SOLUTION

Mathematics

1. $\left.\begin{matrix}7B\\5G\end{matrix}\right\} \rightarrow \binom{3B}{2G}$

Total number of ways of forming a team of 3 boys and 2 girls $= {}^7C_3 \times {}^5C_2 = \frac{7.6.5}{1.2.3} \times \frac{5.4}{1.2} = 350$

Total number of ways of forming a term if two specific boys B_1, B_2 always join the team

$= {}^5C_1 \times {}^5C_2 = 50$

So, the total numbers of ways of forming a team is two specific boys never come together

$= 350 - 50 = 300$

2. $x^2 + 2x + 2 = 0 <^{\alpha}_{\beta}$

$\Rightarrow \alpha, \beta = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

$\Rightarrow \alpha, \beta = \sqrt{2}\left(\frac{-1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}\right)$

$= \sqrt{2}\left(\cos\frac{3\pi}{4} \pm i\sin\frac{3\pi}{4}\right)$

$\Rightarrow \alpha^{15} + \beta^{15} = \left[\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^{15} + \left[\sqrt{2}\left(\cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4}\right)\right]^{15}$

$= 2^{\frac{14}{2}}\left[\left(\cos\frac{45\pi}{4} + i\sin\frac{45\pi}{4}\right) + \left(\cos\frac{45\pi}{4} - i\sin\frac{45\pi}{4}\right)\right]$

$= 2.2^{\frac{15}{2}}.\cos\frac{45\pi}{4} = 2^{\frac{17}{2}}.\cos\frac{5\pi}{4} = -2^{\frac{17}{2}}.\frac{1}{\sqrt{2}}$

$= -256$

3. $I = \int_0^{\pi} |\cos x|^3\, dx = 2\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$

$= 2\int_0^{\frac{\pi}{2}} (1 - \sin^2 x)\cos x\, dx$

$= 2\left[\int_0^{\frac{\pi}{2}} \cos x dx - \int_0^{\frac{\pi}{2}} \sin^2 x \cos x\, dx\right]$

$= 2\left[\sin - \frac{\sin^3 x}{3}\right]_0^{\frac{\pi}{2}}$

$= \left[1 - \frac{1}{3} - 0 + 0\right] = \frac{4}{3}$

4. $\Rightarrow I = \int x \sqrt{\frac{2\sin(x^2-1)-\sin2(x^2-1)}{2\sin(x^2-1)+\sin2(x^2-1)}}dx$

Let $x^2 - 1 = \theta,\ x\,dx = \frac{1}{2}d\theta$

$\Rightarrow I = \frac{1}{2}\int \sqrt{\frac{2\sin\theta-\sin2\theta}{2\sin\theta+\sin2\theta}}d\theta$

$= \frac{1}{2}\int \sqrt{\frac{2\sin\theta-2\sin\theta\cos\theta}{2\sin\theta+2\sin\theta\cos\theta}}d\theta$

$= \frac{1}{2}\int \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}d\theta$

$= \frac{1}{2}\int \tan\frac{\theta}{2}d\theta$

$= \frac{1}{2}\frac{ln\left(\sec\frac{\theta}{2}\right)}{\left(\frac{1}{2}\right)} + c$

$= \ln\sec\left(\frac{x^2-1}{2}\right) + c$

5. Since $\bar{a}.\bar{c} = 0$

$\Rightarrow$ Angle 'θ' between $\bar{a} \times \bar{c}$ is $\frac{\pi}{2}$

Now $\bar{a} \times \bar{c} + \bar{b} = 0$

$\Rightarrow \bar{a} \times \bar{c} = -\bar{b}$

$\Rightarrow |\bar{a} \times \bar{c}| = |-\bar{b}|$

$\Rightarrow |\bar{a}||\bar{c}|\sin\theta = |\bar{b}|$

$\Rightarrow |\bar{c}| = \frac{|\bar{b}|}{|\bar{a}|\sin\frac{\pi}{2}} = \frac{\sqrt{1+1+1}}{\sqrt{1+1.1}} = \sqrt{\frac{3}{2}}$

$\Rightarrow |\bar{c}|^2 = \frac{3}{2}$

6. $f(x) = \begin{cases} 5: x \le 1 \\ a + bx: 1 < x < 3 \\ b + 5x: 3 \le x < 5 \\ 30: x \ge 5 \end{cases}$

For $f(x)$ to be continuous at $x = 5$

$\lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x) = f(5)$

$\Rightarrow \lim_{x\to5}(b + 5x) = 30$

$\Rightarrow b + 25 = 30 \Rightarrow b = 5$

For $f(x)$ to be continuous at $x = 1$

$\lim_{x\to1^+} + f(x) = \lim_{x\to1^-} f(x) = f(1)$

$\Rightarrow \lim_{x\to1}(a+5x) = 5$

$\Rightarrow \lim_{x\to1}(a+5x) = 5$

Now at $x = 3$,

$LHL = \lim_{x\to3}(0+5x) = 15$

$RHL = \lim_{x\to3}(5+5x) = 18$

$\Rightarrow f(x)$ is discontinuous for all $a, b \in R$

7. Let 1^{st} 5 students are
x_1, x_2, x_3, x_4 and x_5
$\therefore 18 = E(x^2) - \left(E(x)\right)^2$

$= \frac{x_1^2+x_2^2+x_3^2+x_4^2+x_5^2}{5} - (150)^2$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 112590$

$\therefore$ new variance $= \frac{x_1^2+x_2^2+....+x_5^2+156^2}{6} - (151)^2$

$\left[\text{Since new mean} = \frac{150\times5+156}{6} - 151\right]$

$= \frac{112590+24336}{6} - 22801$

$= 20$

8. Given a, b, c are in $G.P$ let common ratio of
$G.P$ be r, then $b = ar, c = ar^2$

Given equation can be written as

$a + ar + ar^2 = x.a.r$

$\Rightarrow x = r + \frac{1}{r} + 1$

As we know $r > 0$, then $r + \frac{1}{r} \geq 2 \Rightarrow x \geq 3$

$r < 0$, then $r + \frac{1}{r} \leq -2 \Rightarrow x \leq -1$

$\Rightarrow x \in (-\infty, -1] \cup [3, \infty)$

9. $2^{403} = 2^3.(2^4)^{100}$
$= 8.(15+1)^{100}$

$= 8[\,^{100}C_0 15^{100} + \,^{100}C_1 15^{99} + ... + \,^{100}C_{99} 15 + \,^{100}C_{100}]$

$= 8.[15\lambda + 1]$

$= 8.15\lambda + 8$

$\Rightarrow \frac{2^{403}}{15} = 8\lambda + \frac{8}{15}$

$= k = 8$

10. To solve this questions we need to apply the concept of rationalization two times.

Given limit is $\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$= \lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} \times \frac{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}$

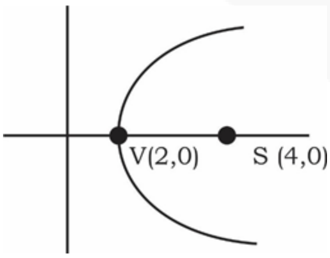
$= \lim_{y\to 0} \frac{\left(\sqrt{1+y^4}-1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)} \times \frac{\left(\sqrt{1+y^4}+1\right)}{\left(\sqrt{1+y^4}+1\right)}$

$= \lim_{y\to 0} \frac{y^4}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$

$= \lim_{y\to 0} \frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$

$= \frac{1}{4\sqrt{2}}$

11. There is an ambiguity is this question. Two possibilities are there
Case1: When vertex is an RHS of y −axis



Clearly in this case vertex is $(2, 0)$.

