Chemistry

Single correct answer type:

41. For the properties mentioned, the correct trend for the different species is in

(A) Strength as Lewis acid $-BCl_3 > AlCl_3 > GaCl_3$

(B) Inert pair effect $-Al > Ga > ln$

(C) Oxidising property $-Al^{3+} > ln^{3+} > Ti^{3+}$

(D) First ionisaiton enthalpy $-B > Al > Ti$

Solution: (D)

As we know on moving down the group first ionization enthalpy decreases top to bottom therefore order of first ionization enthalpy for group 13 element is

$B > Al > Ga > In > Tl$

42. Bohr theory is applicable to

(A) He (B) Li^{2+}

(C) He^{2+} (D) None of these

Solution: (B)

Bohr's theory is applicable to H-like species containing one electron only for e.., Li^{2+}.

43. Using MOT, which of the following pair denotes paramagnetic species

(A) B_2 and C_2 (B) B_2 and O_2

(C) N_2 and C_2 (D) O_2 and O_2^{2-}

Solution: (B)

Among given four pairs, B_2 and O_2 are paramagnetic due to presence of unpaired electron.

$MO(EC)$ of $B_2 = \sigma 1s^2 \sigma^* 1s^2\ \sigma 2s^2 \sigma^* 2s^2\ \pi 2p_2^1\ \equiv \pi 2p_y^1$

$MO(EC)$ of $O_2 = \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2 \sigma^*\ 2s^2\ \sigma 2p_2\ \pi 2p_x^2\ \ \equiv \pi 2p_y^2\ \pi^* 2p_x^1\ \equiv \pi 2p_y^1$

44. $0.1g$ of metal combines with $46.6\ mL$ of oxygen at STP. The equivalent weight of metal is

(A) 12 (B) 24 (C) 18 (D) 36

Solution: (A)

1 mole of $O_2 = 4\ eq$. Of oxygen

$22400\ ml$ of $O_2 = \frac{4}{22400} \times 46.6$

$= 0.00832\ eq.$

Equivalent of metal = Equivalent of oxygen

$$\frac{\text{Weight}}{\text{Equivalent}} = 0.00832$$

$$\frac{0.1}{E} = 0.00832$$

$$\therefore \quad E = \frac{0.1}{0.00832} = 12.0$$

45. Which of the following choice represent correct order of first ionization enthalpy

(A) $B < C < N < O < F$ (B) $B > C > N > O > F$

(C) $B < C < N > O < F$ (D) $B < C < N > O > F$

Solution: (C)

Ionization energy is the minimum amount of energy required to remove the outermost electron from an isolated gaseous atom. Quantitatively, it depends on the attraction between electron present on outermost shell and nucleus. Greater the interaction between outermost electron and nucleus, higher will be its ionization enthalpy. So correct order of first it must be

$B < C < N < O < F$

But due to extra stable half-filled electronic configuration of p orbital of N has more value of first ionization enthalpy than oxygen hence, correct order is

$B < C < N > O > F$

46. Which of the following reaction produces most stable alkene?

(A) 2-chloro butane

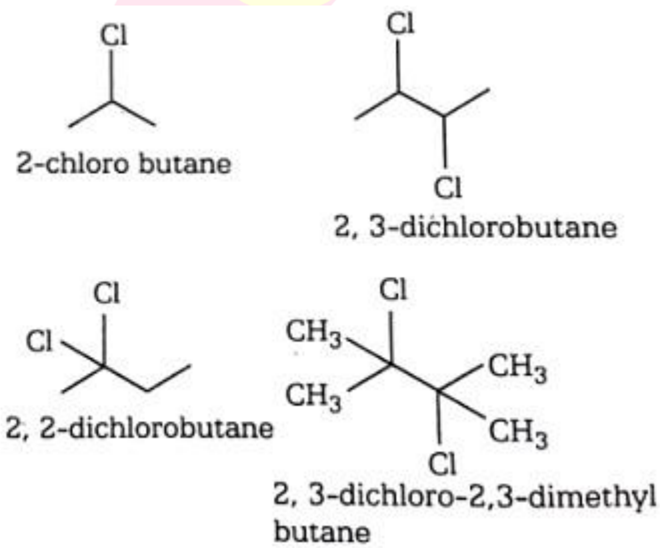
(B) 2, 3-dichloro butane

(C) 2, 2-dichloro butane

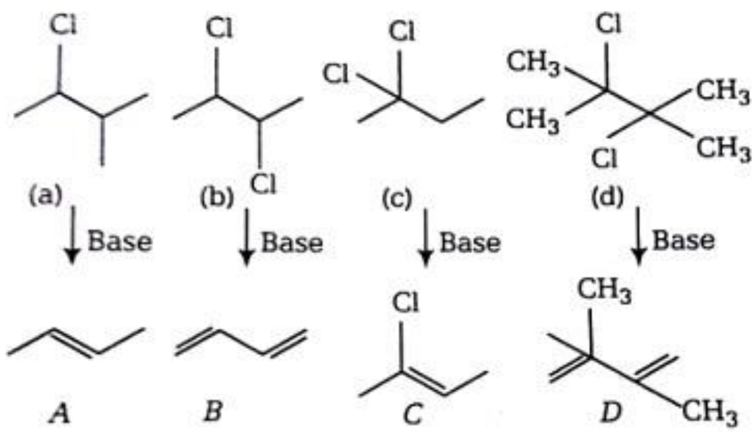
(D) 2, 3-dichloro, 2, 3-dimethyl butane

Solution: (D)

Molecular structure of given names of organic compounds are written as



According to Saytzeff's rule, more substituted (alkylated) alkene are more stable. When the alkyl halide is treated with base, it undergo elimination reaction and produces alkane as follows:



Conjugation Greater the conjugation greater will be the stability of product.

Hence, D has maximum stability, the correct choice is (2, 3-dichloro, 2, 3-dimethyl butane) which is tabilised by conjugation as well as Saytzeff's rule.

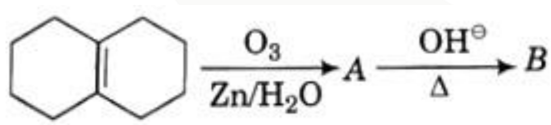
47. Which of the following is less acidic among the given halogen compounds?

(A) CHF_3 (B) CHI_3 (C) $CHCl_3$ (D) $CHBr_3$

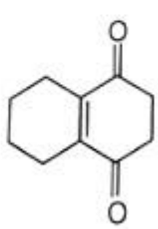
Solution: (B)

Due to stronger $-I$ effect of F than Cl, CHF_3 should be more acidic than $CHCL_3$. But actually reverse is true. This is due to : CCl_3^- left after the removal of a proton from $CHCl_3$ is stabilized by resonance due to presence of d-orbitals in Cl than : CF_3^- left after the removal of a proton from CHF_3 which is not stabilized by resonance due to the absence of d-orbitals on F.

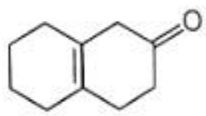
48. What will be the final product of the reaction?



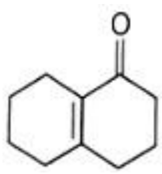
(A)



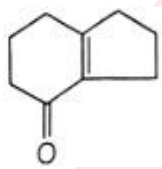
(B)



(C)

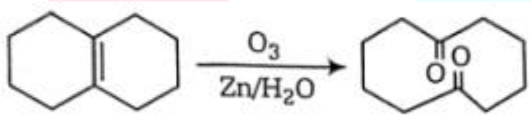


(D)



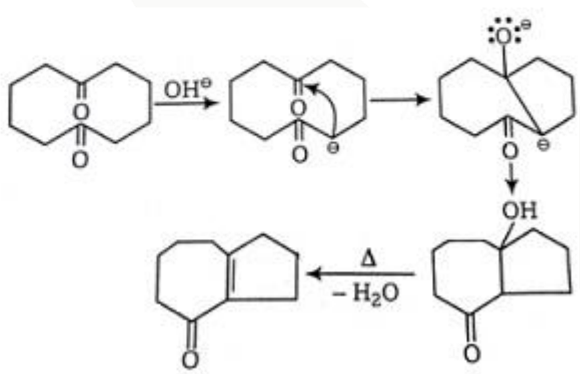
Solution: (D)

Ozonolysis on ozonolysis the given alken undergo ozonide formation followed by reduction to produce bucarbonyl compound as



Bicarbonyl compound

Now, this bicarbonyl compound undergoes interamolecular aldol condensation as follows:



49. The vapour pressure of a solvent decreased by 10 mm of Hg when a non-volatile solute was added to the solvent. The mole fraction of solute in solution is 0.2, what would be the mole fraction of solvent if the decrease in vapour pressure is 20 mm of Hg

(A) 0.8 (B) 0.6 (C) 0.4 (D) 0.3

Solution: (B)

This question is based on Raoult's law. It represent that the partial pressure of each component in the solution is directly proportional to its mole fraction for a solution i.e., $p_A \propto \chi_A$ and $p_B \propto \chi_B$.

FromRaoult's law

$p^o - p_s = p^o \times$ Mole fraction of solute

$10 = p^o \times 0.2$

$20 = p^o \times \chi_2$

$\therefore \quad \chi_2 = 0.4$

and $\chi_1 = 1 - 0.4 = 0.6$

$\chi_1 =$ mole fraction of solvent

50. Chose the law that corresponds to data shown for the following reaction, $A + B \rightarrow$ products

Exp.	$[A]$	$[B]$	Initial rate
1	0.012	0.035	0.1
2	0.024	0.070	0.8
3	0.024	0.035	0.1
4	0.012	0.070	0.8

(A) $Rate = k[B]^3$ (B) $Rate = k[B]^4$

(C) $Rate = k[A]\,[B]^3$ (D) $Rate = k\,[A]^3\,[B]$

Solution: (A)

It is seen that, in experiments (3) and $2i$). $[A]$ is constant and $[B]$ is doubled and rates becomes 8 times, so order w.r.t, $[B] = 3$. In experiments (1)and (3), $[B]$ is constant and $[A]$ is doubled, but rate does not change, so order w.r.t. $[A] = 0$

Thus, rate $= k[B]^3$

51. The magnitude of Δ_o will be highest in which

(A) $[Cr(CN_6)]^{3-}$ (B) $[Cr(H_2O)_6]^{3+}$

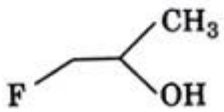
(C) $[Cr(NH_3)_6]^{3+}$ (D) $[Cr(C_2O_4)_3]^{3-}$

Solution: (A)

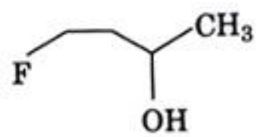
The crystal field splitting, Δ_o depends upon the field produced by the ligand and change on the metal ion. In all these complexes of chromium, charge acquired by metal ion is +3. Therefore Δ_o depends upon the field produced by the ligand. In accordance with the spectrochemical series, the increasing order of field strength is $C_2O_4^{2-} < H_2O < NH_3 < CN^-$

Thus, CN^- is the strong field ligand and will produce highest magnitude of Δ_o.

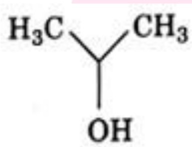
52. Arrange these in correct order of decreasing reactivity.



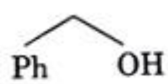
I



II



III



IV

(A) $I > II > III > IV$ (B) $I > III > II > IV$

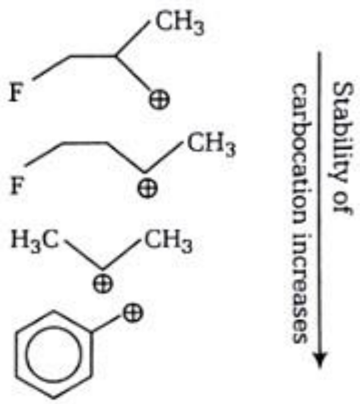
(C) $IV > III > II > I$ (D) $IV > III > I > II$

Solution: (C)

This problem includes conceptual mixing of carbocation stability and reactivity of alcohol.

- Remove the OH group by dehydration and then arrange the carbocation in increasing order of correctly.
- Order of carbocation stability is same as S_N1 reactivity of alkyl halides.

Carbocation stability The carbocation is formed during reaction of alcohol by removal of OH. More stable the carbocation more will be its reactivity of carbocation formed during reaction are as follows



Benzyl carbocation is more stable due to conjugation with phenyl ring.

53. When 2-methyl propan-1-ol is treated with a mixture of conc. HCl and $ZnCl_2$, turbidity appears immediately due to the formation of

(A) 2-methyl propane

(B) 2-methyl propene

(C) 2-methyl-2-chloropropane

(D) 2-chlorobutane

Solution: (C)

When 2-methyl propan-1-ol is treated with a mixture of conc. HCl and $ZnCl_2$ (Lucas reagent) then tert-alkyl halide is formed and produced turbidity due to its less solubility.

54. Gastric juice in human stomach has pH value about 1.8 and pH of small intestine is about 7.8. The pK_a value of aspirin is 3.5. Aspirin will be

(A) Ionised in the small intestine and stomach

(B) Ionised in the stomach and almost unionized in the small intestine

(C) Unionised in small intestine and stomach

(D) Completely ionized in small intestine and stomach

Solution: (A)

Aspirin is a moderate acid $(pK_a = 3.5)$. Therefore, it is almost unionized in stomach due to its strong acidic medium. It happens due to common ion effect. On the other hand, in small intestine, the medium is alkaline, hence, aspirin will be sufficiently ionized in it.

55. When a solution of potassium chromate is treated with an excess of dilute nitric acid

(A) Cr^{3+} and $Cr_2O_7^{2-}$ are formed

(B) $Cr_2O_7^{2-}$ and H_2O are formed

(C) CrO_4^{2-} reduced to Cr^{3+}

(D) CrO_4^{2-} oxidized to $Cr_2O_7^{2-}$ only

Solution: (B)

The reaction of K_2CrO_4 with dilute nitric acid is represent as

$2CrO_4^{2-} + 2H^+ \rightarrow Cr_2O_7^{2-} + H_2O$

56. Calcium carbide reacts with heavy water to form

(A) C_2D_2 (B) CaD_2

(C) CaD_2O (D) CD_2

Solution: (A)

CaC_2 when reacts with water molecule to form acetylene. Similarly, it reacts with D_2O to form C_2D_2

$CaC_2 + 2D_2O \rightarrow C_2D_2 + Ca(OD)_2$

57. Fluorine acts as strongest oxidizing agent because of its high

(A) Electron affinity (B) Ionisation enthalpy

(C) Hydration enthalpy (D) Bond enthalpy

Solution: (C)

Fluorine acts as strongest oxidizing agent due to

(a) low enthalpy of dissociation of $F - F$ bond

(b) high hydration enthalpy of F^-

58. The reaction of P_4 with X leads selectively to P_4O_6. The X is

(A) dry O_2

(B) moist O_2

(C) mixture of O_2 and N_2

(D) O_2 in presence of aquece $NaOH$

Solution: (C)

The reaction of P_4 with X leads selectively to P_4O_6 because N_2 prevents the further reaction of P_4O_6 into P_4O_{10}.

59. The acidic strength for the hydrides of group 15 follows the order

(A) $NH_3 > PH_3 > AsH_3 > SbH_3$

(B) $NH_3 < PH_3 < AsH_3 < SbH_3$

(C) $NH_3 > PH_3 > SbH_3 > AsH_3$

(D) $NH_3 < PH_3 < SbH_3 < AsH_3$

Solution: (B)

The acidic strength of hydrides is inversely proportional to their stability. Since the stability of hydrides decreases from N to Sb. Therefore, the acidic strength increases from N to Sb. Hence, the correct order would be

$$NH_3 < PH_3 < AsH_3 < SbH_3$$

Caution point As the stability decreases from NH_3 to BiH_3, the reducing character of hydrides increases.

60. Which of the following statements are incorrect in context of borax?

(A) It is made up to two triangular BO_3 units and two tetrahedral BO_4 units

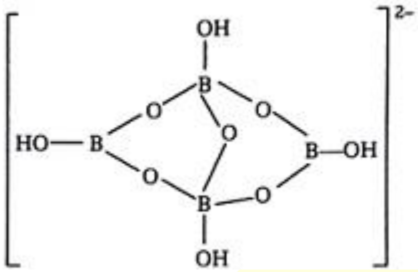
(B) One mole of borax can be used as buffer

(C) It is a useful primary standard for titrating against acids

(D) Aqueous solution of borax can be used as buffer

Solution: (B)

Borax is $Na_2B_4O_7\,.10H_2O$ in which 1 molecules of water among 10 molecules forms a part of structure and exists as $Na_2[B_4O_5(OH)_4]8H_2O$



$Na_2[B_4O_5(OH)_4]\,.8H_2O + 2HCl \rightarrow 2NaCl + 4H_3BO_3 + 5H_2O$

Methyl orange with pH value of 3.7 is used to detect end point. Aqueous solution of borax acts as buffer, because borax is salt is strong base $NaOH$ and weak acid H_3BO_3.

