QUESTION PAPER

Mathematics

1. For the event to be completed is 5^{th} throw, 4^{th} and 5^{th} throw must be 4. Also 3^{rd} throw must be other cases are

$= 4\bar{4}\bar{4}44 + \bar{4}4\bar{4}44 + \bar{4}\bar{4}\bar{4}44$

$= \frac{1}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6}.\frac{1}{6} + \frac{5}{6}.\frac{1}{6}.\frac{5}{6}.\frac{1}{6}.\frac{1}{6} + \frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6}.\frac{1}{6}$

$= \frac{175}{6^5}$

Hence (B) is the correct answer

2. Let the observations be $x_1, x_2, \ldots, x_{50}$

Given $(x_1 - 30) + (x_2 - 30) + \ldots.. + (x_{50-30}) = 50$

$\Rightarrow x_1 + x_2 + \ldots.. + x_{50} = 1550$

Now Mean $= \frac{x_1 + x_2 + \ldots + x_{50}}{50} = \frac{1550}{50} = 31$

3. Given $I = \int \cos(\log_e^x)dx$

Let $\log_e^x = t \Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = e^t dt$

$I = \int \cos t\, e^t dt$

$= \frac{1}{2}\int e^t\big((\cos t + \sin t) + (\cos t - \sin t)\big)dt$

$= \frac{1}{2}e^t(\sin t + \cos t) + C$

$= \frac{x}{2}\left(\sin(\log_e^x) + \cos(\log_e^x)\right) + C$

Hence (A) is the correct answer.

4. $S_K = \frac{K(K+1)}{2.K} = \frac{K+1}{2}$

$\therefore S_1^2 + S_2^2 + \ldots. + S_{10}^2 = \left(\frac{2}{2}\right)^2 + \left(\frac{3}{3}\right)^2 + \ldots + \left(\frac{11}{2}\right)^2$

$= \frac{1}{4}[1^2 + 2^2 + 3^2 + \ldots + 11^2 - 1^2]$

$= \frac{1}{4}\left[\frac{11 \times 12 \times 23}{6} - 1\right]$

$= \frac{1}{4} \times 505$

$\therefore \frac{505}{4} = \frac{5}{12}A$

$\Rightarrow A = \frac{505 \times 3}{5} = 303$

5. $\lim_{x \to \frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{(1 - \tan^4 x)}{\tan^3 x.\left(\frac{1}{12}\cos x - \frac{1}{12}\sin x\right)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{(1 - \tan x)(1 + \tan x)\ \sec^2 x.\sqrt{2}}{\tan^3 x.(\cos x - \sin x)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{(\cos x - \sin x).(1 + \tan x).\sec^2 x.\sqrt{2}}{\cos x.\tan^3 x.(\cos x - \sin x)}$

$= 8$

Hence (C) is the correct answer.

6. Let $I = \int_0^a f(x)g(x)dx = \int_0^a f(a-x)f(a-x)dx$

$= \int_0^a f(x)\big(4 - g(x)\big)dx$

$= 4\int_0^a f(x)dx - \int_0^a f(x)g(x)dx$

$\Rightarrow I = 4\int_0^a f(x)dx - I$

$\Rightarrow 2I = 4\int_0^a f(x)dx$

$\Rightarrow I = 2\int_0^a f(x)dx$

7. Since given vectors are coplanar,

Hence $\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$

$\Rightarrow \mu^3 - 3\mu + 2 = 0$

$\Rightarrow \mu = 1, 1, -2$

Clearly (A) is the correct answer.

8. $\tan^{-1}\left(\frac{2x + 3x}{1 - 2x \times 3x}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1 - 6x^2} = 1$

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow x = -1, \frac{1}{6}$

But, since $x > 0$. Hence $x = \frac{1}{6}$

$\therefore R$ has only one element.

9. The given $D.E$ can be written as

$\frac{dy}{dx} + \frac{y}{x} = ln\, x$ which is a linear $D.E$.

Now $I.F. = e^{\int \frac{1}{x}dx} = e^{ln\, x} = x$

Hence solution of given $D.E$ is

$y.x \int x.\, ln\, x\, dx$

$\Rightarrow y.x = \frac{x^2}{2} ln\, x - \frac{x^2}{4} + C$

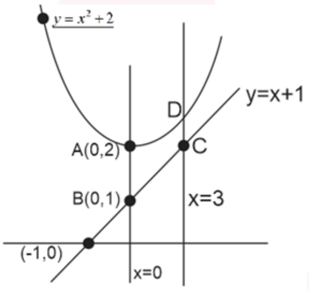
Given $y(2) = ln - 1 \Rightarrow C = 0$

$\Rightarrow y = \frac{x}{2} ln\, x - \frac{x}{4}$

$\Rightarrow y(e) = \frac{e}{4}$

Hence (A) is the correct answer.

10.