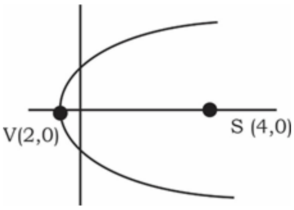
Since focus is $S(4,0) \Rightarrow a = 2$.

Hence equation of parabola is $y^2 = 4.2(x - 2)$

$\Rightarrow y^2 = 8(x - 2)$

$\Rightarrow$ Point $(6, 8)$ does not lie on the parabola

Case 2 when vertex is on LHS of y −axis



Cleary $a = 6$

Equation of parabola $y^2 = 24(x + 2)$.

Since neither of the given points lies on this parabola

This parabola is not considered here

12. Given complex number is $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

$\Rightarrow z = \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1+4\sin^2\theta)}$

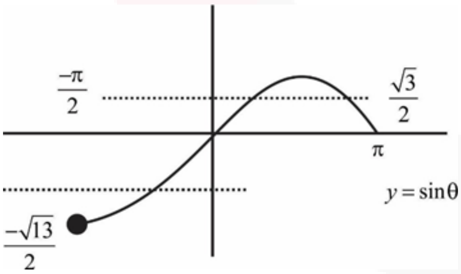
Since given complex number is purely imaginary

$\Rightarrow \text{Re}(z) = 0$

$\Rightarrow 3 - 4\sin^2\theta = 0$

$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{-\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$



$\Rightarrow$ sum of all possible values of $\theta = \frac{2\pi}{3}$.

13. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$

And $adj(A) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{|A|} adj(A) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^{-2} = (A^{-1})^2 = (A^{-1})(A^{-1})$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & \cos^2\theta - \sin 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

It is visible that

$(A^{-1})^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

$\Rightarrow A^{-50} = (A^{-1})^{50} - \begin{bmatrix} \cos 50\theta & \sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{bmatrix}$

Now at $\theta = \frac{\pi}{12}, \sin 50\theta = \sin\frac{50\pi}{12} = \sin\frac{\pi}{6} = \frac{1}{2}$

$\cos 50\theta = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\Rightarrow A^{-50} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

14. 1) $(A \wedge B) \wedge (\sim A \vee B)$

$\equiv (B \wedge A) \wedge (\sim A \vee B)$

$\equiv B \wedge [A \wedge (\sim A \vee B)]$ (using associate property)

$\equiv B \wedge (A \wedge B)$

$\equiv A \wedge B$

2) $= (A \wedge B) \wedge (\sim A \wedge B)$

$\equiv (A \wedge \sim A) \wedge B$

$\equiv F \wedge B \equiv F$

3) $(A \vee B) \wedge (\sim A \vee B)$

$\equiv (A \wedge \sim A) \vee B$

$\Rightarrow F \vee B \equiv B$

4) $(A \vee B) \wedge (\sim A \wedge B)$

$\equiv [(A \vee B) \wedge \sim A] \times B$ (Using associate property)

$\equiv [B \wedge \sim A] \wedge B$

$\equiv \sim A \wedge B$

15. Given DE can be written as
$\frac{dy}{dx} + \frac{2y}{x} = x$, which is a linear differential equation

$\text{If} = e^{\int \frac{2}{x} dx} = e^{2\ln x} = x^2$

Solution is

$y.x^2 = \int x.x^2 dx$

$\Rightarrow y.x^2 = \frac{x^4}{4} + c$

Since given curve passes through $(1, 1)$

$\Rightarrow c = \frac{3}{4}$

Hence $y(x) = \frac{x^2}{4} + \frac{3}{4x^2}$

So $y\left(\frac{1}{2}\right) = \frac{49}{16}$

16. Using the concept of complementary events,
$P(x = 1) + P(x - 2) = 1 - P(x = 0)$

$= 1 - \frac{48}{52} \times \frac{48}{52}$

$= 1 - \frac{144}{169} = \frac{25}{169}$

17. For given hyperbola the eccentricity is given as
$e^2 = 1 + \frac{\sin^2\theta}{\cos^2\theta} = 1 + \tan^2\theta\sec^2\theta$

$\Rightarrow e = \sec\theta$

$\therefore$ length of latus rectum $l = \frac{2\sin^2\theta}{\cos\theta} = \frac{2\tan^2\theta}{\sec\theta}$

$\Rightarrow l = \frac{2(e^2-1)}{ee} = 2\left(e - \frac{1}{e}\right)$

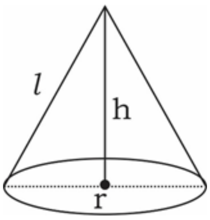
$\frac{dl}{de} = 2\left(1 + \frac{1}{e^2}\right) > 0$

$\therefore l$ is an increasing function

$\therefore l_{\min} = 2\left(2 - \frac{1}{2}\right) = 3$

$\therefore$ range of latus rectum is $(3, \infty)$

18. Given $l = 3$



$\Rightarrow r^2 + h^2 = g$

Volume, $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{1}{3}\pi(9 - h^2)h = \frac{1}{3}\pi(9h - h^3)$

$\frac{dv}{dh} = \frac{1}{3}\pi(9 - 3h^2)$

$\frac{dv}{dh} = 0 \Rightarrow h = \sqrt{3}$

$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h)$

$\therefore$ at $h = \sqrt{3}$, cone has maximum volume

$\therefore V_{\max} = \frac{1}{3}\pi\left(9\sqrt{3} - 3\sqrt{3}\right) = 2\sqrt{3}\pi cm^3$

19. $\because x > \frac{3}{4}$

$\therefore \cos^{-1}\frac{3}{4x} = \sin^{-1}\frac{\sqrt{16x^2-9}}{4x}$

$\therefore$ Given $\cos^{-1}\frac{2}{3x} + \cos^{-1}\frac{3}{4x} = \frac{\pi}{2}$

$\cos^{-1}\frac{2}{3x} + \cos^{-1}\frac{3}{4x} = \frac{\pi}{2}$

$\Rightarrow \frac{2}{3x} + \frac{\sqrt{16x^2-9}}{4x}$

$\Rightarrow \left(\frac{8}{3}\right)^2 = 16x^2 - 9$

$\Rightarrow 16x^2 = \frac{64}{9} + 9 = \frac{145}{9}$

$\Rightarrow x^2 = \frac{145}{144}$

$\Rightarrow x = \frac{\sqrt{145}}{12}$

20. Given condition is $3p + 2q + 4r = 0$
$\Rightarrow \frac{3}{4}p + \frac{1}{2}q + r = 0$ ... (i)

And given family of line is $px + qy + r = 0 \ldots (ii)$

From (i) we can say that equation (ii) always passes through $\left(\frac{3}{4}, \frac{1}{2}\right)$.

