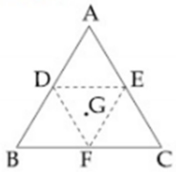
QUESTION PAPER

Physics

1. An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:



(A) $I = \frac{3}{4} I_0$
(B) $I = \frac{15}{16} I_0$
(C) $I = \frac{9}{16} I_0$
(D) $I = \frac{I_0}{4}$

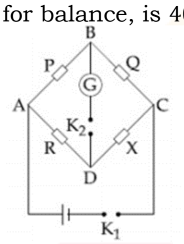
2. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength $980\,\overset{0}{A}$. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be: $\left(hc = 12500\, eV - \overset{0}{A}\right)$
(A) $25\, a_0$
(B) $9\, a_0$
(C) $16\, a_0$
(D) $4\, a_0$

3. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfacing waves is $\frac{1}{8}^{th}$ of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to:
(A) 0.94
(B) 0.80
(C) 0.74
(D) 0.85

4. A particle undergoing simple harmonic motion has tie dependent displacement given by $x(t) = A \sin \frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210\, s$ will be:
(A) $\frac{1}{3}$
(B) $\frac{1}{9}$
(C) 2
(D) 1

5. A body is projected at $t = 0$ with a velocity $10\, ms^{-1}$ at an angle of $60°$ with the horizontal. The radius of curvature of its trajectory at $t = 1s$ is R. Neglecting air resistance and taking acceleration due to gravity $g = 10ms^{-2}$, the value of R is:
(A) $10.3\, m$
(B) $2.8\, m$
(C) $5.1\, m$
(D) $2.5\, m$

6. In a Wheatstone bridge (see figure), Resistance P and Q are approximately equal. When $R = 400\, \Omega$, the bridge is balanced. On interchanging P and Q, the value of R, for balance, is $405\, \Omega$. The value of X is close to:

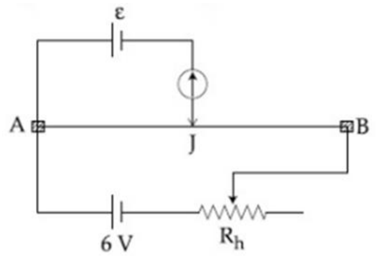


(A) $404.5\, ohm$
(B) $401.5\, ohm$
(C) $403.5\, ohm$
(D) $402.5\, ohm$

7. An object is at a distance of $20\, m$ from a convex lens of focal length $0.3\, m$. The lens forms an image of the object. If the object moves away from the lens at a speed of $5\ m/s$, the speed and direction of the image will be:
(A) $2.26 \times 10^{-3}\, m/s$ away from the lens
(B) $1.16 \times 10^{-3}\ m/s$ towards the lens
(C) $0.92 \times 10^{-3}\ m/s$ away from the lens
(D) $3.22 \times 10^{-3}\ m/s$ towards the lens

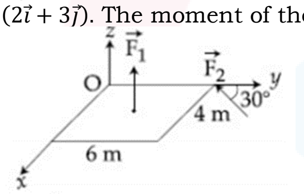
8. If the deBroglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency $6 \times 10^{14} Hz$, then the speed of electron is equal to: (Speed of light $= 3 \times 10^8 m/s$, Plank's constant $= 6.63 \times 10^{-34} J.s$, Mass of electron $= 9.1 \times 10^{-31} kg$)
(A) $1.45 \times 10^6\ m/s$
(B) $1.1 \times 10^6\ m/s$
(C) $1.7 \times 10^6\ m/s$
(D) $1.8 \times 10^6\ m/s$

9. The resistance of the meter bridge AB in given figure is $4\, \Omega$. With a cell of emf $\varepsilon = 0.5\, V$ and rheostat resistance $R_h = 2\Omega$ the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for $R_h = 6\, \Omega$. The emf ε_2 is:
Note: the meter bridge should be read as the potentiometer. In actual paper it is wrongly stated.