61. Salt $A + S \rightarrow B \xrightarrow{BaCl_2}$ white precipitate A is paramagnetic in nature and contains about 55% K. Thus, A is

(A) K_2O (B) K_2O_2 (C) KO_2 (D) K_2SO_4

Solution: (C)

Among the given oxides, only KO_2 i.e., potassium superoxide is paramagnetic in nature. This is because peroxide ion, O_2^- has three electron bond which makes it paramagnetic and coloured.

Hence, A is KO_2

62. When equal volume each of two sols of AgI, one obtained by adding $AgNO_3$ to slight excess of KI and another obtained by adding KI to slight excess of $AgNO_3$ are mixed together. It is observed that

(A) The sol particles acquired more electric charge

(B) The sols coagulated each other mutually

(C) A true solution is obtained

(D) The two sols stabilized each other

Solution: (B)

The two sols prepared contains not only AgI but also KI and $AgNO_3$ as these are taken in excess amounts. When these sols are mixed, the sols being oppositely charged coagulates each other.

63. In the extraction of Ag, Zn is removed from $(Zn - Ag)$ alloy through

(A) Cupellation (B) Fractional crystallization

(C) Distillation (D) Electrolytic refining

Solution: (C)

The extraction of Ag using (Zn – Ag) alloy is called Parke's process.

As zinc is volatile at 920^o while Ag is not. Thus, on heating (Zn + Ag) alloy, zinc vapourises while Ag remains at the bottom of the vessel. Hence, Zn is removed from (Zn-Ag) alloy through distillation.

64. A reaction takes place in three steps. The rate constants are k_1, K_2 and K_3. The overall rate constant $K = \frac{K_1 K_3}{K_2}$. If E_1, E_2 and E_3 (energy of activation) are 60, 30 and 10kJ respectively, the overall energy of activation is

(A) 40 (B) 30 (C) 400 (D) 300

Solution: (A)

$K_1 = ae^{Ea_1/RT}$; $K_2 = Ae^{-Ea_2/RT}$

$K_3 = Ae^{-Ea_3/RT}$

Overall rate $= K = \frac{K_1 K_3}{K_2}$

Therefore, overall $E_a = E_{a_1} + E_{a_3} - E_{a_2}$

$= 60 + 10 - 30$

$= 40kJ$

65. If $E^0_{Fe^{3+}/Fe}$ and $E^0_{Fe^{2+}/Fe}$ are $-0.36V$ and $-0.439V$ respectively, then the value of $E^0_{Fe^{3+}/Fe^{2+}}$ is

(A) $(-0.036 - 0.439)V$ (B) $[3(-0.36) + 2(-0.439)]V$

(C) $(-0.36 - 0.439)V$ (D) $[3(-0.36) - 2(-0.439)]V$

Solution: (D)

Given that, $E^0_{Fe^{3+}/Fe} = -0.36\,V$;

$E^0_{Fe^{2+}/Fe} = -0.439\,V$

$Fe^{3+} + 3e^- \rightarrow Fe; \quad \Delta G_1 = -3F\left(E^0_{Fe^{3+}/Fe}\right)$

$Fe^{2+} + 2e^- \rightarrow Fe; \quad \Delta G_2 = -2F\left(E^0_{Fe^{2+}/Fe}\right)$

$Fe^{3+} + e^- \rightarrow Fe^{2+}; \quad \Delta G_3 = -F\left(E^0_{Fe^{3+}/Fe}\right)$

$E^0_{Fe^{3+}/Fe^{2+}} = 3E^0_{Fe^{3+}/Fe} - 2E^0_{Fe^{2+}/Fe}$

$= [3(-0.36) - 2(-0.439)]V$

66. KCl crystallises in the same type of lattice as does $NaCl$. Given that $r_{Na^+}/r_{Cl^-} = 0.55$ and $r_{K^+}/r_{Cl^-} = 0.74$. Determine the ratio of the side of the unit cell for KCl to that of $NaCl$

(A) 0.124 (B) 1.123 (C) 0.891 (D) 1.414

Solution: (B)

Given that,

$r_{Na^+}/r_{Cl} = 0.55$

$r_{K^+}/r_{Cl^-} = 0.75$

$\frac{r_{KCl}}{r_{NaCl}} = ?$

$\because \quad \frac{r_{Na^+}}{r_{Cl^-}} = 0.55$

$\therefore \quad \frac{r_{Na^+}}{r_{Cl^-}} + 1 = 0.55 + 1$

$\frac{r_{Na^+} + r_{Cl^-}}{r_{Cl^-}} = 1.55$ ......(i)

$\therefore \quad \frac{r_{K^+}}{r_{Cl^-}} = 0.74$

$\therefore \quad \frac{r_{K^+}}{r_{Cl^-}} + 1 = 0.74 + 1$

$$\frac{r_{K^+}+r_{Cl^-}}{r_{Cl^-}} = 1.74 \quad ....(ii)$$

Dividing equations (ii) by (i)

$$\frac{r_{K^+}+r_{Cl^-}}{r_{Na^+}+r_{Cl^-}} = \frac{1.74}{1.55} = 1.1226$$

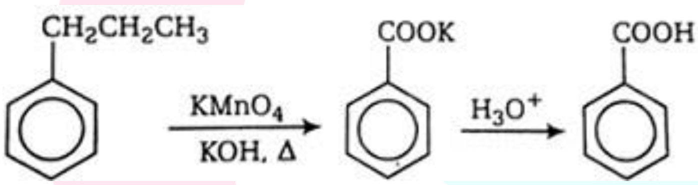
67. The compound formed as a result of oxidation of propyl benzene by $KMnO_4$ is

(A) Benzaldehye (B) Benzyl alcohol

(C) Benzoic acid (D) Acetophenone

Solution: (C)

When alkyl benzene is treated with acidic or alkaline $KMnO_4$, the entire side chain is oxidized to the carboxylic acid irrespective of length of the side chain.



68. Which of the following is an outer d-orbital or high spin complex?

(A) $[Co(NH_3)_6]^{3+}$ (B) $[Ni(CN)_4]^{2-}$

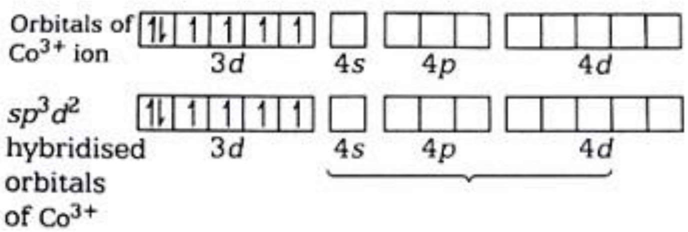
(C) $[NiCl_4]^{2-}$ (D) $[CoF_6]^{3-}$

Solution: (D)

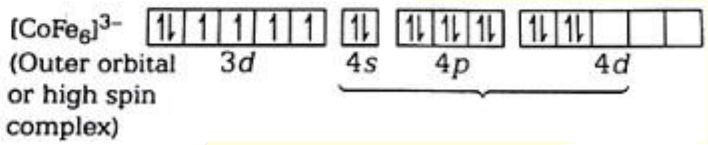
$[Co(NH_3)_6]^{3+}$, $[Ni(CN)_4]^{2-}$ and $[NiCl_4]^{2-}$ are inner d-orbital or high spin complex.

$[CoFe_6]^{3-}, Co = 4s^2, 3d^7, 4p^0$

$Co^{3+} = 3d^6, 4s^0, 4p^0$



Here F^- is a weak legend so no pairing of electron takes place



69. The monosaccharide having anomericcarbon atoms are

(A) Geometrical isomers

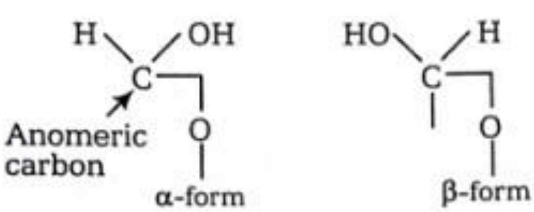
(B) α- and β-optical isomers

(C) Having symmetrical carbon atoms

(D) None of the above

Solution: (B)

C_1 carbon of monosaccharides is called anomeric carbon. When $-OH$ group attached with C_1 cacrbon is towards right, it is called α-form and when $-OH$ group is towards left, it is called β-form, such pair or optical isomers which differ in the configuration only around anomeric carbon are called anomers.



70. Amine is not formed in the reaction

I. Hydrolysis of RCN

II. Reduction of $RCH = NOH$

III. Hydrolysis of RNC

IV. Hydrolysis of $RCONH_2$

The correct answer is

(A) I, II and IV (B) I and IV

(C) II and III (D) I, II and III

Solution: (B)

In I and IV, amine is not formed.

I. Hydrolysis of RCN; $RNC \rightarrow RCOOH + NH_3$

II. Reduction of $RCH = NOH$

$RCH = NOH \rightarrow RCH_2NH_2 + H_2O$

III. Hydrolysis of RNC

$RNC + 2H_2O \xrightarrow{\Delta} RNH_2 + HCOOH$

IV. Hydrolysis of $RCONH_2$

$RCONH_2 \rightarrow RCOOH + NH_3$

71. In vulcanization of rubber

(A) Sulphur reacts to form a new compound

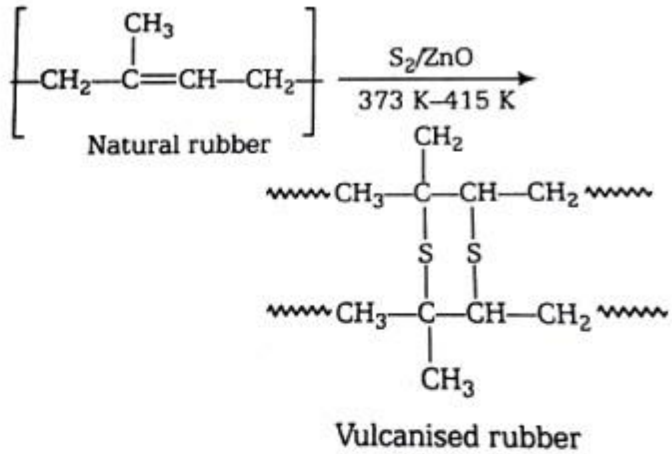
(B) Sulphur cross links are introduced

(C) Sulphur form a very thin protective layer on rubber

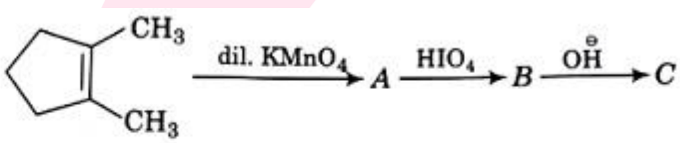
(D) All of the above

Solution: (B)

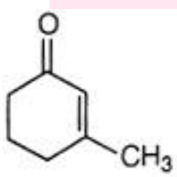
In vulcanization of rubber, sulphur cross-links are introduced at the reactive sites of double bonds.



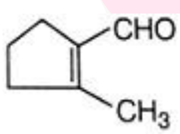
72. What will be the correct structural formula of product for the following reaction?



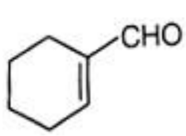
(A)



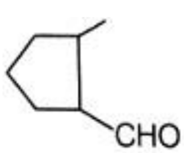
(B)



(C)

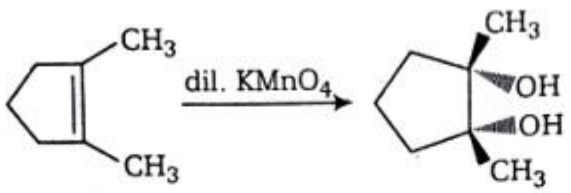


(D)



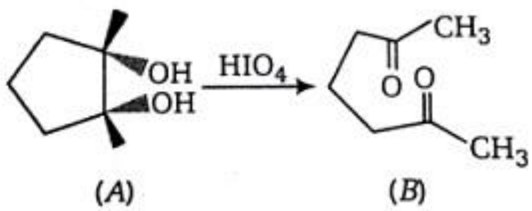
Solution: (A)

Hydroxylation reaction. The alkene on treatment with dil. $KMnO_4$ produces vicinal cis diol.

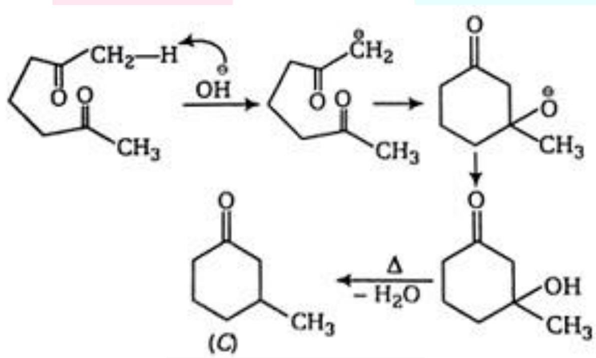


vinial *cis* diol

Malarprade oxidation cis diol undergo Malaprade oxidation in presence of HIO_4 and believe to proceed as



Intramolecular aldol reaction This diketone undergo interamolecular aldol to produce the cyclic α, β-unsaturated ketone as follows



73. What will be the correct reaction between product when 2-methyl cyclohexane is treated with (i) B_2H_6 in presence of H_2O_2/OH and (ii) H_2O/H_2SO_4

(also consider stereochemistry of product)?

(A) They are metamers

(B) They are tautomers

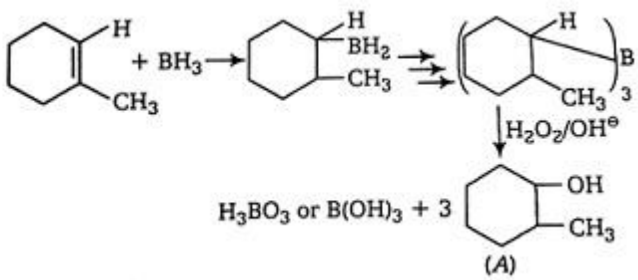
(C) They are functional isomer

(D) They are positional isomer

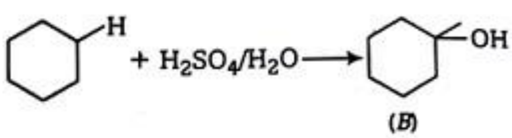
Solution: (D)

This problem includes conceptual mixing of hydroboration. Oxidation and stereochemistry. This problem can be solved by using the skill of electrophilic addition reaction in hydroboration oxidation reaction including isomerism. The steps to solve this problems are complete reaction with acid and then identify the isomerism in the out come products.

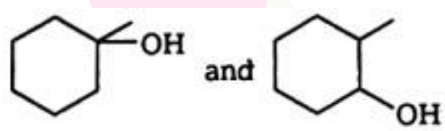
(i) Hydroboration-oxidation When alkene or substituted alkene is treated with B_2H_6 it gives alkyl borane which on treatment with H_2O_2/OH^- causes oxidation of alkyl borone to give alcohol. The above reaction is believed to proceed as



(ii) When 2-methyl cyclohexane is treated with H_2O/H_2SO_4



A and B have difference in position of OH only so A and B are position isomers.



74. The equilibrium constant K_p for the reaction,

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$ is $1.6 \times 10^{-4}(atm)^{-2}$ at 400^o. What will be the equilibrium constant at 500^o if heat of the reaction in this temperature range is $-25.14\ kCal$?