Area of region $ABCDA$ is, $= \int_0^3 [(x^2 + 2) - (x + 1)]dx$

$= \int_0^3 (x^2 - x + 1)dx = \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3$

$= 9 - \frac{9}{2} + 3$

$= 12 - \frac{9}{2} = \frac{15}{2}$

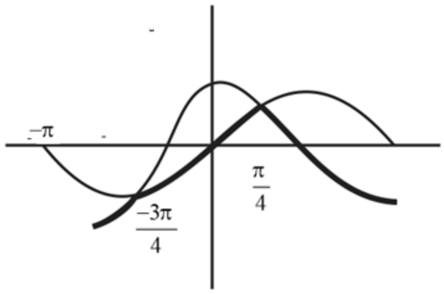
11. The given ratio is $\frac{5^{th}\text{term from beginning}}{5^{th}\text{term from the end}}$

$\frac{T_5}{T_7} = \frac{{}^{10}C_4 \left(2^{\frac{1}{3}}\right)^{10-4} \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^4}{{}^{10}C_6 \left(2^{\frac{1}{3}}\right)^{10-6} \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^6}$

$= 4.(36)^{\frac{1}{3}}$

Hence (A) is the correct answer

12.



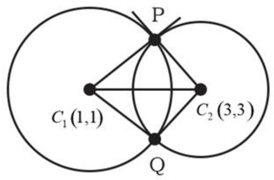
Corner points are

$\frac{-3\pi}{4}$ and $\frac{\pi}{4}$

$\therefore$ of non-differentiable points $\left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$

13. Equation of give circles are
$(x-1)^2 + (y-1)^2 = 4$

$(x-1)^2 + (y-1)^2 = 4$



$C_1(1,1)\ \ r_1 = 2$

$(x-3)^2 + (y-3)^2 = 4$

$C_2(3,3)\ r_2 = 2$

$PC_1 = PC_2 = 2,\ C_1C_2 = \sqrt{8}$

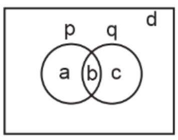
$\Rightarrow PC_1^2 + PC_2^2 = C_1C_2^2$

$\Rightarrow \angle C_1PC_2 = \frac{\pi}{2}$

Hence area of quadrilateral $PC_1\,QC_2 = 2 \times \frac{1}{2} \times 2 \times 2 = 4$

(D) is the correct option.

14. Solution: (B)



$p \vee \sim q\ \equiv a + b + d$

$\sim p \wedge q\ \equiv c$

$\therefore (p \vee \sim q) \wedge (\sim p \wedge q) \equiv \phi$

$\wedge \vee \quad \therefore \big((p \vee \sim q) \wedge (\sim p \wedge q)\big) \vee (\sim p \wedge \sim q) \equiv\ \sim p \wedge \sim q$

15. Given $P\begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\9 & 3 & 1\end{bmatrix} \Rightarrow P^5 = \begin{bmatrix}1 & 0 & 0\\15 & 1 & 0\\135 & 15 & 1\end{bmatrix}$

Now $Q - P^5 = I_3$

$\Rightarrow q_{21} - 15 = 0, q_{31} - 135 = 0,\ q_{32} - 15 = 0$

$q_{21} = 15, q_{31} = 135, q_{32} = 15$

$\Rightarrow \frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$

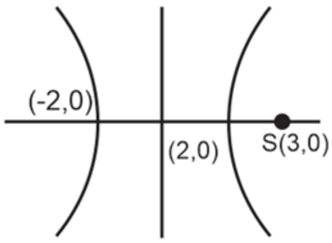
Option (B) is the correct answer.

16. As we know, product of numbers is even when atleast one of the number must be even.

Hence total subsets of A in which product of numbers is even $=$ Total subsets – total subsets in which all the elements are odd.

$= 2^{100} - 2^{50}$.

17.



Since vertex of the hyperbola are $(-2, 0)$ and $(2, 0)$, hence transverse axis of the hyperbola is x –axis and centre is $O(0, 0)$

Equation of hyperbola can be taken as

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow a = 2$ and $ae = 3$

$\Rightarrow e = \frac{3}{2}$

Since $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 9 - 4 = 5$

Hence equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$

Clearly point $(-6, 5\sqrt{2})$ doesn't lies on the hyperbola.

18. If a complex number if purely imaginary, then it must be equal to minus times its conjugate.

$\Rightarrow \frac{z-\alpha}{z+\alpha} = -\left(\frac{\bar{z}-\alpha}{\bar{z}+\alpha}\right)$

$\Rightarrow z\bar{z} + \alpha z - \alpha\bar{z} - \alpha^2 = -(z\bar{z} - \alpha z + \alpha\bar{z} - \alpha^2)$

$\Rightarrow |z|^2 = \alpha^2$

$\Rightarrow \alpha^2 = 4$

$\Rightarrow \alpha = \pm 2$

19. Given $f(\theta) = 3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$

$= 3\cos\theta + 5\left(\sin\theta.\frac{\sqrt{3}}{2} - \cos\theta.\frac{1}{2}\right)$

$= \frac{5\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$

Maximum value of $f(\theta)$ is $\sqrt{\left(\frac{5\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{19}$

20. Let α and β are the roots of given equation then $\frac{\alpha}{\beta} = \lambda$. Given $\lambda + \frac{1}{\lambda} = 1$

$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$

$\Rightarrow (\alpha + \beta)^2 = 3\alpha\beta$

$\Rightarrow \left(\frac{-n(m-4)}{3m^2}\right)^2 = \frac{3.2}{3m^2}$

$\Rightarrow \left(\frac{-n(m-4)}{3m^2}\right)^2 = \frac{3.2}{3m^2}$

$\Rightarrow m = 4 \pm 3\sqrt{2}$

Hence least value of m is $4 - 3\sqrt{2}$.