Hence all lines are concurrent at $\left(\frac{3}{4}, \frac{1}{2}\right)$.

21. Here, $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix}$
$= a^2 - 3$

Hence at $|a| = \sqrt{3}$, $D = 0$. So system has no unique solution.

At $a = 4$ and 3, $D \neq 0$, So, system has unique solution. Hence net inconsistent.

At $|a| = \sqrt{3}, D = 0$ and 2^{nd} & 3^{rd} equation are parallel. Hence no solution are possible.

Hence, system is inconsistence for $|a| = \sqrt{3}$

22. $3(\cos\theta - \sin\theta)^4 + 6(\cos\theta + \sin\theta)^2 + 4\sin^6\theta$

$= 3((\cos\theta - \sin\theta)^2)^2 + 6(\cos\theta + \sin\theta)^2 + 4\sin^6\theta$

$= 3(\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta)^2 + 6(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 - 4\sin\theta\cos\theta + 4\sin^2\theta\cos^2\theta) + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

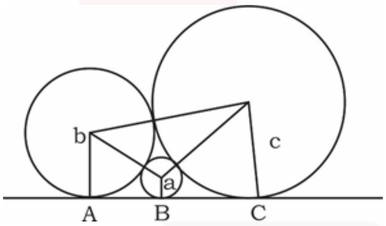
$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$

$= 9 + 12(1 - \cos^2\theta)\cos^2\theta + 4(1 - \cos^2\theta)^3$

$= 9 + 12\cos^2\theta - 12\cos^4\theta + 4(1 - 3\cos^2\theta + 3\cos^2\theta - \cos^2\theta)$

$= 13 - 4\cos^6\theta$

23.



$AB = \sqrt{(a+b)^2(b-a)^2} = 2\sqrt{ab}$

$BC = \sqrt{(a+c)^2 - (c-a)^2} = 2\sqrt{ac}$

$AC = \sqrt{(b+c)^2 - (c-b)^2} = 2\sqrt{bc}$

$\because AC = AB + BC$

$\Rightarrow 2\sqrt{bc} = 2\sqrt{ab} + 2\sqrt{ac}$

$\Rightarrow \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$

24. Given $f_2\left(J\left(f_1(x)\right)\right) = f_3(x)$

$\Rightarrow f_2\left(J\left(\frac{1}{x}\right)\right) = \frac{1}{1-x}$

$\Rightarrow 1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$

$\Rightarrow J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x} = -\frac{x}{1-x}$

$\Rightarrow J(x) = -\frac{\frac{1}{x}}{1-\frac{1}{x}} = -\frac{1}{x-1} = \frac{1}{1-x}$

$\therefore\ J(x) = f_3(x)$

25. A variable point on line $x + y - z = 0 = x + 2y - 3z + 5$ is $(-t + 5, 2t - 5, t)$
$\therefore DR$ of variable point & $(-4, 1, 3)$ is

$(-t + g, 2t - 6, t - 3)$

Since line is parallel to $x + y + z = 3$

$\therefore -t + g + 2t - 6 + t - 3 = 0$

$\Rightarrow t = 0$

$\therefore DR$ of line s $(9, -6, -3)$ or $(-3, 2, 1)$

$\therefore$ Equation of line is $\frac{x+4}{-3} = \frac{y-1}{2} = \frac{z-3}{1}$

26. Intersection points of given curves are $(\pm 2, 6)$ for 1^{st} curve $\frac{dy}{dx} = 2x$
$\therefore m_1 = 4$ or (-4)

For 2^{nd} curve, $\frac{dy}{dx} = -2x$

$\therefore m_2 = -4$ or (4)

$\therefore |\tan\theta| = \left|\frac{4-(-4)}{1+4(-4)}\right|$ or $\left|\frac{-4-4}{1+(-4)4}\right|$

$= \frac{8}{15}$ (In both cases)

27. Let required plane is (using concept of family of plane)
$x + y + z - 1 + t(2x + 3y - z - 4) = 0$

$\Rightarrow (1 + 2t)x + (1 + 3t)y + (1 - t)z - 1 - 4t = 0$

Again DR, of y −axis is $(0, 1, 0)$

Since plane is parallel to y −axis

Hence $(1 + 2t) \times 0 + (1 + 3t) \times 1 + (1 - t) \times 0 = 0$

$\Rightarrow t = -\frac{1}{3}$

$\therefore$ required plane is $\frac{1}{3}x + \frac{4}{3}z + \frac{1}{3} = 0$

$\Rightarrow x + 4z + 1 = 0$

Which satisfy $(3, 1, -1)$

28. Let tangent to parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m} \dots (i)$

If equation (i) is tangent to give circle whose centre is $(3,0)$ and radius is 3 then length of perpendicular from centre of circle to equation (i) is equal to radius of circle.

Hence, $\left|\frac{3m+\frac{1}{m}}{\sqrt{m^2+1}}\right| = 3$

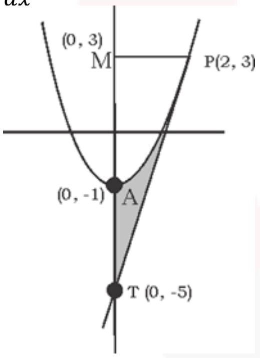
$\Rightarrow |3m^2 + 1| = 3m\sqrt{1 + m^2}$

$\Rightarrow 9m^4 + 6m^2 + 1 = 9m^2 + 9m^4$

$\Rightarrow m = \pm\frac{1}{\sqrt{3}}$

$\therefore$ common tangents are $y = \frac{x}{\sqrt{3}} + \sqrt{3}$ or $y = -\frac{x}{\sqrt{3}} - \sqrt{3}$

29. $\frac{dy}{dx} = 2x$



$\therefore$ slope of tangent at $(2,3)$

$m = 4$

$\therefore$ equation of tangent is $y - 3 = 4(x - 2)$

$\therefore$ point $T \equiv (0, -5)$

$\therefore$ area of $APTM = \frac{1}{2} \times 8 \times 2 = 8$

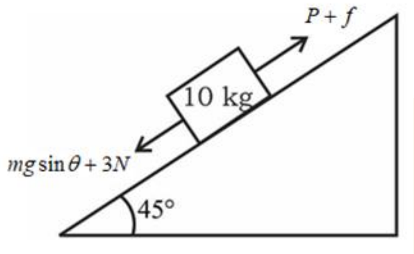
Area of curve $PAMP = \int_{-1}^{3} \sqrt{y + 1}dy$

$= \frac{2}{3}\left[(y + 1)^{\frac{3}{2}}\right]_{1}^{3} = \frac{16}{3}$

$\therefore$ Required shaded area $= 8 - \frac{16}{3} = \frac{8}{3}$

Physics

1.



