tranquillizer. It causes addition.

38. Calculate the pH at the equivalence point during the titration of 0.1M, 25 mL CH_3COOH with $0.05\ M\ NaOH$ solution. $(K_a(CH_3COOH) = 1.8 \times 10^{-5})$

(A) 9.63 (B) 8.63 (C) 10.63 (D) 11.63

Solution: (B)

Since, at equivalence point (for acid) N_1V_1

$= N_2V_2$ (for base)

∴ Volume of NaOH required to reach equivalence point $= \frac{0.1\times25}{0.05} = 50\ mL$

∴ Concentration of salt formed $= \frac{\text{millimolers of acid}}{\text{total volume in mL}} = \frac{25\times0.1}{75} = \frac{0.1}{3}$

Since, $[H^+] = \sqrt{\frac{K_w\times K_a}{C}} = \sqrt{\frac{10^{-14}\times1.8\times10^{-5}\times3}{0.1}}$

∴ $pH = 8.63$

39. The temperature 30.98^oC is called critical temperature (T_c) of carbon dioxide. The critical temperature is the

(A) Lowest temperature at which liquid carbon dioxide is observed

(B) Highest temperature at which gas carbon

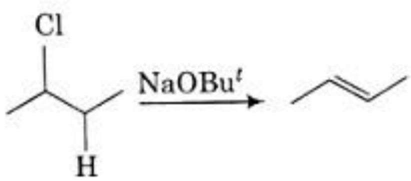
(C) Highest temperature at which solid carbon dioxide is observed

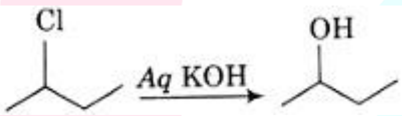
(D) Highest temperature at which liquid carbon dioxide is observed

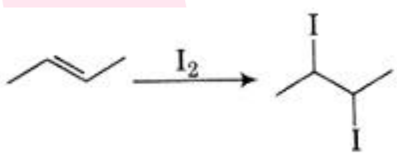
Solution: (D)

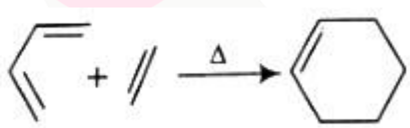
Critical temperature of a gas is highest temperature at which liquification of the gas first occurs. The temperature 30.98^oC is called critical temperature of carbon dioxide because this is the highest temperature at which liquid carbon dioxide is observed. Above this temperature it is gas.

40. The type of reactions for these are

I. 

II. 

III. 

IV. 

(A) Elimination, substitution, addition, addition

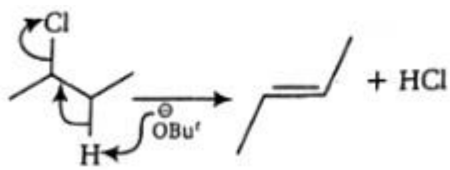
(B) Addition, elimination, addition, substitution

(C) Elimination, addition, substitution, addition

(D) Substitution, elimination, addition, addition

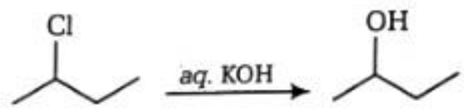
Solution: (A)

I. Elimination reaction



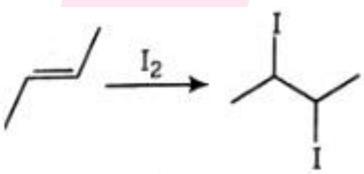
The reaction in which smaller neutral molecule are removed during the reaction is known as elimination reaction.

II. Substitution reaction The reaction in which one nucleophilic group is replaced by another nucleophile is known as nucleophilic substitution reaction.

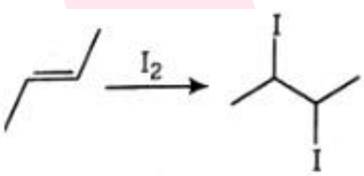


Here, OH replaces Cl.

III. Addition reaction The reaction in which reactant undergo addition with reagent to given a single product.



IV. Addition reaction



Hence,

English

Single correct answer type:

1. Out of the four alternatives, choose the one which express the right meaning of the word.

Augment

(A) Increase (B) Decrease

(C) Save (D) Mention

Solution: (A)

Augment means make bigger, so increase is the correct option.

2. Out of the four alternatives, choose the one which express the right meaning of the word.

Consolation

(A) Comfort (B) Problem

(C) Sadness (D) Solution

Solution: (A)

Consolation means ‘comfort received by a person after a loss’, so comfort is correct option.

3. Out of the four alternatives, choose the one which express the right meaning of the word.

Auxiliary

(A) Chief (B) Supplemental

(C) Negligible (D) Separate

Solution: (B)

Auxiliary means ‘providing additional help’, so supplemental is correct option.

4. Choose the word apposite meaning to the given word.

Auspicious

(A) Prosperous (B) Unfavourable

(C) Improper (D) New

Solution: (B)

Auspicious means ‘favourable’, so ‘unfavourable’ is best opposite word for it.

5. Choose the word apposite meaning to the given word.

Recompense

(A) Emolument (B) Reward

(C) Payment (D) Penalty

Solution: (D)

Recompense means ‘payment’, so ‘penalty’ is the correct opposite word for it.

6. Choose the word apposite meaning to the given word.

Impede

(A) Block (B) Delay

(C) Push (D) Freeze

Solution: (C)

Impede means ‘hinder’ or ‘obstruct’, so ‘push’ is correct opposite word for it.

7. A part of sentence is underlined. Balance are given alternatives to the underlined part a, b, c and d which many improve the sentence. Choose the correct alternative.

They <u>requested</u> me to follow them

(A) Ordered (B) Urged

(C) Asked (D) No improvement

Solution: (A)

Here a sense of command is depicted in sentence, so we should use ‘ordered’ for proper meaning of sentence.

8. A part of sentence is underlined. Balance are given alternatives to the underlined part a, b, c and d which many improve the sentence. Choose the correct alternative.

She did not <u>believed</u> me.

(A) Believing (B) Believe to

(C) Believe (D) No improvement

Solution: (C)

Sentence is in past tense and V_1 is used in those sentence which contain ‘did’, so option (believe) is correct.

9. A part of sentence is underlined. Balance are given alternatives to the underlined part a, b, c and d which many improve the sentence. Choose the correct alternative.

I am fine, what about <u>you?</u>

(A) Your (B) Your’s

(C) Yours (D) No improvement

Solution: (D)

No improvements is needed as sentence is right.

10. Sentence Completion

They were afraid _______ the lion, so they dropped the idea of hunting in jungle.

(A) in (B) to (C) from (D) to

Solution: (D)

Afraid agrees with preposition ‘of’, so option (to) is correct.

11. Sentence Completion

Our company signed a profitable _____ last month.

(A) issue (B) agenda (C) deal (D) paper

Solution: (C)

Normally, company signs a contract or deal, so use of ‘deal’ is proper here.

12. What is your ____ for tonight?

(A) Principle (B) Motto (C) Plan (D) Objective

Solution: (C)

The question gives a sense of query about normal routine of some special/specific day, so use of ‘plan’ is more proper here.

13. Arrange the following sentences in correct pattern and mark at the correct combination.

1. Today we live in modern technology era.

P. We have a lot of problems now.

Q. We want to get everything in one day.

R. Ancient time was quite pleasant.

S. We had no problems then.

C. Perhaps greed is the main cause for this.

(A) $PQRS$ (B) $PRSQ$ (C) $SRQP$ (D) $RPQS$

Solution: (B)

According to the events of sentence, PRSQ is best arrangement.

14. Arrange the following sentences in correct pattern and mark at the correct combination.

1. He is a common man.

P. Yesterday our city saw a brutal crime.

Q. Police is trying to arrest innocent persons.

R. The criminals are well known.

S. Police as well as whole system in corrupt.

C. Police will arrest him as he is an easy target because of being a common man.

(A) $PRSQ$ (B) $PQSR$ (C) $PQRS$ (D) $PSQR$

Solution: (A)

According to events of sentence, PRSQ is best arrangement.

15. Arrange the following sentences in correct pattern and mark at the correct combination.

1. I want to change the room.

P. Last month I got a job.

Q. I had been living there for six months.

R. The office is far from the room.

S. I want to cut expenses of travelling.

C. Hopefully I will do this next week.

(A) $PQRS$ (B) $PRSQ$ (C) $QPRS$ (D) $PQSR$

Solution: (C)

According to sequence of events in the sentence, QPRS best arrangement.

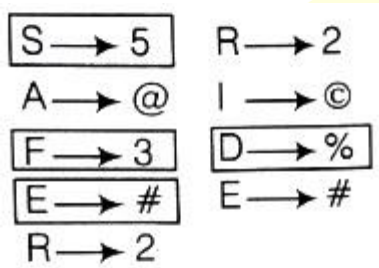
16. In a certain code language, 'SAFER' is written as $'5@3\#2'$ and 'RIDE' is written as $'2©\%\#'$, how would 'FEDS' be written in that code?

(A) $3\#©5$ (B) $3©\%5$ (C) $3\#\%5$ (D) $3\#\%2$

Solution: (C)

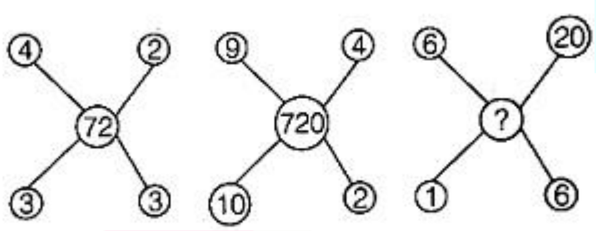
Given,

$SAFER = 5@3@2$ and $RIDE = 2©\%\#$



∴ Code for FEDS $= 3\#\%5$

17. Find the missing number from the given response.



(A) 72 (B) 720 (C) 7200 (D) 38

Solution: (B)

From the given responses,

$4 \times 2 \times 3 \times 3 = 72$

$9 \times 4 \times 2 \times 10 = 720$

Similarly, $6 \times 20 \times 1 \times 6 = 720$

18. If the first and second letters in the word DEPRESSION were interchanged, also the third and fourth letters, the fifth and the sixth letters and so on, then which of the following would be seventh letter from the right.

(A) O (B) P (C) R (D) S

Solution: (B)

Since, consecutive two letters are interchanged. Therefore,

DE PR ES SI ON

↓ ↓ ↓ ↓ ↓

ED R $\boxed{P}$ SE IS NO ← On counting

Now, on counting from right hand side P is the 7^{th} letter from right.