21. Given $(2x)^{2y} = 4.e^{2x-2y}$, taking log both the sides we, get $2y\ ln\ 2x = ln\ 4 + 2x - 2y$

$\Rightarrow 2y = \left(\frac{ln\ 4 + 2x}{ln\ 2x + 1}\right)$

$\Rightarrow \frac{2dy}{dx} = \frac{(ln\ 2x+1).2-(ln\ 4+2x)\frac{1}{x}}{(ln\ 2x+1)^2}$

$\Rightarrow (1 + ln\ 2x)^2\frac{dy}{dx} = \left(\frac{x.ln\ 2x - ln\ 2}{x}\right)$

22. To minimize the calculation, 3 numbers in $G.P$ can be taken as $\frac{a}{r}, a, ar$.

Given product of 3 numbers is 512.

$\Rightarrow \frac{a}{r}.a.ar = 512 \Rightarrow a = 8$

Also, $\frac{a}{r} + 4, a + 4, ar$ are in A.P

$\Rightarrow 2(a + 4) = \frac{a}{r} + 4 + ar$

$\Rightarrow 2r^2 - 5r + 2 = 0$ (as $a = 8$)

$\Rightarrow r = 2$ or $\frac{1}{2}$

Hence numbers may be $4, 8, 16$ or $16, 8, 4$

In both the cases sum $= 28$

23. Each box contains 10 balls numbered from 1 to 10.

n_1, n_2, n_3 are numbers on the balls drawn from the box B_1, B_2 and B_3 respectively such that $n_1 < n_2 < n_3$.

i.e., all 3 numbers n_1, n_2, n_3 must be different and can be arranged only in one way (increasing).

Now n_1, n_2, n_3 can be selected in ${}^{10}C_3$ ways.

Hence total number ways $= {}^{10}C_3.1 = {}^{10}C_3$.

24. For unique solution $\begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$

$C_1 \to C_1 + C_2 + C_3$

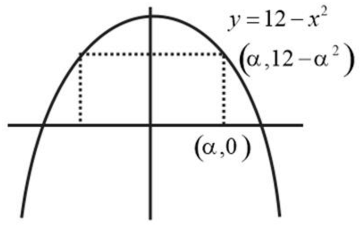
$\Rightarrow \begin{vmatrix} \alpha+\beta+2 & \beta & 1 \\ \alpha+\beta+2 & 1+\beta & 1 \\ \alpha+\beta+2 & \beta & 2 \end{vmatrix} \neq 0$

$\Rightarrow (\alpha+\beta+2) \begin{vmatrix} 1 & \beta & 1 \\ 1 & 1+\beta & 1 \\ 1 & \beta & 2 \end{vmatrix} \neq 0$

$\Rightarrow \alpha + \beta + 2 \neq 0$

Clearly point $(2, 4)$ satisfying the given condition.

25.



Since given parabola is symmetric about the y −axis, hence rectangle will also be symmetric about y −axis.

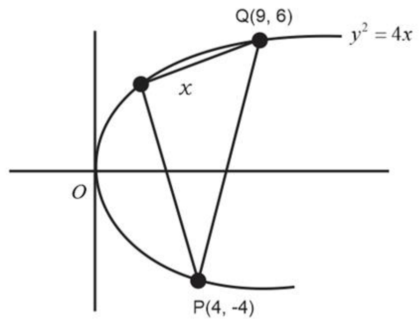
Let one vertex of the rectangle on the x −axis be $(\alpha, 0)$, then

Area of rectangle $A = 2\alpha.(12 - \alpha^2)$

$\Rightarrow \frac{dA}{d\alpha} = 24 - 6\alpha^2 = 0 \Rightarrow \alpha = 2, -2$

$\Rightarrow A = 32$

26.



Two different approaches we can use here.

Approach 1:

Let x be $(t^2, 2t)$, then

Area of $\Delta PXQ = \frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix}$

$\Delta = \frac{1}{2}.10(t^2 - t - 6)$

$\Delta' = 0 \Rightarrow t = \frac{1}{2}$

Hence area of $\Delta PXQ = \frac{125}{4} sq.units$

Approach 2:

For maximum area tangent to the parabola at X must be parallel to PQ. Let $X(t^2, 2t)$, then

$2\frac{dy}{dx} = 4 \Rightarrow \left(\frac{dy}{dx}\right)_{(t^2,2t)} = \frac{1}{t}$

$\Rightarrow \frac{1}{t} = \frac{6+4}{9-4} \Rightarrow \frac{1}{6} = 2$

$\Rightarrow t = \frac{1}{2} \Rightarrow X\left(\frac{1}{2}, 1\right)$

Area of $\Delta PXQ = \frac{125}{4} sq.units$

27. Given, line perpendicular to given line passes through $(7, 15)$ and $(15, \beta)$

Hence $\frac{15-\beta}{7-15} = \frac{-3}{2}$

$\Rightarrow 30 - 2\beta = -21 + 45$

$\Rightarrow \beta = 3$

28. $3x + 4y - \lambda = 0$

$(7 - \lambda)(31 - \lambda) < 0$ {Since centres lie opposite side}

$\lambda \in (7, 31) \ldots\ldots(1)$

$\left|\frac{7-\lambda}{5}\right| \geq 1$ & $\left|\frac{31-\lambda}{5}\right| \geq 2$