(A) 0.5 V
(B) 0.6 V
(C) 0.4 V
(D) 0.3 V

10. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY −plane while force F_1 acts along z −axis at the point $(2\vec{i} + 3\vec{j})$. The moment of these forces about point O will be:



(A) $(3\hat{\imath} + 2\hat{\jmath} + 3\hat{k})F$
(B) $(3\hat{\imath} - 2\hat{\jmath} + 3\hat{k})F$
(C) $(3\hat{\imath} - 2\hat{\jmath} - 3\hat{k})F$
(D) $(3\hat{\imath} + 2\hat{\jmath} - 3\hat{k})F$

11. An amplitude modulated signal is given by $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]\sin(5.5 \times 10^5 t)$. Here t is in seconds. The sideband frequencies (in kHz) are, $\left[\text{Given } \pi = \frac{22}{7}\right]$
(A) 178.5 and 171.5
(B) 892.5 and 857.5
(C) 89.25 and 85.75
(D) 1785 and 1715

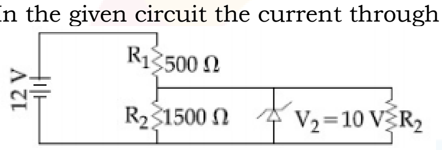
12. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is:
(A) 12 RT
(B) 15 RT
(C) 20 RT
(D) 4 RT

13. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be:
(A) $\frac{1}{2}\rho v^2$

(B) ρv^2
(C) $\frac{3}{4}\rho v^2$
(D) $\frac{1}{4}\rho v^2$

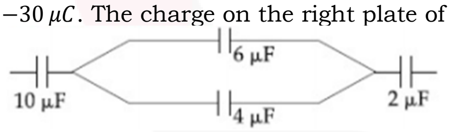
14. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is
(A) $M^0L^2T^{-4}$
(B) $M^2L^2T^{-2}$
(C) M^2LT^{-4}
(D) MLT^{-2}

15. In the given circuit the current through Zener Diode is close to:



(A) 6.0 mA
(B) 0.0 mA
(C) 6.7 mA
(D) 4.0 mA

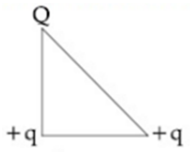
16. In the figure shown below, the charge on the left plate of the $10\,\mu F$ capacitor is $-30\,\mu C$. The charge on the right plate of the $6\,\mu F$ capacitor is:



(A) $+18\,\mu C$
(B) $+12\,\mu C$
(C) $-18\,\mu C$
(D) $-12\,\mu C$

17. Ice at $-20°C$ is added to $50\,g$ of water at $40°C$. When the temperature of the mixture reaches $0°C$, it is found that $20\,g$ of ice is still unmelted. The amount of ice added to the water was close to (specific heat of water $= 4.2\,J/g\,°C$, Specific heat of Ice $=$ $2.1\,J/g\,°C$, Heat of fusion of water at $0°C = 334\,J/g$
(A) 40 g
(B) 100 g
(C) 60 g
(D) 50 g

18. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is:



(A) $+q$

(B) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

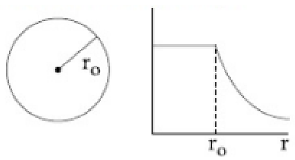
(C) $\frac{-q}{1+\sqrt{2}}$

(D) $-2q$

19. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x =constant, then x is:
(A) $\frac{2}{5}$
(B) $\frac{2}{3}$
(C) $\frac{5}{3}$
(D) $\frac{3}{5}$

20. There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance of the self inductance of the inner-coil is:
(A) $\frac{n_2}{n_1}.\frac{r_2^2}{r_1^2}$
(B) $\frac{n_2}{n_1}.\frac{r_1}{r_2}$
(C) $\frac{n_2}{n_1}$
(D) $\frac{n_1}{n_2}$