Block will be at rest it

$mg\sin\theta + 3N = P + f$

$10 \times 10 \times \frac{1}{\sqrt{2}} + 3 = P + \mu mg\cos\theta$

($\because$ To get P minimum friction must be minimum)

$P_{\min} = 50\sqrt{2} + 3 - 0.6 \times 10 \times 10 \times \frac{1}{\sqrt{2}}$

$= 20\sqrt{2} + 3 = 32N$

2. For path ABC,

$\Delta Q = \Delta U + \Delta W$

$60 = \Delta U + 30$

$\Rightarrow \Delta U = 30J$

For path ADC

$\Delta Q = \Delta U + \Delta W$

As initial and find points are same, change in internal energy is same in both the cases

$\Delta Q = 30 + 10$

$= 40J$

3. From given data

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{10} - \frac{1}{(-10)} = \frac{1}{f}$

$f = 5cm$

When glass plate is placed

Shift $s = t\left(1 - \frac{1}{\mu}\right) = 1.5\left(1 - \frac{2}{3}\right)$

$= 0.5\ cm$

Shift will be in the direction of incoming rays

$\therefore u = 10 - 0.5 = 9.5$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{5} + \frac{1}{(-9.5)}$

$v = \frac{47.5}{4.5}$

Shift $= \frac{47.5}{4.5} - 10 = \frac{2.5}{4.5} = \frac{5}{9}$

4. Angular momentum $L = mr^2w$

Area of sector, $dA = \frac{1}{2}r^2 d\theta$

Areas velocity $\frac{da}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$

(i) Keplers laws

5. $\vec{v} = y\hat{i} + x\hat{j}$

$v_x = y = \frac{dx}{dt}$

$v_y = x = \frac{dy}{dt}$

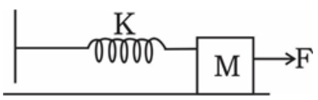
$\frac{dx}{dy} = \frac{y}{x}$

$\int x\,dx = \int y\,dy$

$\frac{x^2}{2} = \frac{y^2}{2} + C$

or $y^2 = x^2 + \text{constant}$

6.



When block attains maximum speed $F = Kx$

Or $x = \frac{F}{K}$

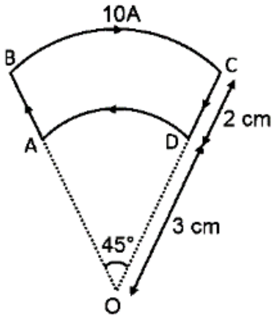
From work energy theorem

$-\frac{1}{2}\left(\frac{K}{F}\right)^2 + F\left(\frac{F}{K}\right) = \frac{1}{2}MV_{\max}^2$

$\frac{F^2}{2K} = \frac{1}{2}MV_{\max}^2$

$V_{\max} = \sqrt{\frac{F}{KM}}$

7.



Magnetic field due to AD and BC are zero at 'O'.

Magnetic field due to AB

$B_{AB} = \frac{\mu_0}{4\pi}\frac{10}{3\times10^{-2}}\frac{\pi}{4}$ (outwards)

Simplifying field due to DC

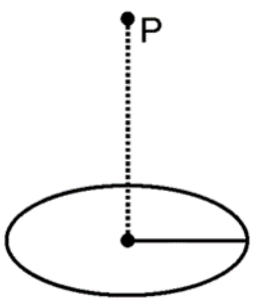
$B_{DC} = \frac{\mu_0}{4\pi}\frac{10}{5\times10^{-2}}\frac{\pi}{4}$ (inwards)

Net field $B = \frac{\mu_0}{4\pi}\frac{10\pi}{4\times10^{-2}}\left(\frac{1}{3}-\frac{1}{5}\right)$

$= 1.05 \times 10^{-5}$

$\approx 10^{-5}T$ (outwards)

8.



Electric field at a distance h from centre on axis

$E = \frac{1}{4\pi\varepsilon_0}\frac{Qh}{(R^2+h^2)^{\frac{3}{2}}}$

To get maximum E, $\frac{dE}{dh} = 0$

$$0 = \frac{1}{4\pi\varepsilon_0}\frac{Q(R^2+h^2)^{\frac{3}{2}} - Qh\frac{3}{2}(R^2+h^2)^{\frac{1}{2}}2h}{(R^2+h^2)^3}$$

$$\Rightarrow h = \pm\frac{R}{\sqrt{2}}$$

9. Activity $A = \lambda N$

$10 = \lambda_A N_A$

$20 = \lambda_B N_B$

$\frac{1}{2} = \frac{\lambda_A}{\lambda_B} 2$

$\frac{\lambda_A}{\lambda_B} = \frac{1}{4}$

10. $B = 0.4\sin(50\pi t)$, $A = 3.5 \times 10^{-3} m^2$

$\varepsilon = \frac{d\phi}{dt}$

$dQ = \frac{\varepsilon}{R}dt = \frac{1}{R}d\phi$

$t = 10\ ms$

$Q = \frac{1}{R}\int_{t=0}^{t=10ms} 0.4 \times 3.5 \times 10^{-3}\sin(50\pi t)dt$

$= 140\mu c$

11. $i = neAv_d$

$1.5 = 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times v_d$

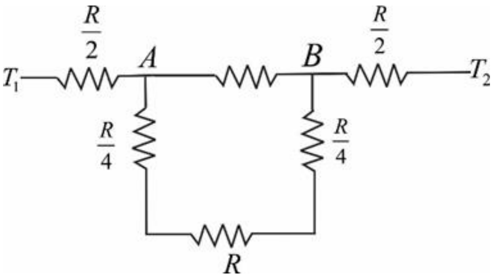
$v_\alpha = 0.02\ mm/s$

12. $T_1 - T_2 = 120°C$

We define thermal resistance

$R = \frac{L}{KA}$

Given circuit can be reduced to



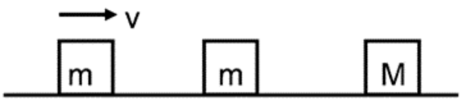
Equivalent resistance $R_{eq} = \frac{8R}{5}$

Thermal current $i = \frac{T_1 - T_2}{\frac{8R}{5}}$

$T_A - T_B = \frac{T_1 - T_2}{\left(\frac{8R}{5}\right)}\left(\frac{3R}{5}\right)$

$= \frac{3}{8} \times 120 = 45$

13.