19. Today is Thursday, The day after 59 days will be

(A) Sunday (B) Monday (C) Tuesday (D) Wednesday

Solution: (A)

Every day of week repeats after seven days.

Hence, $59 = 7 \times 8 + 3 = 56 + 3$

∴ It will be Thursday after 56 days.

∴ 57^{th} day = Thursday ⇒ 58^{th} day = Friday

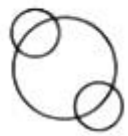
59^{th} day = Saturday ⇒ 60^{th} day = Sunday

∴ It will be Sunday after 59 days.

20. Which of the following represents coal mines, factories and fields?

(A)

(B)

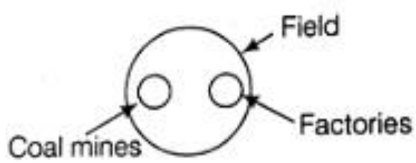


(C)

(D)

Solution: (D)

Both coal mines and factories are located in the fields.



21. Find out the missing term in the series.

1, 8, 27, <u>?</u>, 125, 216

(A) 52 (B) 58 (C) 64 (D) 65

Solution: (C)

From the given series,

$$1^3 \longrightarrow 1$$
$$2^3 \longrightarrow 8$$
$$3^3 \longrightarrow 27$$
$$\boxed{4^3 \longrightarrow 64}$$
$$5^3 \longrightarrow 125$$
$$6^3 \longrightarrow 216$$

Therefore, 64 will come in place of questions mark.

22. If $'+'$ means $'\times'$, $'-'$ means $'+'$, $'\times'$ means $'\div'$ and $'\div'$ means $'-'$, then $6-9+8\times 3\div 20=?$

(A) -2 (B) 6 (C) 10 (D) 12

Solution: (C)

Interchanging the symbols as given in the above question, the above equation becomes

$$6+9\times 8\div 3-20=6+9\times\frac{8}{3}-20$$

$$=6+24-20=10$$

23. What is the water image of

(A)

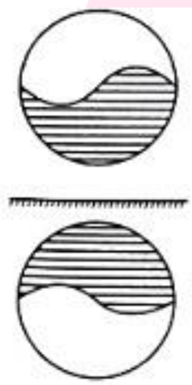
(B)

(C)

(D)

Solution: (B)

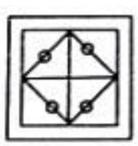
Water image is the reflection of image in water.



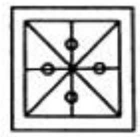
24. A piece of paper is folded and penched as shown in the figure below

How will it appear when unfolded?

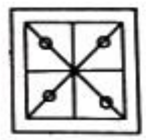
(A)



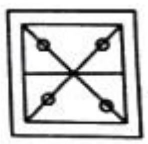
(B)



(C)



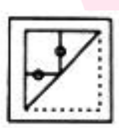
(D)



Solution: (B)

On unfolding layer 1,

On unfolding layer 2,



On unfolding layer 3,

25. Here are some words translated from an artificial language.

mallon piml means blue light

mallon tifl means blue berry

arpan tifl means rasp berry

Which word could means ‘light house’?

(A) tiflmallon (B) pimlarpan (C) mallonarpan (D) pimldoken

Solution: (D)

$\boxed{\text{mallon}}$ piml = $\boxed{\text{blue}}$ light

$\boxed{\text{mallon}}$ tifl = $\boxed{\text{blue}}$ berry

arpan tifl = rasp berry

∴ From the above analysis, we see codes as

Blue = Mallon

Light = Piml

berry = tifl

rasp = arpan

∴ Light i.e., piml is given in two option (b) and (d) but in option (b) other code given is of rasp, hence it cannot be the code of house. So, option (d) is correct.

Mathematics

Single correct answer type:

1. If p, q, r and s are positive real numbers such that $p + q + r + s = 2$, then $M = (p + q)\ (r + s)$ satisfies the relation

(A) $0 < M \leq 1$ (B) $1 \leq M \leq 2$

(C) $2 \leq M \leq 3$ (D) $3 \leq M \leq 4$

Solution: (A)

Since, $AM \geq GM,$ then

$$\frac{(p+q)+(r+s)}{2} \geq \sqrt{(p+q)(r+s)}$$

$$\Rightarrow \frac{2}{2} \geq \sqrt{M} \Rightarrow \sqrt{M} \leq 1 \Rightarrow M \leq 1$$

Also, $(p + q)\ (r + s) > 0$ $(\because \ p, q, r, s > 0)$

$\therefore \quad M > 0$

Hence, $0 < M \leq 1$

2. The complex number $z = x + iy$ which satisfies the equation $\left|\frac{z-3i}{z+3i}\right| = 1$, lie on

(A) The X-axis (B) The straight line $y = 3$

(C) A circle passing through origin (D) None of the above

Solution: (A)

Given, $\left|\frac{z-3i}{z+3i}\right| = 1 \quad \Rightarrow \quad |z - 3i| = |z + 3i|$

(if $|z - z_1| = |z - z_2|$, then it is a perpendicular bisector of z_1 and z_2)

Hence, perpendicular bisector of (0, 3) and (0, -3) is X-axis.

3. If (x) is an odd periodic function with period 2, then $f(4)$ equal to

(A) -4 (B) 4 (C) 2 (D) 0

Solution: (D)

Since, $f(x)$ is an odd periodic function with period 2.

$\therefore \quad f(-x) = -f(x)$ and $f(x+2) = f(x)$

$\therefore \quad f(2) = f(0+2) = f(0)$

and $\quad f(-2) = f(-2+2) = f(0)$

Now, $\quad f(0) = f(-2) = -f(2) = -f(0)$

$\Rightarrow \quad 2f(0) = 0, i.e., f(0) = 0$

$\therefore \quad f(4) = f(2+2) = f(2) = f(0) = 0$

Thus, $\quad f(4) = 0$

4. The solution of the differential equation

$$\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots} = \frac{dx - dy}{dx + dy} \text{ is}$$

(A) $2ye^{2x} = Ce^{2x} + 1$ (B) $2ye^{2x} = Ce^{2x} - 1$

(C) $ye^{2x} = Ce^{2x} + 2$ (D) None of these

Solution: (B)

We have, $\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots} = \frac{dx - dy}{dx + dy}$

On applying componendo and dividend, we get

$$\frac{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right) + \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)}{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right) - \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)}$$

$$= \frac{(dx - dy) + (dx + dy)}{(dx - dy) - (dx + dy)}$$

$$\Rightarrow \quad \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)}{-\left(1 - x + \frac{x^2}{2!} - \frac{x^2}{3!} + \cdots\right)} = \frac{2dx}{-2dy}$$

$$\Rightarrow \quad \frac{e^x}{-e^{-x}} = -\frac{dx}{dy}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-x}}{e^x} \Rightarrow \frac{dy}{dx} = e^{-2x} \Rightarrow \quad dy = e^{-2x} dx$$

On integrating both sides, we get

$$y = \frac{e^{-2x}}{(-2)} + C_1 \Rightarrow \quad 2y = -e^{-2x} + 2C_1$$

$$\Rightarrow \quad 2y = -e^{-2x} + C \qquad \text{(where, } C = 2C_1\text{)}$$

$$\Rightarrow \quad 2ye^{2x} = -1 + e^{2x}\, C \Rightarrow \; 2ye^{2x} = Ce^{2x} - 1$$

5. The value of k such that the lines $2x - 3y + k = 0, 3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent, is

(A) 20 (B) -7 (C) 7 (D) -20

Solution: (B)

Given lines are concurrent,

So, $\begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix} = 0$

$$\Rightarrow \quad 2(132 - 143) + 3(-99 + 104) + k(-33 + 32) = 0$$

$$\Rightarrow \quad -22 + 15 - k = 0 \Rightarrow \quad k = -7$$

6. Two lines, whose equations are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ lie in the same plane. Then, the value of $\sin^{-1} \sin \lambda$ is equal to

(A) 3 (B) $\pi-3$ (C) 4 (D) $\pi-4$

Solution: (D)

Given lines are $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{\lambda}$ .....(i)

and $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$ ....(ii)

These lines lie in the same plane, So, both are coplanar.

$$\therefore \quad \begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix}=0$$

$\Rightarrow \quad 2(-2+3)-3(-3-3)+\lambda\,(-3-2)=0$

$\Rightarrow \quad 2+18-5\lambda=0 \ \Rightarrow \ 5\lambda=20 \Rightarrow \lambda=4$

$\therefore \quad \sin^{-1}\sin\lambda=\sin^{-1}\sin 4$

$=\sin^{-1}\sin(\pi-4)$

$=\pi-4$

7. If $\frac{e^x}{1-x}=B_0+B_1x+B_2x^2+\cdots+B_nx^n+\cdots$, then the value of B_n-B_{n-1} is

(A) 1 (B) $\frac{1}{n}$ (C) $\frac{1}{n!}$ (D) None of these

Solution: (C)

We have,

$e^x=(1-x)\,(B_0+B_1x+B_2x^2+\cdots+B_{n-1}x^{n-1}+B_nx^n+\cdots)$

By the expansion of e^x, we get

$$1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}+\cdots$$

$=(1-x)\,(B_0+B_1x+B_2x^2+\cdots+B_{n-1}x^{n-1}+B_nx^n+\cdots)$

Equating the coefficient of x^n on both sides, we get

$$B_n-B_{n-1}=\frac{1}{n!}$$

8. $2^{3n} - 7n - 1$ is divisible by

(A) 64 (B) 36 (C) 49 (D) 25

Solution: (C)

Let $P(n) = 2^{3n} - 7n - 1 \Rightarrow P(1) = 0, P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) = 2^{3k} - 7k - 1 = 49I$

$P(k+1) = 2^{3k+3} - 7k - 8$

$= 8(49I + 7k + 1) - 7k - 8$

$= 49(8I) + 49k = 49\lambda$

(where, $\lambda = 8I + k$, which is an integer.)

9. If $\int_0^{25} e^{x-[x]} dx = k(e-1)$, then the value of k is equal to

(A) 12 (B) 25 (C) 23 (D) 24

Solution: (B)

We know that, $x - [x]$ is periodic function with period one.

