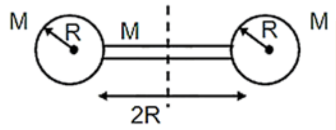
# JEE Main 2019 - 10th Jan Paper - Slot 2 - 02:30PM to 05:30PM

## Physics

**Single correct answer type:**

1. Two identical solid sphere of mass $M$ and radii $R$ are attached with two end of rod of length $2R$ and $M$. Find out moment of inertia about centre of mass of rod.




(A) $\frac{122}{15}MR^2$

(B) $\frac{137}{15}MR^2$

(C) $\frac{153}{15}MR^2$

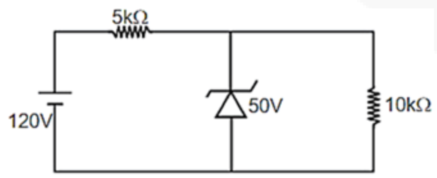
(D) $\frac{44}{5}MR^2$

Solution: (B)

$$I = \left[\frac{2}{5}MR^2 + M(2R)^2\right] \times 2 + \frac{M}{12}(2R)^2$$

$$I = \frac{137}{15}MR^2$$

2. For given circuit find out current through zener diode.



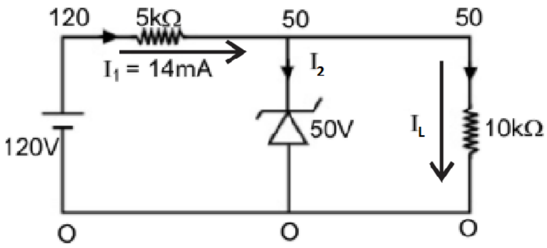

(A) $14mA$

(B) $5mA$

(C) $9mA$

(D) infinite

Solution: (C)




$I = 9mA$

3. Closed pipe of fundamental frequency $1.5KHz$. If hearing range if $20\ KHz$. How many overtone will be observed?

(A) 5

(B) 6

(C) 7

(D) 8

Solution: (B)

$$\frac{20KHz}{1.5KHz} = 13.33$$

Possible harmonics= $1, 3, 5, 7, 9, 11, 13$

No. of overtones= $6$

4. If an object is moving with kinetic energy $3\ J$. A force $F = 3\hat{\imath} + 6\ \hat{\jmath})\ N$ works on the object for displacement= $4\hat{\imath}\ m$ then find final kinetic energy of the block?

(A) $15\ J$

(B) $12\ J$

(C) $27\ J$

(D) $3\ J$

Solution: (A)

Work done by force= $F.S = (3\hat{\imath} + 6\hat{\jmath}) \cdot (4\hat{\imath}) = 12J$

By work energy theorem, $W_{all} = \Delta K$

$12\ J = K_f - K_i \Rightarrow 12 + K_i = K_f$

$\therefore K_f = 12 + 3 + 15\ J$

5. In inductor current $i$ changing from $10A$ to $25A$ in $1\ sec$. Due to this induced $e.m.f$ is 25 volt. Find out change in energy of inductor.

(A) $208.3\ J$

(B) $33.3\ J$

(C) $437.5\ J$

(D) $241.7\ J$

Solution: (C)

$$|\varepsilon| = \frac{Ldi}{dt}$$

$$25 = L \times \frac{(25-10)}{1}$$

Energy of inductor $E = \frac{1}{2}Li^2$

$$\Rightarrow \Delta E = \frac{1}{2} \times \frac{5}{3}[625 - 100]J$$

$$\Delta E = 437.5\ J$$

6. Compass needle of length $l = 12\ cm$ is placed in a region making $45^o$ with the horizontal. If pole strength of needle is $m = 1.8\ A - m$, how much force is required at one end so that needle becomes horizontal. $(B_H = 1.8 \times 10^{-6}T)$

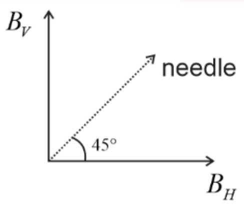
(A) $3.24 \times 10^{-6}N$

(B) $4.54 \times 10^{-6}N$

(C) $9.26 \times 10^{-6}N$

(D) $6.48 \times 10^{-6}N$

Solution: (D)




at $45^o$, $B_H = B_v$

$$\frac{Fl}{2} = \frac{2mlB_v}{l} = 3.6 \times 1.8 \times 10^{-6}N$$

$$= 6.48 \times 10^{-6}N$$

7. A capacitor of capacitance $C = 12pF$ is connected with cell of $e.m.f\ \varepsilon = 10V$. Now cell is disconnected and dielectric of $K = 6.5$ is inserted between plate of capacitor find out done by external agent in this process.

(A) $407\ pJ$

(B) $507\ pJ$

(C) $525\ pJ$

(D) $510\ pJ$

Solution: (B)

$$W = -(U_f - U_i) = \left(\frac{(\varepsilon C)^2}{2KC} - \frac{(\varepsilon C)^2}{2C}\right)$$

$$= \frac{\varepsilon^2 C}{2}\left(\frac{K-1}{K}\right) = \frac{10^2 \times 12 \times 10^{-12}}{2}\left(\frac{5.5}{6.5}\right) = 507\ pJ$$

8. An electromagnetic wave is propagating in free space and electric field of wave is given by $\vec{E} = 10\cos(6x + 8z)\,\hat{j}\ V/m$ at $t = 0$. Then find the expression of magnetic field at any time $t$:

(A) $\vec{B} = \frac{1}{c}(-8\hat{\imath} + 6\hat{k})\cos(6x + 8z + 10ct)\,Tesla$

(B) $\vec{B} = \frac{1}{c}(-8\hat{\imath} + 6\hat{k})\cos(6x - 8z - 10ct)\,Tesla$

(C) $\vec{B} = \frac{1}{c}(-8\hat{\imath} + 6\hat{k})\cos(6x + 8z - 10ct)\,Tesla$

(D) $\vec{B} = \frac{1}{c}(-8\hat{\imath} + 6\hat{k})\cos(6x - 8z + 10ct)\,Tesla$

Solution: (C)

$E = E_0\cos(\omega t - \hat{k}.\hat{r})$

∴ At any time t;

$\hat{k} = 6\hat{\imath} + 8\hat{k}$

$$B_0 = \frac{E_0}{c} = \frac{10}{c}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10} = \frac{\pi}{5}$$

Also, $\hat{E} \times \hat{B} = \hat{c}$

$$\therefore \omega = 2\pi f = \frac{2\pi c}{\lambda}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 0 \\ B_x & B_y & B_z \end{vmatrix} = \frac{6\hat{\imath} + 8\hat{k}}{10}$$

$$\omega = \frac{2\pi \times c}{\left(\frac{\pi}{5}\right)} = 10c$$

$\therefore \hat{\imath}[B_z - 0] + \hat{\jmath}(0) + \hat{k}(-B_x) = 0.6\hat{\imath} + 0.8\hat{k}$

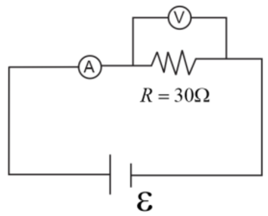
Comparing coefficient both sides

$\therefore B_z = 0.6, B_x = -0.8$

$$|\vec{B_0}| = \frac{10}{c}(-0.8\hat{\imath} + 0.6\hat{k}) = \frac{-8\hat{\imath} + 6\hat{k}}{c}$$

So magnetic field $\vec{B} = \frac{1}{c}(8\hat{\imath} + 6\hat{k})\cos(6x + 8z - 10ct)\,Tesla$

9. To measure the resistance of $R$ by measuring $\frac{V}{I}$, the measured resistance is 5% smaller than original resistance of $R = 30\Omega$. The resistance of the voltmeter is:




(A) $570\Omega$

(B) $500\Omega$

(C) $630\Omega$

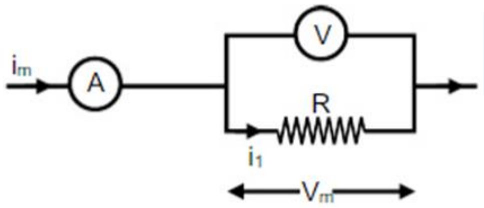
(D) $440\Omega$

Solution: (A)

Let the measured voltage be $V_m$ and

Let the measured current be $i_m$ and

Let the ammeter be ideal, thus




$$\frac{V_m}{i_m} = R_m$$

$$\therefore \frac{i_1 R}{i_m} = R_m$$

$$\therefore R\left(\frac{R_v}{R + R_v}\right) = R_m$$

$$\therefore \frac{1}{R_m} = \frac{1}{R} + \frac{1}{R_v}$$

$$\therefore \frac{1}{R_v} = \frac{1}{R_m} - \frac{1}{R} = \frac{1}{0.95R} - \frac{1}{R}$$

$$\therefore R_V = 19R$$

$$R_V = 570\Omega$$

10. A particle performs $SHM$ with amplitude of $5m$. If at $x = 4m$ magnitude of velocity and acceleration are equal then time period of $SHM$ is:

(A) $\frac{2\pi}{3}$

(B) $\frac{4\pi}{3}$

(C) $\frac{8\pi}{3}$

(D) $\frac{6\pi}{3}$

Solution: (C)