(A) $1.231 \times 10^{-4}\ (atm)^{-2}$ (B) $1.876 \times 10^{-7}\ (atm)^{-2}$

(C) $1.462 \times 10^{-5}\ (atm)^{-2}$ (D) $3.462 \times 10^{-5}\ (atm)^{-2}$

Solution: (C)

Equilibrium constants at different temperature and heat of the reaction are related by the equation.

$$\ln \frac{K_{p_1}}{K_{p_1}} = \frac{\Delta H^o}{R}\left[\frac{1}{T_2} - \frac{1}{T_2}\right]$$

$$2.303 \log \frac{K_{p_2}}{K_{p_1}} = \frac{\Delta H^0}{R}\left[\frac{T_2 - T_1}{T_1 \, T_2}\right]$$

$$\log K_{p_2} = \frac{-25140}{2.303 \times 2}\left[\frac{773 - 673}{773 \times 673}\right] + \log(1.6 \times 10^{-4})$$

$$\log K_{p_2} = -4.835$$

$$\therefore \quad K_{p_2} = 1.462 \times 10^{-5}\,(atm)^{-2}$$

75. At $27^o C, K_p$ value for reaction $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$ is 0.1 atm The K_C value for this reaction is

(A) 4×10^{-3} (B) 6×10^{-3} (C) 2×10^{-3} (D) 9×10^{-3}

Solution: (A)

$$K_p = K_C \ (RT)^{\Delta n}$$

$$\Delta n = 1$$

$$K_C = \frac{K_p}{RT}$$

$$= \frac{0.1}{0.082 \times 300}$$

$$= 4 \times 10^{-3}$$

76. At constant temperature and pressure which one of the following statements is correct for the reaction?

$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$$

(A) $\Delta H = \Delta E$ (B) $\Delta H < \Delta E$

(C) $\Delta H > \Delta E$ (D) ΔH is independent of physical state of reactant

Solution: (B)

As we know, $\Delta H = \Delta E + \Delta n\, RT$

where, $\Delta n =$ gaseous product moles – gaseous reactant moles

For the reaction,

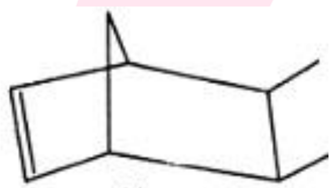
$$CO(g) + \frac{1}{2} O_2(g) \rightarrow CO_2(g)$$

$$\Delta n = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\therefore \quad \Delta H = \Delta E - \frac{1}{2} RT$$

Hence, $\Delta H < \Delta E$

77. IUPAC name and degree of unsaturation of compound X is



X

(A) 2, 3-dimethyl bicyclo [2, 2, 1] hept-5 ene, 2

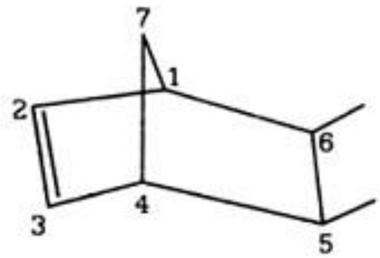
(B) 1, 2-dimethyl bicyclo [2, 2, 1] hept-4 ene, 3

(C) 5, 6-dimethyl bicyclo [2, 2, 1] hept-2 ene, 3

(D) 4, 5-dimethyl bicycle [2, 2, 1] hept-1 ene, 2

Solution: (C)

This problem contains conceptual mixing of nomenclature of cyclic hydrocarbon and degree of unsaturation. This problem can be solved by identifying the parent chain functional group, position of functional group substituent and their position one by one and then write the name of compound according to IUPAC names then calculate degree of unsaturation.



Total carbon atom forming the bicyclic ring (hept.)

Functional group ⇒ double bond (ene)

Position of double bond ⇒ 2, 2-ene

Substituents ⇒ 2-methyl group ⇒ dimethyl

Position of substituents $= 5, 6 \rightarrow$ 5,6-dimethyl

Number of cyclic chain $= 2 \rightarrow$ Bicyclo

3-bridges are of 2 carbons, 2 carbons and one carbon hence,

IUPAC name = 5, 6-dimethyl bicycle [2, 2, 1] hept-2-ene

Molecular formula of compound is C_9H_{14}.

Degree of unsaturation can be calculated as

$$u = (C + 1) - \frac{H}{2} + \frac{N}{2}$$

Where, u=degree of unsaturation

C = number of carbons

H = number of hydrogens

N = number of nitrogen

Hence, for a compound having molecular formula C_9H_{14} the degree of unsaturation may be calculated as

$$u = (9 - 1) - \frac{14}{2} = 10 - 7 = 3$$

78. The oxidation state of sulphur in $Na_2S_4O_6$ is

(A) $+ 6$ (B) $+\frac{3}{2}$ (C) $+\frac{5}{2}$ (D) -2

Solution: (C)

Oxidation number of $Na = +1$

Oxidation number of $O = -2$

Let oxidation number of $S = x$

$\therefore \quad 2(ON \text{ of } Na) + 4(ON \text{ of } S) + 6(ON \text{ of } O) = 0$

$2(+1) + 4x + 6(-2) = 0$

$+2 + 4x - 12 = 0$

$4x = +12 - 2$

$x = +\frac{10}{4}$

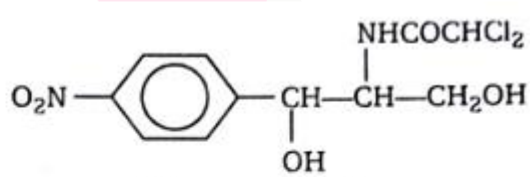
$\Rightarrow \quad x = +\frac{5}{2}$

79. Which of the following antibiotic contains intro group attached to aromatic nucleus in its structure

(A) Tetracyclin (B) Penicillin (C) Streptomycin (D) Chloramphenicol

Solution: (D)

Among the given antibiotics, only chloramphenicol contains a nitro group attached to aromatic ring. Its structure is as follows



Chloramphenicol

General structure of penicillin is

80. The behavior of the gas becomes more ideal at

I. Very low pressure

II. Value of Z is unity

III. Very high pressure

IV. Value of Z is greater than one

Choose the correct option.

(A) I and II are correct (B) I and IV are correct

(C) I and III are correct (D) III and IV are correct

Solution: (A)

The deviation from ideal behavior can be measured in terms of compressibility factor Z.

$$Z = \frac{pV}{nRT}$$

For ideal gas $Z = 1$,

i.e., $pV = nRT$

At very low pressures all gases shown have $Z \approx 1$ and behave as ideal gas. At high pressure all the gases have Z > 1 which show the deviation from ideality. At intermediate pressures, most gases have Z < 1 which also show the deviation from ideality.

English

Single correct answer type:

1. Out of the four alternatives, choose the one which express the right meaning of the word.

Sagacious

(A) Shameless (B) Wise

(C) Powerless (D) Foolish

Solution: (B)

Sagacious means 'judicious', so 'wise' is correct answer.

2. Out of the four alternatives, choose the one which express the right meaning of the word.

Remedial

(A) Corrective (B) Proficient

(C) General (D) Optional

Solution: (A)

Remedial means 'reformative', so 'corrective' is correct answer.

3. Out of the four alternatives, choose the one which express the right meaning of the word.

Reticent

(A) Confident (B) Sad

(C) Truthful (D) Secretive

Solution: (D)

Reticent means 'quiet' so 'secretive' is correct answer.

4. Choose the word apposite is meaning to the given word.

Fidelity

(A) Faith (B) Devotedness

(C) Allegiance (D) Treachery

Solution: (D)

Fidelity means ‘faithfulness in relations’, so ‘treachery’ is correct antonym.

5. Choose the word apposite is meaning to the given word.

Infrangible

(A) Complicated (B) Breakable

(C) Weird (D) Software

Solution: (B)

Infrangible means ‘strong’, so ‘breakable’ is correct antonym.

6. Choose the word apposite is meaning to the given word.

Progeny

(A) Kid (B) Parent (C) Friend (D) Enemy

Solution: (B)

Progeny means ‘child’, so ‘parent’ is correct antonym.

7. A part of sentence is underline. Balance are given alternatives to the underlined part a, b, c and d which may improve the sentence. Choose the correct alternative.

It was not possible to <u>drag</u> any conclusion so he left the case.

(A) Fetch (B) Find (C) Draw (D) No improvement

Solution: (C)

Use of ‘draw’ is more suitable for using before word ‘conclusion’, so option (draw) is correct.

8. A part of sentence is underline. Balance are given alternatives to the underlined part a, b, c and d which may improve the sentence. Choose the correct alternative.

I am <u>looking after</u> my pen which is missing.

(A) Looking for (B) Looking in (C) Looking back (D) No improvement

Solution: (A)

Use of 'looking for' is proper because look for means 'to search for something' which suits here.

9. A part of sentence is underline. Balance are given alternatives to the underlined part a, b, c and d which may improve the sentence. Choose the correct alternative.

<u>"Mind</u> your language !" he shouted.

(A) Change (B) Inspect (C) Hold (D) No improvement

Solution: (D)

'Mind your language' is proper to use here because it gives proper sense of sentence.

10. Sentence Completion

I______ to go there when I was student.

(A) Liked (B) Used (C) Prefer (D) Denied

Solution: (B)

'Use to' is used when any habit is to be shown, so use of option (used) is proper.

11. Sentence Completion

She was angry ______ me.

(A) at (B) about (C) with (D) in

Solution: (C)

'Angry' agrees with preposition 'with', so use of option (with) is correct here.

12. Sentence Completion

You should not laugh _____ the poor.

(A) on (B) at (C) with (D) over

Solution: (B)

Laugh agrees with preposition 'at', so use of option (at) is correct here.

13. Sentence rearrangement

1. He is a famous doctor.

P. Once I had to consult with him.

Q. I never believed him.

R. He suggested me a proper remedy.

S. I become completely fine.

6. Now I also admit this fact.

(A) $P\ Q\ R\ S$ (B) $Q\ R\ S\ R$ (C) $Q\ P\ R\ S$ (D) $R\ Q\ S\ P$

Solution: (C)

According to sequence of events, so option $(Q\ P\ R\ S)$ is best answer.

14. Sentence rearrangement

1. We don't know the plan of Ram.

P. He cares for his friends.

Q. He is a complete person.

R. We want some help and advice.

S. As we are in a trouble.

6. We hope he will do his best for us.

(A) $P\ R\ S\ Q$ (B) $Q\ P\ R\ S$ (C) $P\ Q\ R\ S$ (D) $P\ S\ R\ Q$

Solution: (B)

According to sequence of events, option $(Q\ P\ R\ S)$ best suits here.

15. Sentence rearrangement

1. It is not my problem.

P. All residents of this society are careless.

Q. I am unable to convince anyone.

R. They don't want to do some good.

S. Every one seems to be unwise here.

6. We all have to suffer one day.

(A) $P\ R\ S\ Q$ (B) $P\ R\ Q\ S$ (C) $P\ Q\ R\ S$ (D) $P\ S\ R\ Q$

Solution: (A)

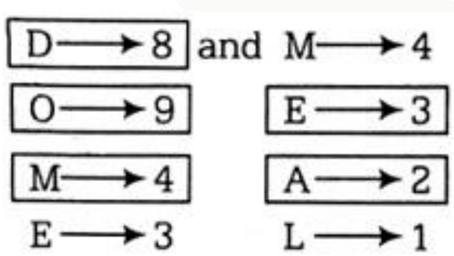
According to sequence of events, so option $(P\ R\ S\ Q)$ is proper here.

16. In a certain code language 'DOME' is written as '8943' and 'MEAL' is written as '4321'. What group of letters can be formed for the code '38249'?

(A) EOADM (B) MEDOA (C) EMDAO (D) EDAMO

Solution: (D)

As,

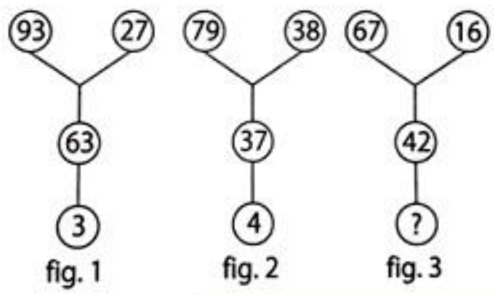


In the same way 38249 will be coded as

Diagram

Hence, option (EDAMO) is correct.

17. Find the missing number from the given response.



(A) 5 (B) 6 (C) 8 (D) 9

Solution: (D)

From fig. $1, 93-(27+63)=93-90=3$

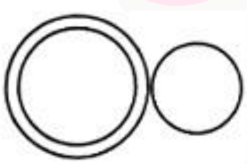
From fig. $2, 79-(38+37)=79-75=4$

From fig. $3, 67\ -(16+42)=67-58=9$

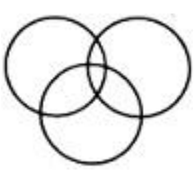
Hence, option (9) is correct.

18. Which of the following correctly represents the relationship among illiterates, poor people and unemployed?

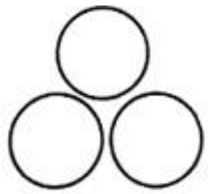
(A)



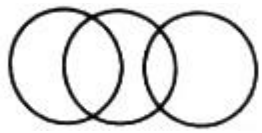
(B)



(C)

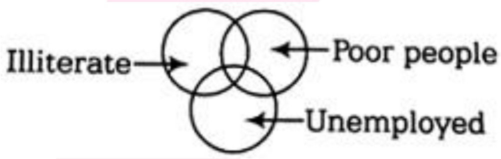


(D)



Solution: (B)

Some poor people can be unemployed, some unemployed people can be illiterates and some illiterates can be poor. Hence, correct diagram is



Hence, option (ii) is correct.

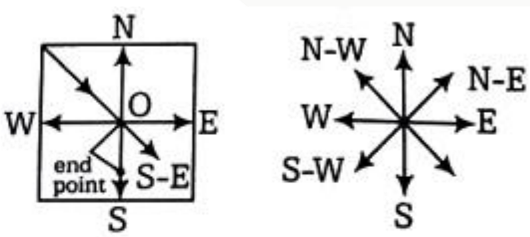
19. $'A'$ starts crossing the field diagonally from North-West. After walking half the distance, he turns right, walks some distance and turns left. Which direction is $'A'$ facing now?

(A) North-East (B) North-West

(C) South-East (D) South-West

Solution: (C)

Staring point



Hence, $'A'$ moving is South-East direction.

Hence option (South-East) is correct.

20. In a classroom, there are 5 rows and 5 children A, B, C, D and E are seated one behind the other in 5 separate rows as follows

- A is sitting behind C but in front of B.
- C is sitting behind E and D is sitting in front of B.
- C is sitting behind E and D is sitting in front of E.
- The order in which they are sitting from the first row to the last is

(A) $D\ E\ C\ A\ B$ (B) $B\ A\ C\ E\ D$

(C) $A\ C\ B\ D\ E$ (D) $A\ B\ E\ D\ C$

Solution: (A)

From the information given in the question the arrangement of students is

$1^{st} \rightarrow D$

$2^{nd} \rightarrow E$

$3^{rd} \rightarrow C$

$4^{th} \rightarrow A$

$5^{th} \rightarrow\ B$

Hence, option ($D\ E\ C\ A\ B$) is correct.

21. Which of the following will fill the series?

$2, 9, 28. ?, 126$

(A) 64 (B) 65 (C) 72 (D) 56

Solution: (B)

The given series follows the pattern

$1^3 + 1 = 2$

$2^3 + 1 = 8 + 1 = 9$

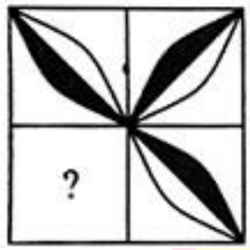
$3^3 + 1 = 27 + 1 = 28$

$4^3 + 1 = 64 + 1 = 65$

$5^3 + 1 = 125 + 1 = 126$

Hence, option (65) is correct.

22. Which one of the following figures completes the original figure?



(A)

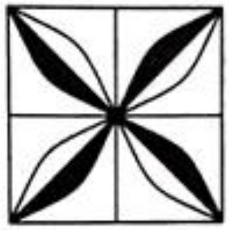
(B)

(C)

(D)

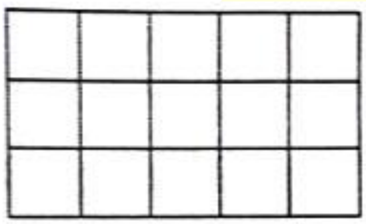
Solution: (B)

Clearly, option figure (ii) completes the original figure which looks like the figure given below



Hence, option (ii) is correct.

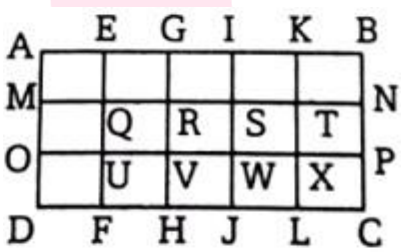
23. How many squares are there in the following figure?



(A) 24 (B) 25 (C) 26 (D) 27

Solution: (C)

On labeling the figure, we get



Each row contains five squares.

∴ Total number of single squares $= 5 \times 3 = 15$

Now, combination of 4 small squares will be = 8 (i.e., AOVG, EUWI, GVXK, IWPB, MDHR, QFJS, RHLT AND SJCN)

Now, combination of 9 small squares will be = 3 (i.e., ADJI, EFLK and GHCB)

∴ Total number of squares

$= 15 + 8 + 3 = 26$ squares

Hence option (26) is correct.