$\therefore \quad e^{x-[x]}$ has period one

Since, $f(x)$ is periodic with period T, then

$$\int_0^{nT} f(x)\,dx = n\int_0^{T} f(x)\,dx$$

$$\therefore \quad \int_0^{25\times 1} e^{x-[x]}\,dx = 25\int_0^{1} e^{x-[x]}\,dx$$

$$= 25\int_0^{1} e^{x-0}\,dx$$

$= 25\,[e^x]_0^1 = 25\,[e-1]$

$\therefore \quad 25(e-1) = k(e-1)$

Hence, $k = 25$

10. A variable chord PQ of the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then the locus of the points of intersection of the normal at P and Q is

(A) A parabola (B) A hyperbola (C) A circle (D) None of these

Solution: (A)

Let P be $(at_1^2, 2at_1)$ and Q be $(at_2^2, 2at_2)$. Since, PQ subtends a right angle at the vertex (0,0).

Hence, $t_1 t_2 = -4$ ….(i)

If (h, k) is the point of intersection of normal at P and Q, then

$h = 2a + a(t_1^2 + t_2^2 + t_1 t_2)$ ……(ii)

and $k = -at_1 t_2(t_1 + t_2)$ ……(iii)

In order to find the locus of (h, k), we have to eliminate t_1 and t_2 between equations (i), (ii) and (iii),

$k = 4a(t_1 + t_2)$ …..(iv)

[from equations (i) and (iii)]

and $h - 2a = a[(t_1 + t_2)^2 - t_1 t_2]$

$\Rightarrow \quad h - 2a = a\left[\frac{k^2}{16a^2} + 4\right]$ [from equation (iv)]

$$\Rightarrow \quad h - 6a = \frac{k^2}{16a}$$

Hence, the required locus is $y^2 = 16a(x - 6a)$.

11. If $\omega \neq 1$ is a cube root of unity, then

$$A = \begin{bmatrix} 1+2\omega^{100}+\omega^{200} & \omega^2 & 1 \\ 1 & 1+2\omega^{100} & \omega \\ \omega & \omega^2 & 2+\omega^{100}+2\omega^{200} \end{bmatrix}$$

(A) A is singular (B) $|A| \neq 0$

(C) A is symmetric (D) None of the above

Solution: (A)

Given matrix can be rewritten as

$$A = \begin{vmatrix} 1+2\omega+\omega^2 & \omega^2 & 1 \\ 1 & 1+\omega^2+2\omega & \omega \\ \omega & \omega^2 & 2+\omega+2\omega^2 \end{vmatrix}$$

$$\therefore \quad |A| = \begin{vmatrix} \omega & \omega^2 & 1 \\ 1 & \omega & \omega \\ \omega & \omega^2 & 1+\omega^2 \end{vmatrix} \qquad (\because \quad 1+\omega+\omega^2=0)$$

$$= \omega \begin{vmatrix} \omega & \omega & 1 \\ 1 & 1 & \omega \\ \omega & \omega & -\omega \end{vmatrix} \qquad (\because \quad 1+\omega+\omega^2=0)$$

$$\Rightarrow \quad \omega(0) = 0$$

12. $\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\ \tan x - 3}{\tan^2 x - 4 \tan x + 3}$ equals to

(A) 1 (B) 2 (C) 0 (D) 3

Solution: (B)

$$\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$$

$$= \lim_{\tan x \to 3} \frac{(\tan x - 3)\,(\tan x + 1)}{(\tan x - 3)\,(\tan x - 1)}$$

$$= \lim_{\tan x \to 3} \frac{\tan x + 1}{\tan x - 1} = \frac{3+1}{3-1} = \frac{4}{2} = 2$$

13. The locus of the points of intersection of the tangents at the extremities of the chords of the ellipse $x^2 + 2y^2 = 6$ which touches the ellipse $x^2 + 4y^2 = 4$, is

(A) $x^2 + y^2 = 4$ (B) $x^2 + y^2 = 6$

(C) $x^2 + y^2 = 9$ (D) None of these

Solution: (C)

The given equation of second ellipse can be rewritten as

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Equation of tangent to this ellipse is

$\frac{x}{2}\cos\theta + y\sin\theta = 1$ .....(i)

Equation of the first ellipse can be rewritten as

$\frac{x^2}{6} + \frac{y^2}{3} = 1$ ......(ii)

Let ellipse (i) meets the first ellipse at P and Q and the tangents at P and Q to the second ellipse intersected at (h, k), then equation (i) is the chord of contact of (h, k) with respect to the ellipse (ii) and thus, its equation is

$\frac{hx}{6} + \frac{ky}{3} = 1$ ....(iii)

Since, equations (i) and (iii) represent the same line

$$\frac{\frac{h}{6}}{\cos\frac{\theta}{2}} = \frac{\frac{ky}{3}}{\sin\theta} = 1$$

$h = 3\cos\theta$ and $k = 3\sin\theta$

Hence, locus is $x^2 + y^2 = 9$.

14. Number of roots of the equation $|\sin x \cdot \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$, where $x \in [0, 4\pi]$, are

(A) 1 (B) 2 (C) 3 (D) None of these

Solution: (D)

We have, $|\sin x \cos x| + |\tan x + \cot x| = \sqrt{3}$

$\Rightarrow \quad |\sin x \cos x| + \dfrac{1}{|\sin x \cdot \cos x|} = \sqrt{3}$

But $|\sin x \cdot \cos x| + \frac{1}{|\sin x \cdot \cos x|} \geq 2$

Hence, there is no solution.

15. If $f(x) = \begin{cases} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$

For what value of $'k', f(x)$ is continuous at $x = 0$?

(A) 1 (B) 0 (C) e (D) e^2

Solution: (D)

$$\lim_{x \to 0} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}} = \lim_{x \to 0} \left[\frac{1 + \tan x}{1 - \tan x}\right]^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left[(1 + \tan x)^{\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}} \times \lim_{x \to 0} \left[(1 - \tan x)^{\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}}$$

$$= e \cdot e = e^2$$

16. The period of $\sin^2 \theta$ is

(A) π^2 (B) π (C) 2π (D) $\frac{\pi}{2}$

Solution: (B)

Since, $\sin^2 \theta = \frac{1 - \cos 2\theta}{2} = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$\therefore$ Period of $\sin^2 \theta = \frac{2\pi}{2} = \pi$

17. Five persons, A, B, C, D and E are in queue of a shop. The probability that A and E are always together, is

(A) $\frac{1}{4}$ (B) $\frac{2}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

Solution: (C)

$\because$ Total number of ways = 5! and favourable number of ways $= 2 \cdot 4!$

$\therefore$ Required probability $= \frac{2 \cdot 4!}{5!} = \frac{2}{5}$

18. $\lim_{x\to\infty} \frac{x^4 \cdot \sin\left(\frac{1}{x}\right) + x^2}{1+|x|^3}$ equal to

(A) 0 (B) -1 (C) 2 (D) 1

Solution: (B)

$$\lim_{x\to\infty} \frac{x^4 \cdot \sin\left[\frac{1}{x}\right] + x^2}{1+|x|^3} = \lim_{x\to\infty}\left[\frac{x\sin\left(\frac{1}{x}\right) + \frac{1}{x}}{\frac{1}{x^3} + \frac{|x|^3}{x^3}}\right]$$

[Dividing numerator and denominator by x^3]

$$= \frac{\lim\limits_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} + \lim\limits_{x\to\infty} \frac{1}{x}}{\lim\limits_{x\to\infty} \frac{1}{x^3} + \lim\limits_{x\to\infty} \frac{|x|^3}{x^3}} = \frac{1-0}{0-1} = -1$$

19. The sum of the $\log_4 2 - \log_8 2 + \log_{16} 2$ .... Is

(A) e^2 (B) $\log_e 2 + 1$

(C) $\log_e 3 - 2$ (D) $1 - \log_e 2$

Solution: (D)

Given that, $\log_4 2 - \log_8 2 + \log_{16} 2 - \cdots$

$$= \frac{1}{\log_2 4} - \frac{1}{\log_2 8} + \frac{1}{\log_2 16} - \cdots$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots$$

$$= 1 - \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots\right]$$

$= 1 - \log_e 2$

20. The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is

(A) $\bar{x} + n$ (B) $\bar{x} + \frac{n}{2}$

(C) $\bar{x} + \frac{n+1}{2}$ (D) None of these

Solution: (C)

Let the observation be $x_1, x_2. x_3 \ldots\ldots, x_n$.

Now, mean $(\bar{x}) = \frac{x_1+x_2+\cdots+x_n}{n}$

When first term increased by 1, second by 2 and so on, then observations will be

$(x_1 + 1), (x_1 + 2), (x_1 + 3), \ldots\ldots, (x_n + n)$

Then, new mean

$$(\bar{x}_1) = \frac{(x_1 + 1) + (x_2 + 2) + \cdots + (x_n + n)}{n}$$

$$= \frac{(x_1 + x_2 + \cdots x_n) + (1 + 2 + 3 + \cdots + n)}{n}$$

$$\Rightarrow \quad \bar{x}_1 = \frac{x_1 + x_2 + \cdots + x_n}{n} + \frac{1 + 2 + 3 + \cdots + n}{n}$$

$$\Rightarrow \quad \bar{x}_1 = \bar{x} + \frac{n(n+1)}{2n} \qquad \left[\because \quad 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}\right]$$

$$\Rightarrow \quad \bar{x}_1 = \bar{x} + \frac{n+1}{2}$$

21. The greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ are respectively

(A) $\frac{\pi^2}{4}$ and 0 (B) $\frac{\pi}{2}$ and $\frac{-\pi}{2}$

(C) $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$ (D) $\frac{\pi^2}{4}$ and $\frac{-\pi^2}{4}$

Solution: (C)

We have,

$(\sin^{-1} x)^2 + (\cos^{-1} x)^2$

$= (\sin^{-1} x + \cos^{-1} x)^2 - 2\sin^{-1} x \cdot \cos^{-1} x$

$= \frac{\pi^2}{4} - 2\sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x\right)$

$= \frac{\pi^2}{4} - \pi \sin^{-1} x + 2(\sin^{-1} x)^2$

$= 2\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x + \frac{\pi^2}{8}\right]$

$= 2\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$

Thus, the least value is $2\left(\frac{\pi^2}{16}\right)$ i.e., $\frac{\pi^2}{8}$ and the greatest value is $2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$ i.e., $\frac{5\pi^2}{4}$.