21. The given graph shows variation (with distance r from centre) of:

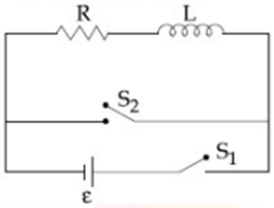


(A) Electric field of a uniformly charged spherical shell
(B) Electric field of a uniformly charged
(C) Potential of a uniformly charged spherical shell
(D) Potential of a uniformly charged sphere

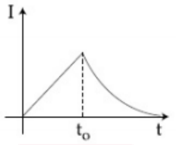
22. A particle is moving along a circular path with a constant speed of $10\, ms^{-1}$. What of the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(A) zero
(B) $10\sqrt{3}\,m/s$
(C) $10\sqrt{2}\,m/s$
(D) $10\ m/s$

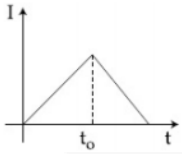
23. In the circuit shown,



The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behavior of the current I as a function of time 't' is given by:



(A)



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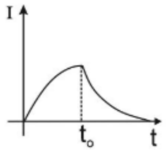
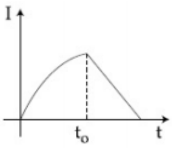


(C)



(D)

24. An electromagnetic wave of intensity $50\,Wm^{-2}$ enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by:
(A) $\left(\sqrt{n}, \sqrt{n}\right)$
(B) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$
(C) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

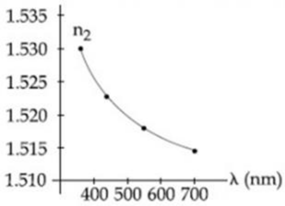
(D) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$

25. A body of mass $1\,kg$ falls freely from a height of $100\,m$, on a platform of mass $3\,kg$ which is mounted on a spring having spring constant $k = 1.25 \times 10^6\,N/m$. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\,ms^{-2}$, the value of x will be close to:
(A) $2\,cm$
(B) $40\,cm$
(C) $4\,cm$
(D) $80\,cm$

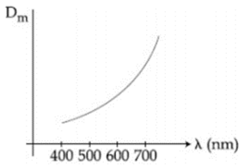
26. Equation of travelling wave on s stretched string of linear density $5\ g/m$ is $y = 0.03\sin(450t - 9x)$ where distance and time are measured in SI units. The tension in the string is:
(A) $7.5\,N$
(B) $10\,N$
(C) $5\,N$
(D) $12.5\,N$

27. Two equal resistances when connected in series to a battery, consume electric power of $60\,W$. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be:
(A) $30\,W$
(B) $60\,W$
(C) $120\,W$
(D) $240\,W$

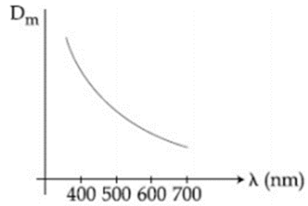
28. The variation of refractive index of a crown glass thin prism with wavelength of incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



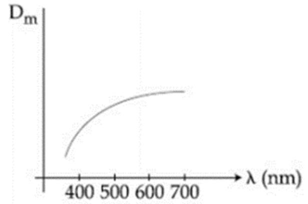
(A)



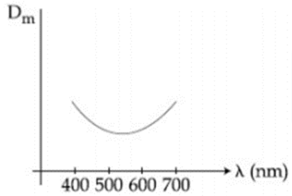
(B)



(C)



(D)

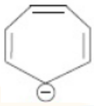


29. A satellite is revolving in a circular orbit at a height h from the earth surface, such that $h << R$ where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is:
(A) $\sqrt{2gR}$
(B) $\sqrt{\frac{gR}{2}}$
(C) $\sqrt{gR}(\sqrt{2}-1)$
(D) $\sqrt{gR}$