$mv = (m + m)v'$

$(m + m)v' = (m + m + M)v_f$

$\therefore$ final velocity $v_f = \frac{m}{2m+M} v$

Initial energy $= \frac{1}{2}mv^2$

Final energy $= \frac{1}{2}(2m + M)\left(\frac{mv}{2m+M}\right)^2$

Given that $\frac{5}{6}\frac{1}{2}mv^2 + \frac{1}{2}mv^2 - \frac{1}{2}(2m + M)\left(\frac{mv}{2m+M}\right)^2$

$\frac{1}{12}mv^2 = \frac{1}{2}\frac{m^2v^2}{(2m+M)}$

$6m = 2m + M$

$\frac{M}{m} = 4$

14. $\frac{(V_{RMS})_{He}}{(V_{RMS})_{Ar}} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$

15. $\lambda_1 = 340 nm,\ \lambda_2 = 540\ nm,\ \frac{V_1}{V_2} = 2$

$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2V)^2$

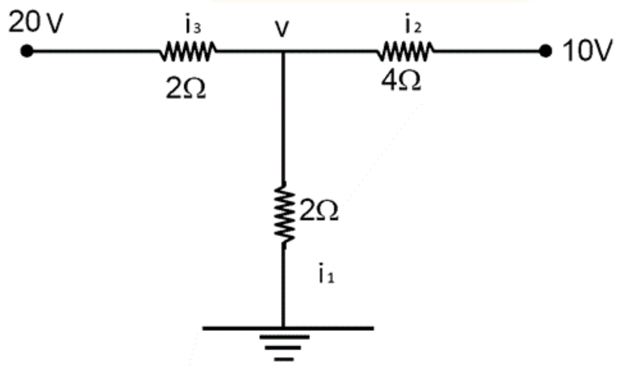
$\frac{hc}{\lambda_2} = \phi + \frac{1}{2}mV^2$

$\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}4 = \phi - 4\phi$

$hc\left(\frac{4}{540nm} - \frac{1}{340nm}\right) = 3\phi$

$\phi = 1.85eV$

16.



Apply KCL

$i + i_2 + i_1 = 0$

$\frac{V}{2} + \frac{V-10}{4} + \frac{V-20}{2} = 0$

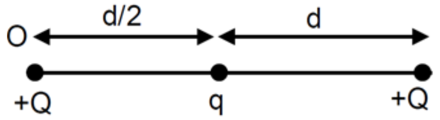
$2V + V - 10 + 2(V - 20) = 0$

$5V - 50 = 0$

$V = 10V$

$\Rightarrow i_1 = \frac{10}{2} = 5A$

17.



Net force on charge at O is zero

$\Rightarrow \frac{KQ^q}{\left(\frac{d}{2}\right)^2} + \frac{KQ^2}{d^2} = 0$

$q = \frac{-Q}{4}$

18. As we know $\frac{E}{B} = C$

$E = 6.3 \times 10^{27} \, V/m$

$B = \frac{E}{C} = \frac{6.3 \times 10^{27}}{3 \times 10^{8}}$

$B = 2.1 \times 10^{19} T$

19. $U = -\vec{P}.\bar{B}$

$F = -\frac{\partial U}{r^r} = P\frac{dB}{dr}$

$F = i_2(\pi a^2) \times \frac{d}{dr}\left(\frac{\mu_0 i_1}{2\pi r}\right)$

$F = \frac{i_1 i_2 \pi_0 a^2}{2} \frac{1}{r^2}$

$F \propto \left(\frac{a}{r}\right)^2$

As $r = d$

$F \propto \frac{a^2}{d^2}$

20. $I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$\Rightarrow \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}$

$\Rightarrow \frac{16}{1} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2}$

$\Rightarrow \frac{4}{1} = \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}$

$\Rightarrow \frac{I_1}{I_2} = \frac{25}{9}$

21. $V = A.e$

$O = \frac{dA}{A} + \frac{dl}{l}$

$\Rightarrow \frac{dA}{A} = -\frac{dl}{l}$

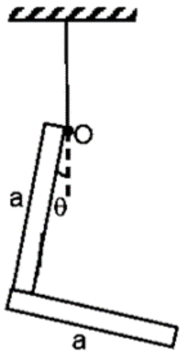
$R = \int \frac{l}{A}$

If dR is change in resistance due to change in l and A, then $\frac{dR}{R} = \frac{dl}{e} - \frac{dA}{A}$

$= 2.\frac{dl}{A}$

$\therefore$ % charge in $R = 2 \times 0.5\% = 1\%$

22.



At equilibrium, the centre of mass will be below the point of suspension.

Taking torque about point O,

$mg\frac{l}{2}\sin\theta + mg\left(l\sin\theta - \frac{l}{2}\cos\theta\right) = 0$

$\Rightarrow \frac{3l}{2}\sin\theta = \frac{l}{2}\cos\theta$

$\Rightarrow \tan\theta = \frac{1}{3}$

$\Rightarrow \theta = \tan^{-1}\frac{1}{3}$

23. Since the length of the rod does not change, the increment due to increase in temperature will be balanced by the compression due to applied force.

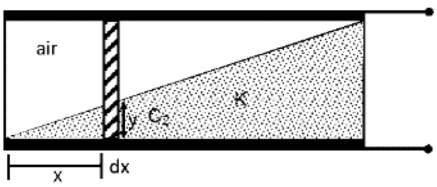
$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} \Rightarrow \frac{\Delta l}{l} = \frac{F}{A.Y}$

Also $\frac{\Delta l}{l} = \alpha \Delta T$

$\therefore \frac{F}{AY} = \alpha\ \Delta T$

$Y = \frac{F}{A.\alpha\Delta T}$

24.



Consider a small element at a distance x. It is a two capacities in series.