22. The probability of simultaneous occurrence of atleast one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then $P(A') + P(B')$ is equal to

(A) $2 - 2p + q$ (B) $2 + 2p - q$

(C) $3 - 3p + q$ (D) $2 - 4p + q$

Solution: (A)

Since, P (exactly one of A, B occurs) = q (given),

We get

$P(A \cup B) - P(A \cap B) = q$

$\Rightarrow \quad p - P(A \cap B) = q$

$\Rightarrow \quad P(A \cap B) = p = q$

$\Rightarrow \quad 1 - P(A' \cup B') = p - q$

$\Rightarrow \quad P(A' \cup B') = 1 - p + q$

$\Rightarrow \quad P(A') + P(B') - P(A' \cap B') = 1 + q - p$

$\Rightarrow \quad P(A') + P(B') = (1 - p + q) + [1 - P(A \cup B)]$

$= (1 - p + q) + (1 - p)$

$= 2 - 2p + q$

23. The solution of the differential equation $x = 1 + xy\frac{dy}{dx} + \frac{x^2y^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{x^3\,y^3}{3!}\left(\frac{dy}{dx}\right)^3 + \cdots$ is

(A) $y = \log x + C$ (B) $y^2 = (\log x)^2 + C$

(C) $y = \log x + xy$ (D) $xy = x^y + C$

Solution: (B)

The given equation is reduced to $x = e^{xy\left(\frac{dy}{dx}\right)}$

$\Rightarrow \quad \log x = xy\frac{dy}{dx}$

$\Rightarrow \quad y\,dx = \frac{\log x}{x}\,dx$

$\Rightarrow \quad \int y\,dx = \int \frac{1}{x}\log x\,\,dx$

$\Rightarrow \quad \frac{y^2}{2} = \frac{(\log x)^2}{2} + C'$

$\Rightarrow \quad y^2 = (\log x)^2 + 2C'$

$\Rightarrow \quad y^2 = (\log x)^2 + C$ (where, $C = 2C'$)

24. If n is a positive integer, then $n^3 + 2n$ is divisible by

(A) 2 (B) 6 (C) 15 (D) 3

Solution: (D)

Let $P(n) = n^3 + 2n$

$P(1) = 1 + 2 = 3$

$P(2) = 8 + 4 = 12$

$P(3) = 27 + 6 = 33$

Clearly, we see that all these numbers are divisible by.

25. The sum of the coefficients in the expansion of $(5x - 4y)^n$, where n is a positive integer, is

(A) 0 (B) n (C) 1 (D) -1

Solution: (C)

∵ Using Binomial theorem,

$$(5x - 4y)^n = {}^nC_0(5x)^n + {}^nC_1(5x)^{n-1}(-4y) + {}^nC_2(5x)^{n-2}(-4y)^2 + \cdots + {}^nC_n(-4y)^n$$

Sum of coefficients

$= {}^nC_0\, 5^n + {}^nC_1\, 5^{n-1}(-4) + {}^nC_2\, 5^{n-2}.(-4)^2 + \cdots + {}^nC_n(-4)^n$

$= (5 - 4)^n$

$= 1^n = 1$

26. The distance of the point $(1, -5, 9)$ from the plane $x + y + z = 5$ measured along a straight line $x = y = z$ is $2\sqrt{3}k$, then the value of k is

(A) 5 (B) 6 (C) $\sqrt{3}$ (D) 4

Solution: (A)

Given equation of plane is $x + y + z = 5$.

The distance measured along the line $x = y = z$.

Direction ratio's of the given line is (1, 1, 1).

So, the equation of line PQ is

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}$$

Now, let $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

$x=\lambda+1, y=\lambda-5, z=\lambda+9$

Lies on the plane $x+y+z=5$

$\lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow \quad \lambda=-10$

The coordinate of Q is $(-99,-15,-1)$ and the coordinate of P is $(1,-5,9)$.

$PQ=\sqrt{(10)^2+(10)^2+(10)^2}=10\sqrt{3}$

$\therefore \quad 2\sqrt{3}k=10\sqrt{3} \quad \Rightarrow \quad k=5$

27. $\lim_{x\to 1}\frac{x^m-1}{x^n-1}$ is equal to

(A) $\frac{n}{m}$ (B) $\frac{m}{n}$ (C) $\frac{2m}{n}$ (D) $\frac{2n}{m}$

Solution: (B)

$\lim_{x\to 1}\frac{x^m-1}{x^n-1}=\lim_{x\to 1}\frac{mx^{m-1}}{nx^{n-1}}$ (by L'Hospital rule)

$=\frac{m}{n}$

28. The value of $x>1$ satisfying the equation $\int_1^x t\log t dt=\frac{1}{4}$, is

(A) $\sqrt{e}$ (B) $e^{\frac{3}{2}}$ (C) e^2 (D) $2e-1$

Solution: (A)

Consider that, $I=\int_1^x t\log t \; dt$

$$= \left[\log t \cdot \frac{t^2}{2}\right]_1^x - \int_1^x \frac{1}{t} \cdot \frac{t^2}{2} dt$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\left[\frac{t^2}{2}\right]_1^x$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\left[\frac{x^2}{2} - \frac{1}{2}\right]$$

$\Rightarrow \quad \frac{1}{4} = \frac{x^2}{2}\log x - \frac{1}{4}(x^2 - 1)$

$\Rightarrow \quad \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 = 0$ (as $x > 1$)

$\Rightarrow \quad x^2(2\log x - 1) = 0$

$\Rightarrow \quad 2\log x - 1 = 0$

$\Rightarrow \quad \log x = \frac{1}{2}$

$\Rightarrow \quad x = e^{\frac{1}{2}}$

$\Rightarrow \quad x = \sqrt{e}$

29. $a = \sum_{n=0}^{\infty} \frac{x^{3n}}{3n!}, b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$ And $c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3 + b^3 + c^3 - 3abc$ is

(A) 1 (B) 0 (C) -1 (D) -2

Solution: (A)

We have,

$$a = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}, b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!} \text{ and } c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$$

Now, $a + b + c = \sum_{n=0}^{\infty} \frac{x^{3n}}{3n!} + \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!} + \sum_{n=1}^{\infty} \frac{x^{3}n-1}{(3n-1)!}$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = e^x$$

$$a + b\omega + c\omega^2 = 1 + \omega x + \frac{\omega^2 x^2}{2!} + \frac{\omega^3 x^3}{3!} + \cdots = e^{\omega x}$$

and $a + b\omega^2 + c\omega = e^{\omega^2 x}, \omega$ is imaginary cube root of unity.

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)\,(a + b\omega + c\omega^2)\,(a + b\omega^2 + c\omega)$

$= e^x \,.\, e^{\omega x} \,.\, e^{\omega^2 x} = e^{x(1+\omega+\omega^2)} = e^{0.x} = 1$

30. The unit vector perpendicular to the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + \hat{\jmath}$ forming a right handed system is

(A) $\hat{k}$ (B) $-\hat{k}$

(C) $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$ (D) $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

Solution: (A)

Required unit vector is $\frac{(\hat{\imath}-\hat{\jmath})\times(\hat{\imath}+\hat{\jmath})}{|(\hat{\imath}-\hat{\jmath})\times(\hat{\imath}+\hat{\jmath})|}$

$$= \frac{\hat{k} + \hat{k}}{2} = \frac{2\hat{k}}{2} = \hat{k}$$

31. The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ are

(A) 0 (B) 1 (C) 2 (D) 3

Solution: (A)

$f(x) = x^3 + 2x^2 + 5x + 2\cos x$

$f'(x) = 3x^2 + 4x + 5 - 2\,.\sin x$

$$= 3\left(x + \frac{2}{5}\right)^2 + \frac{11}{3} - 2\,.\sin x$$

$\Rightarrow \quad f'(X) > 0, \forall\, x$

$f(x)$ is increasing for all $x \in R$.

Also, $f(0) = 2 \Rightarrow \quad f(x) = 0$

So, $f(x)$ has no solution.

32. $\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}$ is equal to

(A) 8 (B) 6 (C) 4 (D) 2

Solution: (C)

$$\lim_{x\to 0}\frac{(1+x)^8\ -\ 1}{(1+x)^2-1}$$

$$=\ \lim_{x\to 0}\frac{[(1+x)^4+1]\,[(1+x)^2+1]\,[(1+x)^2-1]}{(1+x)^2-1}$$

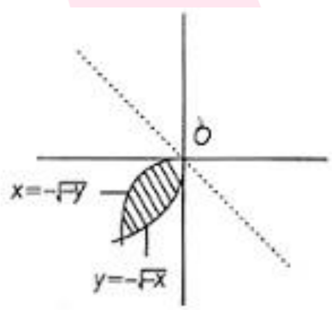
$= 2\times 2 = 4$

33. The area bounded by the curves $y=-\sqrt{-x}$ and $x=\ -\sqrt{-y}$, where $x,y\ \leq 0$, is

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{2}$

Solution: (A)

Given that, $y=\ -\sqrt{-x}$



$\Rightarrow\quad y^2=-x$, where x and y both negative.

Now, $x=-\sqrt{-y}$

$\Rightarrow\quad x^2=-y$, where x and y both negative.