24. Two signs in the equations have been interchanged, find out the two signs to make equation correct.

$3 \div 5 \times 8 + 2 - 10 = 13$

(A) $+ \text{ and } -$ (B) $\times \text{ and } \div$

(C) $\div \text{ and } -$ (D) $\div \text{ and } +$

Solution: (D)

Interchanging symbols + and – as given in option $(+ \text{ and } -)$ the above equation becomes

$3 \div 5 \times 8 - 2 + 10 = \frac{3}{5} \times 8 - 2 + 10 = \frac{24}{5} + 8 \neq 13$

Interchanging symbols × and ÷ as given in option $(\times \text{ and } \div)$, we get

$3 \times 5 \div 8 + 2 - 10$

$= 3 \times \frac{5}{8} + 2 - 10 = \frac{15}{8} - 10 \neq 13$

Interchanging symbols $\div \text{ and } -$ as given in option $(\div \text{ and } -)$, we get

$3 - 5 \times 8 + 2 \div 10$

$= 3 + 5 \times \frac{8}{2} - 10$

$= 3 + 20 - 10 = 13$

Hence, option $(\div \text{ and } +)$ is correct.

25. Assertion $[A]$ = India is a democratic country.

Reason $[R]$ = India has a constitution of its own.

Choose the correct alternative from the given options.

(A) Both (A) and (R) are true and (R) is correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true (R) is false

(D) (A) is false (R) is true

Solution: (B)

Both Assertion and Reason are correct but India is a democratic country because the government is elected by its citizens and not because India has its own constitution.

Hence, option Both (A) and (R) are true but (R) is not the correct explanation of (A) is correct.

Mathematics

Single correct answer type:

106. The value of $\sum_{k=1}^{6}\left(\frac{\sin 2\pi k}{7}-\frac{i\cos 2\pi k}{7}\right)$ is

(A) -1 (B) 0 (C) $-i$ (D) i

Solution: (D)

$$\sum_{k=1}^{6}\left(\frac{\sin 2\pi k}{7}-\frac{i\cos 2\pi k}{7}\right)$$

$$=\sum_{k=1}^{6}-i\left(\frac{\cos 2\pi k}{7}+\frac{i\sin 2\pi k}{7}\right)$$

$$=-i\sum_{k=1}^{6}e^{i2\pi k/7}$$

$$=-i\left[e^{i2\pi/7}+e^{i4\pi/7}+\cdots e^{i12\pi/7}\right]$$

$$=-i\left[e^{i2\pi/7}\frac{\left(1-e^{i12\pi/7}\right)}{1-e^{i2\pi/7}}\right]$$

$$=-i\left[\frac{e^{i2\pi/7}-e^{i14\pi/7}}{1-e^{i2\pi/7}}\right]$$

$$=-i\left[\frac{e^{i2\pi/7}-e^{2\pi i}}{1-e^{i2\pi/7}}\right]=-i\left[\frac{e^{i2\pi/7}-1}{1-e^{i2\pi/7}}\right]=i$$

107. The mean life of a sample of 60 bulbs was 650 and the standard deviation was 8 h. A second sample of 80 bulbs has a mean life of 660 h and standard deviation 7h. Find the over all standard deviation.

(A) 8.97 (B) 8.98 (C) 8.94 (D) None of the above

Solution: (C)

Given, $n_1=60, \bar{x}_1=650, \sigma_1=8,$

$n_2=80, \bar{x}_2=660, \sigma_2=7$

∴ Combined SD

$$= \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2\,(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

$$= \sqrt{\frac{60 \times 64 + 80 \times 49}{60 + 80} + \frac{60 \times 80\,(650 - 660)^2}{(60 + 80)^2}}$$

$$= \sqrt{\frac{3840 + 3920}{140} + \frac{(4800 \times 100)}{(140)^2}}$$

$$= \sqrt{\frac{7760}{140} + \frac{480000}{19600}} = \sqrt{\frac{776}{14} + \frac{4800}{196}}$$

$$= \sqrt{55.42 + 24.49}$$

$$= \sqrt{79.91} = 8.94$$

108. Let R be the relation on the set R of all real numbers, defined by aRb iff $|a - b| \leq 1$. Then, R is

(A) Reflexive and symmetric only

(B) Reflexive and transitive only

(C) Equivalence

(D) None of the above

Solution: (A)

Since, $|a - a| = 0 < 1$, so $aRa, \forall\, a \in R$

∴ R is reflexive.

Now, $aRb \Rightarrow |a - b| \leq 1 \Rightarrow |b - a| \leq 1 \Rightarrow bRa$

∴ R is symmetric.

But R is not transitive as

$1R2, 2R3$ but $1R3$ $\quad (\because \ |1 - 3| = 2 > 1)$

109. The value of $\int_0^{10\pi}([\sec^{-1} x] + [\cot^{-1} x])\, dx$, where $[\cdot]$ denotes the greatest integer function, is

(A) $10\pi - \tan^{-1} x$ (B) $8\pi - \sec 1$

(C) $10\pi - \sec 1$ (D) $10\pi + \sec 1$

Solution: (C)

Given that,

$$I = \int_0^{10\pi} ([\sec^{-1} x] + [\cot^{-1} x])dx$$

$$= \int_0^{\sec 1} ([\sec^{-1} x] + [\cot^{-1} x])\, dx + \int_{\sec 1}^{10\pi} ([\sec^{-1} x] + [\cot^{-1} x])\, dx$$

$$= \int_0^{\sec 1} (0 + 0)dx + \int_{\sec 1}^{10\pi} (10 + 0)\, dx = 0 + [x]_{\sec 1}^{10\pi}$$

$$= 10\pi - \sec 1$$

110. The value of the expression $\sin[\cot^{-1}(\cos(\tan^{-1} 1))]$ is

(A) 0 (B) 1 (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{\frac{2}{3}}$

Solution: (D)

$$\sin\left[\cot^{-1}\left(\cot\frac{\pi}{4}\right)\right]$$

$$= \sin\left[\cot^{-1}\frac{1}{\sqrt{2}}\right] = \sin\left[\sin^{-1}\sqrt{\frac{2}{3}}\right] = \sqrt{\frac{2}{3}}$$

111. The sum of the series $1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 100 \cdot 2^{99}$ is

(A) $100 \cdot 2^{100} + 1$ (B) $99 \cdot 2^{100} + 1$

(C) $99 \cdot 2^{99} - 1$ (D) $100 \cdot 2^{100} - 1$

Solution: (B)

Let

$S = 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 100 \cdot 2^{99}$ ......(i)

It is an arithmetic-geometric series.

On multiplying equation (i) by 2 and then subtracting,

We get

$S = 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + 100 \cdot 2^{99}$

$2S = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + 99 \cdot 2^{99} + 100 \cdot 2^{100}$

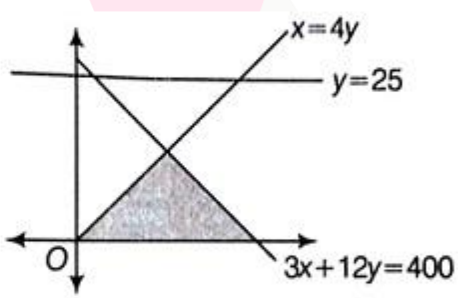
$-S = 1 + 2 + 2^2 + 2^3 + \cdots + 2^{99} - 100 \cdot 2^{100}$

$\Rightarrow \quad -S = \frac{1(2^{100} - 1)}{2 - 1} - 100 \cdot 2^{100}$

$\Rightarrow \quad -S = 2^{100} - 1 - 100 \cdot 2^{100}$

$\Rightarrow \quad -S = -1 - 99 \cdot 2^{100} \Rightarrow \quad S = 99 \cdot 2^{100} + 1$

112. The shaded region given below represents the constraints (other than $x \geq 0, y \geq 0$)



(A) $3x + 4y \leq 400, y \leq 25, x \leq 4y$

(B) $3x + 12y \geq 400, y \leq 25, x \geq 4y$

(C) $3x + 12y \leq 400, y \leq 25, x \geq 4y$

(D) None of the above

Solution: (C)

Consider a point (2, 0) on the x-axis.

Substituting $x = 2, y = 0$ in $3x + 12y = 6 < 400$.

Hence, one constraints is $3x + 12y \leq 400$

Again, substituting $x = 2, y = 0$ in $x - 4y = 2 - 0 > 0$

$\therefore$ $x - 4y \geq 0$ is other constraints and also the third constraint from the figure is $y \leq 25$.

Hence, the correct alternative is $3x + 12y \leq 400, y \leq 25, x \geq 4y$.

113. The coefficient of x^n in the expansion of $\log_e \left(\frac{1}{1+x+x^2+x^3}\right)$, when n is odd, is

(A) $-\frac{2}{n}$

(B) $-\frac{1}{n}$

(C) $\frac{1}{n}$

(D) None of these

Solution: (B)

$$\log\left(\frac{1}{1 + x + x^2 + x^3}\right) = \log\left(\frac{1 - x}{1 - x^4}\right)$$

$$= \log_e(1 - x) - \log(1 - x^4)$$

$$= -\sum_{r=1}^{\infty} \frac{x^r}{r} + \sum_{r=1}^{\infty} \frac{x^{4r}}{r}$$

When n is odd, there is no term in the second series containing x^n, therefore the coefficient x^n is zero in the second series and in the first series the coefficient of x^n is $-\frac{1}{n}$. Hence, when n is odd, then, the coefficient of x^n in the whole expansion is $-\frac{1}{n} + 0 = -\frac{1}{n}$.

114. The maximum value of $f(x) = \frac{\log x}{x}$ is

(A) 1

(B) $\frac{2}{e}$

(C) e

(D) $\frac{1}{e}$

Solution: (D)

For maximum value, find $f'(x)$

$f'(x) = \frac{1}{x^2}(1 - \log x)$

$f'(x) > 0$ for $x < e$ and $f'(x) < 0$ for $x > e$

$\Rightarrow$ $f(x)$ is increasing for $x < e$ and decreasing for $x > e$

$\Rightarrow$ $x = e$ is the point of local maxima.

$\therefore$ Maximum value of $f(x) = \frac{1}{e}$

Hence, the answer is $\frac{1}{e}$.

115. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that no two are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\,\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ is equal to

(A) $\frac{2\sqrt{2}}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

Solution: (A)

We have, $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$

$\Rightarrow$ $(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\,\vec{a} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\vec{a}$

$\Rightarrow$ $(\vec{a}\cdot\vec{c})\,\vec{b} - \left\{(\vec{b}\cdot\vec{c}) + \frac{1}{3}|\vec{b}||\vec{c}|\right\}\vec{a} = 0$

$\Rightarrow$ $(\vec{a}\cdot\vec{c}) = 0$ and $\vec{b}\cdot\vec{c} + \frac{1}{3}|\vec{b}|\,|\vec{c}| = 0$

($\because$ θ is the angle between $\vec{b}$ and $\vec{c}$)

$\Rightarrow$ $|\vec{b}|\,|\vec{c}|\cos\theta + \frac{1}{3}|\vec{b}||\vec{c}| = 0 \Rightarrow \cos\theta = -\frac{1}{3}$

$\therefore$ $\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

116. The value of $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$ is

(A) e^2 (B) e (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

Solution: (A)

$$\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} = \lim_{x\to 0}\left(1+\frac{2x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$$

$$= e^{\lim_{x\to 0}\left(\frac{2x^2}{1+3x^2}\right)} = e^2$$

117. An object is observed from the points $A.B$ and C lying in a horizontal straight line which passes directly underneath the object. The angular elevation at B is twice that at A and at C three times that at A. If $AB = a, BC = b$, then the height of the object is

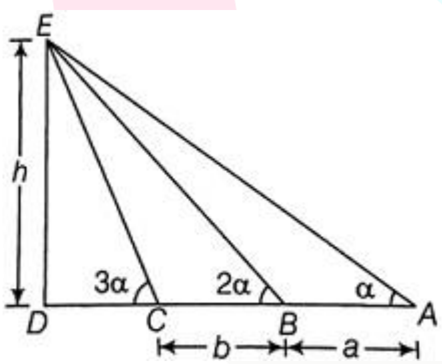
(A) $\frac{3a}{2b}\sqrt{(a+b)(3b-a)}$ (B) $\frac{3b}{2a}\sqrt{(a+b)\,(3a-b)}$

(C) $\frac{a}{2b}\sqrt{(a+b)\,(3b-a)}$ (D) None of the above

Solution: (C)

Let $ED = h, \angle EAB = \alpha$

$\therefore \quad \angle EBD = 2\alpha, \quad \angle ECD = 3\alpha$



Now, $\angle DBE = \angle EAB + \angle BEA$

$\Rightarrow \quad 2\alpha = \alpha + \angle BEA$

$\Rightarrow \quad \angle BEA = \alpha = \angle EAB$

$\Rightarrow \quad AB = EB = a$

Similarly, $\angle BEC = \alpha$

From $\Delta EBC, \frac{BC}{\sin\alpha} = \frac{EB}{\sin(180^o - 3\alpha)}$

$$\Rightarrow \quad \frac{b}{\sin\alpha} = \frac{a}{\sin 3\alpha} \quad \Rightarrow \frac{a}{b} = \frac{\sin 3\alpha}{\sin\alpha}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{3\sin\alpha - 4\sin^3\alpha}{\sin\alpha} = 3 - 4\sin^2\alpha$$

$$\Rightarrow \quad 4\sin^2\alpha = 3 - \frac{a}{b} = \frac{3b - a}{b}$$

$$\Rightarrow \quad \sin\alpha = \sqrt{\frac{3b - a}{4b}}$$

From $\Delta EBD, \sin 2\alpha = \frac{ED}{EB}$

$$\Rightarrow \quad ED = a \cdot 2\sin\alpha \cdot \cos\alpha$$

$$\Rightarrow \quad h = 2a\sqrt{\frac{3b - a}{4b}} \cdot \sqrt{1 - \frac{3b - a}{4b}}$$

$$= 2a\sqrt{\frac{3b - a}{4b}}\sqrt{\frac{b + a}{4b}}$$

$$= \frac{a}{2b}\sqrt{(a + b)(3b - a)}$$

118. Function $f: (-\infty, -1] \rightarrow (0, e^5]$ defined by $f(x) = e^{x^3 - 3x + 2}$ is

(A) many-one and onto (B) many-one and into

(C) one-one and onto (D) one-one and into

Solution: (D)

We have, $f(x) = e^{x^3 - 3x + 2}$

Let $h(x) = x^3 - 3x + 2$

$\therefore \quad h'(x) = 3x^2 - 3 = 3(x^2 - 1)$

$\Rightarrow \quad h'(x) \geq 0$ for $x \in (-\infty, -1]$

$\therefore \quad f(x)$ is increasing function.

$\therefore \quad f(x)$ is one-one.

Now, range of $f(x) = (0, e^4]$

But codomain of $f(x) = (0, e^5]$

$\therefore \quad f(x)$ is an into function.

119. The foci of the conic section $25x^2 + 16y^2 - 150x = 175$ are

(A) $(0, \pm 3)$ (B) $(0, \pm\ 2)$

(C) $(3, \pm 3)$ (D) $(0, \pm\ 1)$

Solution: (C)

Given equation can be rewritten as

$$\frac{(x-3)^2}{16} + \frac{y^2}{25} = 1$$

$\therefore \quad a^2 = 16$ and $b^2 = 25$

Now, $e = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25-16}{25}}$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Hence, the foci of conic section are $(3, \pm\ be)$ i.e., $(3, \pm 3)$.

So, option $(3, \pm 3)$ is correct.

120. The system of equations

$x - y + 3z = 4$

$x + z = 2$

$x + y - z = 0$ has

(A) A unique solution

(B) Finitely many solution

(C) Infinitely many solutions

(D) None of the above

Solution: (C)

Let $D = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= 1\,(10 - 1) - 1(1 + 1) + 3(1)$

$= -1 - 2 + 3 = 0$

$D_1 = \begin{vmatrix} 4 & -1 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 4(0 - 1) - 1(0 + 2) + 3(2)$

$= -4 - 2 + 6 = 0$

$D_2 = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{vmatrix}$

$= 1(-2) + 4(1 + 1) + 3(0 - 2)$

$= -28 - 6 = 0$

$D_3 = \begin{vmatrix} 1 & -1 & 4 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{vmatrix}$

$= 1(0 - 2) - 1(2 - 0) + 4(1 - 0)$

$= -2 - 2 + 4 = 0$

Hence, the given system of equations has infinitely many solutions.