$v = \omega\sqrt{5^2 - 4^2} = 3\omega$

$a = \omega^2 \times 4$

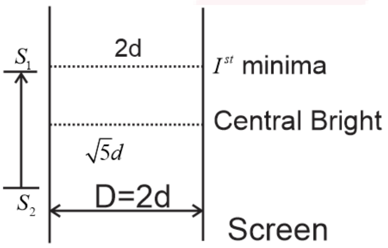
$|a| = |v|$

$4\omega^2 = 3\omega$

$\omega = \frac{3}{4} = \frac{2\pi}{T}$

$T = \frac{8\pi}{3}\ sec$

11. For Given $YDSE$ experiment find the value of $d$ is $1^{st}$ minima lie directly in front of one of the slit.



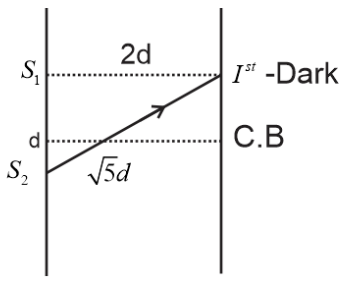

(A) $d = \frac{\lambda}{2(\sqrt{5}-2)}$

(B) $d = \frac{\lambda}{2(\sqrt{5}+2)}$

(C) $d = \frac{3\lambda}{2(\sqrt{5}-2)}$

(D) $d = \frac{2\lambda}{2(\sqrt{5}+2)}$

Solution: (A)




Path diff. in front of a slit on screen

$\Delta = \sqrt{5}d - 2d = (2n-1)\frac{\lambda}{2}$

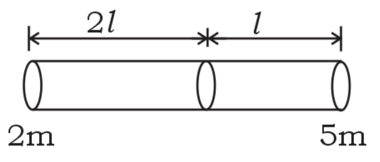
(for minima)

For $1^{st}$ minima $n = 1$

$d(\sqrt{5}-2)=\frac{\lambda}{2}$

$d=\frac{\lambda}{2(\sqrt{5}-2}$

12. A light rod of length $3l$ is hinged as given in figure with two-point masses $2m$ and $5m$. Angular acceleration will be:



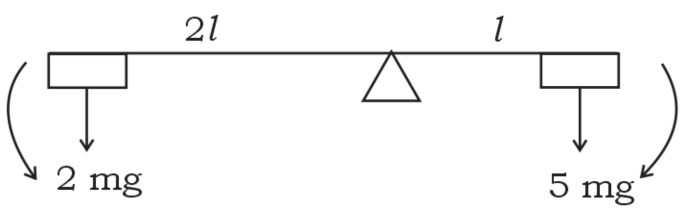

(A) $\left(\frac{3g}{13l}\right)$

(B) $\left(\frac{g}{13l}\right)$

(C) $\left(\frac{5g}{13l}\right)$

(D) $\left(\frac{2g}{13l}\right)$

Solution: (C)



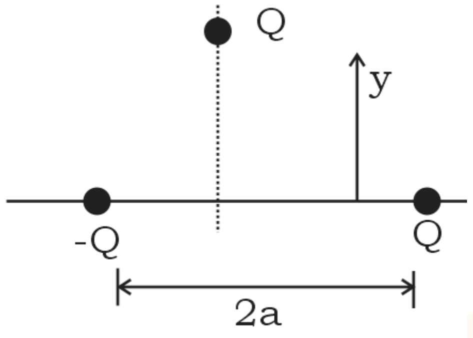

Both the weight will provide torque in opposite direction:-

Use:- $2_{net}=I_{\propto}$ [where $'I'\rightarrow M.I$ about axis of rotation

$(5mg)l-(2mg)(2l)=(2m(2l)^2+5m(l)^2]\propto$

$mgl=\frac{g}{13l}$

13. There are two charge $+Q$ and $-Q$ and the separation between them is $2a$. A third charge $Q$ is place at distance $y$ on perpendicular bisector of line joining of $Q$ & $-Q$ then the force on third charge is $F$. Now if we place the third charge at distance $y/3$ on perpendicular bisector of line joining $Q$ & $-Q$ (Giving $y\gg 2a$. Then force on third particle will be:




(A) $9F$

(B) $18F$

(C) $27F$

(D) $36F$

Solution: (C)

Electric field intensity at distance '$y$'

Along the perpendicular bisector is :-

$$|E_1| = \frac{KP}{y_3}$$

So force at distance '$y$' on '$Q$'

$F = Q(E_1)$ ....(1)

$F' = Q.\frac{KP}{y^3}$ ....(2)

$\frac{(2)}{(1)}$ i.e., $\frac{F'}{F} = (3)^3$

$\therefore F' = 27F$

14. There is a hoop and solid cylinder of same mass and same dimensions. The magnetic moment of hoop of double of cylinder. Now both hoop and cylinder are oscillated in uniform magnetic field then the relation between time period of hoop & cylinder is

(A) $T_H = 2T_C$

(B) $2T_H = T_C$

(C) $T_H = 3T_C$

(D) $T_H = T_C$

Solution: (D)

Time period of any magnetic body place in an external magnetic field is:-

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

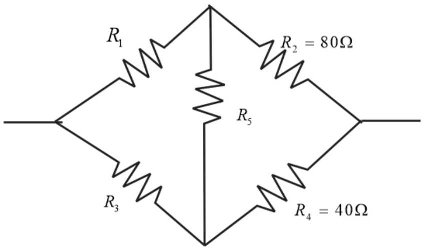
$T_{hoop} = 2\pi\sqrt{\frac{mR^2}{MB}}$ ......(1) $I_{hoop} = mR^2$

$T_c = 2\pi\frac{\sqrt{\frac{mR^2}{2}}}{\frac{M}{2}(B)}$ ......(2) $I_{Solid\ cylinder} = \frac{mR^2}{2}$

From (1) and (2)

$T\pi = T_c$

15. There is a balanced wheatstone bridge as shown in figure. If the colour strips of $R_1$ is Orange-Red-Brown. Then the color code of $R_3$ is




(A) Brown-Blue-Brown

(B) Brown-Green-Black

(C) Black-Blue-Brown

(D) Black-Orange-Brown

Solution: (A)

Condition for Balance wheat stone Bridge:-

$\frac{R_1}{R_3} = \frac{R_2}{R_4}$

For $R_1$:- orange-Red-Brown

$3 \quad 2 \quad 10^1$

So, $\frac{320}{R^3} = \frac{80}{40}$ $R_1 = 320\Omega$

$\therefore R_3 = 160\Omega$

Brown Blue Brown

$(1) \quad (6) \quad (10^1)$

16. There is a cylindrical bottle of negligible mass and radius 2.5 cm floating in the water of density $10^3$ kg/m$^3$ filled with 310 ml of water inside it. Now the bottle is slightly dipped into water and released. The frequency of oscillation is:

(A) 3.75 $\sec^{-1}$

(B) 1.25 $\sec^{-1}$

(C) 2.75 $\sec^{-1}$

(D) 3.00 $\sec^{-1}$

Solution: (B)

Let at equilibrium, $'y'$ length of bottle is immerged in water:-

So, $mg = f_{buo}$

$mg = \rho(Ay)g$ .....(1) [Equilibrium Condition]

Now it is further paused by distance $'x'$

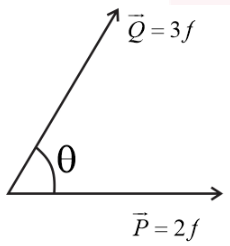
$F_{restoring} = \rho[A(x+y]g - mg$

$mw^2x = \rho Axg + \rho Ayg - mg$

$$w = \sqrt{\frac{\rho Ag}{m}} = \sqrt{\frac{\rho Ag}{\rho V}} = \sqrt{\frac{\pi r^2 g}{V}} = 2\pi f$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{\pi r^2 g}{V}} = 1.25\ sec^{-1}$$

17. Given two force $\vec{P} = 2F$ and $\vec{Q} = 3F$ and the angle between and $P$ and $Q$ is $\theta$. Now we double the value of $Q$ then the magnitude of new resultant force is double of initial resultant force. Then find the value of $\theta$?




(A) $120^o$

(B) $60^o$

(C) $180^o$

(D) $90^o$

Solution: (A)

$$|R|^2 = (\sqrt{P^2 + Q^2 + 2PQcos\theta})^2$$

$$|R|^2 = \left(\sqrt{(2F)^2 + (2F)^2 + 2(2F)(3F)cos\theta}\right)^2 \text{ ......... } (1)$$

On doubling $'Q'$

$$|2R|^2 = \sqrt{(2F)^2 + (6F)^2 + 2(2F)(6F)cos\theta} \text{ ..........}(2)$$

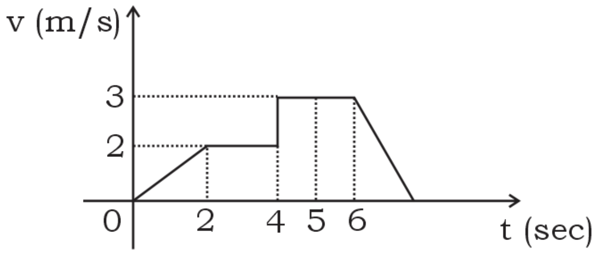
Dividing $(2)/(1)$

$$4 = \frac{40 + 24cos\theta}{13 + 12cos\theta}$$

Solving:

$cos\theta = \frac{-1}{2}$

18. For given $V - t$ graph find out displacement from $t = 0$ to $t = 5$ second




(A) $9m$

(B) $7m$

(C) $6m$

(D) $12m$

Solution: (A)

Area enclosed by $V - T$ graph=displacement (As particle moving in same direction)

$\frac{1}{2} \times 2 \times +2 \times 2 + 1 \times 3$ =Displacement

∴So displacement= $9m$

19. If diameter of a cylinder is $12.6 \pm 0.1\ cm$ and height is $34.2 \pm 0.1$ cm. Find volume of cylinder to nearest significant figure.