$\therefore \quad Area = \left| \int_{-1}^{0} -\sqrt{-x}\, dx - \int_{-1}^{0} -x^2\, dx \right|$

$= \frac{1}{3}$

34. If α and β are the roots of the equation $x^2 - px + q = 0$, then the value of $(\alpha + \beta)x - \left(\frac{\alpha^2+\beta^2}{2}\right)x^2 + \left(\frac{\alpha^3+\beta^3}{3}\right)x^3 + \cdots$, is

(A) $\log(1 - px + qx^2)$

(B) $\log(1 + px - qx^2)$

(C) $\log(1 + px + qx^2)$

(D) None of these

Solution: (A)

Given series

$\left[\alpha x - \frac{1}{2}(\alpha x)^2 + \frac{1}{3}(\alpha x)^3 - \cdots\right] + \left[\beta x - \frac{1}{2}(\beta x)^2 + \frac{1}{3}(\beta x)^3 - \cdots\right]$

$= \log(1 - \alpha x) + \log(1 - \beta x)$

$= \log[1 - (\alpha + \beta\ )\ x + \alpha\beta x^2]$

Now, $\alpha + \beta = p$ and $\alpha\beta = q$

Given series $= \log(1 - px + qx^2)$

35. If $\cos^{-1} x > \sin^{-1} x$, then

(A) $\frac{1}{\sqrt{2}} < x \leq 1$

(B) $0 \leq x < \frac{1}{\sqrt{2}}$

(C) $-1 \leq x < \frac{1}{\sqrt{2}}$

(D) $x > 0$

Solution: (C)

We know that,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \forall x \in [-1, 1]$

$\therefore \quad \cos^{-1} x > \sin^{-1} x$

$\Rightarrow \quad \frac{\pi}{2} - \sin^{-1} x > \sin^{-1} x$

$\Rightarrow \quad \frac{\pi}{2} > 2 \sin^{-1} x$

$\Rightarrow \quad \sin^{-1} x < \frac{\pi}{4}$

$\Rightarrow \quad x < \frac{1}{\sqrt{2}}$

$\therefore \quad -1 \leq x < \frac{1}{\sqrt{2}}$

36. A student is allowed to select atmost $n$books from a collection of $(2n+1)$ books. If the number of ways in which he can do this, is 64, then the value of n is

(A) 6 (B) n (C) 3 (D) None of these

Solution: (C)

Number of ways

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \cdots + {}^{2n+1}C_n$

$= \frac{1}{2}(2^{2n+1}) = 2^{2n}$

Thus, $2^{2n} = 64$

i.e., $2^{2n} = 2^6$

On comparing, $2n = 6 \Rightarrow \ n = 3$

37. Let $R = \{(3,\ 3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,12), (3,6)\}$ be a relation on the set $A = \{3,6,9,12\}$. The relation is

(A) An equivalence relation

(B) Reflexive and symmetric

(C) Reflexive and transitive

(D) Only reflexive

Solution: (C)

R is reflexive as (3, 3), (6, 6), (9, 9), (12, 12) ∈ R. R is not symmetric as $(6, 12) \in R$ but $(12, 6) \notin R$. R is transitive as the only pair which needs verification is (3, 6) and (6, 12) $\in R$

$\Rightarrow \quad (3, 12) \in R$.

38. The value of a, so that the sum of squares of the roots of the equation $x^2 - (a-2)x - a + 1 = 0$ assume the least value, is

(A) 2 (B) 0 (C) 3 (D) 1

Solution: (D)

Let α and β be the roots of the equation

$x^2 - (a-2)\,x - a + 1 = 0$

$\therefore \quad \alpha + \beta = a - 2$ and $\alpha\beta = -(a-1)$

$s = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (a-2)^2 + 2(a-1)$

$= a^2 - 4a + 4 + 2a - 2 = a^2 - 2a + 2$

Now, $\frac{ds}{da} = 2a - 2$

For maximum and minimum, $\frac{ds}{da} = 0$

$\Rightarrow \quad 2a - 2 = 0$

$\Rightarrow \quad a = 1$

Also, $\frac{d^2s}{da^2} = 2 > 0$

Hence, at $a = 1, s$ will have minimum value.

39. The form of the differential equation of the central conics $ax^2 + by^2 = 1$ is

(A) $x = y\frac{dy}{dx}$

(B) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} - y\frac{dy}{dx} = 0$

(C) $x + y\frac{d^2y}{dx^2} = 0$

(D) None of the above

Solution: (D)

We have, $ax^2 + by^2 = 1$

On differentiating both sides w.r.t. x, we get

$$2ax + 2by\frac{dy}{dx} = 0$$

$\Rightarrow \quad ax + by\frac{dy}{dx} = 0$ .....(i)

$\Rightarrow \quad \frac{-a}{b} = \frac{y}{x}\frac{dy}{dx}$ ......(ii)

Again, differentiating equation (i) w.r.t x, we get

$$a + b\left(\frac{dy}{dx}\right)^2 + by\frac{d^2y}{dx^2} = 0$$

$\Rightarrow \quad -\frac{a}{b} = \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}$ ....(iii)

From equations (ii) and (iii), we get

$$\frac{y}{x}\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2}$$

40. A furniture dealer deals in only two items namely tables and chairs. He has Rs. 5000 to invest and space to store at the most 60 pieces. A table cost him rs 250 and a chair rs 60. He can sell a table at a profit of rs 15. Assume that the can sell all the items that he produced. The number of constraints in the problem are

(A) 2 (B) 3 (C) 4 (D) 5

Solution: (C)

If x tables and y chairs are purchased for maximum profit.

Then, $x + y \leq 60$

$5x + \frac{6y}{5} \leq 100$

$x \geq 0, y \geq 0$

So, number of constraints are four.

41. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

(A) 7 (B) 5 (C) 9 (D) 17

Solution: (C)

For $n = 1, 10^n + 3 \cdot 4^{n+2} + 5 = 10 + 3 \cdot 4^3 + 5$

$= 207$, which is divisible by 99

So, by induction, the result is divisible by 9.

42. The total number of subsets of a finite set A has 56 more elements than the total number of subsets of another finite set B. What is the number of elements in the set A?

(A) 5 (B) 6 (C) 7 (D) 8

Solution: (B)

Let set A and B have m and n elements, respectively.

$2^m - 2^n = 56$

$2^n(2^{m-n} - 1) = 56 = 8 \times 7 = 2^3 \times 7$

Comparing both sides, we get

$2^n = 2^3$ and $2^{m-n} = 7$

$\Rightarrow \quad n = 3$ and $2^{m-n} = 8$

$\Rightarrow \quad 2^{m-n} = 2^3 \Rightarrow \quad m - n = 3$

$\Rightarrow \quad m - 3 = 3 \Rightarrow \quad m = 6$

Number of the elements in A is 6.

43. A sample of 35 observation has the mean 80 and standard deviation as 4. A second sample of 65 observations from the same population has mean 70 and standard deviation 3. Then, the standard deviation of the combined sample is

(A) 5.85 (B) 5.58 (C) 34.2 (D) None of the above

Solution: (A)

Mean of the combined sample

$$\bar{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{35 \times 80 + 65 \times 70}{35 + 65}$$

$= 73.5$

Standard deviation of the combined sample is given by

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

Where, $d_1^2 = (\bar{x}_1 - \bar{x})^2 = (80 - 73.5)^2 = 42.25$

$d_2^2 = (\bar{x}_2 - \bar{x})^2 = (70 - 73.5)^2 = 12.25$

$$\therefore \quad \sigma^2 = \frac{35(4^2 + 42.25) + 65(3^2 + 12.25)}{35 + 65}$$

$\Rightarrow \quad \sigma^2 = 34.2 \quad \Rightarrow \quad \sigma = 5.85$

44. Three straight lines $2x + 11y - 5 = 0, 24x + 7y - 20 = 0$and $4x - 3y - 2 = 0$

(A) From a triangle

(B) Are only concurrent

(C) Are concurrent with one line bisecting the angle between the other two

(D) None of the above

Solution: (C)

For the two lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$, the angle bisectors are given by $\frac{24x+7y-20}{25} = \pm\frac{4x-3y-2}{5}$

Taking positive sign, we get $2x + 11y - 5 = 0$

So, the given three lines are concurrent with one line bisecting the angle between the other two.

45. If the plane $x + y + z = 1$ is rotated through an angle 90^o about its line of intersection with the plane $x - 2y + 3z = 0$, then the now position of the plane is

(A) $x - 5y + 4z = 1$

(B) $x - 5y + 4z = -1$

(C) $x - 8y + 7z = 2$

(D) $x - 8y + 7z = -2$

Solution: (D)

The new position of plane is

$(x - 2y + 33) + \lambda(x + y + z - 1) = 0$

$\Rightarrow \quad (1 + \lambda)x + (\lambda - 2)y + (3 + \lambda)z - \lambda = 0$

Given that this is perpendicular to $x + y + z = 1$

$\therefore \quad (1 + \lambda) \cdot 1 + (\lambda - 2) \cdot 1 + (3 + \lambda) \cdot 1 = 0$

$\Rightarrow \quad 1 + \lambda + \lambda - 2 + 3 + \lambda = 0$

$\Rightarrow \quad 3\lambda + 2 = 0$

$\Rightarrow \quad \lambda = \frac{-2}{3}$

Hence, the new position of the plane is $(x - 2y + 3z) - \frac{2}{3}(x + y + z - 1) = 0$

$\Rightarrow \quad 3x - 2x - 6y - 2y + 9z - 2z + 2 = 0$

$x - 8y + 7z = -2$

Physics

Single correct answer type:

1. In process of amplitude modulation of signal to be transmitted.

Signal to be modulated is given by $m(t) = A_m \sin \omega_m t$, carrier wave is given by $c(t) = A_c \sin \omega_c t$, modulated signal $c_m(t)$ is given by

(A) $C_m(t) = A_c \sin \omega_c t + A_m \sin \omega_m t$

(B) $C_m(t) = (A_c + A_m) \sin \omega_c t$

(C) $C_m(t) = [A_c + m(t)] \sin \omega_c t$

(D) None of the above

Solution: (C)

In amplitude modulation, amplitude of carrier wave varies with signal to be modulated so modulated signal $C_m(t)$ is given by

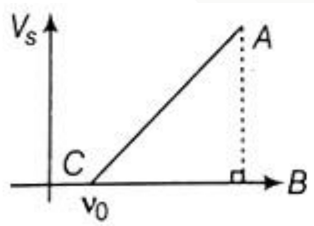
$C_m(t) = [A_c + m(t)] \sin \omega_c t$

$\Rightarrow \quad C_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$

As $m(t) = A_m \sin \omega_m t$ (given)

2. Graph of stopping potential for most energetic emitted photoelectron (V_S) with frequency of incident radiation on metal is given below

Value of $\frac{AB}{BC}$, in graph is



(h = Planck’s constant, e – electronic charge)

(A) h (B) e (C) $\frac{h}{e}$ (D) $\frac{e}{h}$

Solution: (C)

By Einstein's photoelectric effect equation

$KE_{max} = eV_S = h\text{v} - h\text{v}_0$

$\Rightarrow \quad V_S = \left(\frac{h}{e}\right)\text{v} - \frac{h\text{v}_0}{e}$

Graph of V_s with v is straight line whose slope $= \frac{h}{e}$

From given graph slope $= \frac{AB}{BC} \Rightarrow \frac{AB}{BC} = \frac{h}{e}$

3. A block of mass 0.18 kg is attached to a spring of force constant $2\,N/m$. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block.