121. The sum of the sequence $5, 55, 555, \ldots$ upto n infinite terms is

(A) $\frac{5}{9}\left[\frac{10(10^n-1)+n}{9}\right]$

(B) $\frac{5}{9}\left[\frac{10(10^n-1)}{9} - n\right]$

(C) $\frac{5}{9}\left[\frac{10\left(10^{n\mp1}-1\right)}{9} - n\right]$

(D) $\frac{5}{9}\left[\frac{10\left(10^{n-1}-1\right)}{9} - n\right]$

Solution: (B)

$S_n = 5 + 55 + 555 + \cdots \text{upto n terms}$

$= 5\,[1 + 11 + 111 + \cdots \text{upto n terms}]$

$= \frac{5}{9}$ [9 + 99 + 999 … . upto n temrs]

$= \frac{5}{9}[(10-1)+(10^2-1)+(10^3-1)+\cdots \text{ upto n terms}]$

$= \frac{5}{9}[(10+10^2+10^3+\cdots \text{ upto n terms})-(1+1+1+1+\cdots \text{ upto n terms})]$

$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$

$= \frac{5}{9}\left[\frac{10(10^n-1)}{9}-n\right]$

122. A plane passes through the point $(1,-2,3)$ and is parallel to the plane $2x-2y+z=0$. The distance of the point $(-1,2,0)$ from the plane is

(A) 2 (B) 3 (C) 4 (D) 5

Solution: (D)

Let the parallel plane to $2x-2y+z=0$ is $2x-2y+z+\lambda=0$

It passes through (1, -2, 3)

$\therefore \quad 2+4+3+\lambda=0 \quad \Rightarrow \quad \lambda=-9$

The distance of (-1, 2, 0) from the plane

$2x-2y+z-9=0$ is $\left|\frac{-2-4-9}{\sqrt{5+4+1}}\right| = \left|\frac{-15}{3}\right| = 5$

123. The distance between the pair of lines represented by the equation $x^2-6xy+9y^2+3y-9y-4=0$ is

(A) $\frac{15}{\sqrt{10}}$ (B) $\frac{1}{2}$ (C) $\sqrt{\frac{5}{2}}$ (D) $\frac{1}{\sqrt{10}}$

Solution: (C)

Clearly, we have

$a=1.h=-3, b=9, g=\frac{3}{2}, f=\frac{-9}{2}$ and $c=\ -4$

Required distance $= \left|2\sqrt{\frac{t^2-bc}{b(a+b)}}\right|$

$$= \left|2\sqrt{\frac{\left(\frac{-9}{2}\right)^2 + 9 \times 4}{9(9+1)}}\right|$$

$$= \left|2\sqrt{\frac{225}{4 \times 90}}\right| = \left|\frac{2\sqrt{5}}{2\sqrt{2}}\right| = \sqrt{\frac{5}{2}}$$

124. If $A = \{x \in \mathbb{C} : x^4 - 1 = 0\}$

$B = \{x \in \mathbb{C} : x^2 - 1 = 0\}$

$C = \{x \in \mathbb{C} : x^2 + 1 = 0\}$

Where $\mathbb{C}$ is complex plane.

(A) $A = B \cup C$ (B) $C = A \cap B$

(C) $B = A \cap C$ (D) $A = B \cap C$

Solution: (A)

$A = \{1, -1, i, -i\}$

$B = \{1, -1\}$

$C = \{i, -i\}$

Now, $B \cup C = \{1, -1, i, -i\} = A$

125. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is

(A) $\log\tan\left(\frac{y}{2}\right) = C - 2\sin x$ (B) $\log\tan\left(\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$

(C) $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = C - 2\sin x$ (D) None of the above

Solution: (B)

We have, $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$

$$\Rightarrow \quad \frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = -2\cos\left(\frac{x}{2}\right)\sin\left(\frac{y}{2}\right)$$

$$\Rightarrow \quad \frac{dy}{\sin\left(\frac{y}{2}\right)} = -2\cos\left(\frac{x}{2}\right)dx$$

On integrating both sides, we get

$$\int \frac{dy}{\sin\left(\frac{y}{2}\right)} = -2\int \cos\left(\frac{x}{2}\right)dx$$

$$\Rightarrow \quad \frac{1}{2}\int cosec\ \left(\frac{y}{2}\right)dy = -\int \cos\left(\frac{x}{2}\right)dx$$

$$\Rightarrow \quad \frac{1}{2}\cdot\left[\frac{\log\left\{cosec\ \left(\frac{y}{2}\right) - \cot\left(\frac{y}{2}\right)\right\}}{\frac{1}{2}}\right] = -\frac{\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$$

$$\Rightarrow \quad \log\left[\frac{1}{\sin\left(\frac{y}{2}\right)} - \frac{\cos\left(\frac{y}{2}\right)}{\sin\left(\frac{y}{2}\right)}\right] = -2\sin\left(\frac{x}{2}\right) + C$$

$$\Rightarrow \quad \log\left[\frac{2\sin^2\left(\frac{y}{4}\right)}{2\sin\left(\frac{y}{4}\right)\cos\left(\frac{y}{4}\right)}\right] = -2\sin\left(\frac{x}{2}\right) + C$$

$$\left(\because \quad 1-\cos x = 2\sin^2\frac{x}{2} \text{ and } \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}\right)$$

$$\Rightarrow \quad \log\tan\left(\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$$

126. The set of all real x satisfying the inequality $\frac{3-|x|}{4-|x|} \geq 0$

(A) $[-3,3] \cup (-\infty,-4) \cup (4,\infty)$

(B) $(-\infty,-4) \cup (4,\infty)$

(C) $(-\infty,-3) \cup (4,\infty)$

(D) $(-\infty,-3) \cup (3,\infty)$

Solution: (A)

Given, $\frac{3-|x|}{4-|x|} \geq 0$

$\Rightarrow \quad 3-|x| \leq 0$ and $4-|x|<0$

or $\quad 3-|x| \geq 0$ and $4-|x|>0$

$\Rightarrow \quad |x| \geq 3$ and $|x|>4$

or $\quad |x| \leq 3$ and $|x|<4$

$\Rightarrow \quad |x|>4$ or $|x| \leq 3$

$\Rightarrow \quad x \in(-\infty,-4) \cup[-3,3] \cup(4, \infty)$

127. If N is the any four digit number say x_1, x_2, x_3, x_4, then the maximum value of $\frac{N}{x_1+x_2+x_3+x_4}$ is equal to

(A) 1000 (B) $\frac{1111}{4}$

(C) 800 (D) None of these

Solution: (A)

$$\frac{N}{x_1+x_2+x_3+x_4}$$

$$=\frac{1000 x_1+100 x_2+10 x_3+x_4}{x_1+x_2+x_3+x_4}$$

$$=1000-\left(\frac{900 x_2+990 x_3+999 x_4}{x_1+x_2+x_3+x_4}\right)$$

$\Rightarrow$ Maximum value is 1000.

128. If A and B are two events such that $P(A)=0.6, P(B)=0.2$ and $P\left(\frac{A}{B}\right)=0.5$, then $P\left(\frac{A'}{B'}\right)$ equal to

(A) $\frac{1}{10}$ (B) $\frac{3}{10}$ (C) $\frac{3}{8}$ (D) $\frac{6}{7}$

Solution: (C)

$\because \quad P(A \cap B)=P\left(\frac{A}{B}\right) \cdot P(B)$

$=0.5 \times 0.2=0.1$

$\therefore \quad P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{P(B')}$

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$= \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - 0.2}$

$= \frac{1 - 0.6 - 0.2 + 0.1}{0.8} = \frac{3}{8}$

129. The quartile deviation for the data

x :	2	3	4	5	6
y :	3	4	8	4	1

is

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1

Solution: (D)

Here, $N = \Sigma f = 20$

$Q_1 = \frac{(N+1)th}{4}$ observation

$= \left(\frac{21}{4}\right) th$ observation

Similarly, $Q_3 = \frac{3(N+1)th}{4}$ observation

$= \left(\frac{63}{4}\right) th$ observation

Now, $QD = \frac{1}{2}(Q_3 - Q_1)$

$= \frac{1}{2}(5 - 3) = 1$

130. If $\int f(x) \cos x \, dx = \frac{1}{2} f^2(x) + C$, then $f(x)$ can be

(A) x (B) 1 (C) $\cos x$ (D) $\sin x$

Solution: (D)

Given that, $\int f(x) \cos x \, dx = \frac{1}{2} f^2(x) + C$

On differentiating w.r.t. x, we get

$$f(x)\cos x = \frac{1}{2}.2f(x).f'(x)$$

$$\Rightarrow \quad \cos x = f'(x)$$

$$\Rightarrow \quad \cos x = \frac{d}{dx}\left(f(x)\right)$$

$$\int \cos x \, dx = f(x)$$

$$f(x) = \sin x + C$$

131. There are 10 points in a plane, out of these 6 are collinear. If $'n'$ is the number of triangles formed by joining these points, then

(A) $n \leq 100$ (B) $100 < n < 140$

(C) $140 < n \leq 190$ (D) $n > 190$

Solution: (A)

Case 1 Taking 2 points from collinear points and one from non-collinear.

i.e., number of triangles so formed $= {}^6C_2 \times {}^4C_1$

$$= \frac{6 \cdot 5}{1 \cdot 2} \times \frac{4}{1} = 60$$

Case 2 Taking 1 point from collinear and two from non-collinear points

i.e., number of triangles so formed $= {}^6C_1 \times {}^4C_2$

$$= \frac{6}{1} \times \frac{4 \cdot 3}{1 \cdot 2} = 36$$

Case 3 All the three points from non-collinear points.

i.e., number of triangles so formed

$$= {}^4C_3 = {}^4C_1$$

$$= \frac{4}{1} = 4$$

Total number of triangles $= 60 + 36 + 4 = 100$

Alternatively Number of triangles

$$= {}^{10}C_3 - {}^{6}C_3 = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} - \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3}$$

$$= 120 - 20 = 100$$

Hence, option ($n \leq 100$) is correct answer.

132. $\frac{{}^{8}C_0}{6} - {}^{8}C_1 + {}^{8}C_2 \cdot 6 - {}^{8}C_3 \cdot 6^2 + {}^{8}C_4 \cdot 6^3 + \cdots + {}^{8}C_8 \cdot 6^7$ equals to

(A) 0 (B) 6^7 (C) 6^8 (D) $\frac{5^8}{6}$

Solution: (D)

$$\frac{{}^{8}C_0}{6} - {}^{8}C_1 + {}^{8}C_2 \cdot 6 - {}^{8}C_3 \cdot 6^2 + {}^{8}C_4 \cdot 6^3 + \cdots + {}^{8}C_8 \cdot 6^7$$

$$= \frac{1}{6}[{}^{8}C_0 - {}^{8}C_1 \cdot 6^1 + {}^{8}C_2 \cdot 6^2 - {}^{8}C_3 \cdot 6^3 + \cdots + {}^{8}C_8 \cdot 6^8]$$

$$= \frac{1}{6}[1-6]^8 = \frac{1}{6} \times (-5)^8 = \frac{5^8}{6}$$

Hence, the answer is $\frac{5^8}{6}$.

133. A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is atleast one girl on the committee, then the probability that there are exactly 2 girls on the committee is

(A) $\frac{68}{125}$ (B) $\frac{56}{165}$ (C) $\frac{63}{625}$ (D) $\frac{168}{425}$

Solution: (D)

Let A denote the event that atleast one girl will be chosen and B be the event that exactly 2 girls will be chosen. We required $P\left(\frac{B}{A}\right)$.

Since, A denotes the vent that atleast one girl will be chosen, A' denotes that no girl is

Then, $P(A') = \frac{{}^{8}C_4}{{}^{12}C_4} = \frac{70}{495} = \frac{14}{99}$

$$\Rightarrow \quad P(A) = 1 - \frac{14}{99} = \frac{85}{99}$$

Now, $P(A \cap B) = P$ (2 boys and 2 girls)

$$= \frac{{}^8C_2 \cdot {}^4C_2}{{}^{12}C_4} = \frac{28 \times 6}{495} = \frac{56}{165}$$

$$\therefore \quad P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{56 \times 99}{165 \times 85} = \frac{168}{425}$$

134. What are the values of c for which Rolle's theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in the interval $[0, 2]$ is verified?

(A) $c = \pm 1$ (B) $c + 1 \pm \frac{1}{\sqrt{3}}$

(C) $c = \pm 2$ (D) None of these

Solution: (B)

Here, we observe that

(a) $f(x)$ is a polynomial, so it is continuous in the interval [0, 2].

(b) $f'(x) = 3x^2 - 6x + 2$ exists for all $x \in (0, 2)$.

So, $f(x)$ is differentiable for all $x \in (0, 2)$

(c) $f(0) = 0, f(2) = 2^3 - 3(2)^2 + 2(2) = 0$

$\therefore \quad f(0) = f(2)$

Thus, all the three conditions of Rolle's theorem are satisfied.

So, there must exist $c \in (0, 2)$ such that $f'(c) = 0$

$\Rightarrow \quad f'(c) = 3c^2 - 6c + 2 = 0$

$$\Rightarrow \quad c = 1 \pm \frac{1}{\sqrt{3}} \quad \Rightarrow \quad C \in (0, 2)$$

135. If $\int \frac{4}{\sin^4 x + \cos^4 x} dx = a \tan^{-1}\left(\frac{\tan x - \frac{1}{\tan x}}{b}\right) + C$, then find the value of a and b, respectively.

(A) $2\sqrt{2}, \sqrt{2}$ (B) $\sqrt{2}, 2$ (C) $\sqrt{3}, \sqrt{2}$ (D) $\sqrt{2}, 4$

Solution: (A)

Consider that, $I = \int \frac{4}{\sin^4 x + \cos^4 x} dx$

$$I = \int \frac{4}{\cos^4 x \ (\tan^4 x + 1)} dx$$

$$= \int \frac{4 \sec^4 x}{1 + \tan^4 x} dx$$

$$= 4 \int \frac{\sec^2 x \,(1 + \tan^2 x)}{1 + \tan^4 x} dx$$

Put $\tan x = t \ \Rightarrow \ \sec^2 x \ dx = dt$

$$I = 4 \int \frac{1 + t^2}{1 + t^4} dt = 4 \int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} dt$$

$$\Rightarrow \quad I = 4 \int \frac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2 + 2} dt$$

Now, put $t - \frac{1}{t} = z \Rightarrow \ \left(1 + \frac{1}{t^2}\right) dt = dz$

$$\therefore \quad I = 4 \int \frac{dz}{z^2 + \left(\sqrt{2}\right)^2} = \frac{4}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + C$$

$$\Rightarrow \quad I = 2\sqrt{2} \tan^{-1}\left(\frac{\tan x - \frac{1}{\tan x}}{\sqrt{2}}\right) + C$$

$\therefore \quad a = 2\sqrt{2}$ and $b = \sqrt{2}$

136. If $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, then A is

(A) Iidempotent (B) nilpotent (C) involutory (D) periodic

Solution: (C)

Given, $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 25-24+0 & 40-40+0 & 0+0+0 \\ -15+15+0 & -24+25+0 & 0+0+0 \\ -5+6-1 & -8+10-2 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

As $A^2 = I$

$\therefore$ A is involuntary.

137. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{19} = 1$ and having its centre (0, 3) is

(A) 4 (B) $\frac{3}{7}$ (C) $\sqrt{12}$ (D) $\frac{7}{2}$

Solution: (A)

Given, $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$$\therefore \quad e = \sqrt{1 - \frac{9}{16}} = \sqrt{\frac{16-9}{16}} = \sqrt{\frac{7}{16}} = \frac{\sqrt{17}}{4}$$

$\therefore$ Coordinates of foci are $(\pm\sqrt{7}, 0)$.

Since, centre of circle is (0, 3) and passing through foci $(\pm\sqrt{7}, 0)$

$\therefore$ Radius of the circle $= \sqrt{(0 \pm \sqrt{7})^2 + (3-0)^2}$

$= \sqrt{7+9} = 4$

Hence, option 4 is correct.

138. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ is equal to

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$

Solution: (D)

The two normal vectors are $m = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $n = \hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

The line L is along, $m \times n = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix}$

$= \hat{\imath}\,(6-3) - \hat{\jmath}(4-1) + \hat{k}(6-3)$

$= 3\hat{\imath} - 3\hat{\jmath} + 3\hat{k} = 3(\hat{\imath} - \hat{\jmath} + \hat{k})$

Now, the direction cosines of X-axis are (1, 0, 0).

$$\therefore \quad \cos\alpha = \frac{3(\hat{\imath} - \hat{\jmath} + \hat{k}) \cdot \hat{\imath}}{\sqrt{3^2(1^2+1^2+1^2)}\sqrt{1}}$$

$$= \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

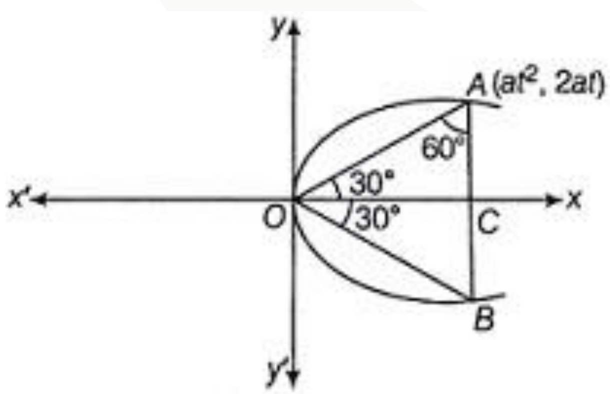
$$\Rightarrow \quad \cos\alpha = \frac{1}{\sqrt{3}}$$

139. If OAB is an equilateral triangle inscribed in the parabola $y^2 = 4ax$ with O as the vertex, then the length of the side of the ΔOAB is

(A) $8a\sqrt{3}$ (B) $4a\sqrt{3}$ (C) $2a\sqrt{3}$ (D) $a\sqrt{3}$

Solution: (A)

In ΔOCA, $\tan 30^o = \frac{AC}{OC}$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{2at}{at^2} \quad \Rightarrow \quad t = 2\sqrt{3}$$

Again, in ΔOCA,

$$OA = \sqrt{OC^2 + AC^2} = \sqrt{(at^2)^2 + (2at)^2}$$

$$= \sqrt{\left[\left(2\sqrt{3}\right)^2\right]^2 a^2 + 4a^2\left(2\sqrt{3}\right)^2}$$

$$= \sqrt{192a^2} = 8a\sqrt{3}$$

Hence, option $8a\sqrt{3}$ is correct.

140. If $f(x+y) = f(x) \cdot f(y)$ for all real x, y and $f(0) \neq 0$, then function $g(x) = \frac{f(x)}{1+\{f(x)\}^2}$ is

(A) even function

(B) odd function

(C) odd, if $f(x) > 0$

(D) neither even nor odd

Solution: (A)

Given, $f(x+y) = f(x) \cdot f(y)$

Put $x = y = 0$, then $f(0) = 1$

and put $y = -x$, then $f(0) = f(x)\, f(-x)$

$$\Rightarrow \quad f(-x) = \frac{1}{f(x)}$$

Now consider,

$$g(x) = \frac{f(x)}{1 + \{f(x)\}^2}$$

$$\Rightarrow \quad g(-x) = \frac{f(-x)}{1 + \{f(-x)\}^2}$$

$$= \frac{\frac{1}{f(x)}}{1 + \left\{\frac{1}{f(x)}\right\}^2}$$

$$= \frac{f(x)}{1 + \{f(x)\}^2} = g(x)$$

141. If $f(x)=(\tan^{-1}x)^2+\frac{2}{\sqrt{x^2+1}}$, then $f(x)$ is increasing in

(A) $(0,\infty)$ (B) $(-\infty,0)$

(C) $(-\infty,-5)$ (D) None of these

Solution: (A)

$$f(x)=(\tan^{-1}x)^2+\frac{2}{\sqrt{x^2+1}}$$

$$f'(x)=\frac{2}{1+x^2}\left[\tan^{-1}x-\frac{x}{\sqrt{1+x^2}}\right]$$

Let $g(x)=\tan^{-1}-\frac{x}{\sqrt{x^2+1}}$

$\Rightarrow \quad g'(x)=\frac{1}{1+x^2}\left[1-\frac{1}{\sqrt{x^2+1}}\right]>0$ for all $x\in R$

$\Rightarrow \quad g(x)$ is increasing for all $x\in R$.

But $g(0)=0 \;\Rightarrow\; g(x)>0$ for $x>0$

So, $f'(x)>0$ for $x>0$

Hence, $f(x)$ is increasing in $(0,\infty)$.

142. The number of solutions of $\cos x=|1+\sin x|, 0\le x\le 3\pi$ is

(A) 1 (B) 2 (C) 3 (D) 4

Solution: (C)

Clearly, $1+\sin x\ge 0$

$\therefore$ The given equation becomes

$$\cos x-\sin x=1$$

$$\Rightarrow \quad \cos x\cdot\frac{1}{\sqrt{2}}-\sin x\cdot\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad x+\frac{\pi}{4}=\frac{\pi}{4},\frac{7\pi}{4},\frac{9\pi}{4},\frac{15\pi}{4}$$

$\Rightarrow \quad x = 0, \frac{3\pi}{2}, 2\pi, \frac{7\pi}{2}$

$\because \quad 0 \leq x \leq 3\pi$

$\Rightarrow \quad x = 0, \frac{3\pi}{2}, 2\pi$

143. If a, b, c are in GP and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then x, y, z are in

(A) AP (B) GP (C) HP (D) None of these

Solution: (A)

Given, $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$

Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$\therefore \quad a = k^x, b = k^y, c = k^z$

$\because \quad a, b, c$ are in GP.

Therefore, $b^2 = ac$

$\Rightarrow \quad (k^y)^2 = k^x \cdot k^2$

$\Rightarrow \quad k^{2v} = k^{x+z}$

$\Rightarrow \quad 2y = x + z$

$\therefore \quad x, y$ and z are in AP.

144. The acute angle between the lines, whose direction cosines are given by $2l - m + 2n = 0, lm + mn + nl = 0$, is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Solution: (D)

Given that, $2l - m + 2m = 0$ .....(i)

and $lm + mn + nl = 0$ ......(ii)

From equation (i), $m = 2(l + n)$ put in equation 9ii),

$2l(l+n)+2n(l+n)+nl=0$

$\Rightarrow \quad 2l^2(l+n)+2n(l+n)+nl=0$

$\Rightarrow \quad 2l^2+5nl+2n^2=0$

$\Rightarrow \quad 2l^2+4nl+nl+2n^2=0$

$\Rightarrow \quad 2l(l+2n)+n(l+2n)=0$

$\Rightarrow \quad (l+2n)\,(n+2l)=0$

$\Rightarrow \quad l=\ -2n$ and $n=-2l$

If $l=-2n$, then $m=2(-2n+n)=-2n$

and if $n=-2l$, then $m=2(l-2l)=-2l$

The DR's are $1,-2,-2$ and $-2,-2,1$.

Now, $1(-2)-(2)(-2)-2(1)$

$=-2+4-2=0$

Hence, lines are perpendicular, so angle between then is $\frac{\pi}{2}$.

145. The equation of the lines through (1, 1) and making angles of 45^o with the line $x+y=0$ are

(A) $x-1=0, x-y=0$ (B) $x-y=0, y-1=0$

(C) $x+y-2=0, y-1=0$ (D) $x-1=0, y-1=0$

Solution: (D)

Let m be the slope of required line.

$\therefore \quad \left|\frac{m-(-1)}{1+m(-1)}\right|=1$

$\Rightarrow \quad \frac{m+1}{1-m}=\ \pm\ 1$

$\Rightarrow \quad m+1=1-m$

and $\quad m+1=-1+m$

$\Rightarrow \quad m=0$ and $m=\infty$

$\therefore$ Equation of line through (1, 1) is $y-1=0, x-1=0$

Hence, option $x-1=0, y-1=0$ is correct.

146. The area of the figure bounded by two branches of the curve $(y-x)^2 = x^3$ and straight line $x=1$ is

(A) $\frac{4}{5}$ sq unit (B) $\frac{4}{7}$ sq unit (C) $\frac{4}{9}$ sq unit (D) $\frac{4}{11}$ sq unit

Solution: (A)

Given curve is

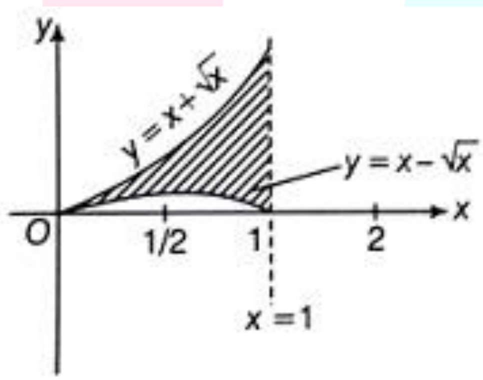
$(y-x)^2 = x^3$

$\Rightarrow \quad y - x = \pm x\sqrt{x}$

$\Rightarrow \quad y = x + x\sqrt{x}$ ....(i)

$\Rightarrow \quad y = x - x\sqrt{x}$ ....(ii)

and $x = 1$



From the figure, required area

$$= \int_0^1 \{(x + x\sqrt{x}) - (x - x\sqrt{x})\}\, dx$$

$$= \int_0^1 2x\sqrt{x}\, dx = 2\int_0^1 x^{\frac{3}{2}}\, dx$$

$$= 2\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1 = \frac{4}{5}[1-0] = \frac{4}{5} \text{ sq unit}$$

147. If $(x+iy)^{\frac{1}{3}} = 2+3i$, then $3x+2y$ is equal to

(A) -20 (B) -60 (C) -120 (D) 60

Solution: (C)

$(x+iy)^{\frac{1}{3}} = 2+3i$

On cubing both sides, we get

$x+iy = (2+3i)^3$

$\Rightarrow \quad x+iy = (2)^3 + (3i)^3 + 3\times 2\times 3i(2+3i)$

$\Rightarrow \quad x+iy = 8-27i+18i\ (2+3i)$

$\Rightarrow \quad x+iy = 8-27i+36i-54$

$\Rightarrow \quad x+iy = -46+9i$

On comparing real and imaginary both sides,

We get

$x=-46, y=9$

Then, $3x+2y = 3(-46)+2(9)$

$= -138+18 = -120$

148. In a town of 10000 families it was found that 40% family buy newspaper A, 20% buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspaper, then the number of families which buy A only is

(A) 3100 (B) 3300 (C) 2900 (D) 1400

Solution: (B)

$n(A) = 40\%\ of\ 10000 = 4000$

$n(B) = 20\%\ of\ 10000 = 2000$

$n(C) = 10\% \; of \; 10000 = 1000$

$n(A \cap B) = 5\% \; of \; 10000 = 500$

$n(B \cap C) = 3\% \; of \; 10000 = 300$

$n(A \cap C) = 4\% \; of \; 10000 = 400$

$n(A \cap B \cap C) = 2\% \; of \; 10000 = 200$

To find $d[A \cap B^C \cap C^C]$

$= n[A \cap (B \cup C)^C]$

$= n(A) - n[A \cap (B \cup C)]$

$= n(A) - [n(A \cap B) \cup n(A \cap C)]$

$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$

$= 4000 - [500 + 400 - 200] = 3300$

149. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Solution: (D)

Since, $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|} = \frac{8}{10} = \frac{4}{5}$

$\therefore \quad \cos\theta = \pm \frac{3}{5}$

$\therefore \quad \vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$= 2 \times 5 \times \left(\pm \frac{3}{5}\right) = \pm 6$

$\Rightarrow \quad |\vec{a} \cdot \vec{b}| = 6$

150. The equation of circle which passes through the origin and cuts off intercepts 5 and 6 from the positive parts of the x-axis and y-axis respectively is $\left(x - \frac{5}{2}\right)^2 + (y - 3)^2 = \lambda$, where λ is

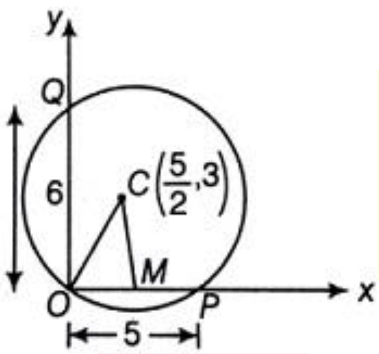
(A) $\frac{61}{4}$ (B) $\frac{4}{6}$ (C) $\frac{1}{4}$ (D) 0

Solution: (A)

From figure, we have

$OP = 5, OQ = 6$

and $OM = \frac{5}{2}, CM =$



$\therefore$ In $\Delta OMC, OC^2 = OM^2 + MC^2$

$\Rightarrow \quad OC^2 = \left(\frac{5}{2}\right)^2 + (3)^2 \Rightarrow OC = \frac{\sqrt{61}}{2}$

Thus, the required circle has its centre $\left(\frac{5}{2}, 3\right)$

and radius $\frac{\sqrt{61}}{2}$.

Hence, its equation is $\left(x - \frac{5}{2}\right) + (y-3)^2 = \left(\frac{61}{4}\right)$.

Hence, $\lambda = \frac{61}{4}$

Physics

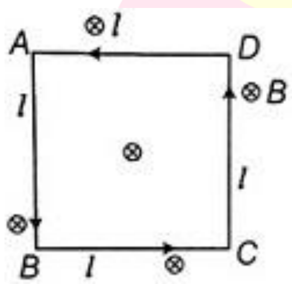
Single correct answer type:

1. A square shape current loop of side length l and carrying current I lies in a uniform magnetic field B acting perpendicular to the plane of square loop and directed inward. The net magnetic force acting on current loop is

(A) lBL (B) $4lBL$ (C) $zero$ (D) $2lBL$

Solution: (C)

Let the current is flowing in anti-clockwise direction as shown in figure.



Now, magnetic force on $AD = F_1 = ilB$ inwards (By Fleming's left hand rule)

Similarly, magnetic force on $BC = F_2 = \ i\ lB$ inwards

Since two forces are equal in magnitude and opposite in direction, therefore, they cancel out each other.

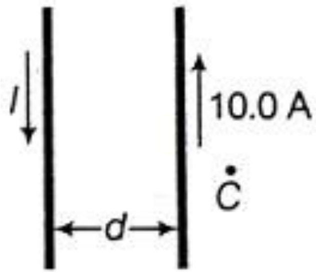
Also, magnetic force on $CD = F_3 = ilB$ inwards (By Fleming's left hand rule)

Similarly, magnetic force on $AB = F_4 = ilB$ inwards

Again, two forces are equal in magnitude and opposite in direction, therefore, they cancel out each other.

So, the net force on the current loop is 0.

2. Two parallel conductors carry current in opposite directions, as shown in figure. One conductor carries a current of $10.0\ A$. Point C is a distance $\frac{d}{2}$ to the right of the $10.0\ A$ current. If $d = 18$ cm and l is adjusted so that the magnetic field at C is zero, the value of the current I is



(A) $10.0A$ (B) $30.0A$ (C) $8.0A$ (D) $18.0A$

Solution: (B)

The magnetic field at C due to first conductor is $B_1 = \frac{\mu_0}{2\pi}\frac{i}{\frac{3d}{2}}$ (Since Point C is separated by $d + \frac{d}{2} = \frac{3d}{2}$ from 1^{st} conductor)

The direction of field is perpendicular to the plane of paper and directed outward. The magnetic field at C due to second conductor is $B_2 = \frac{\mu_0}{2\pi}\frac{10}{\frac{d}{2}}$ (Since, Point C is separated by $\frac{d}{2}$ from 2^{nd} conductor)

The direction of field is perpendicular to the plane of paper and directed inward.

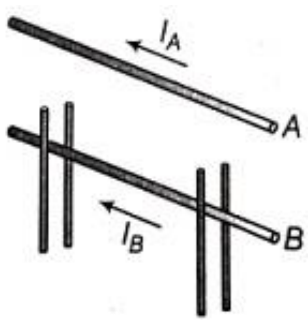
Since, direction of B_1 and B_2 at point C is in opposite direction and the magnetic field at C is zero, therefore,

$B_1 = B_2$

$$\frac{\mu_0}{2\pi}\frac{i}{\frac{3d}{2}} = \frac{\mu_0}{2\pi}\frac{10}{\frac{d}{2}}$$

On solving $i = 30.0A$

3. Two long, parallel conductors carry currents in the same direction, as shown in figure. Conductor A carries a current of 100A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of non-conducting guides. The mass per unit length of conductor B is 0.1 g/cm and the distance between the two conductors is 5cm. If system of conductors is in equilibrium, the value of current I_B is



(A) 250 A (B) 240 A (C) 220 A (D) 230 A

Solution: (A)

When system of conductors is in equilibrium,

The magnetic force of attraction per unit length between conductors = weight of conductor B per unit length.

$$\frac{\mu_0}{2\pi}\frac{I_A \times I_B}{d} = \frac{mg}{L} = \left(\frac{m}{L}\right)g$$

$$\frac{\mu_0}{2\pi}\frac{I_A \times I_B}{d} = \left(\frac{m}{L}\right)g$$

$$2 \times 10^{-7} \times \frac{10 \times I_B}{0.05} = (0.01\ kg/m) \times 10$$

On solving $I_B = 250\ A$

4. The number of photo electrons in a photoelectric effect experiment depends on the

(A) Frequency of light

(B) Intensity of light

(C) Both (frequency of light) and (intensity of light) are correct

(D) Both (frequency of light) and (intensity of light) are incorrect

Solution: (B)

Number of free electrons depends on the intensity of light only.