$dc = \frac{dc_1 + dc_2}{dc_1 + dc_2}$

$$dc = \frac{\varepsilon_0 . adx}{\frac{x\tan\theta}{k} + \frac{d - x\tan\theta}{1}}$$

$$\therefore dc = \frac{\varepsilon_0 K.adx}{-x\tan\theta(k-11) + d.k}$$

$$\therefore C = \int_0^a \frac{\varepsilon_0 k.adx}{-x\tan\theta(k-1) + d.k}$$

$$= \frac{\varepsilon_0 k.a}{\tan\theta(k-1)} ln \frac{-a\tan\theta(k-1) + d.k}{d.k}$$

$$= \frac{\varepsilon_0 k a^2}{d(k-1)} ln\, k$$

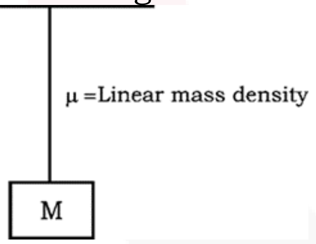
25. $\rho = \frac{E}{J} = \frac{E}{nev_d} = \frac{1}{ne\mu}$

μ = mobility

$$\rho = \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6} = \frac{1}{256} \Omega - m$$

$$= 0.4\Omega - m.$$

26. For standing car



For accelerating car

$$v' = \sqrt{\frac{\sqrt{(Mg)^2 + (Ma)^2}}{\mu}}$$

$$\Rightarrow \frac{60.5}{60} = \sqrt{\frac{\sqrt{(Mg)^2 + (Ma)^2}}{Mg}}$$

$$\Rightarrow \left(1 + \frac{1}{120}\right)^2 = \frac{\sqrt{(Mg)^2 + (Ma)^2}}{Mg}$$

$$\Rightarrow 1 + \frac{2}{120} = \sqrt{1 + \frac{a^2}{g^2}}$$

$$\approx 1 = +\frac{1}{2}\frac{a^2}{g^2}$$

$$\therefore a = g\sqrt{\frac{4}{120}} = \frac{g}{\sqrt{30}}$$

Chemistry

1. k_b values are a measure of basicity. Higher the k_b value, greater will be basicity.

   Also basicity implies the tendency to donate electrons Basic character $\alpha \frac{1}{\% S\, character}$

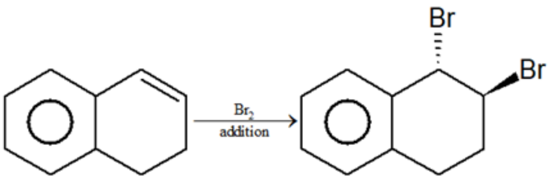
   $Nsp^2 > Nsp^3$ (% s −character=more%s-character more will be electro negativity)

   Than Nsp^2 has more % $s - character$ than Nsp^3 is less basic as compare to Nsp^3 So $R > p$.

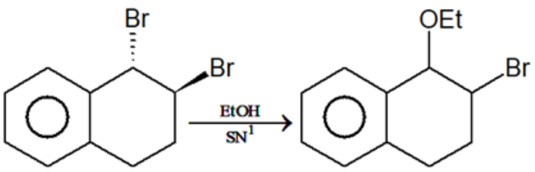
   In Q lone pair involved in Aromaticity it so difficult to donate as it is very less basic

   Thus, the correct order will be: $R > P > Q$

2. The first step is anti addition of Br_2 across the double bond.



   In the second step, $^{-}OEt\ nu^{-}$) gets added by SN^1 mechanism. The Br in dashed position acts as the having group as this will add to the formation of a stable benzylic carbocation, where OEt gets added.



3. Acidic character α stability of conjugate base (anion)

   As stability of an $\alpha -M$ and $-I$

   $-Cl$ and -F have ($-I > +M$ effect) in which F is more electron withdrawing due to greater electronegativity. NO_2 and CN have both $-I$ and $-M$ effect with NO_2 being more electron withdrawing.

   Order of -I-effect $-NO_2$>CN>-F>-Cl Hence, order of acidity (k_a) will be

   $O_2NCH_2COOH > NC\,CH_2COOH > FCH_2\,COOH > Cl\,CH_2\,COOH$
   $(R) \quad (S) \quad (P) \quad (Q)$

4. The concentration of the ions for drinking water are

   $Fe \le 0.2\,ppm$ $\quad Mn \le 0.05\,ppm$ $\quad Cu \le 3.0\,ppm$ $\quad Zn \le 5.0\,ppm$

   Hence, only Mn^+ concentration is greater than allowed limit.

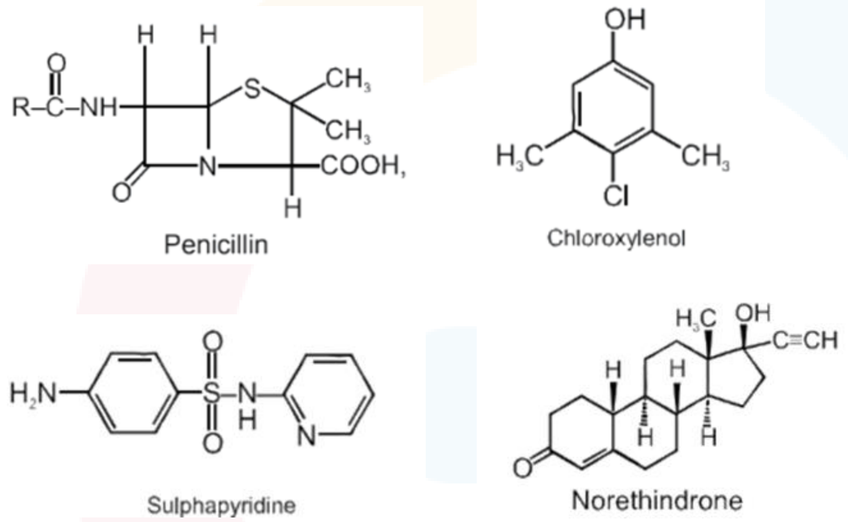
5. Acidic character α stability of conjugate base (anion)

As stability of α $-M$ and $-I$

Electron withdrawing groups increase acidic strength. Among the given compounds, the electron withdrawing strength of substituents is $CN > Cl > Br > I$

Hence, $CH\ (CN)_3$ will be most acidic

6.

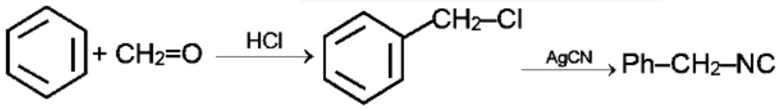


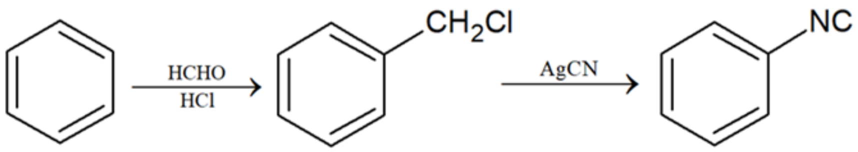
Chloroxylenol is a phenol derivative and hence can be identified with $FeCl_3$ test.