$|7 - \lambda| \geq 5$ & $|31 - \lambda| \geq 10$

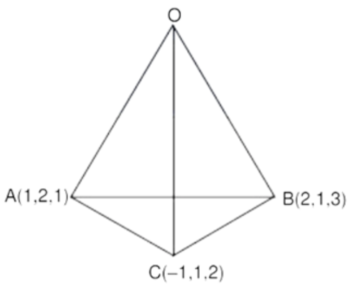
$\lambda \leq 2$ *or* $\lambda \geq 12 \ldots\ldots(2)$

and $\lambda \leq 21$ *or* $\lambda \geq 41 \ldots\ldots(3)$

$(1) \cap (2) \cap (3)$

$\lambda \in [12, 21]$

29.



Vector perpendicular to face $OAB = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{\imath} - \hat{\jmath} - 3\hat{k}$

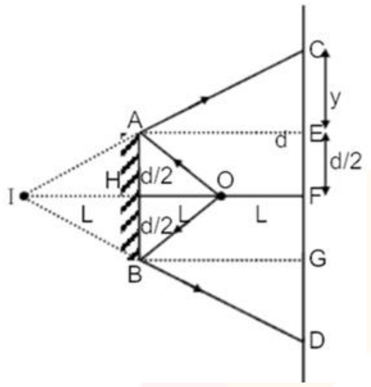
Vector perpendicular to face $ABC = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

Angle between two faces $\cos\theta = \left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right| = \frac{19}{35}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$

Physics

1.



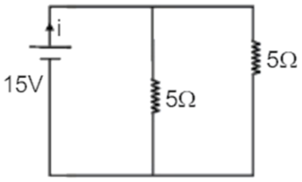
By similar triangles

$\Delta AEC \sim \Delta IHA$

$$\frac{y}{2L} = \frac{\frac{d}{2}}{L}$$

Total distance $(CD) = y + \frac{d}{2} + \frac{d}{2} + y = 3d$

2.

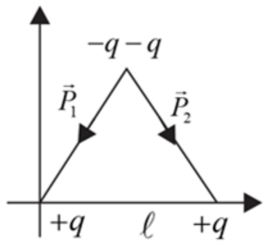


After along time

So $I = \frac{V}{R_G} = \frac{15}{2.5} = 6A$

3. $|\vec{P}_1| = q\ell$

$|\vec{P}_2| = q\ell$



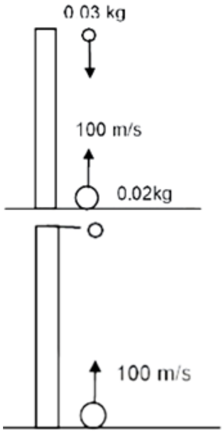
Angle between them 60°

∴ Resultant dipole moment $P = \sqrt{(q\ell)^2 + (q\ell)^2 + 2(q\ell)^2 \cos 60°}$

$= \sqrt{3}$

As $|\vec{P}_1| = |\vec{P}_2|$direction of resultant is along -y axis this option is (A)

4.



$a_{rel} = 0$

$v_{rel} = 100$

$100 - v_{rel} \times t$

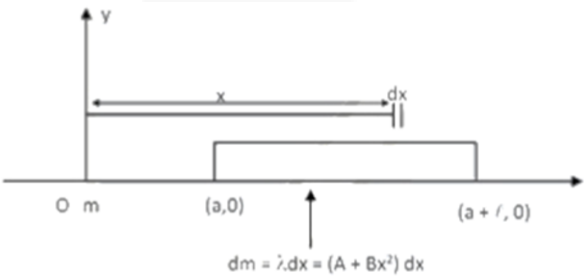
$t = \frac{100}{100} = 1s$

$v_{\text{bullet}} = 100 - 1 \times 10 = 90\ m/s$

$v_{\text{particle}} = 10 \times 1 = 10\ m/s$

$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95 \text{ meter}$

5.



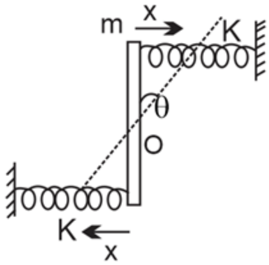
$$F = \int_a^{a+1} \frac{Gm\,dM}{x^2} - GM \int_a^{a+1} \frac{(A + Bx^2)dx}{x^2}$$

$$= GM\left[\int_a^{a+1} \frac{A}{x^2}dx + \int_a^{a+1} B\ dx\right]$$

$$= GM[A[\frac{-1}{x}]_a^{a+1} + B_1]$$

$$= GM\left[A\left(\frac{1}{a} - \frac{1}{a+2}\right)\right] + B_1$$

6. Torque on the rod about O,



$\tau = kx.\frac{\ell}{2} \times 2$

$= k.\frac{\ell}{2}.\theta.\frac{\ell}{2} \times 2$

$= k\frac{\ell^2}{2}.\theta$

ℓ_0 = moment of inertia of rod about O,

$\therefore\ \omega^2 = \frac{k\frac{\ell^2}{2}}{M\frac{\ell^2}{2}} = \frac{6k}{M}$

$f = \frac{1}{2\pi}\sqrt{\frac{M}{6k}}$

7. $R_A = \frac{V^2}{P} = \frac{220\times220}{25} = 44 \times 44$

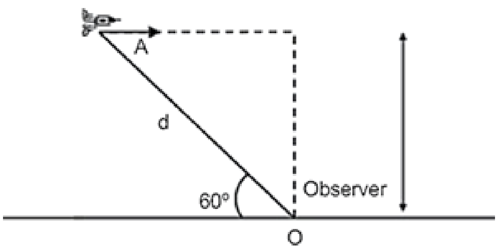
$R_B = \frac{220 \times 220}{100} = 22 \times 22$

As resistance are in series current through them will be same.