5. In hydrogen atom, if $\lambda_1, \lambda_2, \lambda_3$ are shortest wavelengths in Lyman, Balmer and Paschen series respectively then $\lambda_1 : \lambda_2 : \lambda_3$ equals

(A) 1 : 4 : 9 (B) 9 : 4 : 1 (C) 1 : 2 : 3 (D) 3 : 2 : 1

Solution: (A)

For hydrogen atom

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), n_2 > n_1$$

For Lyman $n_1 = 1,\ n_2 = \infty \Rightarrow \frac{1}{\lambda_1} = R$

For Balmer $n_1 = 2, n_2 = \infty \Rightarrow \frac{1}{\lambda_2} = \frac{R}{4}$

For Paschen series $n_1 = 3,\ n_2 = \infty \Rightarrow \frac{1}{\lambda_2} = \frac{R}{9}$

So, $\lambda_1 = \frac{1}{R}, \lambda_2 = \frac{4}{R}, \lambda_3 = \frac{9}{R}$

$\lambda_1 : \lambda_2 : \lambda_3 = 1 : 4 : 9$

6. Half-live of elements A and B are 1h and 2h respectively. Which of the following is correct?

(A) Element A decays slower

(B) Decay constant of A is smaller

(C) If initial number of nuclei are same then activity of A is more

(D) Mean life of A is more

Solution: (C)

Let, initial number of nuclei of each element $= N_0$

Decay constants $\lambda_A = \frac{0.693}{1}\ hr^{-1}, \lambda_B = \frac{0.693}{2} hr^{-2}$

$\lambda_A > \lambda_B$

Activities $R_A = \lambda_A\ N_0$

$R_B = \lambda_B\ N_0$

$\Rightarrow \quad R_A > R_B$ as $\lambda_A > \lambda_B$

Less half-life of element A implies faster decay

Mean life $\tau = \frac{1}{\lambda}$

$$\tau_A = \frac{1}{\lambda_A}, \tau_B = \frac{1}{\lambda_B}$$

$\tau_A < \tau_B \; as \; \lambda_A > \lambda_B$

7. A glass piece is dipped in a liquid of refractive index $\frac{4}{3}$, it gets disappeared in the liquid. The refractive index of the glass piece is?

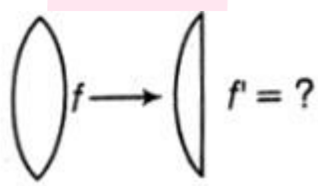
(A) $\frac{3}{4}$ (B) $\frac{5}{3}$ (C) $\frac{4}{5}$ (D) $\frac{4}{3}$

Solution: (D)

The glass piece will disappear only, if the refractive index of the glass and liquid is same.

So, refractive index of glass piece must be $= \frac{4}{3}$.

8.



If the bio-convex lens is cut as shown in the figure, the new focal length f' is

(A) $2f$ (B) f (C) $\frac{f}{2}$ (D) Infinite

Solution: (A)

$$f = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$f = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$f = (\mu - 1)\frac{2}{R}$ ....(i)

and $f' = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$f' = \frac{(\mu-1)}{R}$ ....(ii)

From equations (i) and (ii), we get

$f' = 2f$

9. Refractive index of a medium depends

(A) On the medium only

(B) On the incident light only

(C) On both the conditions given in options (on the medium only) and (on the incident light only)

(D) None of the above

Solution: (C)

Refractive index of a medium depends on the medium as well as on the wavelength of the incident light.

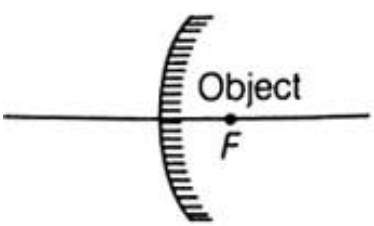
10. A point object is placed at the focus of a convex mirror the image will be formed at

(A) Infinity (B) Centre of curvature

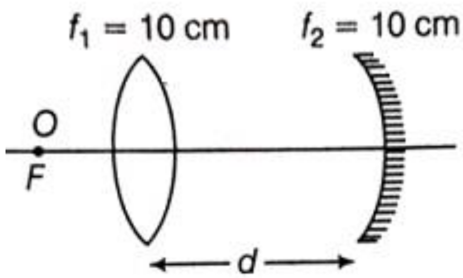
(C) At focus itself (D) None of these

Solution: (D)

Image will not form, because object is placed on the side from where reflection is not possible.

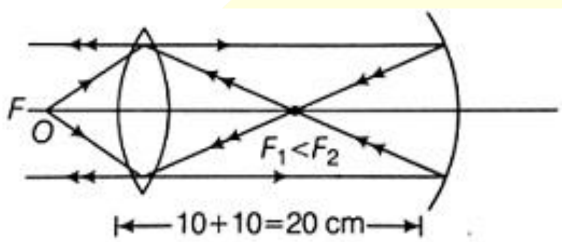


11. A point object is placed at the focus of the bio-convex lens. What should be the value of X, so the final image form at infinity?



(A) 10 cm (B) 20 cm (C) 15 cm (D) None of these

Solution: (B)



The final image will be at infinity only, if the foci of lens and mirror coincides. The situation could be understood on the basis of given diagram.

12. The image formed by a concave spherical mirror,

(A) Is always virtual (B) Is always real

(C) Is always inverted (D) May be erect

Solution: (D)

The image formed by a concave mirror could be real, virtual, erect and inverted.

13. The total energy of a revolving satellite around the earth is $-K\ J$. The minimum energy required to throw it out of earth's gravitational fields is,

(A) $K\ J$ (B) $\frac{K}{2} J$ (C) $2K\ J$ (D) None of these

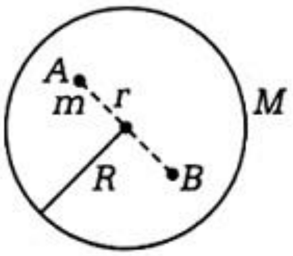
Solution: (A)

If the satellite could be through out side the earth's gravitational field, its minimum total energy = 0.

So, $-K + K' = 0$

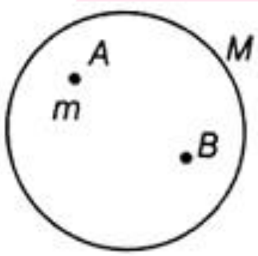
$K' = K\,J$

14. There is a shell of mass M and density of shell is uniform. The work done to take a point mass from point A to B is, $(AB = r)$



(A) $\frac{GmM}{r}$ (B) $\frac{GmM}{R}$ (C) $-\frac{GmM}{r}$ (D) $zero$

Solution: (D)



The gravitational field at A and B are zero. So no work required to make charge between the points A and B.

15. A body of mass $m = 20g$ is attached to an elastic spring of length $L = 50\ cm$ and spring constant $k = 2\ Nm^{-1}$. The system is revolved in a horizontal plane with a frequency $v = 30\ \text{rev/min}$. Find the radius of the circular motion and the tension in the spring.

(A) $0.25\ m, 0.1\ N$ (B) $0.5\ m, 0.52\ N$

(C) $0.55\ m, 0.1\ N$ (D) $0.9\ m, 0.2\ N$

Solution: (C)

Angular velocity $\omega = 2\pi r = 2\pi \times \frac{30}{60} = \pi\, rad/s$

For an elastic spring force $F = k_x$ where x is the extension.

Radius of circular motion $r = L + x$

Centripetal force $= mr\,\omega^2 = F$

$\Rightarrow \quad m(L + x)\omega^2 = Kx$

$\Rightarrow \quad x = \dfrac{mL\omega^2}{k - m\,\omega^2} = \dfrac{0.02 \times 0.5 \times (3.14)^2}{2 - 0.02 \times (3.14)^2}$

$\approx 0.05\ m$

Radius of the circular motion (r)

$= L + x = 0.5 + 0.05$

$= 0.55\ M$

Tension in the spring

$T = kx = 2 \times 0.05\ \approx 0.1\ N$

16. A gramophone record of mass M and radius R is rotating at an angular velocity ω. A coin of mass m is gently placed on the record at a distance $r = \frac{R}{2}$ from its centre. The new angular velocity of the system is

(A) $\frac{2\omega M}{(2M+m)}$ (B) $\frac{2\omega M}{(M+2m)}$ (C) ω (D) $\frac{\omega M}{m}$

Solution: (A)

The initial angular momentum of the rotating record is $L = I\omega$

Where $I = \frac{1}{2}MR62$

Let ω' be the angular velocity of the record when the coin of mass m is placed on it at a distance r from its centre.

The angular momentum of the system becomes

$L' = (I + mr^2)\omega'$

Since, no external torque acts on the system, the angular momentum is conserved i.e.,

$L' = L$ or $(I + mr^2)\omega' = I\omega$

or $\quad \omega' = \frac{I\omega}{I+mr^2} = \frac{\frac{1}{2}MR^2\omega}{\frac{1}{2}Mr^2+mr^2}$

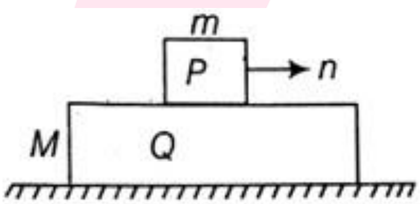
or $\omega' = \frac{\omega}{1+\frac{2mr^2}{MR^2}}$ .....(i)

Putting $r = \frac{R}{2}$ in equation (i), we get

$$\omega' = \left[\frac{\omega}{1 + \frac{2m \times \left(\frac{R}{2}\right)^2}{Mr^2}}\right]$$

$$\Rightarrow \qquad \omega' = \frac{2\omega M}{2M + m}$$

17. A block of mass $m = 1kg$ is placed over a plank Q of mass $M = 6\ kg$, placed over a smooth horizontal surface as shown in figure. Block P is given a velocity $v = 2\ m/s^2$ to the right. If the coefficient of friction between P and Q is $\mu = 0.3$. Find the acceleration of Q relative to P.



(A) $4\,m/s^2$ (B) $3.5\,m/s^2$ (C) $2\,m/s^2$ (D) $10.0\,m/s^2$

Solution: (B)

Frictional force between P and Q is $f = \mu mg$ which will retard P and accelerate Q.

Retardation of P is $a_P = -\frac{f}{m} = \frac{-\mu mg}{m} = -\mu g$

Acceleration of Q is $a_Q = \frac{+f}{M} = \frac{\mu mg}{M}$

Acceleration Q relative to P is

$$a_{QP} = a_Q - a_P = \frac{\mu mg}{M} - (-\mu g)$$

$$= \mu g \left[1 + \frac{m}{M}\right]$$

$$= 0.3 \times 10\ \left[1 + \frac{1}{6}\right]$$

$$= 3.5\ m/s^2$$

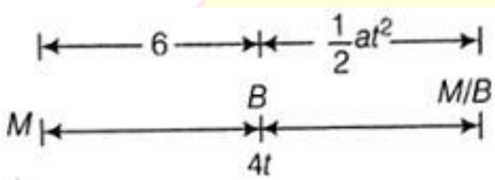
18. A man runs at a speed of $4\,m/s$ to overtake a standing bus. When he is 6 m behind the door at $t = 0$, the bus moves forward and continuous with a constant acceleration of $1.2\ m/s^2$. The man reaches the door in time t. Then

(A) $4t = 6 + 0.6t^2$ (B) $1.2t^2 = 4t$

(C) $4t^2 = 1.2t$ (D) $6 + 4t = 0.2t^2$

Solution: (A)

Let us draw the figure for given situation,



$\Rightarrow \quad 4t = 6 + \frac{1}{2} \times 1.2 \times t^2$

$\Rightarrow \quad 4t = 6 + 0.6\,t^2$

19. In completely inelastic collision

(A) The complete KE of the medium must lost

(B) The linear momentum of the system must remain conserved during collision

(C) Both (the complete KE of the medium must lost) and (the linear momentum of the system must remain conserved during collision) are correct

(D) Both (the complete KE of the medium must lost) and (the linear momentum of the system must remain conserved during collision) are incorrect

Solution: (B)

In any type of collision the linear moments of the system remain conserved even during collision.

20. The number of particles pen unit volume is given by $n = -\frac{Dn_2 - n_1}{x_2 - x_1}$ are crossing a unit area perpendicular to x-axis in unit time, when n_1 and n_2 are the number of particles per unit volume for the values x_1 and x_2 of x respectively. Then the dimensional formula of diffusion constant D is

(A) $[M^0LT^0]$ (B) $[M^0L^2T^{-4}]$

(C) $[M^0LT^{-3}]$ (D) $[M^0L^2T^{-1}]$

Solution: (D)

From the given relation, $D = \frac{n(x_2-x_1)}{n_2-n_1}$

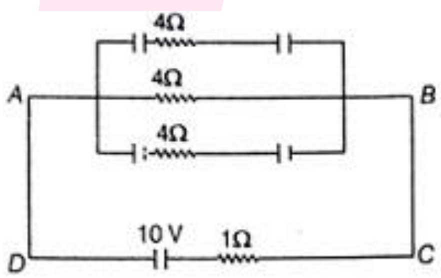
Here, $[n] = \left[\frac{1}{\text{area}\times\text{time}}\right] = \frac{1}{[L^2\,T]} - [L^{-2}\,T^{-1}]$

$x_2 - x_1 = [L]$ and $n_2 - n_1 = \left[\frac{1}{\text{volume}}\right] = \left[\frac{1}{L^3}\right]$

$= [L^{-3}]$

So, $[D] = \frac{[L^{-2}\,T^{-1}\,L]}{[L^{-3}]} = [L^2\,T^{-1}]$

21. In the given circuit (as shown in figure). Each capacitor has a capacity of $3\mu F$. What will be the net charge on each capacitor?



(A) 48 μC (B) 24 μC (C) 12 μC (D) None of these

Solution: (C)

Net resistance between ABCD is $R = 4 + 1 = 5\Omega$

∴ Current $I = \frac{V}{R} = \frac{10}{5} = 2A$

Potential difference across A and B

$= I \times 4 = 2 \times 4 = 8V$

∵ Two capacitors of $3\mu F$ each are in series

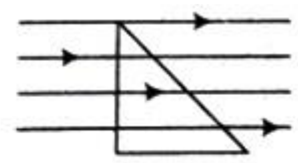
∴ Potential difference across each capacitor $= \frac{8}{2} = 4V$

Charge on each capacitor $q = CV = 3 \times 4$

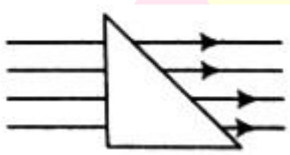
$= 12\ \mu C$

22. A solid conductor is placed in an uniform electric field as shown in figure. Which path will the lines of force follow?

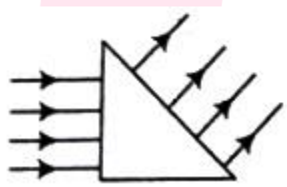
(A)



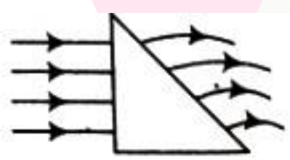
(B)



(C)



(D)



Solution: (C)

The electric field inside a conductor is zero and is always perpendicular to the surface of a conductor.

23. A bomb at rest explodes into three parts of the same mass. The linear momentum of two parts are $-2P\ \hat{i}$ and $P\hat{j}$. The magnitude of momentum of third part is $P\sqrt{x}$. Find x.

(A) P (B) $\sqrt{5}P$ (C) $2P$ (D) $10P$

Solution: (B)

Given, $P_1 = -2P\hat{i} = 2\ P$ along negative x-axis.

$P_2 = P\ \hat{\imath} = P$ along y-axis.