The primary amine of sulphapyridine can be identified with earbylamine test.

The aliphatic alkyne of Nore thindrone can be identified by the Baeyer's reagent.

7.



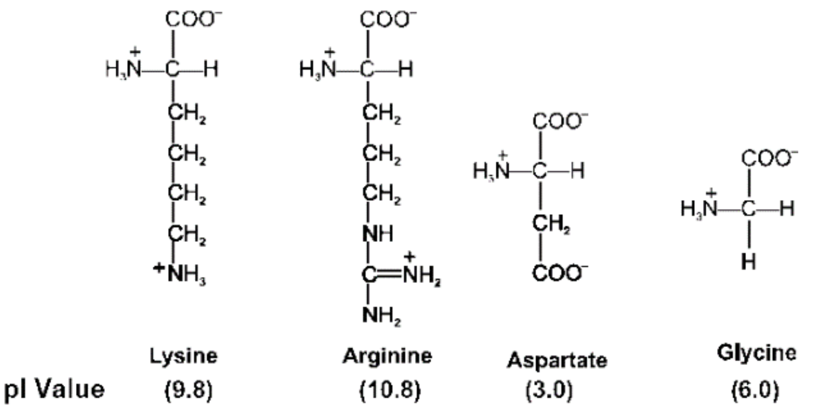


The final product is an isocyanide as the $AgCN$ bond has covalent nature, due to which C is unavailable for bond. Hence, N bonds with the benzylic carbon leading to formation of isocyanide.

8. In the first step, R CN gets reduced to $RCH = NH_2$ which then gets hydrolyzed to $RCH = O$

$$R - C \equiv N \xrightarrow[reduction]{DIBAL-H} R - CH = NH \xrightarrow{H_2O} R - CH = 0$$

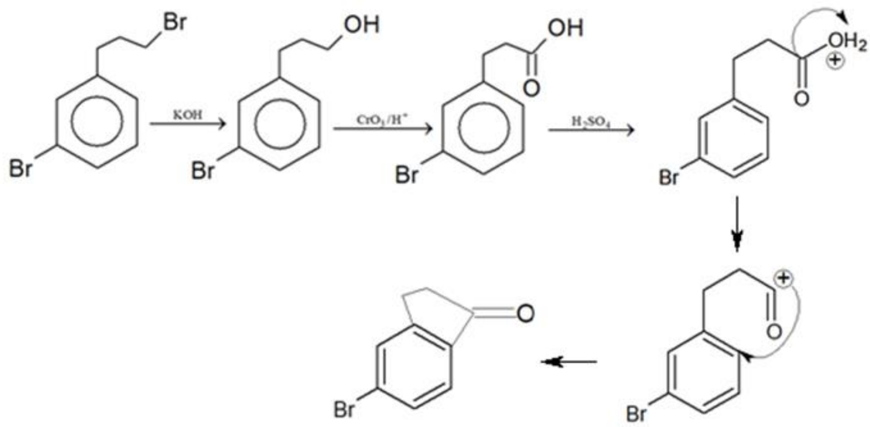
9.



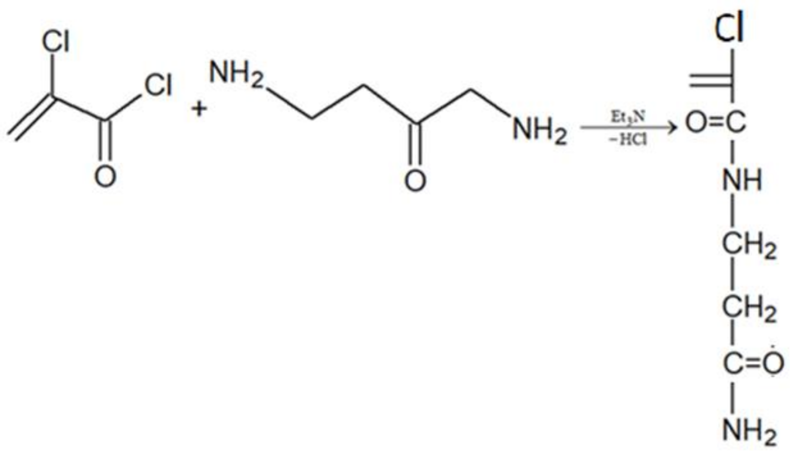
Hence, the correct order is

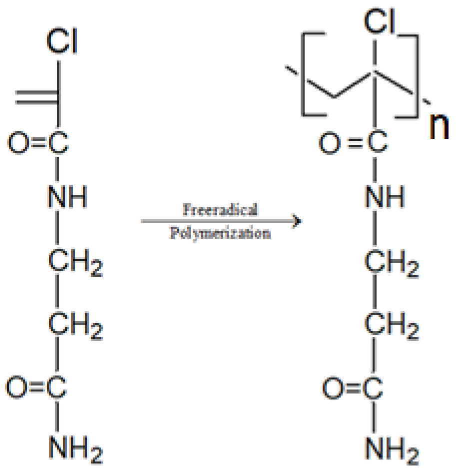
$Arg > Lys > Gly > Asp$

10.



11.





12. Moles of $Na^{+} = \frac{\text{Weight of Na}^{+}\text{ ions}}{\text{Molecular weight}} = \frac{92}{23} = 4$

4, mole of Na^{+} present in 1 kg of water Hence molarity Na^{+} ions will be 4

13. $Ba\,(NO_3)_2$ does not have any water of crystallization. This happens because the large size of Ba leads to poor polarizing power and hence, if unit able to attract water molecules effectively.

14. According to the Freundlich adsorption isotherm, $\frac{x}{m} = Kp^{\frac{1}{n}}$

Taking log on bot sides.

$\log\left(\frac{x}{m}\right) = \log K + \frac{1}{n}\log P$

Slope $= \frac{1}{n}$

Slope from graph $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{4} = \frac{1}{2}$

or, $\frac{1}{n} = \frac{1}{2}$

$\therefore \frac{x}{m} = Kp^{\left(\frac{1}{2}\right)}$

or $\frac{x}{m} \propto p^{\left(\frac{1}{2}\right)}$

15. Iron and copper are both present in

Copper pyrite – $CuFeS_2$

Malachite – $CuCO_3.Cu(OH)_2$

Azurite –$2CuCO_3.Cu(OH)_2$

16. Millimoles of $H_2SO_4 = 20 \times 0.1 = 2$

Millimoles of $NH_4OH = 30 \times 0.2 = 6$

	H_2SO_4 +	$2NH_4OH \rightarrow$	$(NH_4)_2SO_4$ +	H_2O
$t = 0$	2	6	0	0
Upon completion	0	2	2	4

Hence, the solution will act as a basic buffer.