(A) $4264.4 \pm 81\ \text{cm}^3$

(B) $4260 \pm 81\ \text{cm}^3$

(C) $4300 \pm 80\ \text{cm}^3$

(D) $4260 \pm 80\ \text{cm}^3$

Solution: (D)

$$V = \frac{\pi d^2 h}{4} = \frac{\pi}{4}(12.6)^2(34.2)cm^3$$

$$\frac{dV}{V} = \frac{2d(d)}{d} + \frac{d(h)}{h}$$

Converting to errors

$$\frac{\Delta V}{V} = 2\frac{\Delta d}{d} + \frac{\Delta h}{h}$$

$$\frac{\Delta V}{4260} = 2\left(\frac{0.1}{12.6}\right) + \left(\frac{0.1}{34.2}\right)$$

$$\Rightarrow \Delta V = 80\ cm^3$$

$$V = 4260 \pm 80 cm^3$$

20. $2kg$ monoatomic gas of density $8\,\mathrm{kg/m^3}$ and pressure $4\times10^3\ \mathrm{N/m^2}$, is enclosed in a container. Internal energy of gas will be:

(A) $1.5\times10^4 J$

(B) $1.5\times10^3 J$

(C) $3\times10^3 J$

(D) $3\times10^4 J$

Solution: (B)

Internal Energy of an g Ideal gas given by

$U=\frac{f}{2}nRT$ (for n-moles)

$U=\frac{f}{2}(PV)=\frac{f}{2}\times P\times\frac{M}{P}$ [$f=3$ for monoatomic gas]

$=\frac{3}{2}\times4\times10^3\times\frac{2}{8}$

$=1.5\times10^3 T$

21. A light of intensity $16$ mW and energy $10$eV incident on a metal plate of work function $5eV$ and area $10^{-4}m^2$ then find maximum kinetic energy of emitted electrons and number of photoelectrons emitted per second if photon efficiency is $10\%$

(A) $5eV, 10^{11}$

(B) $10\ eV, 10^{12}$

(C) $5\ eV, 10^{13}$

(D) $10\ eV, 10^{14}$

Solution: (A)

Maximum $KE=\frac{hC}{\lambda}-\phi$

$=10-5$

$=5eV$

Intensity $I=\frac{(N_p)\frac{hC}{\lambda}}{tA}\Rightarrow\frac{N_p}{t}=\frac{IA}{\left(\frac{hC}{\lambda}\right)}$

and number of electrons per second. $=\left(\frac{N_p}{t}\right)\times10\%$

$$=\left(\frac{IA}{\frac{hC}{\lambda}}\right)\times\frac{1}{10}=\frac{16\times10^{-3}\times10^{-4}}{10\times1.6\times10^{-19}\times10}$$

$=10^{11}$ per sec.

22. If four charges of same magnitude $Q$ are placed at $(0,2)$, $(0,-2)$, $(4,-2)$ and $(4,2)$, then find work done by external agent in placing a fifth change $Q$ at origin.

(A) $\frac{Q^2}{4\pi\varepsilon_0}$

(B) $\frac{Q^2}{4\pi\varepsilon_0}\left[1+\frac{1}{\sqrt{5}}\right]$

(C) $\frac{Q^2}{4\pi\varepsilon_0}\left[1-\frac{1}{\sqrt{5}}\right]$

(D) $-\frac{Q^2}{4\pi\varepsilon_0}\left[1-\frac{1}{\sqrt{5}}\right]$

Solution: (B)

$W_{ext} = \Delta U$

$= U_f - U_i$ if $U_i = 0$ (assumed)

$W_{ext} = U_f - 0$

$$= \left[\frac{KQ^2}{2}+\frac{KQ^2}{2}+\frac{KQ^2}{2\sqrt{5}}+\frac{KQ^2}{2\sqrt{5}}\right]$$

$$W_{ext} = \frac{Q^2}{4\pi\varepsilon_0}\left[1+\frac{1}{\sqrt{5}}\right]$$

23. There are two identical starts of mass $3\times10^{31}\ kg$ each with distance $2\times10^{11} m$ between them. Now a material is projected with velocity $v$ from the midpoint of the line joining the stars. What should be the minimum value of $v$ so that it goes out by gravitational field of the given stars?

(A) $9\times10^3\ \mathrm{m/s}$

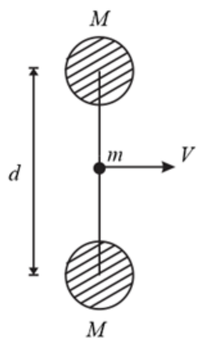
(B) $6\times10^3\ \mathrm{m/s}$

(C) $18\times10^3\ \mathrm{m/s}$

(D) $12\times10^3\ \mathrm{m/s}$

Solution: (A)

To escape out the total energy of small particle must be zero.




$TE = KE + U$

$$O = \frac{1}{2}mV^2 + \left(\frac{-GMm}{\frac{d}{2}}\right)$$

$\Rightarrow V = \sqrt{\dfrac{8GM}{d}}$

$= \sqrt{\dfrac{8 \times 6.6 \times 10^{-11} \times 3 \times 10^{31}}{2 \times 10^{11}}}$

$= 9 \times 10^3\ m/s = 9K\, m/s.$

24. $Ne\ _e^{20} \rightarrow\ _2He^4 +\ _2He^4 +\ _6C^{12}$

If binding energy per nucleon of $Ne^{20}$ is 8.03 $MeV$ $_2He^4$ is 7.07 $MeV$ and $_6C^{12}$ is 7.86 $MeV$ then

(A) $-5.72$

(B) $-9.72$

(C) $-12.72$

(D) $-6.72$

Solution: (B)

$Q = E_f - E_i$

$= (12 \times 7.78 + 4 \times 7.07 + 4 \times 7.07) - 20 \times 8.03$

$Q = -9.72\, MeV$

25. Eye can be considered as single refracting surface radius of curvature of eye is 7.8 mm medium outside the eye is air $(n = 1)$ and inside eye is $n = 1.34$ then distance at which parallel beam of light from air will be conserved.

(A) 3 $cm$

(B) 4 $cm$

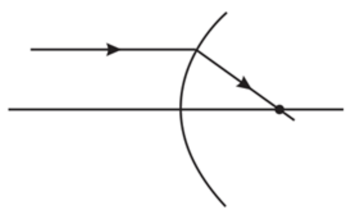
(C) 2 $cm$

(D) 5 $cm$

Solution: (A)

We know eye lens is convex refracting surface.

Use expression for refraction from curves surface



$\mu_1 \rightarrow$ Refractive index of medium having incident ray

$\mu_2 \rightarrow$ Refractive index of medium having reflected ray

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\frac{1.34}{v}-\frac{1}{\infty}=\frac{1.34-1}{7.8}$$

$v=30.7\ mm$

$v=3.07\ cm$

26. $|\vec{A}+\vec{B}|=n|\vec{A}-\vec{B}|$ and $|A|=|B|$ then $\theta=?$

(A) $\theta=\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

(B) $\theta=\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

(C) $\theta=\sin^{-1}\left(\frac{n-1}{n+1}\right)$

(D) $\theta=\cos^{-1}\left(\frac{n-1}{n+1}\right)$

Solution: (A)

Given $|\vec{A}+\vec{B}|=n|\vec{A}-\vec{B}|$

$\sqrt{A^2+B^2+2AB\cos\theta}=n\left(\sqrt{A^2+B^2-2AB\cos\theta}\right)$

Also $|A|=|B|$

$\sqrt{2A^2+2A^2\cos\theta}=n\sqrt{2A^2-2A^2\cos\theta}$

Squaring both sides:

$2A^2(1+\cos\theta)=n^2 2A^2(1-\cos\theta)$

$$\cos\theta=\frac{n^2-1}{n^2+1}$$

i.e., $\theta=\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

27. A calorimeter of Brass is of 128 gms, contains 240 gm of water. 192 $gm$ of unknown material initially at $100^oC$ is placed in calorimeter. Initial temperature of water is $8.4^oC$. The final temperature of mixture is $21.5^oC$. Then specific heat of unknown material is. ($S_{brass}=346\ \mathrm{J/kg-k}$)

(A) $600\ \mathrm{J/kg-k}$

(B) $900\ \mathrm{J/kg-k}$

(C) $850\ \mathrm{J/kg-k}$

(D) $750\ \mathrm{J/kg-k}$

Solution: (C)

Conservation of energy:

$m_bS_b\Delta T_b=(m_cS_c+m_wS_w)\Delta T_c$

$(192)(s_b)(100-21.5)=(128\times346+240\times4200)\times(21.5-8.4)$

$$s_b=914.6\ J/kg-k$$

JEE Main 2019 - 10th Jan Paper - Slot 2 - 02:30PM to 05:30PM

# Chemistry

**Single correct answer type:**

1. Last electron in atom with atomic number 71 goes into which orbital?

(A) $6s$

(B) $5p$

(C) $5d$

(D) $4f$

Solution: (C)

$_{71}Lu$ : $[Xe]\ 4f^{14}6s^{2}5d^{1}$ (Last electron enters $5d$ orbital)

2. Charge on gold sol and hemoglobin is respectively:

(A) $+ve, +ve$

(B) $-ve, -ve$

(C) $+ve, -ve$

(D) $-ve, +ve$

Solution: (D)

Fact.