The block slides a distance of 0.06m and comes to rest for the first time the initial velocity of the block in m/s is $v = N/10$. Then N is

(A) 2 (B) 2 (C) 4 (D) 6

Solution: (C)

Here, $m = 0.18\,kg, K = 2\,N/m\,, \mu = 0.1,\; x = 0.06m.$

According to conservation of mechanical energy principle, we know

Decrease in mechanical energy = Work done against friction

$$\frac{1}{2}mv^2 - \frac{1}{2}kx^2 = \mu mgx$$

$$\Rightarrow \quad v = \sqrt{\frac{2\mu mgx + Kx^2}{m}}$$

Substituting the values of m, μ, g, x and K, we get

$$v = \sqrt{\frac{2 \times 0.1 \times 0.18 \times 9.8 \times 0.06 + 2 \times 0.066}{0.18}}$$

$$v = \left(\frac{4}{10}\right)m/s.$$

So, N = 4

4. Ohm's law says

(A) $V = IR$

(B) V/I = constant

(C) Both ($V = IR$) and (V/I = constant) are correct

(D) Both ($V = IR$) and (V/I = constant) are incorrect

Solution: (B)

Ohm's law says

$$\frac{V}{I} = R = constant$$

5. A train accelerating uniformly from rest attains a maximum speed of $40\ ms^{-1}$ in 20s. It travels at this speed for 20s and is brought to rest with uniform retardation in further 40s. What is the average velocity during this period?

(A) $80\ m/s$ (B) $25\ m/s$ (C) $40\ m/s$ (D) $30\ m/s$

Solution: (B)

(i) $v = u + at_1$

$40 = 0 + a \times 20$

$\therefore a = 2\ m/s^2$

$v^2 - u^2 = 2as$

$40^2 - 0 = 2 \times 2s_1$

$\therefore \quad s_1 = 400\ m$

(ii) $s_2 = v \times t_2 = 40 \times 20 = 800\ m$

(iii) $v = u + at$

$= 0 = 40 + a \times 40$

$\therefore \quad a = -1\ m/s^2$

$0^2 - 40^2 = 2(-1)s_3$

$\therefore \quad s_3 = 800\,m$

Total distance travelled

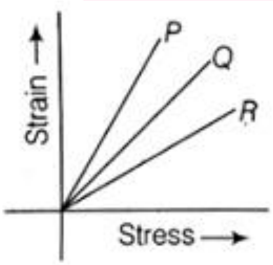
$= s_1 + s_2 + s_3$

$= 400 + 800 + 800 = 2000\,m$

Total time taken $= 20 + 40 = 80s$

$\therefore$ Average velocity $= \frac{2000}{80} = 25\,m/s$

6. The strain-stress curves of three wires of different material are shown in the figure. P, Q, R are the elastic limits of the wires, the figure shows that



(A) Elasticity of wire P is maximum

(B) Elasticity of wire Q is maximum

(C) Tensile strength of R is maximum

(D) None of the above

Solution: (C)

As stress is shown on x-axis and strain on y-axis.

So, we can say that $y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{slope}$

So, elasticity of wire P is minimum and R is maximum.

7. For the equation $F \propto A^a v^b d^c$, where F is the force, A is the area, v is the velocity and d is the density, the values of a, b and c are respectively.

(A) 1, 2, 1 (B) 2, 1, 1 (C) 1, 1, 2 (D) 0, 1, 1

Solution: (A)

$[MLT^{-2}] = [L^{2a}] \times [L^bT^{-b}]\ [M^cL^{-3c}]$

$= [M^cL^{2a+b-3c}T^{-b}]$

Comparing powers of M, L and T, we get

$c = 1, 2a + b - 3c = 1$

$-b = -2\ \ or\ \ b = 2$

Also, $2a + 2 - 3(1) = 1$

$\Rightarrow \quad 2a = 2\ \ or\ a = 1$

$\therefore$ This is 1, 2, 1

8. In hydrogen atom, an electron jumps from bigger orbit to smaller orbit so that radius of smaller orbit is one-fourth of radius of bigger orbit. If speed of electron in bigger orbit was v then speed in smaller orbit is

(A) $\frac{v}{4}$ (B) $\frac{v}{2}$ (C) v (D) $2v$

Solution: (D)

Radius of nth orbit $r_n \propto n^2$

$\frac{r_{n\ big}}{r_{n\ small}} = \frac{n_{big}^2}{n_{small}^2} = \frac{4}{1}$ (given)

$\Rightarrow \quad \frac{n_{big}}{n_{small}} = 2 \quad \Rightarrow \quad \frac{n_{small}}{n_{big}} = \frac{1}{2}$

Velocity of electron in nth orbit

$v_n \propto \frac{1}{n}$

$\frac{v_{n\ big}}{v_{n\ small}} = \frac{n_{small}}{n_{big}} = \frac{1}{2}$

$v_{n\ small} = 2(v_{n\ big)}\ = 2v$

9. A steel wire of length 4.7m and cross-section $3.0 \times 10^{-5}\ m^2$ stretches by the same amount as a copper wire of length 3.5m and cross-section $4.0 \times 10^{-5}\ m^2$ under a given load. What is the ratio of the Young's modulus of steel so that of copper?

(A) 1.5 : 2 (B) 1.8 : 2 (C) 1.5 : 1 (D) 1. 8 : 1

Solution: (D)

As given for steel wire

$A_1 = 3 \times 10^{-5}\ m^2, l_1 = 4.7m, \Delta l_1 = \Delta l; F_1 = F$

For copper wire,

$A_2 = 4 \times 10^{-5} m^2, l_2 = 3.5m, \Delta l_2 = \Delta l, F_2 = F$

Let Y_1 and Y_2 be the Young's modulus of steel wire and copper wire respectively.

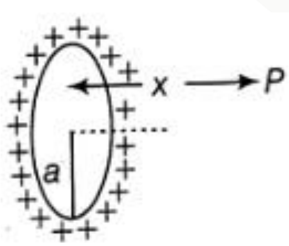
So, $Y_1 = \frac{F_1}{A_1} \times \frac{l_1}{\Delta l_1} = \frac{F}{3\times10^{-5}} \times \frac{4.7}{\Delta l}$

and $Y_2 = \frac{F_2 \times l_2}{A_2 \times \Delta l_2} = \frac{F\times3.5}{4\times10^{-5}\times\Delta l}$

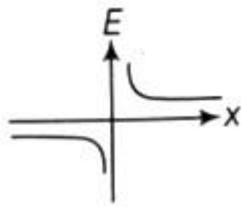
$$\frac{Y_1}{Y_2} = \frac{4.7 \times 4 \times 10^{-5}}{3.5 \times 3.0 \times 10^{-5}} = 1.8$$

So, $Y_1 : Y_2 = 1.8 : 1$

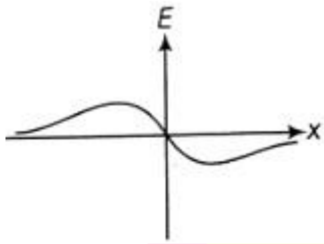
10. A ring shaped conductor with radius a carries a net positive charge q uniformly distributed on it as shown in figure. A point P is situated at a distance x from its centre. Which of following graph shows the correct variation of electric field (E) with distance (x)?



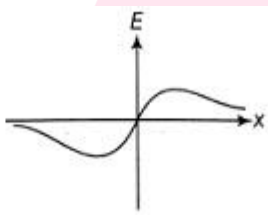
(A)



(B)



(C)



(D) None of the above

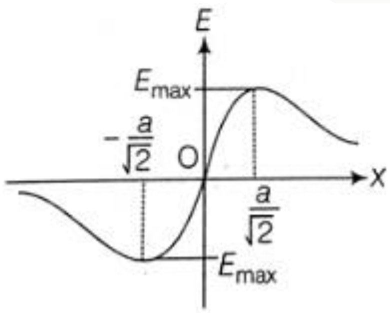
Solution: (B)

The net electric field at point P is given by

$$E = \frac{qx}{4\pi\varepsilon_0 (x^2 + a^2)^{\frac{3}{2}}}$$

∴ At centre of ring $x = 0 \Rightarrow E_0$

IT $x \gg a,\ E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$



and E will be maximum where,

$$\frac{dE}{dx} = 0$$

or $x = \pm\frac{a}{\sqrt{2}}$

and $E_{max} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}a^2}$

11. ......A..... is the essential condition for coherent sources. Here A refers to

(A) Constant phase difference

(B) Equal amplitude

(C) Both (Constant phase difference) and (Equal amplitude) are correct

(D) Both (Constant phase difference) and (Equal amplitude) are incorrect

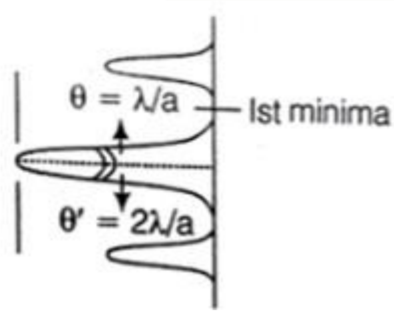
Solution: (A)

Constant phase difference is the essential condition for coherent sources.

12. The angular size of the central maxima due to a single slit diffraction is (a → slit width)

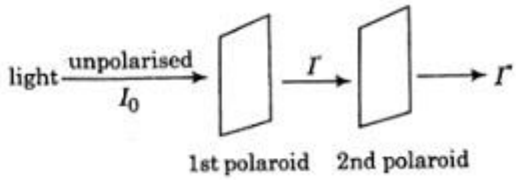
(A) $\frac{\lambda}{a}$ (B) $\frac{2\lambda}{a}$ (C) $\frac{3\lambda}{2a}$ (D) $\frac{\lambda}{2a}$

Solution: (B)



So, angular size of central maxima is $= 2\left(\frac{\lambda}{a}\right) = \frac{2\lambda}{a}$

13.



Find the final intensity of light (I''), if the angle between the axes of two polaroids is 60^o.

(A) $\frac{3I_0}{2}$ (B) $\frac{I_0}{2}$ (C) $\frac{I_0}{4}$ (D) $\frac{I_0}{8}$

Solution: (D)

From first polaroid the unpolarised light will become polarized with half the intensity.