$pOH = pK_b + \log \frac{[salt]}{[Base]}$

$[NH^+] = 2 \times [(NH_4)_2SO_4]$

$2 \times 2 = 4$

$pOH = pK_b + \log \frac{[NH_4^+]}{[NH_3]}$

$= 4.7 + \log \frac{4}{2}$

$= 4.7 + 0.3010 = 5.010$

$pH = 14 - pOH \sim 9$

17. According to molecular orbital theory Li_2^+ has electrons $-\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^1$

Li_2^- has 7 electrons $-\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^1$

bond order $= \frac{\text{electron in bonding molecular orbital} - \text{Electron in Antibonding molecular orbital}}{2}$

Bond order of $Li_2^+ = \frac{3-2}{2} = \frac{1}{2}$

Bond order of $Li_2^- = \frac{4-3}{2} = \frac{1}{2}$

As both have similar bond order, both Li_2^+ and Li_2^- will be stable.

Note: Li_2^- will be relatively unstable to Li_2^+ as it has more electrons in anti bonding orbitals, so have more energy and having more repulsion so Li_2^- is less stable as Li_2^+

18. Henry's constant increases with increase in temperature while it decrease with increase in solubility. Hence value of K_H increases as solubility of gas increases is a wrong statement.

19. Let Assume x & y are the order of reaction w,r,t of A & B respectively

From equation (1) & (2)

$$\left(\frac{.1}{.1}\right)^x \left(\frac{.2}{.25}\right)^y = \frac{6.93 \times 10^{-3}}{6.93 \times 10^{-3}} \Rightarrow y = 0$$

From equation (1) & (3) and put the value $y = 0$

$\left(\frac{.1}{.2}\right)^x \left(\frac{.2}{.3}\right)^0 = \frac{6.93\times10^{-3}}{1.386\times10^{-2}}$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right) \Rightarrow x = 1$

the rate law will be $R = K[A]^1\,[B]^0$

Hence, the reaction is of first order.

$K = \frac{R}{[A]} = \frac{6.93\times10^{-3}}{0.1} = 6.93\times10^{-2}$

$t_{\frac{1}{2}}$ for first order reaction is given by

$t_{\frac{1}{2}} = \frac{0.693}{K}$

$= \frac{0.693}{6.93\times10^{-2}} = 10$

20. In a group decreased down and in a group increase.

(i) → Electronegativity: Top to bottom decrease

(radii $\alpha\, n^2$ so generally top to bottom increases)

(ii) Electronegativity: - Top to bottom decreases

Electron affinity: - generally top to bottom decreases not increases

(iii) radii: - $r\alpha n^2$ top to bottom generally increases

→ Electronegativity: - top to bottom increases

(iv) Electron gain enthalpy: - top to bottom decreases

Electronegativity: - Top to bottom decreases

21. $|W| = nRT\ \ \ell n\frac{v_f}{v_i}$

$|W| = nRT\ \ \ell n v_f - nRT\ \ v_i$

Intercept will be $-ve$

22. (A) True. $(\Delta_0)_A < (\Delta_0)_B$, both complex have $CN = 6$; B has strong field ligand and A has weak field ligand

so $(\Delta_0)_A <\ (\Delta_0)_B$

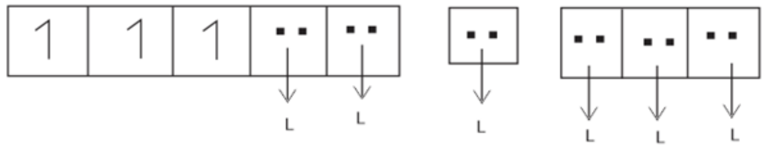
(B) True. By the crystal field splitting parameter can be measured by wavelength of complementary colours for A and B respectively

(C) True. In both complexes, Cr has 3 unpaired electrons and hence, will be paramagnetic

$[Cr(H_2O)]Cl_3$

$Cr^{+3} = 1S^2\ 2S^2\ 2p^6\ 3S^2\ 3p^6\ 3d^3 Cr^{+3}m$ in strong field ligand or weak field ligand

$t_{2g}{}^{111}, eg^{\circ}$



L = ligand = H_2O/NH_3

Hybridization is $d^2Sp^3$and

Having 3 top lame pairs

(D) False. The crystal field splitting cannot be measured by wavelength of yellow and violet colours for A & B respectively.

23. Silicones with medium length chains behave as viscous oils, jellies and greases. They are chemically inert, i.e., resistant

to oxidation, thermal decomposition or to attach by organic reagents. Since silicones are surrounded by non-polar alkyl groups, they are hydrophobic by nature. Being biocompatible, they are used in surgical and cosmetic implants. Hence, all options are correct.

24. Change of one mole of e^- = 1faraday

$Pb(s) + SO_4^{-2} \rightarrow PbSO_4 + 2e^-$

2 mole of e^- involved the above reaction = 2 faraday

2 mole of electron = 2 faraday current pass than $PbSO_4$ deposited = 303 gm/mol

$\therefore$ 0.05 F faraday current pass than $PbSO_4$ deposited $= \frac{303}{2} \times .05$

$= 7.575$

≈ 7.6

25. Piezoelectric material are those that produce an electric current when they are placed under mechanical stress. It was first discovered in 1880 by two French Physicists, brother Pierre and Paul- Jacques curie, in crystals of quartz, tourmaline and Rochelle salt.

26. Isotopes have same atomic number and different atomic mass ie. Same number of proton & different number of Neutron relative atomic mass

Protein	$1H^1 = H$	1.00782
Deuterium	$1H^2 = D$	2.01410
Trillian	$1H^3 = T$	3.01604

27. According to Rydberg's equation

$\bar{V} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$= R_H \times 1^2 \left(\frac{1}{n_f^2} - \frac{1}{8^2}\right)$

$\bar{V} = \frac{R_H}{n_f^2} - \frac{R_H}{64}$

Hence, slope for $\bar{V} \& = \frac{1}{n_f^2} is + R_H$

28. The sum of the first three ionization enthalpies of at considerably decreases and is therefore after to form at^{+3} ions. However down the group due to poor shielding, effective nuclear charge holds ns electron tightly (responsible for in emf pair effect) & thereby, restricting results of this only P-orbital electrons may be involved in banding so till show $+1 \& +3$ oxidation states.

29. Maximum number of unpaired electrons for transition metals = 5

or $n = 5$

Spin only magnetic moment $= \sqrt{n(n+2)} BM$

$= \sqrt{5(5+2)} = \sqrt{35}$

$= 5.92 BM$

30. According to ideal gas equation

$pV = nRT$

$200 \times 10 = (0.5 + x)R \times 1000$

$2 = 0.5R + xR$

$x = \frac{2 - 0.5R}{R} = \frac{4 - R}{2R}$