3. Which of the following reactions requires catalyst?

(A) $H_2 + F_2 \rightarrow 2HF$

(B) $H_2 + Cl_2 \rightarrow 2HCl$

(C) $H_2 + Br_2 \rightarrow 2HBr$

(D) $H_2 + l_2 \rightarrow 2Hl$

Solution: (D)

$I_2$ is least reactive halogen. So its reaction with $H_2$ required catalyst.

4. In $B_2H_6$, how many $3C - 2e^{-}$& $2C - 2e^{-}$ bonds are present?

(A) $2, 4$

(B) $4, 2$

(C) $2, 2$

(D) $2, 1$

Solution: (A)

$B_2H_6$ has 2 bridge bonds $(3C - 2e^-)$ and 4 terminal bonds $(2C - 2e^-)$.

5. Energy of electron in ground state of Hydrogen atom is $-13.6\ eV$. Find energy of electron in $2^{nd}$ excited state $He^+$.

(A) $-6.04\ eV$

(B) $-3.4\ eV$

(C) $-55.5\ eV$

(D) $-27.2\ eV$

Solution: (A)

$$E = -13.6\left(\frac{z^2}{n^2}\right) = -13.6\left(\frac{2^2}{3^2}\right) = -6.04\ eV$$

6. $2l, 0.1\ M\ C_{12}H_{22}O_{11}$ solution is to be prepared. How much of sucrose is required?

(A) $68.4\ g$

(B) $34.2\ g$

(C) $136.8\ g$

(D) $180\ g$

Solution: (A)

Moles of sucrose required $= 2 \times 0.1 = 0.2$

$wt. = 0.2 \times 342\ g$

$= 68.4\ g$

7. $Co(III)$ chloride & $en$ react in 1: 2 mole ratio. Two products are obtained $-(A)$ Violet & $(B)$ Green. One is optically active and other is inactive. Find relation between $A$ & $B$.

(A) Geometrical isomer

(B) Linkage isomer

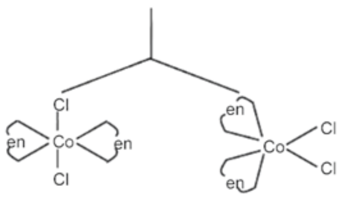
(C) Ionisation isomer

(D) Coordination isomer

Solution: (A)

$CoCl_3 + en \rightarrow [Co(en)_2Cl_2]\ Cl$

1 mole  2 mole




Geometrical Isomers

8. In reaction of $C_2O_4^{2-}$ with $KMnO_4$ in acidic medium, no. of $e^-$ transferred for formation of 1 molecule of $CO_2$ is:

(A) 1

(B) 10

(C) 5

(D) 2

Solution: (A)

$2\ MnO_4^- + 5C_2O_4^{2-} \rightarrow 10\ CO_2 + 2\ Mn^{2+}$

Total $10e^-$ transfer for 10 molecules of $CO_2$

Hence for 1 molecule of $CO_2$, only one $e^-$ is transferred.

$2\ MnO_4^- + 5C_2O_4^{2-} \rightarrow 10CO_2 + 2\ Mn^{2+}$

9. $A_2B_3$ has $HCP$ lattice. Which atom forms lattice & other atom occupies what?

(A) $B$ forms $HCP$ lattice; $A$ occupies $\frac{1}{3}$ tetrahedral voids.

(B) $A$ forms $HCP$ lattice; $B$ occupies $\frac{1}{3}$ tetrahedral voids.

(C) $B$ forms $HCP$ lattice; $B$ occupies $\frac{2}{3}$ tetrahedral voids.

(D) $A$ forms $HCP$ lattice; $B$ occupies $\frac{2}{3}$ tetrahedral voids.

Solution: (A)

$B$ forms $HCP$ $\quad\therefore$ No of $B = 6$

$A$ occupies $\frac{1}{3}TV$ $\quad\therefore$ No. of $A = \frac{1}{3} \times 12 = 4$

$\therefore$ Formula = $A_4B_6 = A_2B_3$

10. Which of the following has 2 $P - H$ bonds?

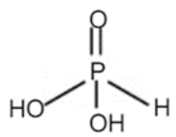
(A) $H_3PO_3$ and $H_4P_2O_5$

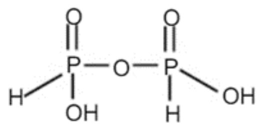
(B) $H_3PO_2$ and $H_4P_2O_5$

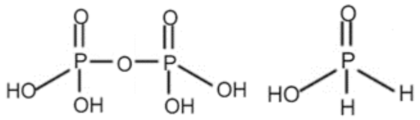
(C) $H_4P_2O_7$ and $H_4P_2O_5$

(D) $H_3PO_2$ and $H_3PO_3$

Solution: (B)





11. In which of the following, entropy decreases?

(A) Sublimation of dry ice

(B) $N_2 + 3H_2 \rightarrow 2NH_3$

(C) $CaCO_3 \rightarrow CaO + CO_2$

(D) Dissolution of $l_2$

Solution: (B)

$CO_2(s) \rightarrow CO_2(g) \quad ; \Delta S = +ve$

$N_2 + 3H_2 \rightarrow 2NH_3 \; ; \; \Delta S = -ve$

$CaCO_3 \rightarrow CaO + CO_2 \; ; \Delta S = +ve$

Dissolution of $I_2 I_2 \; ; \Delta S = +ve$

12. If elevation in boiling point of 1 molal solution of a non volatile solute is $2^o C$ and depression in freezing point of 2 molal solution of the same solute in same solvent is $2^o C$. Identify the relation between $k_b$ & $k_f$.

(A) $k_b = k_f$

(B) $k_b = 0.5\, k_f$

(C) $k_b = 4\, k_f$

(D) $k_b = 2\, k_f$

Solution: (D)

$\Delta T_b = ik_b m \Rightarrow 2 = ik_b \times 1$ ....(i)

$\Delta T_f = ik_f m \Rightarrow 2 = ik_f \times 2$ .....(ii)

$$1 = \frac{k_b}{2k_f}$$

$k_b = 2\, k_f$

13. Sodium in liquid ammonia shows deep blue colour. Which of the following is reason for the same?

(A) Ammoniated electron

(B) Complex formation

(C) Sodamide

(D) Ammoniated sodium ion

Solution: (A)

Reason of colour:

$$\underset{\text{Ammoniated electron}}{\underset{\downarrow}{\left[e^{-}(NH_3)_y\right]}}$$

14. For electroplating of $Ag$ & $Au$, which electrolytes are used?

(A) $[AgCl_2]^{-}\&[Au(NH_3)_2]^{+}$

(B) $[Ag(CN)_2]^{-}\&[Au(NH_3)_2]^{+}$

(C) $[Ag(CN)_2]^{-}\&[Au(CN)_2]^{-}$

(D) $[Ag(NH_3)_2]^{+}\&[Au(NH_3)_2]^{+}$

Solution: (C)

Fact

15. $A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$ ; $\frac{d[A]}{dt} = ?$

(A) $2k_1[A_2] - 2k_{-1}[A]^2$

(B) $k_1[A_2] + k_{-1}[A]^2$

(C) $k_1[A_2] - k_{-1}[A]^2$

(D) $2k_1[A_2] + k_{-1}[A]^2$

Solution: (A)

$$\frac{d[A]}{dt} = 2k_1[A_2] - 2k_{-1}[A]^2$$

16. 5.1 $g$ $NH_4HS$ decomposes partially with degree of dissociation 30% according to reaction:

$NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$

If $V_{\text{container}} = 3L, T = 327^oC$ & $R = 0.0821$ $L$ atm $K^{-1}$ $\text{mol}^{-1}, K_P = ?$

(A) 0.242

(B) $2.42 \times 10^{-4}$

(C) $2.42 \times 10^{-2}$

(D) $2.42 \times 10^{-3}$

Solution: (A)

$NH_4HS \rightarrow \quad H_2S + NH_3$

$0.1\ (1-0.3)\ 0.03 \quad 0.03$

$K_C = \frac{0.03}{3} \times \frac{0.03}{3} = 10^{-4}$

$K_P = K_C(RT)^{\Delta n}$

$= 10^{-4} \times (0.821 \times 600)^2 = 0.242$

17. A gas undergoes isothermal compression from $5m^3$ to $1m^3$ against constant external pressure of $4\ N/m^2$. Heat released in this process is used to heat 1 mole $Al$. Then calculate rise in temperature. [Specific heat of $Al = 24\ J/mol$]

(A) $\frac{2}{3}K$

(B) $\frac{1}{3}K$

(C) $2K$

(D) $1K$

Solution: (A)

$W = -4\ [1-5] = 16J$

$16 = 1 \times 24 \times \Delta T$

$\Delta T = \frac{2}{3}K$

18. Consider the cell

$Pt|H_2(1\ \text{atm})|\ HCl\ (10^{-6}M)|AgCl|Ag|Pt$

Given: $E_{cell} = 0.92$ Volt

Calculate the $E^o$ for $Cl^-/AgCl/Ag$

(A) 0.76

(B) 0.21

(C) 0.36

(D) 0.46

Solution: (B)