$\frac{P_A}{P_B} = \frac{i^2R_A}{i^2R_B} = \frac{44 \times 44}{22 \times 22} = \frac{4}{1}$

∴ Possible option is 'C'

8.



$\frac{d}{V_s} = \frac{d\cos 60^o}{V_a}$

$V_a = \frac{V_s}{2} = \frac{V}{2}$

9. $B = 2\left(\frac{\mu_0 i}{4\pi d}\right)$

$$10^{-4} = 2\frac{4\pi \times 10^{-7} \times i}{4\pi \times \left(\frac{4}{100}\right)}$$

$i = 20A$

10. $r = \frac{\sqrt{2mqv}}{qB} \Rightarrow r \propto \sqrt{\frac{m}{q}}$

$$\sqrt{\frac{1}{4} \times \frac{2}{1}} = \frac{1}{\sqrt{2}}$$

11. $v = \frac{\omega}{k} = \frac{50}{2} = 25\ m/s$

12. $Y = \overline{A.\overline{A.B}}.\overline{B.\overline{AB}}$

$= \left(A.\overline{AB}\right) + \left(B.\ \overline{AB}\right)$

$= A.\ \left(\overline{A}.\overline{B}\right) + B.\left(\overline{A} + \overline{B}\right)$

$= \ A.B + B.\overline{A}$

It is XOR gate

13. $F_r = \frac{-dU}{d} = -kr$

For circular motion

$$|F_r| = kr = \frac{mv^2}{r} \Rightarrow kr^2 = mv^2 \ldots\ldots\ldots (1)$$

Bohr's quantization $= mvr = \frac{nh}{2\pi} \ldots . (2)$

From (1) & (2)

$$\frac{m^2v^2}{m} = kr^2$$

$$\Rightarrow \frac{1}{m}\left(\frac{nh}{2\pi r}\right)^2 = kr^2 \Rightarrow \frac{n^2h^2}{4\pi^2 mk} = r^4 \Rightarrow r = \left(\frac{h^2}{4\pi^2 mk}\right)^{1/4} \quad n^{1/2}$$

$r \propto \sqrt{n}$

From equation (1) $U \propto \sqrt{n}$

$$KE = \frac{1}{2}mv^2 PE = \frac{1}{2}kr^2$$

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kr^2 = kr^2 \propto n$$

14. $A_C = 100$

$A_C + A_m = 160$

$A_C - A_m = 40$

$A_C = 100, A_m = 60$

$\mu = \frac{A_m}{A_c} = 0.6$

15. $Q_0 \propto i_G \Rightarrow Q_0 C = i_G$

I-case $CQ_0 = \frac{v}{220+R}$ ......(1)

II-case $C\frac{\theta_0}{5} \frac{v}{\left(220+\frac{5R}{5+R}\right)} \times \frac{5}{5+R}$ ....(2)

From (1) and (2) $R = 22\Omega$

16. $dR = \frac{cd\ell}{\sqrt{\ell}}$

According to Questions

$\int_0^\ell C\frac{d\ell}{\sqrt{\ell}} = \int_\ell^1 c\frac{d\ell}{\ell}$

Acording to Quation

$\int_0^\ell C\frac{d\ell}{\sqrt{\ell}} = \int_\ell^1 c\frac{d\ell}{\ell}$

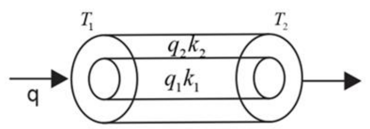
$(2\sqrt{\ell})_0^\ell = (2\sqrt{\ell})_\ell^1$

$2\sqrt{\ell} = 2 - 2\sqrt{\ell}$

$4\sqrt{\ell} = 2$

$\ell = \frac{1}{4} = 0.25\, m$

17. Let $q_1$and q_2 be the rate of flow of heat through inner part from outer part respecting net flow of heat



$q = q_1 + q_2$

$= \frac{K_1 \pi R^2}{\ell} . \Delta T + \frac{K_1 . \pi[(2R)^2 - R^2]}{\ell} . \Delta T$

$\frac{\pi R^2}{\ell} \Delta T (K_1 + 3K_2)$

If the cylinder is replaced by a single
Material of thermal conductivity K then

$q = \frac{K.(2R)^2}{\ell} . \Delta T$

On comparison

$K = \frac{K_1 + 3\prime K_2}{4}$

18. $\frac{hc}{\lambda} = (KE)_{Max} + \phi$

$\frac{12400}{4000} = \frac{\frac{1}{2} \times 9.1 \times 10^{-31} \times 36 \times 10^{10}}{1.6 \times 10^{-19}} + \phi$

$3.1 = 102.375 \times 10^{-2} + \phi$

$\phi = 2.076 eV$

19. $\frac{1}{V} + \frac{1}{20} = \frac{1}{5}$

$\frac{1}{V} = \frac{1}{5} - \frac{1}{20} = \frac{4-1}{20}$

$v_1 = \frac{20}{3}$

Distance of first image $B = \frac{20}{3} - 2 = \frac{14}{3} cm$

$\frac{1}{V} - \frac{3}{14} = -\frac{1}{5} = \frac{15-14}{70} = \frac{1}{70}$

Real image 70 cm right of B.