So, $I' = \frac{I_0}{2}$

From second polaroid

$$I'' = I' \cos^2 \theta = \frac{I_0}{2} \cos^2(60) = \frac{I_0}{2} \frac{1}{4} = \frac{I_0}{8}$$

14. Resolving power of a telescope will be more, if the diameter (a) of the objective is

(A) Larger (B) Smaller

(C) Resolving power does not depend on a (D) None of the above

Solution: (C)

$RP \propto a$

15. Let binding energy per nucleon of nucleus is denoted as E_{bn} and radius of nucleus is denoted as r. If mass number of nuclei A, B are 64 and 125 respectively, then

(A) $r_A < r_B, E_{bnA} < E_{bnB}$ (B) $r_A > r_B, E_{bnA} > E_{bnB}$

(C) $r_A = \frac{4}{5} r_B, E_{bnA} < E_{bnB}$ (D) $r_A < r_B, E_{bnA} > E_{bnB}$

Solution: (D)

$r = r_0(A)^{\frac{1}{3}}$

r increases with increasing A = mass number So, $r_A < r_B$ as mass number of A is smaller E_{bn} decreases with increasing A for $A > 56$. ^{56}Fe has highest E_{bn} value.

So, E_{bn} for A = 64 is larger as compared to E_{bn} for nucleus with A = 125

$E_{bnA} > E_{bnB}$

16. The heat energy

(A) Is a state variable

(B) Does not depend on the state of the system

(C) Is equal to internal energy of the system

(D) None of the above

Solution: (B)

Heat energy is not a state variable, it just energy in transition.

17. A current 4.0 A exist in a wire of cross-sectional area 2.0 mm^2. If each cubic metre of the wire contains 12.0×10^{28} free electrons then the drift speed is

(A) $2 \times 10^{-8}\, m/s$ (B) $0.5 \times 10^{-3}\, m/s$

(C) $1.04 \times 10^{-4}\, m/s$ (D) None of these

Solution: (C)

The current density in the wire is

$$J = \frac{i}{A} = \frac{4}{2 \times 10^{-6}} = 2 \times 10^{6}\ Am^{-2}$$

The drift speed is

$$v = \frac{j}{hc} = \frac{2 \times 10^{6}}{12 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= \frac{10^{6}}{6 \times 1.6 \times 10^{9}} = 1.04 \times 10^{-4}\, m/s$$

18. In an experiment on the specific heat of a metal a 0.20 kg block of the metal at $150^o C$ is dropped in a copper calorimeter (of water equivalent 0.025kg) containing 150 cc of water at $27^o C$. The final temperature is $40^o C$. Calculate the specific heat of the metal. If heat losses to the surroundings are not negligible, is our answer greater or smaller than the actual value of specific heat of the metal?

(A) 0.02 (B) 0.02 (C) 0.01 (D) 0.1

Solution: (D)

Mass of metal $m = 0.2\ kg = 200g$

Fall in temperature of metal

$\Delta T = 150 - 40 = 11^o C$

If s is specific heat of metal, the heat lost by the metal

$\Delta Q = ms\ \Delta T = 200 \times 110 \times s$

Volume of water = 150 cc

Mass of water $m' = 150g$

Water equivalent of calorimeter

$w = 0.025kg = 25g$

Rise in temperature of water in calorimeter

$\Delta T' = 40 - 27 = 13^o C$

Heat gained by water and calorimeter

$\Delta Q' = (m' + w)\Delta T'$

$= (150 + 25) \times 13$

$\Delta Q' = 175 \times 13 = \Delta Q$

So, $200 \times s \times 100 = 175 \times 13$

$\Rightarrow \quad s = \dfrac{175 \times 13}{200 \times 100} \approx 0.1$

19. Electric field in a region is given by $E = \left(\frac{M}{x^3}\right)\hat{\imath}$, then the correct expression for the potential in the region is (assume potential at infinity is zero).

(A) $\frac{M}{2x^2}$ (B) Mx^2 (C) $\frac{M}{3x^4}$ (D) None of these

Solution: (A)

$$v(x, y, z) = -\int_{\infty}^{(x,y,z)} E \cdot dr$$

$$= -\int_{\infty}^{(x,y,z)} \frac{Mdx}{x^3} = \frac{M}{2x^2}$$

20. A ball is projected upwards from the top of tower with a velocity $50\,m/s$ making an angle 30^o with the horizontal. The height of tower is 70m. After how many seconds from the instant of throwing will the ball reach the ground?

(A) 2 s (B) 5 s (C) 7 s (D) 9 s

Solution: (C)

Taking vertical downward motion of projectile from point of projection to ground we have,

$u = -50 \sin 30^o = -25\,m/s$

$a = +10\,m/s^2, s = 70m, t = ?$

$s = ut + \frac{1}{2}at^2$

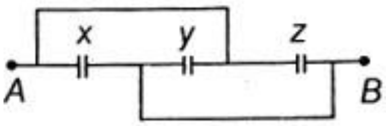
So, $70 = -25 \times t + \frac{1}{2} \times 10 \times t^2$

or $5t^2 - 25t - 70 = 0$

or $t^2 - 5t - 14 = 0$

On solving $t = 7s$

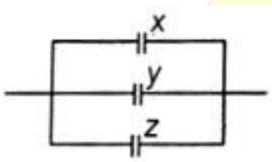
21. Three capacitors $X = 1\mu F, Y = 2\mu F$ and $Z = 3\mu F$ are connected as shown in figure, then the equivalent capacitance between points A and B is



(A) $6\ \mu F$ (B) $12\ \mu F$ (C) $3\ \mu F$ (D) None of these

Solution: (A)

The equivalent circuit of the following figure is as follow.



$\therefore \quad C_{eq} = X + Y + Z$

$= 1 + 2 + 3$

$= 6\mu F$

22. The work done in blowing a soap bubble of surface tension 0.06 Nm^{-1} from 2cm radius to 5cm radius is

(A) 0.004168 J (B) 0.003168 J (C) 0.003158 J (D) 0.004568 J

Solution: (B)

As given $s = 0.06\ Nm^{-1}$,

$r_1 = 2cm = 0.02m, r_2 = 5cm = 0.05m$

Since, bubble has two surface

Initial surface area of the bubble $= 2 \times 4\pi\ r_2^2$

$= 2 \times 4\pi \times (0.02)^2 = 32\pi \times 10^{-4} m^2$

Final surface area of the bubble $= 2 \times 4\pi\ r_2^2$

$= 2 \times 4 \times \pi \times (0.05)^2$

$= 200 \times \pi \times 10^{-4} m^2$

So, work done $=$ $s \times$ increase in surface

$= 0.06 \times (200\pi \times 10^{-4} - 32\,\pi \times 10^{-4})$

$= 0.06 \times 168\pi \times 10^{-4}$

$= 0.003168\,J$

23. The sum of the magnitudes of two forces acting at a point is 16N. The resultant of these forces is perpendicular to the smaller force which has a magnitude of 8N. If the smaller force is magnitude x, then the value of x is

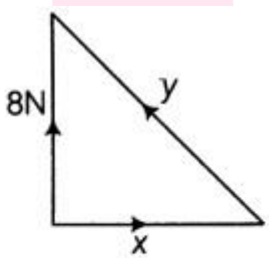
(A) 2N (B) 4N (C) 6N (D) 7N

Solution: (C)

$x + y = 16,$

Also, $y^2 = 8^2 + x^2$

or $y^2 = 64 + (16 - y)^2$



$(\because \quad x = 16 - y)$

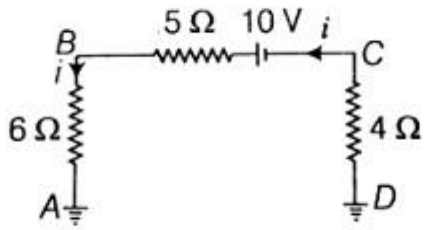
or $y^2 = 64 + 256 + y^2 - 32y$

or $32y = 320$

$Y = 10\,N$

$\therefore \quad x + 10 = 16 \Rightarrow \quad x = 6N$

24. What will be the value of current i in the circuit shown?



(A) 0.67A (B) 1A (C) 0.32 A (D) None of these

Solution: (A)

$V_A - V_D = -6i - 5i + 10 - 4i$

Here, $V_A = V_D$

Since, points A and D are centred.

$-6i - 5i + 10 - 4i = 0 \Rightarrow \quad 15i = 10$

$$\Rightarrow \quad i = \frac{10}{15} = 0.67A$$

25. The average depth of Indian Ocean is about 300m. Bulk modulus of water is $2.2 \times 10^4\ N/m^2$, $g = 10\, m/s^2$, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of the Indian Ocean will be

(A) 1.36% (B) 20.6% (C) 13.9% (D) 0.52%

Solution: (A)

The pressure exerted by a 3000m column of water on the bottom layer.

$p = h\rho g = 3000 \times 1000 \times 10$

$= 3 \times 10^7\ kg\, m^{-1} s^{-2} = 3 \times 10^7\ \ N/m^2$

Fractional compression $\left(\frac{\Delta V}{V}\right)$

$$= \frac{Stress}{B} = \frac{3 \times 10^7}{2.2 \times 10^9}$$

$= 1.36 \times 10^{-2}$

$$\frac{\Delta V}{V} \times 100 = 1.36\%$$

26. If A and B denote the sides of parallelogram and its area is $\frac{1}{2}\,AB$ (A and B are magnitude of A and B respectively), the angle between A and B is

(A) 30^o (B) 45^o (C) 60^o (D) 90^o

Solution: (A)

Area of parallelogram $= |A \times B|$

$$AB\sin\theta = \frac{1}{2}AB$$

$$\sin\theta = \frac{1}{2}, \theta = 30^o$$

27. If edge length of a cuboid are measured to be 1.2cm, 1.5cm and 1.8cm, then volume of the cuboid is

(A) $3.240\ cm^3$ (B) $3.24\ cm^3$ (C) $3.2\ cm^3$ (D) $3.0\ cm^3$

Solution: (C)

Volume of cuboid $= l \times b \times h$

$= 1.8 \times 1.5 \times 1.2\ cm^3$

$= 2.70 \times 1.2$

$= 3.240\ cm^3$

Using concept of significant figures, product is reported in number of significant figures present in measurement which has least number of significant figures, here all measurement have 2 significant figures.

So, volume $= 3.2\ cm^3$

(keeping 2 significant figures only.)

28. A ball thrown upward from the top of a tower with speed v reaches the ground in t_1 sec. If this ball is thrown downward from the top of the same tower with speed v it

reaches the ground in t_2 sec. In what time the ball shall reach the ground, if it is allowed to fall freely under gravity from the top of the tower?