$A: \frac{1}{2}H_2(g) \rightarrow H^+\ (aq.) + e^- \quad E^o = 0.0\ V$

$C: AgCl\ (s) + e^- \rightarrow Ag\ (s) + Cl^-\ E^o = xV$

$E_{cell} = E^o_{cell} - \frac{0.0591}{1}\log\{[H^+][Cl^-]\}$

$0.92 = x - \frac{0.591}{1}\log(10^{-12})$

$0.92 = x + 0.7092$

$x = 0.92 - 0.7092$

$= 0.2108$ Volt

19. Metal forms octahedral complex having difference in unpaired electrons in high spin complex and low spin complex as 2. Then metal ion will be:

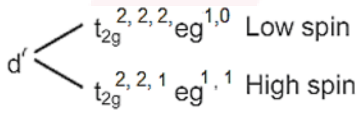
(A) $Fe^{+2}$

(B) $Co^{+2}$

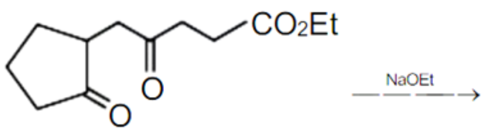
(C) $Ni^{+2}$

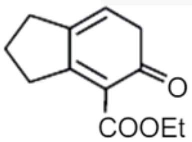
(D) $Mn^{+2}$

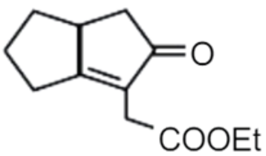
Solution: (B)

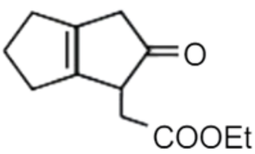


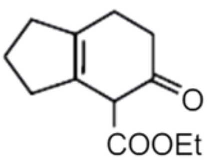

20. The product of the following reaction



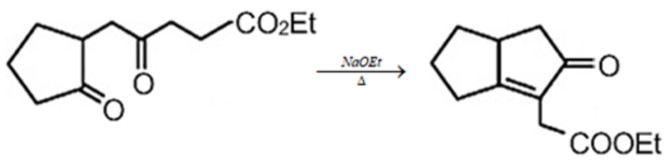

(A) 


(B) 


(C) 


(D) 


Solution: (B)



Intramolecular aldol condensation

21. Find the product in the following reaction

$$H_3C-N=CH-CH=CH-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{NaBH_4}$$

(A) $H_3C-N=CH-CH_2-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3$

(B) $H_3C-NH-CH_2-CH_2-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3$

(C) $H_3C-NH-CH_2-CH-CH-\underset{\underset{O}{\|}}{C}-CH_3$

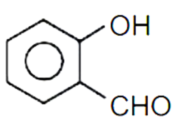
(D) $H_3C-N=CH-CH=CH-\underset{\underset{OH}{|}}{CH}-CH_3$

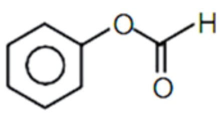
Solution: (D)

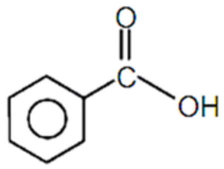
$NaBH_4$ selectively reduce $-\underset{\underset{O}{\|}}{C}-$ group $-\underset{\underset{OH}{|}}{CH}-$ to

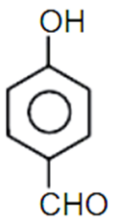
22. In the following reaction sequence the first reactin $(X)$ will be:

$X(C_7H_6O_2) \xrightarrow{NH_3/\Delta} C_7H_7ON \xrightarrow{Br_2/NaOH} C_6H_7N$

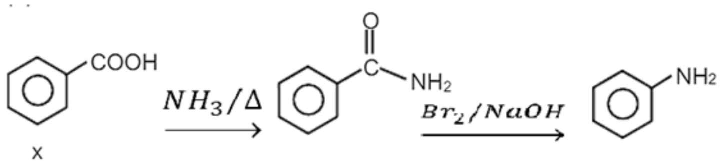
(A) 

(B) 

(C) 

(D) 

Solution: (C)




23. Which of the following is not the test of amino acid.

(A) Ninhydrine test

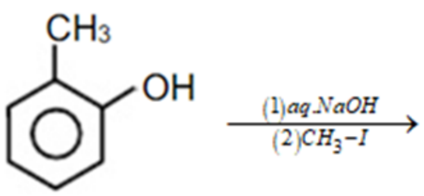
(B) Biuret test

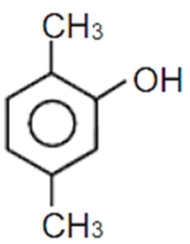
(C) Xanthoproteic test

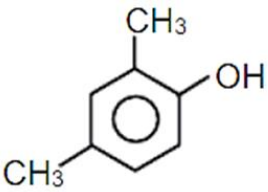
(D) Bourdieu test

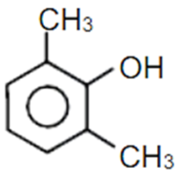
Solution: (D)

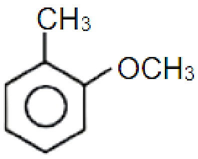
24. The final product in the following reaction



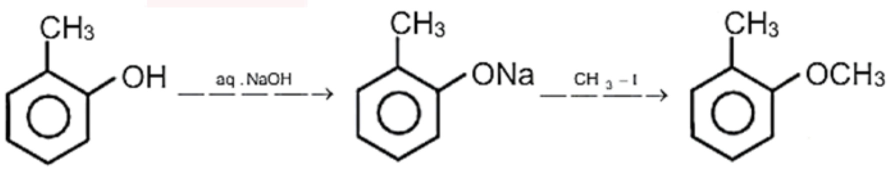

(A) 

(B) 

(C) 

(D) 

Solution: (D)



25. Which reaction do not show ozone depletion?

(A) $CH_4 + 2O_3 \rightarrow 3CH_2 + 3H_2O$

(B) $HOCl \rightarrow HO^{\cdot} + Cl^{\cdot}$

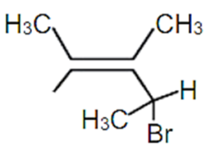
(C) $CF_2Cl_2 \rightarrow Cl^{\cdot} + \dot{C}\,F_2a$

(D) $ClO^{\cdot} + [O] \rightarrow Cl^{\cdot} + O_2$

Solution: (A)

Factual

26. $IUPAC$ name of the compound?



(A) 4 −Bromo−3 −methyl pent −2 −ene

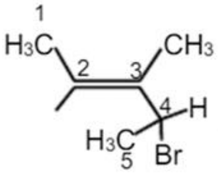
(B) 2 −Bromo−3 −methyl pent −3 −ene

(C) 1 −Methyl−1 −bromo but−2 −ene

(D) 4 −Bromo−4 −methyl but−2 −ene

Solution: (A)

IUPAC refer



(4-Bromo-3-methyl pent-2-ene)

27. The suitable reagent for the following conversion:

$$CH_3-CH_2-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3 \longrightarrow CH_3-CH_2-CH_2-COOH$$

(A) $2,4-DNP$

(B) Lucas-reagent

(C) $KMnO_4/H^{\oplus}$

(D) $I_2/NaOH$

Solution: (D)

$$CH_3-CH_2-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3 \longrightarrow CH_3-CH_2-CH_2-COOH$$

It is iodoform reaction.

28. Which of the following is correct for matching the compounds of column$-I$ with the test of column$-II$

| **Column – I** | | **Column – II** | |
|---|---|---|---|
| $(A)$ | Lysine | $(P)$ | Cerric ammonium nitrate |
| $(B)$ | Carbohydrate | $(Q)$ | Furfural |
| $(C)$ | Benzyl alcohol | $(R)$ | $KMnO_4$ |
| $(D)$ | Styrene | $(S)$ | Ninhydrin |

(A) $A-S, B-Q, C-P, D-R$

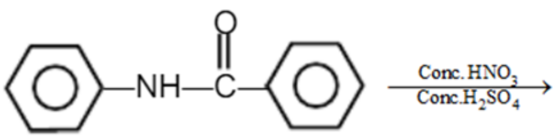
(B) $A-Q, B-R, C-S, D-P$

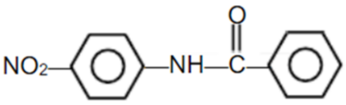
(C) $A-S, B-R, C-Q, D-P$

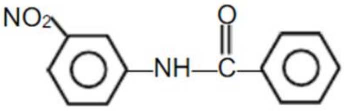
(D) $A-P, B-R, C-Q, D-S$

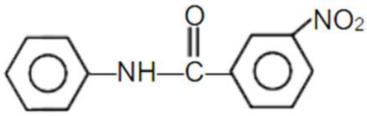
Solution: (A)

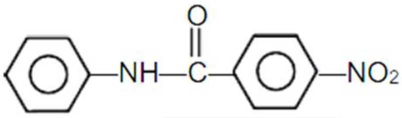
29. The major product in the following reaction:



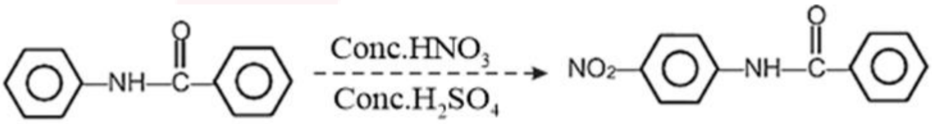
(A) 

(B) 

(C) 

(D) 

Solution: (A)



JEE Main 2019 - 10th Jan Paper - Slot 2 - 02:30PM to 05:30PM

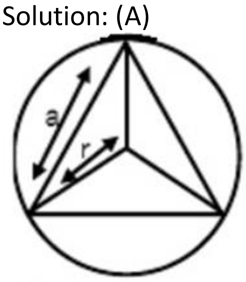
# Mathematics

**Single correct answer type:**

1. If $z = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then
(A) $Im(z) = 0$
(B) $Re(z) > 0, Im(z) > 0$
(C) $Re(z) > 0, Im\ (z) < 0$
(D) $Re(z) = 3$
Solution: (A)
Let $\alpha = \frac{\sqrt{3}}{2} + \frac{i}{2}$, then $z = \alpha^5 + \bar{\alpha}^5 = \alpha^5 + \overline{\alpha^5} = 2\ Re\ (\alpha^5)$
Hence $Im\ (z) = 0$

2. If $\sum_{r=0}^{25} {}^{50}C_r \left({}^{50-r}C_{25-r}\right) = k\left({}^{50}C_{25}\right)$, then $k$ is equal to
(A) $2^{25}$
(B) $2^{25} - 1$
(C) $2^{24}$
(D) $25^2$
Solution: (A)
$\sum_{r=0}^{25} {}^{50}C_r\ {}^{5-r}C_{25-r}$
$= \sum_{r=0}^{25} \frac{50!}{r!(50-r)!} \frac{(50-r)!}{(25-r)!(25)!}$
$= \sum_{r=0}^{25} \frac{50!\ 25!}{r!(25-r)!(25)!(25)!}$
$= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r$
$= {}^{50}C_{25} \times 2^{25}$
$\Rightarrow k = 2^{25}$

3. If area of an equilateral triangle inscribed in the circle $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$, then the value of $c$ is
(A) 25
(B) $-25$
(C) 36
(D) $-36$
Solution: (A)




$\frac{\sqrt{3}}{4} a^2 = 27\sqrt{3}$

$\Rightarrow a = 6\sqrt{3}$

$\Rightarrow r = a \times \dfrac{\sqrt{3}}{2} \times \dfrac{2}{3}$

$\Rightarrow r = 6$

$\Rightarrow \sqrt{5^2 + 6^2 - c} = 6$

$\Rightarrow 25 + 36 - c = 36$

$\Rightarrow c = 25$

4. A curve which satisfies the differential equation $(x^2 - y^2)\, dx + 2xy\, dy = 0$ passes through $(1, 1)$, then curve is

(A) A circle with centre on $x$ −axis

(B) A circle with centre on $y$ −axis

(C) A hyperbola with transverse axis as $x$ −axis

(D) An ellipse with major axis as $y$ −axis

Solution: (A)

$x^2dx - y^2dx + 2xy\, dy = 0$

$x^2dx = y^2dx - 2x\, ydy$

$\Rightarrow -dx = d\left(\dfrac{y^2}{x}\right)$

$\Rightarrow\ -x = \dfrac{y^2}{x} + C$

$\Rightarrow x^2 + y^2 + Cx = 0$

$\because$ It passes through $(1, 1)$ hence $C = -2$

$\Rightarrow x^2 + y^2 - 2x = 0$

5. In a $\Delta ABC, \angle A + \angle B = 120^o, a = \sqrt{3} + 1, b = \sqrt{3} - 1$, then the ratio of $\angle A$ to $\angle B$ is

(A) $7: 1$

(B) $5: 1$

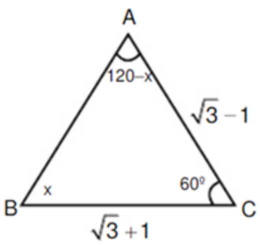
(C) $3: 1$

(D) $5: 3$

Solution: (A)

$\frac{\sqrt{3}+1}{\sin(120-x)} = \frac{\sqrt{3}-1}{\sin x}$ (Using sine rule)

$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sin(120^o-x)}{\sin x} = \sin 120^o \cos x - \cos 120^o \sin x$




$\Rightarrow \dfrac{\sqrt{3}+1}{\sqrt{3}-1} - \dfrac{1}{2} = \dfrac{\sqrt{3}}{2}\cot x$

$\Rightarrow \dfrac{3 + 2\sqrt{3}}{2} = \dfrac{\sqrt{3}}{2}\cot x$

$\Rightarrow \cot x = \sqrt{3} + 2$

$\Rightarrow \tan x = 2 - \sqrt{3}$

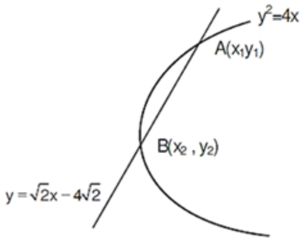
$\Rightarrow x = 15^o$

$\Rightarrow 120 - x = 105^o$

$\therefore \frac{\angle A}{\angle B} = \frac{7}{1}(7:1)$

6. The mean and standard deviation of five observations $x_1, x_2, x_3, x_4, x_5$ are 10 and 3 respectively, then variance of the observations $x_1, x_2, x_3, x_4, x_5, -50$ is equal to
(A) 437.5
(B) 507.5
(C) 537.5
(D) 487.5
Solution: (B)
Mean $= \frac{x_1+x_2+x_3+x_4+x_5}{5} = 10 \Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 50$ ...(i)
Variance $= \frac{\Sigma x_1^2}{5} - (\bar{x})^2 = 9$
$\Rightarrow \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - 100 = 9$
$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 545$ ...(ii)
Now $\bar{x}_{\text{new}} = \frac{x_1+x_2+x_3+x_4+x_5-50}{6} = 0$
Variance new $= \frac{\sum_{i-1}^{6} x_1^2}{6} - (\bar{x}_{\text{new}})^2$
$= \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + 2500}{6} - 0$
$= \frac{545 + 2500}{6} = \frac{3045}{6} = 507.5$

7. The length of intercept made by the line $\sqrt{2}x - y - 4\sqrt{2} = 0$ on the parabola $y^2 = 4x$ is equal to
(A) $6\sqrt{3}$
(B) $4\sqrt{3}$
(C) $8\sqrt{2}$
(D) $6\sqrt{2}$
Solution: (A)
Solving the equations of the given parabola and the line, we get
$(\sqrt{2}x - 4\sqrt{2})^2 = 4x \Rightarrow x^2 - 10x + 16 = 0 \Rightarrow |x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1x_2} = \sqrt{100 - 64} = 6$




$\Rightarrow$ Length of intercept $= AB = \sqrt{1 + m^2} \cdot |x_1 - x_2|$
$= 6\sqrt{1 + 2} = 6\sqrt{3}$

8. The probability that a shooter hits a target is $\frac{1}{3}$. The minimum number of trials such that the probability of hitting the target at-least once, is greater than $\frac{5}{6}$ is equal to
(A) 4
(B) 5
(C) 6
(D) 7

Solution: (B)

$P(\text{hitting target}) = \dfrac{1}{3}$

Let no. of trials $= n$

$P(\text{hitting target atleast once})$

$= 1 - P$ (not hit)

$= 1 - \left(\dfrac{2}{3}\right)^n > \dfrac{5}{6}$

$\Rightarrow \left(\dfrac{1}{6}\right) > \left(\dfrac{2}{3}\right)^n$

So minimum number of trials $= 5$

9. If two vertices of a triangle are $(0, 2)$ and $(4, 3)$ and its orthocenter is $(0, 0)$, then the third vertex of the triangle lies in

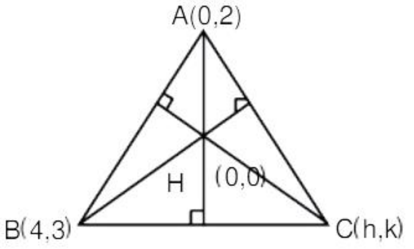
(A) $1^{\text{st}}$ quadrant

(B) $2^{\text{nd}}$ quadrant

(C) $3^{\text{rd}}$ quadrant

(D) $4^{\text{th}}$ quadrant

Solution: (B)




Since $H$ is orthocenter of $\Delta ABC$

Then $C$ is also orthocentre of $\Delta AHB$

Since the slope of $BH$ is $\frac{3}{4}$ hence the slope of $AC$ will be $-\frac{4}{3}$

$\Rightarrow$ Equation of line $AC$ is

$4x + 3y = 6 \quad \ldots(i)$

Similarly, since $AH$ is parallel to y -axis hence $BC$ will be parallel to x - axis and

Equation of line $BC$ is

$y = 3 \qquad \ldots(ii)$

On solving $(i)$ and $(ii)$

$h = -\dfrac{3}{4}, k = 3$

$\Rightarrow$ point $C(h, k)$ lies in $2^{nd}$ quadrant

10. The value of $\cot \sum_{n=1}^{19} \left(\cot^{-1}\left(1 + \sum_{p=1}^{n} 2p\right)\right)$ is equal to

(A) $\frac{21}{19}$
(B) $\frac{19}{21}$
(C) $-\frac{19}{21}$
(D) $-\frac{21}{19}$