21. Energy dissipated when switch is thrown from 1 to 2.

22.

$2mv_x = mv$

$v_x = \frac{v}{2}$

$v_y = \frac{v}{2}$

$V_{net} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)} = \frac{v}{\sqrt{2}}$

So path will be elliptical

23. $t_1 = \frac{x}{v-u} = \frac{x}{50}$ (here total of two trains is x)

$t_2 = \frac{x}{v+u} = \frac{x}{110}$

$\frac{t_1}{t_2} = \frac{11}{5}$

24. Least count of screw gauge = 5 μm

$L.C = \frac{\text{pitch}}{\text{no. of div on cirxular scale}}$

$5\mu m = \frac{1mm}{N}$

$N = 200$

25. $I = \frac{1}{2}\varepsilon_0 E_0^2 . C$

$I' = \frac{1}{2}\varepsilon E^2 V$

$I' = 0.96I$

$\frac{1}{2}\varepsilon E^2 v = 0.96\frac{1}{2}\varepsilon_0 E_0^2 C$

$E = \sqrt{0.96}\sqrt{\frac{\epsilon_0}{\epsilon_0 \epsilon_0}}\sqrt{\frac{c}{v}}E_0$

$= \sqrt{0.96}\ \mu_r \approx 1E_0$

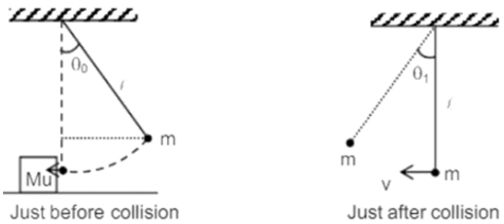
$= \sqrt{\mu_r \varepsilon_r}$

(For most of the transparent mediu $\mu_r \approx 1$)

Therefore $\sqrt{\frac{0.96}{\mu}}$

Hence $E = \sqrt{\frac{0.96}{\mu}}E_0 = 24$

26.



$u = \sqrt{2\ g\ell(1 - \cos\ \theta_0)}\ldots\ldots(i)$

V = velocity of ball after collision

$v = \left(\frac{m-M}{m+M}\right)u$

Since ball rises up to angle θ_1

$v = \sqrt{2g\ell(1 - \cos\ \theta_1)} = \left(\frac{m-M}{m+M}\right)u\ldots\ldots(ii)$

From (i) & (ii)

$$\frac{m-M}{m+m} = \sqrt{\frac{1-\cos\ \theta_1}{1-\cos\ \theta_0}} = \frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)} \Rightarrow \frac{M}{m} = \frac{\theta_0 - \theta_1}{\theta_0 + \theta_1} \Rightarrow M = \left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)m$$

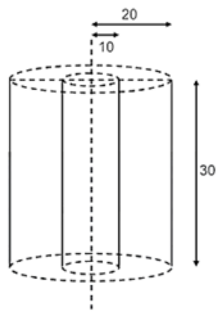
27. $U_i + K_i = U_f + K_f$

$$\frac{KQ^2}{2r_0} + 0 = \frac{KQ^2}{2r} + \frac{1}{2}mv^2$$

$$v^2 = \frac{KQ^2}{m}\left(\frac{1}{r_0} - \frac{1}{r}\right)$$

$$v = \sqrt{\frac{KQ^2}{m}\left(\frac{1}{r_0} - \frac{1}{r}\right)}$$

28.



$$\frac{m(20^2 + 10^2)}{2} = mk^2$$

$$K = \sqrt{\frac{400+100}{2}} K = \sqrt{250}$$

$K = 5\sqrt{10}\ cm$

29. Potential gradient $= x = \frac{5\times10^{-3}}{10\times10^{-2}} = \left(\frac{4}{R+5}\times\right)\times\frac{1}{1}$

$\Rightarrow \frac{1}{20} = \frac{20}{R+5}$

$\Rightarrow 400 = R + 5$

$R = 395\,\Omega$

Chemistry

1. $(\Delta T_f)_X = (\Delta T_f)_Y$

$k_f m_x = k_f m_y$

$\frac{4\times1000}{A\times96} = \frac{12\times1000}{M\times88}$

$M = 3.27A$

$\simeq 3A$

2. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more

3. Iodine gets oxidized to IO_3^- when it reacts with an oxidizing agent (HNO_3). The oxidation number of I will be

$-1 = x + 3\times(-2)$

$-1 = x - 6$

$x = -1 + 6 = +5$

Oxidation $\rightarrow I \rightarrow IO_3^-$

Reduction $\rightarrow HNO_3\,(dil) \rightarrow NO$

$HNO_3(\text{conc}) \rightarrow NO_2$

4. $\Delta G = -nFE_{cell} = -2\times96500\times2 = -386\,kJ$

$\Delta S = nF\frac{dE}{dT} = 2\times96500\times-5\times10^{-4}J/\,^oC$

$= -96.5J$

$at\ \ 298\,K$

$T\Delta S = 298\times(-96.5J) = -28.8\,kJ$

At constant $T(=248K)$ and pressure

$\Delta G = \Delta H - T\Delta S$

$\Delta H = \Delta G + T\Delta S$

$= -386 - 28.8 = -414.8\,kJ$

5. $[Og_{118}]8s^2$ is configuration for $Z = 120$

$\therefore$ it will belong to II^{nd} group

6. Boiling point $\propto$ force of attraction in b, c, d -Hydrogen bonding takes place hence maximum force of attraction so high Boiling point. In (a) only vanderwall force of attraction (feable force) so having low Boiling point