(A) $\frac{t_1+t_2}{2}$ (B) $\frac{t_1-t_2}{2}$ (C) $\sqrt{t_1 t_2}$ (D) $t_1 + t_2$

Solution: (C)

$h = -vt_1 + \frac{1}{2}gt_1^2 \; or \; \frac{h}{t_1} = v + \frac{1}{2}gt_1$ ......(i)

$h = vt_2 + \frac{1}{2}gt_2^2 \; or \; \frac{-h}{t_2} = -v + \frac{1}{2}gt_2$ ......(ii)

$\therefore \quad \frac{h}{t_1} + \frac{h}{t_2} = \frac{1}{2}g\,(t_1 + t_2)$

or $\quad h = \frac{1}{2}gt_1 t_2$

For falls under gravity from the top of the tower $h = \frac{1}{2}gt^2$

$\therefore \quad \frac{1}{2}gt_1 t_2 = \frac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{t_1 t_2}$

29. One end of steel wire is fixed to ceiling of an elevator moving up with an acceleration $2\ m/s^2$ and a load of 10kg hands from other end. Area of cross-section of the wire is $2\ cm^2$. The longitudinal strain in the wire is $(g = 10\,m/s^2$ and $y = 2 \times 10^{11}\ Nm^{-2})$

(A) 4×10^{11} (B) 3×10^{-6} (C) 8×10^{-6} (D) 2×10^{-6}

Solution: (B)

As $T = m(g + a_0) = 10(10 + 2) = 120\ N$

Stress $= \frac{T}{A} = \frac{120}{2\times 10^{-4}} = 60 \times 10^4 Nm^{-2}$

and $\quad Y = \frac{\text{Stress}}{\text{Strain}}, \text{Strain} = \frac{\text{Stress}}{Y}$

$= \frac{60 \times 10^4}{2 \times 10^{11}}$

$= 30 \times 10^{-7} = 3 \times 10^{-6}$

30. the electrostatic force of repulsion between two positively charged ions carrying equal charge is $3.7 \times 10^{-9} N,$ when they are separated by a distance of 5 Å. How much electrons are missing from each ion?

(A) 10 (B) 8 (C) 2 (D) 1

Solution: (C)

Here, $F = 3.7 \times 10^{-9} N$

Let, $q_1 = q_2 = q$

$r = 5\text{Å} = 5 \times 10^{-10} m$

$\because \quad F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

$\Rightarrow \quad 3.7 \times 10^{-9} = 9 \times 10^9 \frac{q \times q}{(5 \times 10^{-10})^2}$

$q^{22} = \frac{3.7 \times 10^{-9} \times 25 \times 10^{-20}}{9 \times 10^9}$

$= 10.28 \times 10^{-38}$

or $q = 3.2 \times 10^{-19} C$

Now, $q = ne$

$\therefore \quad n = \frac{q}{e} = \frac{3.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 2$

31. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. The ratio of the radii of the circular paths described by them is

(A) $1 : 2$ (B) $1 : 1$ (C) $2 : 1$ (D) $1 : 3$

Solution: (B)

Since the radius of circular path of charge particle of in magnetic field $r = \frac{mv}{qB} = \frac{p}{qB}$

Now, the radius of circular path of charge particle of given momentum p and magnetic field B is given by

$$r \propto \frac{1}{q}$$

But charge on both charge particles protons and deuterons, is same. Therefore,

$$\frac{r_p}{r_D} = \frac{q_D}{q_p} = \frac{1}{1}$$

32. A solenoid of length 1.0m has a radius of 1cm and has a total of 1000 turns wound on it. It carries a current of 5A. If an electron were to move with a speed of $10^4\ ms^{-1}$ along the axis of this current carrying solenoid the force experienced by this electron is

(A) 2N (B) 1.2N (C) zero (D) 2.5N

Solution: (C)

Here, $L = 1m, N = 1000$

The number of turn per unit length $n = N/L = 1000\ \ turn/m$

Magnetic field inside the solenoid

$$B = \mu_0 nI = \mu_0 \times 1000 \times 5 = 2\pi \times 10^{-3} T$$

The direction of magnetic field is along the solenoid.

For electron $q = \ -e, v = 10^4\ ms^{-1}$

Magnetic Lorentz force $F = -evB \sin 0^o\ = 0$ as the angle between B and v is 0^o.

33. Magnetic field

(A) Can increase the speed of charge particle

(B) Can accelerate a charge particle

(C) Both (can increase the speed of charge particle) and (can accelerate a charge particle) are correct

(D) Both (can increase the speed of charge particle) and (can accelerate a charge particle) are incorrect

Solution: (B)

Magnetic field can accelerate a charge particle by charging the direction of its velocity but it cannot change the speed of charged particle as magnetic force always acts perpendicular to the velocity of charged particle.

34. A square coil of side 10cm has 20 turn and carries a current of 12A. The coil is suspended vertically and the normal to the plane of the coil, makes an angle θ with the direction of a uniform horizontal magnetic field of 0.80 T. If the torque, experienced by the coil, equals 0.96N-m, the value of θ is

(A) 0^o (B) $\frac{\pi}{2}$ radian (C) $\frac{\pi}{3}$ radian (D) $\frac{\pi}{6}$ radian

Solution: (D)

Area of coil $A = side^2 = (0.1)^2 - 0.01m^2,$

Number of turns N = 20, current $I = 12A,$

Normal to the coil make an angle θ with the direction of B, magnetic field $B = 0.80T$
Torque, experienced by the coil, $\tau = 0.96\ N - m$

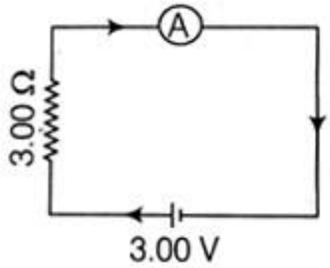
Since, total torque on the coil $\tau = (NIA)B \sin\theta$

Substituting the values in above formula $0.96\ N - m = 20 \times 12\ A \times 0.01m^2 \times 0.80T \times \sin\theta$

$$\sin\theta = \frac{0.96}{1.92} = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}\ \text{radian}$$

35. In the circuit (figure) the current is to be measured. The ammeter shown is a galvanometer with a resistance $R_G = 60.00\Omega$ converted to an ammeter by a shunt resistance $r_s = 0.02\Omega$. The value of the current is



(A) 0.79 A (B) 0.29 A (C) 0.99 A (D) 0.8 A

Solution: (C)

$R_g = 60.00\Omega$ shunt resistance $r_s = 0.02\Omega$

Total resistance in the circuit is,

$R_G + 3 = 63\Omega$. Hence, $I = \frac{3}{63} = 0.048\ A$

Resistance of the galvanometer converted to an ammeter is,

$$\frac{R_G r_s}{R_G + r_s} = \frac{60\Omega \times 0.02\Omega}{(60 + 0.02)\Omega} = 0.02\Omega$$

Total resistance in the circuit is,

$= 0.02 + 3 = 3.02\Omega$

Hence, $l = \frac{3}{3.02} = 0.99\ A$

36. The susceptibility of a magnetism at 300K is 1.2×10^{-5}. The temperature at which the susceptibility increases to 1.8×10^{-5} is

(A) 150 K (B) 200 K (C) 250 K (D) 20 K

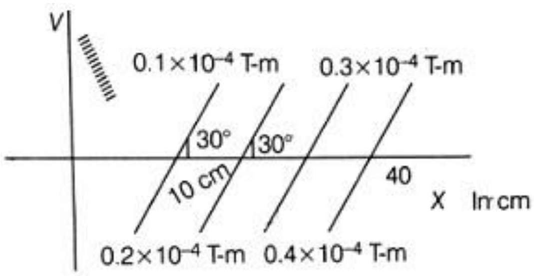
Solution: (B)

$$x = \frac{C}{T} \Rightarrow \frac{x_1}{x_2} = \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{1.2 \times 10^{-5}}{1.8 \times 10^{-5}} = \frac{T_2}{300}$$

$$\Rightarrow \quad T_2 = \frac{12}{18} \times 300 = 200K$$

37. Figure shows some of the equipotential surface of the magnetic scalar potential. The magnetic field B at a point in the region is



(A) $1 \times 10^{-4}T$ (B) $2 \times 10^{-4}T$ (C) $3 \times 10^{-4}T$ (D) $4 \times 10^{-4}T$

Solution: (B)

Since perpendicular distance between two equipotential is

$dx = 10 \sin 30^o \ cm = 5cm$

Since the potential gradient gives magnetic field

$(B)\ as$

$$\frac{dV}{dx} = B$$

Substituting the values

$$\frac{dV}{dx} = B = \frac{0.1 \times 10^{-4}T - m}{5 \times 10^{-2}m}$$

$B = 2 \times 10^{-4}T$

Since B is perpendicular to equipotential surface. Here, it is at angle 120^o with (+ ve) x-axis.

38. In a thermodynamic process, the pressure of a fixed mass of a gas is changed in such a manner that the gas release 20J of heat and 8J of work is done on the gas. If the initial internal energy of the gas was 30J, then the final internal energy will be

(A) 2 J (B) 42 J (C) 18 J (D) 58 J

Solution: (C)

Given, $Q = -20\,J, W = -8\,J$

Using I^{st} law

$Q = \Delta U + W$

$\Rightarrow \quad -20 = \Delta U + (-8)$

$\Rightarrow \quad \Delta U = -12\,J$

$\Rightarrow \quad U_f - U - i = -12\ J$

$\Rightarrow \quad U_f = 30 = -12\,J$

$\Rightarrow \quad U_f = 18\,J$

39. Two balloons are filled one with pure He gas and other with air respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is

(A) More in He filled balloon

(B) Same in both balloons

(C) More in air filled balloon

(D) In the ratio 1 : 4

Solution: (B)

Assuming the balloons have the same volume, as $pV = nRT$. If p, V and T are the same, n the number of moles present will be the same, whether it is He or air. Hence, number of molecules per unit volume will be same in both the balloons.

40. The equation of a simple harmonic motion is given by $y = 3\sin\frac{\pi}{2}(50t - x)$, where x and y are in metres and t is in seconds, the ratio of maximum particle velocity to the wave velocity is

(A) 2π (B) $\frac{3}{2}\pi$ (C) 3π (D) $\frac{2}{3}\pi$

Solution: (B)

The given wave equation is

$y = 3\sin\frac{\pi}{2}\ (50t - x)$

$\Rightarrow \qquad y = 3\sin\left(25\pi t - \frac{\pi}{2}x\right)$ …..(i)

Comparing with standard equation

$y = a\sin(\omega t - kx)$ …..(ii)

$\omega = 25\pi, k = \frac{\pi}{2}$

Wave velocity $v = \frac{\omega}{K} = \frac{25\pi}{\frac{\pi}{2}} = 50\ m/s$