Solution: (A)

$\cot\sum_{n=1}^{19}\left(\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$

$$= \cot\sum_{n=1}^{19}\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)$$

$$= \cot\sum_{n=1}^{19}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)$$

$$= \cot\sum_{n=1}^{19}(\tan^{-1}(n+1)-\tan^{-1}(n))$$

$$= \cot(\tan^{-1}20-\tan^{-1}1)$$

$$= \frac{1}{\tan(\tan^{-1}20-\tan^{-1}1)}$$

$$= \frac{1}{\frac{20-1}{1+(20).1}} = \frac{21}{9}$$

Alternate Solution:

$$\cot\sum_{n=1}^{19}(\cot^{-1}(1+\sum_{p=1}^{n}2p)) = \cot\sum_{n=1}^{19}\cot^{-1}(1+2)(2+3+4\ldots n)]$$

$$= \cot\sum_{n=1}^{19}\left(\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right) = \cot\sum_{n=1}^{19}\left(\cot^{-1}(n^2+n+1)\right)$$

11. A helicopter flying along the path $y = 7 + x^{\frac{3}{2}} \cdot A$ soldier standing at point $\left(\frac{1}{2}, 7\right)$ wants to hit the helicopter when it is closest from him, then minimum distance is equal to

(A) $\frac{1}{6}\frac{\sqrt{2}}{3}$
(B) $\frac{1}{2}$
(C) $\frac{1}{3}\sqrt{\frac{2}{3}}$
(D) $\sqrt{\frac{5}{2}}$

Solution: (A)

$$y = 7 + x^{\frac{3}{2}}$$

Let the point on curve be $P\left(x_1 7 + x_1^{\frac{3}{2}}\right)$ and given point be $A(\frac{1}{2}, 7)$

For nearest point normal at P passes through $A$

So slope of line $AP$ = Slope of normal at $P$

$$\Rightarrow \frac{x_1^{\frac{3}{2}}}{x_1-\frac{1}{2}} = -\frac{dx}{dy}\bigg|(x_1, y_1) = -\frac{2}{3\sqrt{x_1}}$$

$\Rightarrow 3x_1^2 = 1 - 2x_1$

$\Rightarrow 3x_1^2 + 2x_1 - 1 = 0$

$\Rightarrow (x_1 + 1)(3x_1 - 1) = 0$

$\Rightarrow x_1 = \frac{1}{3}$ ($x_1 = -1$ is not possible as $x_1 > 0$)

Hence point $P$ is $\left(\frac{1}{3}, 7 + \frac{1}{3\sqrt{3}}\right)$

So $AP = \sqrt{\frac{1}{36} + \frac{1}{27}} = \frac{1}{6}\sqrt{\frac{7}{3}}$

12. The value of $\lambda$ for which the sum of squares of roots of the equation $x^2 + (3 - \lambda)\,x + 2 = \lambda$ is minimum, then $\lambda$ is equal to
(A) 2
(B) $-1$
(C) $-3$
(D) $-2$
Solution: (A)

$$x^2 + (3 - \lambda)x + 2 = \lambda <_{\beta}^{\alpha}$$

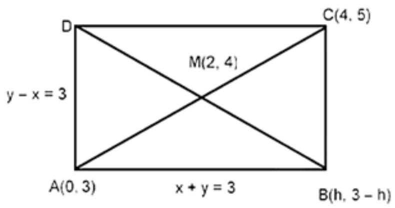
$$f(\lambda) = \alpha^2 + \beta^2$$

$$= (\alpha + \beta)^2 - 2\alpha\beta = (3 - \lambda)^2 - 2(2 - \lambda) = \lambda^2 = 6\lambda + 9 - 4 + 2\lambda = \lambda^2 - 4\lambda + 5$$

$f'(\lambda) = 2\lambda - 4 = 0$ {for minimum value of $f(\lambda)$}

$\Rightarrow \lambda = 2$

13. Two sides of a parallelogram are $x + y = 3$ and $y - x = 3$. If the diagonals meet at $(2, 4)$, then which of the following can be one of the vertex of parallelogram.
(A) $(3, 6)$
(B) $(0, 0)$
(C) $(1, -2)$
(D) $(2, 3)$
Solution: (A)



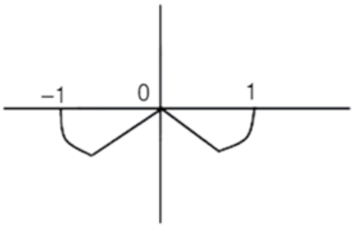

Slope of $BC = 1 \Rightarrow \frac{3-h-5}{h-4} = 1$

$\Rightarrow \ -h - 2 = h - 4 \Rightarrow \ 2h = 2 \Rightarrow \ h = 1$

$\Rightarrow B(1, 2)$ and $D(3, 6)$

14. Let $f: (-1, 1) \to R$ be defined as $f(x) = \max\left(-|x|, -\sqrt{1 - x^2}\right)$, then number of points where it is non-differentiable are equal to

(A) 1
(B) 2
(C) 3
(D) 5
Solution: (C)




From the graph it can be concluded than $f(x)$ is non-differentiable at $x = 0,\ \pm\frac{1}{\sqrt{2}}$ in $(-1, 1)$

i.e., at 3 points in $(-1, 1)$

15. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$ is equal to
(A) $\frac{3}{20}(4\pi - 3)$
(B) $\frac{3}{10}(4\pi - 3)$
(C) $\frac{1}{12}(7\pi - 5)$
(D) $\frac{1}{12}(7\pi - 3)$
Solution: (A)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4} = \int_{-\frac{\pi}{2}}^{0} \frac{dx}{[x]+3} + \int_{0}^{\frac{\pi}{2}} \frac{dx}{[x]+4} = \int_{-\frac{\pi}{2}}^{-1} \frac{dx}{1} + \int_{-1}^{0} \frac{dx}{2} + \int_{0}^{1} \frac{dx}{4} + \int_{1}^{\frac{\pi}{2}} \frac{dx}{5}$$

$$= [x]_{-\frac{\pi}{2}}^{-1} + \left[\frac{x}{2}\right]_{-1}^{0} + \left[\frac{x}{4}\right]_{0}^{1} + \left[\frac{x}{5}\right]_{1}^{\frac{\pi}{2}} = \left(-1 + \frac{\pi}{2}\right) + \left(0 + \frac{1}{2}\right) + \frac{1}{4} + \frac{\pi}{10} - \frac{1}{5}$$

$$= \frac{-20 + 10\pi + 10 + 5 + 2\pi - 4}{20} = \frac{12\pi - 9}{20} = \frac{3}{20}(4\pi - 3)$$

16. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$, where $c$ is constant of integration then $f(x)$ equals to
(A) $-4x^3 - 1$
(B) $-1 - 2x^3$
(C) $4x^3 + 1$
(D) $1 - 2x^3$
Solution: (A)

$\int x^5 e^{-4x^3} dx = I$ (say)

Let $-4x^3 = t \Rightarrow -12x^2 dx = dt$

$$\Rightarrow I = \int \left(-\frac{t}{4}\right) . e^t \left(-\frac{dt}{12}\right) = \frac{1}{48} \int te^t dt$$

$$= \frac{1}{48}\left[te^t - \int e^t dt\right] = \frac{(t-1)e^t}{48} + C = \frac{(-4x^3 - 1)e^{-4x^3}}{48} + C$$

Hence $f(x) = -(4x^3 + 1)$

17. If $f(x)$ is a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}, f(1) \neq 4$, then $\lim_{x\to 0} x.f\left(\frac{1}{x}\right)$ is equal to

(A) Does not exist

(B) Exist and equal to 4

(C) Exist and is equal to $\frac{4}{7}$

(D) Exist and equal to 0

Solution: (B)

$\frac{dy}{dx} + \frac{3y}{4x} = 7$ (where $y = f(x)$)

This is a linear differential equation

Integrating factor $= e^{\int \frac{3\,dx}{4x}} = e^{\frac{3}{4}lnx} = x^{\frac{3}{4}}$

So integrating the given differential equation using I.F.

$$\Rightarrow y.x^{\frac{3}{4}} = 7.\frac{4x^{\frac{7}{4}}}{7} + C$$

$$\Rightarrow y\left(x^{\frac{3}{4}}\right) = 4x^{\frac{7}{4}} + C$$

$$\Rightarrow f(x) = 4x + Cx^{\frac{-3}{4}}$$

$$\Rightarrow \lim_{x\to 0^+} xf\left(\frac{1}{x}\right) \Rightarrow \lim_{x\to 0^+} x\left(\frac{4}{x} + Cx^{\frac{3}{4}}\right) = 4$$

18. The equation $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$

(A) Represents a hyperbola of eccentricity equal to $\frac{2}{\sqrt{r+1}}$ if $r \in (0,1)$

(B) Represents a hyperbola of eccentricity equal to $\sqrt{\frac{1-r}{1+r}}$ if $r \in (0,1)$

(C) Represents an ellipse of eccentricity equal to $\sqrt{\frac{2}{r+1}}$ if $r > 1$

(D) Represents an ellipse of eccentricity equal to $\sqrt{\frac{r+1}{2}}$ if $r > 1$