7. $a > b > c$

Acidic character $\propto$ % S character

$\propto \frac{1}{+I}$

Because acidic character $\propto$ stability of conjugate base (Anion)

(a) $CH \equiv C^{\ominus}$ Anion $(-ve)$ on Csp

(b) $CH_3 - C \equiv C^- - ve$ on Csp & $+I$ of $-CH_3$

(d) $CH_2 = \bar{C}H(-ve)$ on Csp^2

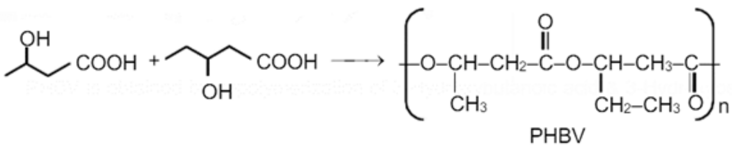
Acidic character order

$\underset{Csp}{CH \equiv C^-} > \underset{Csp \text{ with} +I \text{ of } CH_3}{CH_3 - C \equiv C^-} > \underset{Csp^2}{CH_2 = C^-}$

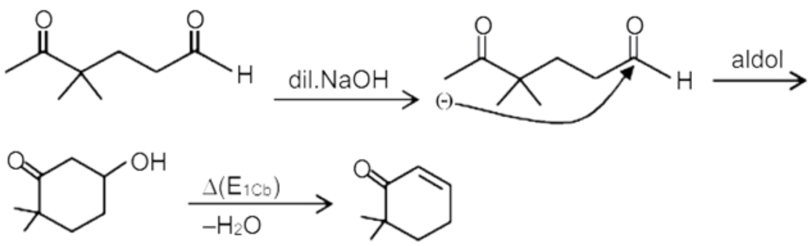
So acidic character order $= a > b > c$

8. Cp is a molar heat capacity at constant pressure. It is a function of temperature it does not varies with pressure

9. PHBV is obtained by copolymerization of 3 −Hydroxybutanoic acid & 3 − hydroxypentanoic acid.



10.



11. $A + 2B \rightleftharpoons 2C + D$

Initially conc. a 1.5a

At eq. $a - x$ 1.5 $(a - 2x)$ $2x$ x

At equilibrium $a - x = 1.5a - 2x$

$0.5\,a = x$

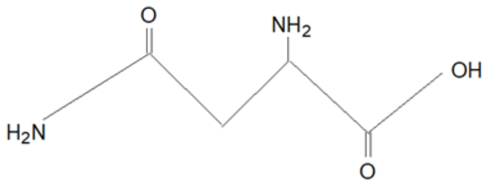
$a = 2x$

$$k_C = \frac{(2x)^2(x)}{(a-x)(1.5a-2x)^2} = \frac{4x^2.x}{(x)(x)^2} = 4$$

12. Lysine

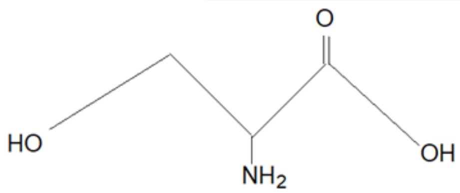
$P^I = 9.7$ (basic)

Aspargin



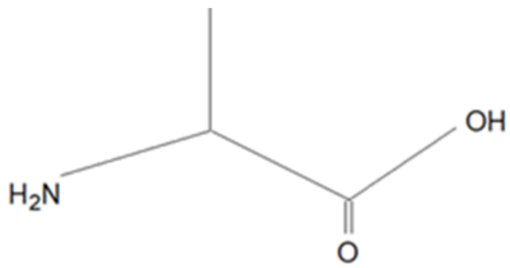
Neutral amino acid $P^I = 5.4$

Serine



$P^I = 5.7$ neutral with polar, but non ionisable side chain

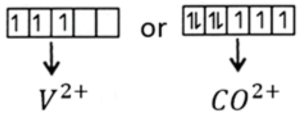
Alanine



$P^I = 6$ (neutral)

13. The value of adsorption is dependent on the inter molecular forces of attraction. Among the given options, H_2 will have the weakest van der waal's force (London Dispersion), in which only 2 electrons are involved.
Hence, as force of attraction is weakest for H_2, it will have the lowest adsorption value.

14. If the complex is $[M(H_2O)_6]Cl_2, M$ will have +2 oxidation state and hence, it has already lost 2 electrons from s –orbital since it has $\mu = 3.9 \left(\sqrt{3(3+2)}\right)$ this tells us that there will be 3 unpaired so, the configuration can either be:



15. The electrolytic cell which is used for extraction of aluminum is a steel vessel. The vessel is lined with carbon, which acts as cathode and graphite is used at the anode.