30. In an experiment, electrons are accelerated from rest, by applying a voltage of $500\,V$. Calculate the radius of the path if a magnetic field $100\,mT$ is then applied. [Charge of the electron $= 1.6 \times 10^{-19} C$, Mass of the electron $= 9.1 \times 10^{-31} kg$]
(A) $7.5 \times 10^{-4} m$
(B) $7.5\ m$
(C) $7.5 \times 10^{-2} m$
(D) $7.5 \times 10^{-3} m$

Chemistry

1. Which compound (s) out of the following is/are not aromatic?

(A)  (B)  (C)  (D) 

(A) (B)

(B) (A) and (C)

(C) $(B), (C)$and (D)

(D) (C) and (D)

2. Peroxyacetyl nitrate (PAN), an eye irritant, is produced by:

(A) Classical smog

(B) Acid rain

(C) Photochemical smog

(D) Organic waste

3. The concentration of dissolved oxygen (DO) in cold water can go up to:

(A) 10 ppm

(B) 8 ppm

(C) 14 ppm

(D) 16 ppm

4. Match the following

(A)	H_2O + Sugar	(P)	Sublimation
(B)	H_2O +Aniline	(Q)	Recrystallization
(C)	H_2O +Toluene	(R)	Steam distillation
		(S)	Differential extraction

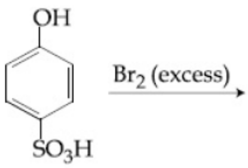
(A) $(A) \rightarrow (Q); (B) \rightarrow (R); (C) \rightarrow (S)$

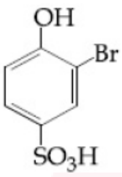
(B) $(A) \rightarrow (S); (B) \rightarrow (R); (C) \rightarrow (P)$

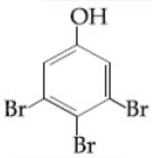
(C) $(A) \rightarrow (Q); (B) \rightarrow (R); (C) \rightarrow (P)$

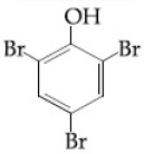
(D) $(A) \rightarrow (R); (B) \rightarrow (P); (C) \rightarrow (S)$

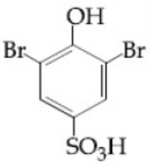
5. The major product of the following reaction is:



(A) 

(B) 

(C) 

(D) 

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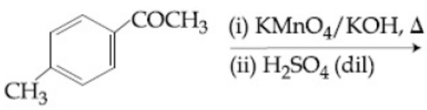
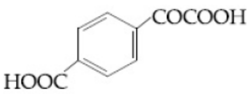
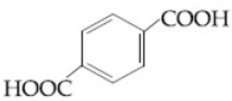
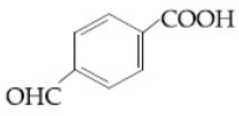
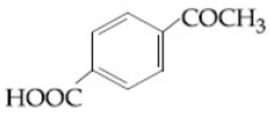


(A) 

(B) 

(C) 

(D) 

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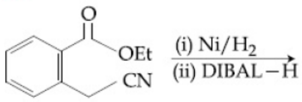
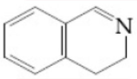
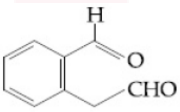
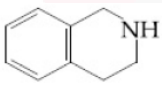
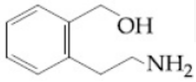


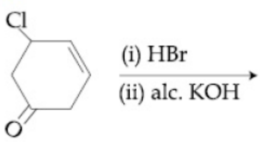
(A) 

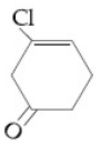
(B) 

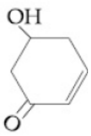
(C) 

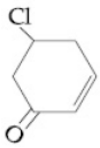
(D) 

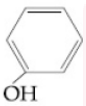
8. The major product of the following reaction is:



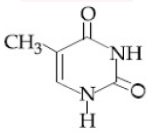
(A) 