The resultant momentum of two parts

$$P' = \sqrt{P_1^2 + P_2^2} = \sqrt{(2P)^2 + P^2} = P\sqrt{5}$$

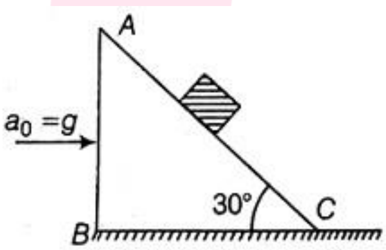
As the bomb was initially at rest final momentum of all the three parts must be zero

$P_3 + P' = 0$

$P_3 = -P' = -P\sqrt{5} = \sqrt{5}P$

$P_3 = \sqrt{5}\ P$

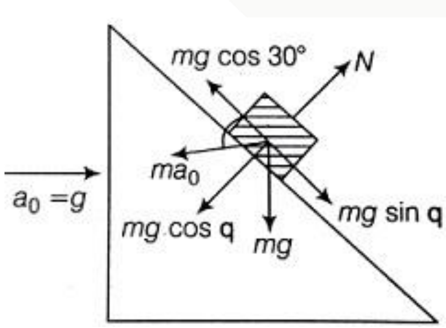
24. Block is placed on an inclined plane. The block is moving towards right horizontally with an acceleration $a_0 = g$. The length of the inclined plane (AC) is equal to $1\ m$. Whole the situation are shown in the figure. Assume that all the surfaces are frictionless. The time taken by the block to reach from C to A is (take $g = 10\ m/s^2$)



(A) $0.74\ s$ (B) $0.9\ s$ (C) $0.52\ s$ (D) $1.24\ s$

Solution: (A)

The forces on smaller block is given as



For the motion of the block along the incline plane in upward direction.

Net force on the block = mass × acceleration of the block

$\Rightarrow \quad mg\cos 30^o - mg\sin 30^o = ma \qquad (\because \quad a_0 = g)$

$\Rightarrow \quad a = \left(\frac{\sqrt{3}-1}{2}\right)g = 3.66\ m/s^2$

Now, from equation of motion $s = \frac{1}{2}\,at^2$

$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 1}{3.66}} = 0.74\ s$

25. Pseudo force is

(A) Electromagnetic in nature

(B) A nuclear force

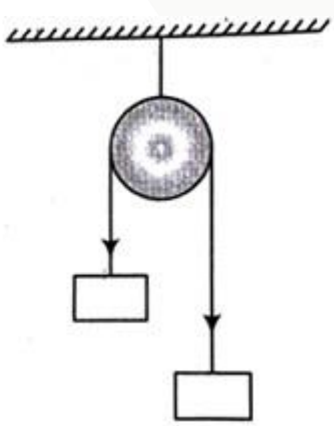
(C) A gravitational force

(D) None of the above

Solution: (D)

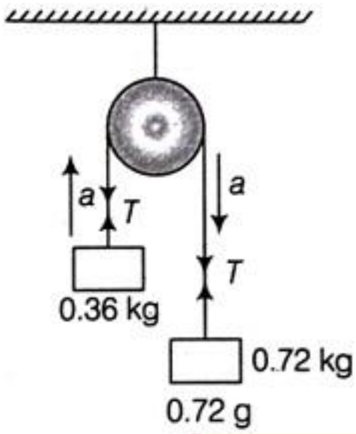
Pseudo force is not a real force.

26. A light in extensible string that goes over a smooth fixed pulley as shown in the figure connect two blocks of masses $0.36kg$ and $0.72kg$. Taking $g = 10\ m/s^2$. Find the work done by string on the block of mass $0.36\ kg$ during the first second after the system is released from rest.



(A) $4\ J$ (B) $2\ J$ (C) $8\ J$ (D) $10\ J$

Solution: (C)



So, acceleration $= \frac{\text{Net pulling force}}{\text{Total mass}}$

$$= \frac{0.72g - 0.36\,g}{0.72 + 0.36} = \frac{g}{3}$$

Distance, $s = ut + \frac{1}{2}at^2$

$$= 0 + \frac{1}{2} \times \frac{g}{3} \times (1)^2 = \frac{g}{6}$$

So, $T - mg = ma$

$T - 0.36g = 0.36\,a$

$T = 0.48\,g$

Now, work done by string on the block $W_T = Ts\cos 0^o$ (on 0.36 kg of mass)

$= (0.48g)\left(\frac{g}{6}\right)(1) = 0.08\,g^2$

$= 0.08 \times (10)^2 = 8\,J$

27. In the equation $A = 3BC^2$, A and C have dimensions of capacitance and magnetic induction respectively. In $MKSQ$ system, the dimensional formula of B is

(A) $[M^{-3}\,L^{-2}\,T^{-2}\,Q^{-4}]$ (B) $[ML^{-2}]$

(C) $[M^{-3}\,L^{-2}\,Q^4\,T^8]$ (D) $[M^{-3}\,L^{-2}\,Q^4\,T^4]$

Solution: (D)

[Capacitance A] = $[M^{-1}L6 - 2\,T^2Q^2]$

[Magnetic induction C] = $[MT^{-1}Q^{-1}]$

$[C]^2 = [M^2T^{-2}Q^{-2}]$

Given, $A = 3BC^2$ or $B = \frac{A}{3C^2} \Rightarrow \quad [B] = \frac{[A]}{[C]^2}$

$$\therefore \quad [B] = \frac{[M^{-1}\, L^{-2}\ T^2Q^2]}{[M^2\, T^{-1}\, Q^{-2}]} = [M^{-3}\, L^{-2}\, T^4\, Q^4]$$

28. Infinite number of masses each $1\ kg$ are placed along the x- axis at $x = \pm 1m, \pm 2m, \pm 4m, \pm 8m, \pm 16m$..... The magnitude of the resultant gravitational potential in terms of gravitational constant G at the origin $(x = 0)$ is

(A) $\frac{G}{2}$ (B) G (C) $2G$ (D) $4G$

Solution: (C)

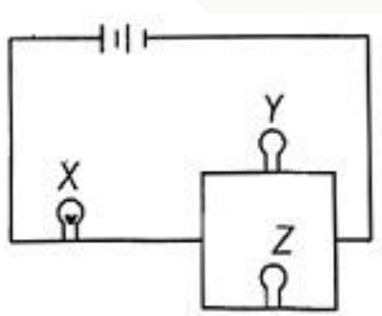
As, $V = GM\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \ldots.\right)$

$$= G \times 1\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right)$$

Sum of $GP = \frac{a}{1-r}$

$$V = G\left(\frac{1}{1-\frac{1}{2}}\right) = 2G$$

29. Three bulbs X, Y and Z are connected as shown in figure. The bulbs Y and Z are identical. If bulb Z gets fused then,



(A) Both X and Y will glow more brightly

(B) Both X and Y will glow less brightly

(C) X will glow less brightly and Y will glow more brightly

(D) X will glow more brightly and Y will glow less brightly

Solution: (C)

If bulb Z is fused, the current stops flowing through Z. The effective resistance of the circuit due to bulbs X and Y in series becomes more as compared to before. Due to which, the current in the circuit decrease.

$\therefore$ brightness $\propto (\text{current})^2$

So, the brightness of bulb $\times$ decreases,

Now, bulb Y gets more current than before fusing the bulb, Z.

$\therefore$ Brightness of bulb Y will increase.

30. Active state of n-p-n transistor, in circuit is achieved by

(A) Low input voltage

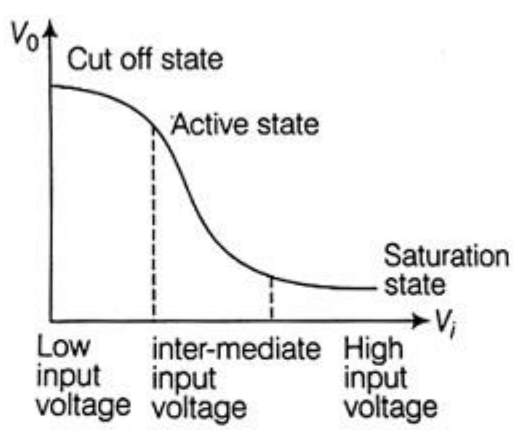
(B) High input voltage

(C) Both (low input voltage) and (high input voltage)

(D) Neither (low input voltage) nor (high input voltage)

Solution: (D)

Transfer characteristic of transistor is



Active state is achieved at intermediate input voltage.

31. A turntable of radius $R = 10\ m$ is rotation making 98 rev in 10 s with a boy of mass $m = 60\ kg$ standing at its centre. He starts running along a radius. Find the frequency of the turntable when the boy is 4m from the centre. The moment of inertia of the turntable about its axis $1000\ kg - m^2$.

(A) $10\ Hz$ (B) $2.5\ Hz$ (C) $5\ Hz$ (D) $4\ Hz$

Solution: (C)

Initial moment of inertia of the system is

$M_1 =$ moment of inertia of turntable + Moment of inertia of boy at the centre

$= 1000 + 0 = 1000\ kgm^2$

Initial frequency $v_1 = 9.8\ \text{rev/s}$

Final moment of the system

$M_2 = MI$ of turntable + MI of boy at a distance 4m from the centre of turntable.

$= 1000 + 60 \times (4)^2$

$= 1960\ kgm^2$

Since no external torque acts, the angular momentum of the system is conserved i.e.,

$I_2\omega_2 = I_1\omega_1$

$\Rightarrow \quad I_2 v_2 = I_1 v_1$

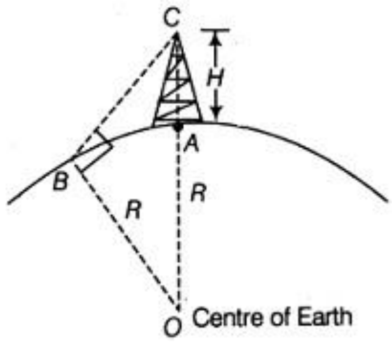
$$\Rightarrow \quad v_2 = \frac{I_1 v_1}{I_2} = \frac{1000 \times 9.8}{1960}$$

$= 5\ \text{rev/s} = 5Hz$

32. To transmit a signal, if height of transmitting signal above surface of the earth is H, this signal can be received on surface of the earth upto distance d from transmitter. Then

(A) $d \propto H$ (B) $d \propto H^2$ (C) $d \propto H^{\frac{1}{2}}$ (D) $d \propto H^{\frac{3}{2}}$

Solution: (C)



$OB^2 + BC^2 = OC^2$

$R^2 + BC^2 = (R + H)^2$

$\Rightarrow \quad BC = \sqrt{2RH + H^2}$

$= \sqrt{RH\left(2 + \frac{H}{R}\right)}$

Here, $\frac{H}{R} << 2$

So, $BC = \sqrt{2RH}$

$BC \approx AB = d =$ distance of reach

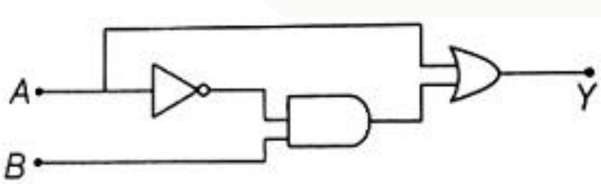
$d = \sqrt{2RH} \propto H^{\frac{1}{2}}$

R = constant

$= 6400\ km$

= radius of the earth

33. The circuit is equivalent to



(A) AND gate (B) OR gate (C) Not gate (D) None of these

Solution: (B)

$Y = A + (\bar{A} \cdot B) = (A + \ \bar{A}) \cdot (A + B)$

$= t \ \cdot (A + B) = A + B$ OR gate

Here, t has value 1 for all inputs.

34. Length of 20 cm (exact) long pipe is measured by two instruments and reported as $19.65\ cm$ and $20.1\ cm$

(A) 19.65 cm is more accurate

(B) Both measurements are equally precise

(C) Both measurements are equally accurate

(D) 20.1 cm is less precise

Solution: (D)

Accuracy is closeness to true/exact value.

Precision is based on instrument, more decimal places in measurement indicate more precision.

So, 20.1 cm is less precise as compared to 19.65 cm, because 20.1 cm has lesser number of decimal places.

35. An electric pump on the ground floor of a building takes 10 min to fill a tank of volume 2000 L with water. If the tank is 40 m above the ground and the efficiency of the pump is 40%, how much electric power is consumed by the pump in filling the tank?

Take $g = 10\ m/s^2$

(A) 2 kW (B) 3.33 kW (C) 4 kW (D) 6 kW

Solution: (B)

Volume of tan $V = 2000\ L$

$= 2000 \times 10^{-3}\ m^3 = 2m^3$

Mass of water $m = \rho_v' = 1000 \times 2$

$= 2 \times 10^3\ kg$

Work done to lift this mass to a height i.e., h = 40 m is

$W = mgh = 2 \times 10^3 \times 10 \times 40 = 8 \times 10^5 J$

Power needed $= \frac{\text{Work done}}{\text{Time taken}} = \frac{8\times10^5}{10\times60}$

$P' = \frac{4}{3} \times 10^3 W$

It P is the total power consumed, the useful power available = 40%, if $P' = 0.4\,P$, then

$= 0.4\,P = \frac{4}{3} \times 10^3$

$\Rightarrow \quad P = 3.33 \times 10^3 W$

$= 3.33\,kW$

36. A vessel containing 1 mole of O_2 gas (molar mass 32) at temperature T. The pressure of the gas is p. An identical vessel containing one mole of He gas (molar mass 4) at temperature. $2T$ has a pressure of

(A) $\frac{p}{8}$ (B) p (C) $2p$ (D) $8p$

Solution: (C)

Applying gas equation $pV = nRT$

We can write; $p_1 V = n_1 R T_1$

and $p_2 V = n_2 R T_2$

$\Rightarrow \quad \frac{p_2}{p_1} = \frac{n_2}{n_1} \times \frac{T_2}{T_1}$

$= \frac{1}{1} \times \frac{2T}{T} = 2$

$\Rightarrow \quad p_2 = 2p$

37. The temperature of an ideal gas is increased from $27^o C$ to $127^o C$, then percentage increase in v_{rms} is

(A) 37% (B) 11% (C) 33% (D) 15.5%

Solution: (D)

We know $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \quad \text{% increase in } v_{rms} = \frac{\sqrt{\frac{3RT_2}{M}} - \sqrt{\frac{3RT_1}{M}}}{\sqrt{\frac{3RT_1}{M}}} \times 100$$

$$= \frac{\sqrt{T_2} - \sqrt{T_1}}{\sqrt{T_1}} \times 100$$

$$= \frac{\sqrt{400} - \sqrt{300}}{\sqrt{300}} \times 100$$

$$= \frac{20 - 17.32}{17.32} \times 100$$

$$= 15.5\%$$

38. A particle of mass $m = 5g$ is executing simple harmonic motion with an amplitude $0.3\ m$ and time period $\frac{\pi}{5}\ sec$. The maximum value of force acting on the particle is

(A) $5\ N$ (B) $4\ N$ (C) $0.5\ N$ (D) $0.15\ N$

Solution: (D)

We know,

Maximum acceleration $a_{max} = \omega^2 A = \frac{4\pi^2}{T^2}\ A$

$$= \frac{4\pi^2}{\left(\frac{\pi}{5}\right)^2} \times 0.3 = 30\ m/s^2$$

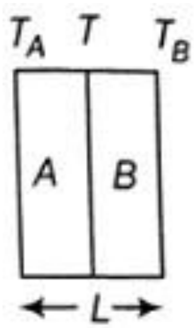
Maximum force

$$F_{max} = ma_{max} = \frac{5}{1000} \times 30 = 0.15\ N$$

39. A partition wall has two layers of different materials A and B in contact with each other. They have the same thickness but the thermal conductivity of layer A is twice that of B. At steady state if the temperature difference across the layer B is 50 K, then the corresponding difference across the layer A is

(A) 50 K (B) 12.5 K (C) 25 K (D) 60 K

Solution: (C)



Let T be the junction temperature

Here, $K_A = 2K_B, T - T_B = 50K$

At the steady state $H_A = H_B$

$$\Rightarrow \quad \frac{K_A A(T_A - T)}{L} = \frac{K_B A\,(T - T_B)}{L}$$

$$\Rightarrow \quad 2K_B\,(T_A - T) = K_B\,(T - T_B)$$

$$\Rightarrow \quad T_A - T = \frac{T - T_B}{2} = \frac{50}{2} = 25\ K$$

40. Pulse rate of a normal person is 75 per min. The time period of heart is

(A) $0.8\ s$ (B) $0.75\ s$ (C) $1.25\ s$ (D) $1.75\ s$

Solution: (A)

The beat frequency of heart is

$$v = \frac{75}{(1\ min)} = \frac{75}{60\ s} = 1.25\ s^{-1}$$

$$= 1.25\ Hz$$

The time period of heart is

$$T = \frac{1}{v} = \frac{1}{1.25\ s^{-1}} = 0.8\ s$$