Solution: (C)

$$\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$$

If $r \in (0,1)$ then this curve is a hyperbola whose

Eccentricity is $\sqrt{1 + \frac{1-r}{1+r}} = \sqrt{\frac{2}{r+1}}$

If $r > 1$, then this curve is an ellipse whose

Eccentricity is $\sqrt{1 - \frac{r-1}{r+1}} = \sqrt{\frac{2}{r+1}}$

19. If matrix $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$, then minimum value of $\frac{|A|}{b}$ is equal to

(A) $2\sqrt{3}$
(B) $-2\sqrt{3}$
(C) $\sqrt{3}$
(D) $-\sqrt{3}$

Solution: (A)

$|A| = \begin{vmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix}$

$= 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$

$= 2(b^2 + 2) - b^2 - 1$

$= 2b^2 + 4 - b^2 - 1$

$= b^2 + 3$

$\frac{|A|}{b} = b + \frac{3}{b} = \left(\sqrt{b} - \sqrt{\frac{3}{b}}\right)^2 + 2\sqrt{3}$

Hence minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$

20. Let $\bar{a}$ and $\bar{b}$ are non-collinear vectors. If vectors $\vec{\alpha} = (\lambda - 2)\,\bar{a} + \bar{b}$ and $\vec{\beta} = (4\lambda - 2)\,\bar{a} + 3\bar{b}$ are collinear, then $\lambda$ is equal to

(A) $-4$
(B) 4
(C) 2
(D) $-2$

Solution: (A)

$\because\ \bar{\alpha}, \bar{\beta}$ are collinear

$\Rightarrow \frac{\lambda - 2}{4\lambda - 2} = \frac{1}{3} \Rightarrow \lambda = -4$

21. Tangent drawn at point $(1, e)$ on the curve $y = xe^{x^2}$, also passes through the point

(A) $(4/3, 2e)$
(B) $(5/3, e)$
(C) $(4/3, 3e)$
(D) $(3/4, 3e)$

Solution: (A)

$y = xe^{x^2}$

$\Rightarrow \frac{dy}{dx} = 2x^2e^{x^2} + e^{x^2}$

$\Rightarrow \frac{dy}{dx}$ at $(1, e) = 3e$

$\Rightarrow$ Equaton of tangent is $y - e = 3e(x - 1)$ which passes through $\left(\frac{4}{3}, 2e\right)$

22. A plane bisects the line segment joining $(-3,-3,4)$ and $(3,7,6)$ and also perpendicular to this line, then a point lying on this plane can be
(A) $(1,-2,3)$
(B) $(1,2,2)$
(C) $(3,-5,2)$
(D) $(3,-1,0)$
Solution: (B)
The midpoint of line joining the given points is $(0,2,5)$ and it lies on the required plane.

Normal to the plane, the line joining the given points, has direction ratio $6,10,2$.

$\Rightarrow$ equation of plane is $3(x-0)+5(y-2)+(z-5)=0$

$\Rightarrow 3x+5y+z=15$

23. The number of possible values of $\theta$ which lies in $(0,\pi)$, such that system of equation $x+3y+7z=0, -x+4y+7z=0, x\sin 3\theta + y\cos 2\theta + 2z = 0$ has a non-trivial solution, is/are equal to
(A) 2
(B) 3
(C) 5
(D) 4
Solution: (A)
For non-trivial solutions

$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$\Rightarrow 4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{2}$ or $\sin\theta = -\frac{3}{2}$

Since $\theta \in (0,\pi)$

$\Rightarrow \theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$ will satisfy the equation

$\Rightarrow$ Number of $\theta's = 2$

24. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then $f\left(\frac{1}{2}\right)$ is equal to
(A) $\frac{24}{25}$
(B) $\frac{4}{25}$
(C) $\frac{4}{5}$
(D) $\frac{2}{5}$
Solution: (C)
Differentiating both side

$f(x) = 2x - x^2 f(x)$

$$\Rightarrow f(x) = \frac{2x}{1+x^2}$$

$\Rightarrow f\left(\frac{1}{2}\right) = \frac{2.\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2} = \frac{4}{5}$

25. Which of the following line is passing through the point of intersection of the line $\frac{x-4}{2}=\frac{y-3}{2}=\frac{z-5}{1}$ and the plane $x+y+z=2$ ?
(A) $\frac{x}{2}=\frac{y+1}{3}=\frac{z-3}{4}$
(B) $\frac{x-2}{3}=\frac{y-1}{3}=\frac{z-3}{2}$
(C) $\frac{x}{2}=\frac{y+1}{3}=\frac{z-2}{4}$
(D) $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z+3}{4}$
Solution: (A)
A general point on the given line can we taken as
$$\frac{x-4}{2}=\frac{y-3}{2}=\frac{z-5}{1}=\lambda$$
If it also lies on the given plane then

$(2\lambda+4)+(2\lambda+3)+(\lambda+5)=2$

$\Rightarrow 5\lambda+12=2$

$\Rightarrow \lambda=-2$

$\Rightarrow$ Point of intersection is $(0,-1,3)$

26. If $a_1, a_2 \ldots\ldots a_{10}$ are in G.P., where $a_i>0$ and $S$ is a set of ordered pairs $(r,k)$ such that $\begin{vmatrix} \ln a_1^r a_2^k & \ln a_2^r a_3^k & \ln a_3^r a_4^k \\ \ln a_4^r a_5^k & \ln a_5^r a_6^k & \ln a_6^r a_7^k \\ \ln a_7^r a_8^k & \ln a_8^r a_9^k & \ln a_9^r a_{10}^k \end{vmatrix}=0$, then number of ordered pairs $(r,k)$ is
(A) Infinitely many
(B) 1
(C) 5
(D) 3
Solution: (A)
$C_3 \to C_3-C_2,\ C_2 \to C_2-C_1$ (Let $\alpha$ is common ratio of $GP$)

$$\begin{vmatrix} \ln a_1^r & \ln(\alpha^{r+k}) & \ln(\alpha^{r+k}) \\ \ln a_4^r a_5^k & \ln(\alpha^{r+k}) & \ln(\alpha^{r+k}) \\ l\, n a_7^r a_8^k & \ln(\alpha^{r+k}) & \ln(\alpha^{r+k}) \end{vmatrix}=0 \quad \text{which is always true}$$

27. Given three statements
$P$: 5 is a prime number
$Q$: 7 is a factor of 192
$R$: The LCM of 5 & 7 is 35
Then which of the following statements are true
(A) $P \vee (\sim Q \wedge R)$
(B) $\sim P \wedge (\sim Q \wedge R)$
(C) $(P \vee Q) \wedge \sim R$
(D) $\sim P \wedge (\sim Q \vee R)$
Solution: (A)
$P$ is true

$Q$ is False

$R$ is True

(1) $P \vee (\sim Q \wedge R) = T \vee (T \wedge T) = T$

(2) $F \wedge (T \wedge T) = F$

(3) $(T \vee F) \wedge F = T \wedge F = F$

(4) $F \wedge (T \vee T) = F \wedge T = F$

28. The value of $\cos\frac{\pi}{2^2} \cdot \cos\frac{\pi}{2^3} \cdot \cos\frac{\pi}{2^4} \ldots . \cos\frac{\pi}{2^{10}} \cdot \sin\frac{\pi}{2^{10}}$ is

(A) $\frac{1}{1024}$

(B) $\frac{1}{512}$

(C) $\frac{1}{256}$

(D) $\frac{1}{128}$

Solution: (B)

$$\cos\frac{\pi}{2^2} . \cos\frac{\pi}{2^3} \ldots . \cos\frac{\pi}{2^{10}} . \sin\frac{\pi}{2^{10}}$$

$$= \frac{\sin\left(2^9 . \frac{\pi}{2^{10}}\right)}{2^9 . \sin\frac{\pi}{2^{10}}} . \sin\frac{\pi}{2^{10}} = \frac{1}{2^9}$$

29. If coefficient of $x^2$ in expansion of $x^2\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$ is 720, then $\lambda$ can be equal to

(A) 8

(B) 4

(C) 12

(D) 2

Solution: (D)

Coefficient of $x^2$ in $x^2(\sqrt{x} + \frac{\lambda}{x^2})^{10}$

$=$ coefficient of $x^0$ in $\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$

A general term in the expansion $\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$ is ${}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r = {}^{10}C_r(\lambda)^r(x)^{5-5\frac{r}{2}}$

$\Rightarrow$ coefficient of $x^0 = {}^{10}C_2\lambda^2$ (for $r = 2$)

$= 45\lambda^2$

Now $45\lambda^2 = 720$

$\Rightarrow \lambda = \pm 4$

30. If $f: N \to N$ is given by $f(x) = \begin{cases} \frac{x+1}{2} & \text{if } x \text{ is odd} \\ \frac{x}{2} & \text{if } x \text{ is even} \end{cases}$

and $g: N \to N$ is given by $g(x) = x - (-1)^x$, then $f\left(g(x)\right)$ is

(A) One one and onto

(B) Many one and onto

(C) One one and not onto

(D) Neither one-one nor onto

Solution: (B)

$$g(n) = \begin{cases} n+1 \; if \; n \; odd \\ n-1 \; if \; n \; even \end{cases}$$

$f\big(g(1)\big) = f(2) = 1$

$f\big(g(2)\big) = f(1) = 1$

$\therefore f\big(g(x)\big)$ is many one

$f\big(g(2k)\big) = f(2k-1) = k$

$f\big(g(2k+1)\big) = f(2k+2) = k+1$

$\therefore f(g(x))$ is onto