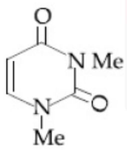
(B) 

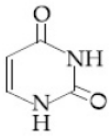
(C) 

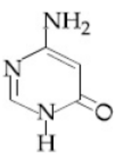
(D) 

9. Among the following compounds, which one is found in RNA?

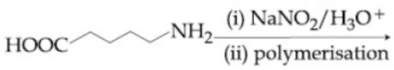
(A) 

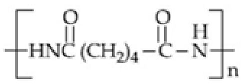
(B) 

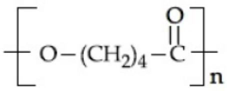
(C) 

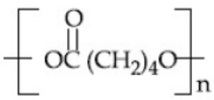
(D) 

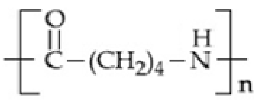
10. The polymer obtained from the following



(A) 

(B) 

(C) 

(D) 

Commented [UK1]: Put H beside N in the extreme right N

Commented [UK2]: Put H beside N in the extreme right N

11. The correct matches between item (I) and item (II) are:

(A)	Norethindrone	(P)	Anti-biotic
(B)	Ofloxacin	(Q)	Anti-fertility
(C)	Equanil	(R)	Hypertension
		(S)	Analgesic

(A) $(A) \rightarrow (R); (B) \rightarrow (P); (C) \rightarrow (S)$

(B) $(A) \rightarrow (R); (B) \rightarrow (P); (C) \rightarrow (R)$

(C) $(A) \rightarrow (Q); (B) \rightarrow (P); (C) \rightarrow (R)$

(D) $(A) \rightarrow (Q); (B) \rightarrow (R); (C) \rightarrow (S)$

12. The element that usually does NOT show variable oxidation state is:

(A) Sc

(B) Ti

(C) V

(D) Cu

13. If a reaction follows the Arrhenius equation, the plot lnk vs $\frac{1}{(RT)}$ gives straight line with a gradient (-y) unit. The energy required to activate the reactant is:

(A) $-y$ unit

(B) y unit

(C) $\frac{y}{R}$ unit

(D) yR unit

14. The chloride that CANNOT get hydrolyzed is:

(A) $SiCl_4$

(B) CCl_4

(C) $SnCl_4$

(D) $PbCl_4$

15. An example of solid is:

(A) Butter

(B) Gem stone

(C) Paint

(D) Hair cream

16. A solid having density of $9 \times 10^3\ kg\ m^{-3}$ forms face centered cubic crystals of edge length $200\sqrt{2}pm$. What is the molar mass of the solid?

[Avogadro constant $\cong 6 \times 10^{23} mol^{-1}, \pi \cong 3$]

(A) $0.0305\ kg\ mol^{-1}$

(B) $0.0216\ kg\ mol^{-1}$

(C) $0.0432\ kg\ mol^{-1}$

(D) $0.4320\ kg\ mol^{-1}$

17. Consider the reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$. The equilibrium constant of the above reaction is K_p. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by (Assume that $p_{NH_3} << p_{\text{total}}$ sequilibrium)

Commented [UK3]: Reversible sign not appearing in reaction

(A) $\frac{K_P^{\frac{1}{2}} p^2}{4}$

(B) $\frac{3^{3/2}K_P^{\frac{1}{2}}p^2}{4}$

(C) $\frac{K_P^{\frac{1}{2}}p^2}{16}$

(D) $\frac{3^{3/2}K_P^{\frac{1}{2}}p^2}{16}$

18. NaH is an example of

(A) Saline hydride

(B) Molecular hydride

(C) Electron-rich hydride

(D) Metallic hydride

19. The amphoteric hydroxide is:

(A) $Mg(OH)_2$

(B) $Be(OH)_2$

(C) $Ca(OH)_2$

(D) $Sr(OH)_2$

20. Two block of the same metal having same mass and at temperature T_1 and T_2 respectively, are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is:

(A) $2C_p ln\left(\frac{T_1+T_2}{4T_1T_2}\right)$

(B) $C_p ln\left(\frac{(T_1+T_2)^2}{4T_1T_2}\right)$

(C) $2C_p ln\left(\frac{T_1+T_2}{2T_1T_2}\right)$

(D) $2C_p ln\left(\frac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right)$

21. Match the (column A) with the metals (column-B):

	Column-A		Column-B
	Ores		Metals
(I)	Siderite	(a)	Zinc
(II)	Kaolinite	(b)	Copper
(III)	Malachite	(c)	Iron
(IV)	Calamine	(d)	Aluminium

(A) (I) – (c); (II) – (d); (III) – (b); (IV) – (a)

(B) (I) – (c); (II) – (d); (III) – (a); (IV) – (b)

(C) (I) – (a); (II) – (b); (III) – (c); (IV) – (d)

(D) (I) – (b); (II) – (c); (III) – (d); (IV) – (a)

22. The correct order of the atomic radii of C, Cs, Al, and S is:

(A) $S < C < Cs < Al$

(B) $S < C < Al < Cs$

(C) $C < S < Cs < Al$

(D) $C < S < Al < Cs$

23. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H-atom is suitable for this purpose?

$[R_H = 1 \times 10^5\ cm^{-1}, h = 6.6 \times 10^{-34}\ Js, c = 3 \times 10^8\ ms^{-1}]$

(A) Paschen, $\infty \to 3$

(B) Paschen, $5 \to 3$

(C) Balmer, $\infty \to 2$

(D) Lyman, $\infty \to 1$

24. The freezing point of a diluted milk sample is found to $-0.2°C$, while it should have been $-0.5°C$, for pure milk. How much water has been added to pure milk to make the diluted sample?

(A) 3 Cups of water to 2 cups of pure milk

(B) 1 cup of water to 3 cups of pure milk

(C) 1 cup of water to 2 cups of pure milk

(D) 2 cups of water to 3 cups of pure milk

25. A 10 mg effervescent tablet containing acid releases 0.25 ml of CO_2 at $T = 298.15\ K$ and $p = 1$ bar. If molar volume of CO_2 is 25.0 L under such condition, what is the percentage of sodium bicarbonate in each tablet?

[Molar mass of $NaHCO_3 = 84 g\ mol^{-1}$]

(A) 16.8%

(B) 33.6%

(C) 0.84%

(D) 8.4%

26. For the cell $Zn(s)|Zn^{2+}(aq)||M^{x+}(aq)|M(s)$, different half cells and their standard electrode potential are given below:

$M^{x+}(aq)/M(s)$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}(aq)/Ag(s)$	$Fe^{3+}(aq)/Fe^{2}(aq)$	$Fe^{2}(aq)/Fe(s)$
$E°M^{x+}/M^{(V)}$	1.40	0.80	0.77	-0.44

If $E°_{Zn^{2+}/Zn} = -0.76$, which cathode will give a maximum value of $E°_{cell}$ per electron transferred?

(A) Fe^{2+}/Fe

(B) Fe^{3+}/Fe^{2+}

(C) Ag^{+}/Ag

(D) Au^{3+}/Au

27. For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as $\Delta_r G°(in\ kJ\ mol^{-1}) = 120 - \frac{3}{8}T$. The major component of the reaction mixture at T is

(A) Y if $T = 300\ K$

(B) Y if $T = 280\ K$

(C) X if $T = 315\ K$

(D) X if $T = 350\ K$

28. An organic compound is estimated through Dumus method and was found to evolve 6 moles of CO_2, 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is:

(A) C_6H_8N

(B) $C_6H_8N_2$

(C) $C_{12}H_8N_2$

(D) $C_{12}H_8N$

29. Match the following

	Column - I		Column - II
(A)	Co	(i)	Wilkinson
(B)	Zn	(ii)	Chlorophyll
(C)	Rh	(iii)	Vitamin B-12
(D)	Mg	(iv)	Carbonic anhydrate

(A) $(\mathrm{A}-\mathrm{iii}),(\mathrm{B}-\mathrm{iv}),(\mathrm{C}-\mathrm{i}),(\mathrm{D}-\mathrm{ii})$

(B) $(\mathrm{A}-\mathrm{i}),(\mathrm{B}-\mathrm{ii}),(\mathrm{C}-\mathrm{iii}),(\mathrm{D}-\mathrm{iv})$

(C) $(\mathrm{A}-\mathrm{ii}),(\mathrm{B}-\mathrm{i}),(\mathrm{C}-\mathrm{iv}),(\mathrm{D}-\mathrm{iii})$

(D) $(\mathrm{A}-\mathrm{iv}),(\mathrm{B}-\mathrm{iii}),(\mathrm{C}-\mathrm{i}),(\mathrm{D}-\mathrm{ii})$

30. The correct statements among (a) to (d) regarding H_2 as a fuel are:

(a) It produces less pollutants than petrol.

(b) A cylinder of compressed dihydrogen weighs ~ 30 times more than a petrol tank producing the same amount of energy

(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$

(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively

(A) (a) and (C) only

(B) (b), (c) and (d)only

(C) (a), (b) and (c) only

(D) (b) and (d) only

Mathematics

1. The value of r for which ${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots.. + {}^{20}C_0\,{}^{20}C_r$ is maximum, is:

(A) 20

(B) 10

(C) 15

(D) 11

2. Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ $(i = \sqrt{-1})$, where x and y are real numbers, then $y - x$ equals:

(A) 91

(B) -91

(C) -85

(D) 85

3. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$ (where $[x]$ denotes the greatest integer less than or equal to x) is:

(A) $\sin 4$

(B) $4 - \sin 4$

(C) 4

(D) 0

4. let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1,2,3, \dots$. Then for $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to:

(A) $\frac{-1}{12}$

(B) $\frac{1}{12}$

(C) $\frac{1}{4}$

(D) $\frac{5}{12}$

5. Let $f: R \to R$, be defined $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is

(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) $R - [-1,1]$

(C) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$

(D) $(-1,1) - \{0\}$

6. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of the series is:

(A) $\frac{2}{3}$

(B) $\frac{4}{9}$

(C) $\frac{1}{3}$

(D) $\frac{2}{9}$

7. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(A) -300

(B 100

(C) 144

(D) -81

8. Two circles with equal radii are intersecting at the points $(0, 1)$ and $(0, -1)$. The tangent at the point $(0, 1)$ to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is

(A) 2

(B) 1

(C) $2\sqrt{2}$

(D) $\sqrt{2}$

9. Let $a_1, a_2, \ldots\ldots a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals:

(A) 5^3

(B) 5^4

(C) $2(5^2)$

(D) $4(5^2)$

10. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x).\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals:

(A) $\frac{-1}{27x^9}$

(B) $\frac{-1}{3x^3}$

(C) $\frac{1}{9x^4}$

(D) $\frac{1}{27x^6}$

11. Equation of common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is

(A) $x - 2y + 4 = 0$

(B) $x + y + 1 = 0$

(C) $4x + 2y + 1 = 0$

(D) $x + 2y + 4 = 0$

12. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve:

(A) $\frac{x^2}{2} + \frac{y^2}{4} = 1$

(B) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(C) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

(D) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

13. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is

(A) $\sqrt{137}$

(B) 6

(C) $\sqrt{41}$

(D) 13

14. Let $[x]$ denote the greatest integer less than or equal to x. Then $\lim_{x\to 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$:

(A) does not exist

(B) equals 0

(C) equals $\pi + 1$

(D) equals π

15. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equal

(A) $\frac{\sqrt{5}}{2}$

(B) $\sqrt{2}$

(C) 2

(D) $\frac{2}{3}$

16. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ equals 5670 is

(A) 6

(B) 4

(C) 0

(D) 8

17. Two integers are selected at random from the set {1, 2,....11}. Given that the sum of selected number is even, the conditional probability that both the number are even is:

(A) $\frac{2}{5}$

(B) $\frac{7}{10}$

(C) $\frac{1}{2}$

(D) $\frac{3}{5}$

18. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(A) $\frac{3}{4}$

(B) $\frac{9}{8}$

(C) $\frac{7}{8}$

(D) $\frac{5}{4}$

19. The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is:

(A) $\frac{\sqrt{5}}{2}$

(B) $\frac{\sqrt{5}}{4}$

(C) $4\sqrt{5}$

(D) $2\sqrt{5}$

20. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

(A) $p \vee r$

(B) $p \wedge r$

(C) $(p \wedge r) \rightarrow (p \vee r)$

(D) $(p \vee r) \rightarrow (p \wedge r)$

21. If $x\log_e(\log_e x) - x^2 + y^2 = 4$ $(y > 0)$, then $\frac{dy}{dx}$ at $x = e$ is equal to:

(A) $\frac{e}{\sqrt{4+e^2}}$

(B) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(C) $\frac{(1+2e)}{\sqrt{4+e^2}}$

(D) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

22. Let $A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$. If $AA^T = I_3$, then $|p|$ is:

(A) $\frac{1}{\sqrt{6}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{1}{\sqrt{3}}$

23. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is

(A) $\frac{y}{\sqrt{3}}$

(B) $\frac{c}{3}$

(C) $\frac{c}{\sqrt{3}}$

(D) $\frac{3}{2}y$

24. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, x > 0$ where $y(1) = \frac{1}{2}e^{-2}$, then:

(A) $y(\log_e 2) = \log_e 4$

(B) $y(\log_e 2) = \frac{\log_e 2}{4}$

(C) $y(x)$ is decreasing in $(\frac{1}{2}, 1)$

(D) $y(x)$ is decreasing in $(0,1)$

25. Let $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 \leq x \leq 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2), g$ is:

(A) differentiable at all point

(B) not differentiable at two points

(C) not continuous

(D) not differential at one point

26. Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 4\hat{k}$, $\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + 4\hat{k}$ and $\vec{c} = 2\hat{\imath} + 4\hat{\jmath} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is:

(A) $-14\hat{\imath} + 5\hat{\jmath}$

(B) $-14\hat{\imath} - 5\hat{\jmath}$

(C) $-10\hat{\imath} - 5\hat{\jmath}$

(D) $-10\hat{\imath} + 5\hat{\jmath}$

27. The direction ratios of normal to the plane through points $(0, -1, 0)$ and $(0,0,1)$ and making an angle $\frac{\pi}{4}$ with

the plane $y - z + 5 = 0$ are:

Note: In actual JEE paper two options were correct. So we have changed one option.

(A) $2\sqrt{3}, 1, -1$

(B) $2, -1, 1$

(C) $\sqrt{2}, 1, -1$

(D) $2, -\sqrt{2}, -\sqrt{2}$

28. If the system of linear equations $2x + 2y + 3z = a, 3x - y + 5z = b, x - 3y + 2z = c$ where a, b, c are non zero real numbers, has more than one solution, then:

(A) $b - c - a = 0$

(B) $a + b + c = 0$

(C) $b + c - a = 0$

(D) $b - c + a = 0$

29. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R: x^2 + 30 \leq 11x\}$ is

(A) 122

(B) 222

(C) -122

(D) -222

30. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which of the following points?

(A) $(0, -2, 2)$

(B) $(2, 0, -2)$

(C) $(2, 2, 0)$

(D) $(-2, 2, 2)$