

MODEL TEST PAPER-IV

Time: 3 hours

Maximum Marks: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) This questions paper contains 29 questions.
- (iii) Questions nos. 1-4 in section A are very short answer type questions carrying 1 mark each
- (iv) Question nos. 5 -12 in section B are short answer type questions carrying 2 marks each
- (v) Questions non. 13 -23 in section C are long answer-I type questions carrying 4 marks each
- (vi) Question 24 -29 in section D are long answer –II type questions carrying 6 marks each.

SECTION A

1. Write the domain and range of the real function $f(x) = -|x|$.
2. What is the real value of a for which
$$3i^3 - 2ai^2 + (1 - a)i + 5$$
is real
3. Write the intercepts of line $2x - 3y = 7$ on coordinate axes. 4.
Write the converse of the following statements
'I go to a beach whenever it is a sunny day'.

SECTION B

5. Draw the Venn diagram to illustrate 'All the students who study Mathematics study English but some students who study English do not study Mathematics'. If E is the set of students studying English in a school, M is the set of students studying Mathematics in the same school, U is the set of all students in that school.
6. Prove that

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \left(\frac{x-y}{2} \right).$$

7. If $x + iy = \frac{(a^2+1)^2}{2a-i}$, what is the value of $x^2 + y^2$?
8. Find the sum of all two digit numbers which when divided by 4, yield 1 as remainder.
9. If $|x| < 1$ and $y = x + x^2 + x^3 + \dots \infty$ show that :

$$X = \frac{y}{1+y}$$

10. $\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}$ when a, b, $a + b \neq 0$.
11. (i) Write the contra positive of the following conditional statements :

'If my grandmother had wheels, then she would be a bus'.

(iii) Write the negation of the following statements :

'Australia is a continent'.

12. Find the probability of having exactly one girl in a family of three children.
13. Let $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$, $C = \{4, 5, 6, 7\}$ Verify the identity :

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

14. If $2f(x) - 3f\left(\frac{1}{x}\right) = x^2$, $x \neq 0$, then find $f(2)$.

15. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, find the value of :

$$\sin \frac{x}{2}, \cos \frac{x}{2} \text{ and } \tan \frac{x}{2}$$

16. Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$.

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

17. Find the square root of $16 - 30i$.

18. Solve the following linear inequalities graphically :

$$3x + 4y \leq 60$$

$$X + 3y \leq 30$$

$$X \geq 0, y \geq 0.$$

19. A line is such that its segment between the lines

$$5x - y + 4 = 0 \text{ and } 3x + 4y - 4 = 0$$

is bisected at the point $(1, 5)$. Obtain its equation.

OR

Find the image of the point $(3, 8)$ with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

20. Find the equation of the ellipse having axes along the coordinate axes and passing through the points $(4, 3)$ and $(-1, 4)$.

21. Determine the values of a and b so that the points $(a, b, 3)$, $(2, 0, -1)$ and $(1, -1, -3)$ are collinear.

22. Find the derivative of $\frac{x \sin x}{1 + \cos x}$ with respect to x .

OR

Find the derivative of $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ with respect to x .

23. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75. What is the probability of passing Hindi examination?

SECTION C

24. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed :

(i) In English and Mathematics but not in Science?

(ii) in more than one subject only?

25. Prove that :

$$\cos^3 A + \cos^3 (120^\circ + A) + \cos^3 (240^\circ + A) = \frac{3}{4} \cos 3A.$$

OR

If $x \cos \theta = y \cos \left(\theta + \frac{2\pi}{3} \right) = z \cos \left(\theta + \frac{4\pi}{3} \right)$ prove that :

$$xy + yz + zx = 0.$$

26. A committee of 10 is to be formed from 8 gentlemen and 8 ladies. In how many ways this can be done if at least five ladies have to be included? In how many of these committees :

(i) The ladies are in majority?

(ii) The gentlemen are in majority?

27. The second, third and fourth terms in the binomial expansion $(x + a)^n$ are 240, 720 and 1080 respectively. Find x, a and n.

OR

Show that the middle terms in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$ is

$$\frac{1.3.5. \dots(2n-1)}{n!} 2^n$$

28. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that :

$$(q + p) : (q - p) = 17 : 15.$$

OR

Let S be the sum, P the product and R the sum of reciprocals of n terms is a G.P. Prove that :

$$P = \left(\frac{S}{R}\right)^n$$

29. Calculate mean deviation about median for the following data :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	16	4	2