16. K, Rb and Cs form super oxides on reaction with excess air
$Rb + O_2 \rightarrow RbO_2$ (excess)
$2RbO_2 + 2H_2O \rightarrow 2RbOH = H_2O_2 + O_2 \uparrow$

17. $A(s) \rightleftharpoons B(g) + C(g) k_{P_1} = x\, atm^2$

$P_1 P_1 + P_2$

$D(s) \rightleftharpoons C(g) + E(g) k_{P_2} = y\, atm^2$

$P_1 + P_2 P_2$

$k_{p_1} = P_1(P_1 + P_2)$

$k_{p_2} = P_2(P_1 + P_2)$

$k_{P_1} + k_{P_2} = (P_1 + P_2)^2$

$x + y = (P_1 + P_2)^2$

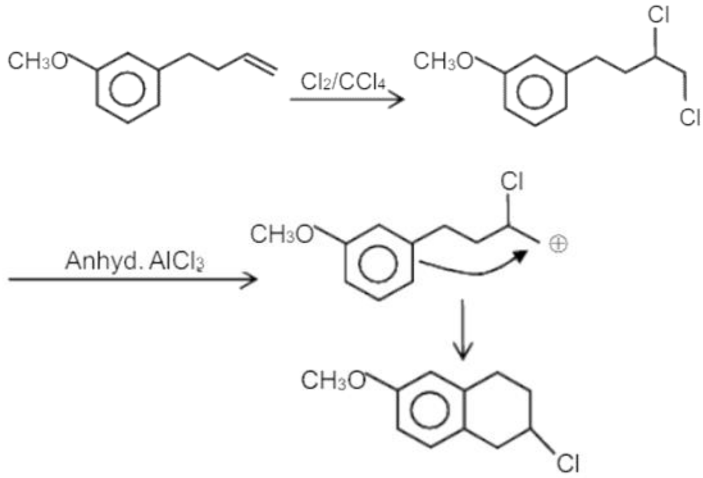
$P_1 + P_2 = \sqrt{x+y}$

$2(P_1 + P_2) = 2\sqrt{x+y}$

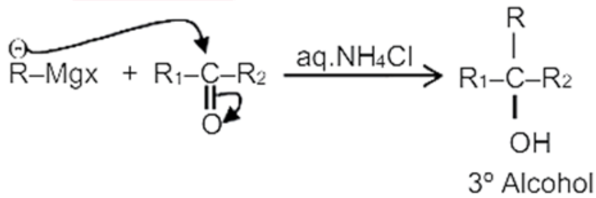
$P_{Total} = P_B + P_C + P_E$

$= 2(P_1 + P_2) = 2\sqrt{x+y}$

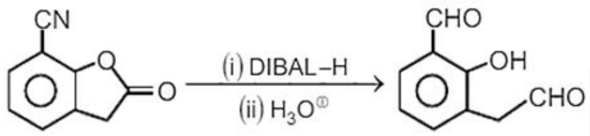
18.



20. Ketone react with $RMgx$ gives 3^o Alcohol



21. Nitriles are selectively reduced by DIBAL-H to imines followed by hydrolysis to aldehydes similarly, esters are also reduced to aldehyde with DIBAL-H



22. According to unit of rate constant it is a zero order reaction then half life of reaction will be

$$t_{\frac{1}{2}} = \frac{C_0}{2k} = \frac{5\mu g}{2 \times 0.05\,\mu g/\text{year}} = 50\text{year}$$

23. $PV = ZnRT$

$$P = \frac{ZnRT}{V}$$

At constant T and mol $P \propto \frac{Z}{V}$

24. As 1L solution have $10^{-3}\, mol\, CaSO_4$

$Eq.\ of\ CaSO_4 = eq.\ of\ CaCO_3$

In 1L solution

$n_{CaSO_4} \times v.f = n_{CaCO_3} \times v.f.$

$10^{-3} \times 2 = n_{CaCO_3} \times 2$

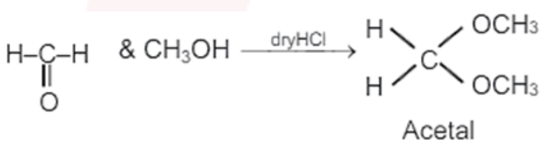
$n_{CaCO_3} = 10^{-3}\, mol\, in\, 1L$

$\therefore W_{CaCO_3} = 100 \times 10^{-3} g$ in 1 L solution

$\therefore$ hardness in terms of $CaCO_3$

$= \frac{W_{CaCO_2}}{W_{total}} \times 10^6 = \frac{100 \times 10^{-3} g}{1000g} \times 10^6 = 100ppm$

25.



26. Now $n_{NaOH}$is $50\, ml = M \times V = 2 \times \frac{50}{1000} = 0.1\, mol$

Mass of $NaOH\ is\ 50\, ml = 4\, g$

27. Give $K_3[Co(CN)_6]$ is inner orbital complex with hybridization d^2sp^3 and octahedral geometry. Ligands are approaching metal along the axes. Hence $d_{x^2-y^2}, d_{z^2}$ orbitals are directly in front of the ligands.

29. More nucleophilic nitrogen, more reactive with alkyl halide.