Symmetry in Math

Symmetry is something all human beings look for and seem to intuitively understand. One way to describe symmetry is to say that it is harmony or beauty of form that results from balanced proportions. Mathematics is the language in which God has written the codes of his creation. We, as a student should develop the ability to see the application of symmetry in Math. It will help to increase our speed in competitive exams. I'll use the problems based on syllabus of JEE Main and Advanced. (Relatively easier problem to begin with).

Single Option Correct

Q.1 In any Δ ABC, (where a, b and c are sides of the triangle and A, B and C are the respective angles),

then, 2 [bc cos A + ca cos B + ab cos C] =

(1) $-a^2+b^2+c^2$ (2) $a^2+b^2-c^2$ (3) $a^2-b^2+c^2$ (4) $a^2+b^2+c^2$

Explanation - From the question it is clear that sides a, b and c (also angles A, B and C) are on the same status. That is question is symmetric about a, b and c. That means the very essence of question will not change if I interchange sides a and b (also angles A and B), similarly the question will not change if I interchange sides b and c or c and a (also corresponding angles). The same pattern would be observed in the answers. So Option (1) - $a^2+b^2+c^2$ ***is out rightly rejected as it is behaving differently as it does not allow a and b to get interchanged (question allows it). Similarly Option (2)*** $a^2+b^2-c^2$ ***and (3)*** $a^2-b^2+c^2$ ***are rejected as they are also not symmetric about a, b, and c. So the correct answer from the logic of Symmetry is (4)*** $a^2+b^2+c^2$***. As we can interchange a, b and c with the same freedom as in the Question.***

Q.2 In any triangle ABC, (where a, b and c are sides of the triangle and A, B, and C are the respective Angles),

$$\frac{a^2\sin(B-C)}{\sin B+\sin C}+\frac{b^2\sin(C-A)}{\sin C+\sin A}+\frac{c^2\sin(A-B)}{\sin A+\sin B}=$$

(1) - a + b + c (2) a + b – c (3) a – b + c (4) 0

Explanation - This question is also symmetric about a, b and c. On interchanging a and b (also corresponding angles) it will not affect the essence of the question, so the correct answer is (4), as option (1), (2), and (3) are not symmetric.

Q.3 In a ΔABC, (where a, b and c, are sides of the triangle and A, B, and C are the respective angles),

then, $2ac\sin\left(\frac{A-B+C}{2}\right)$ is equal to [IIT 2000]

(1) $a^2+b^2-c^2$ (2) $c^2+a^2-b^2$ (3) $-b^2-c^2-a^2$ (4) $c^2-a^2-b^2$

Explanation - This question is symmetric about side a and c and not b, so the correct answer is (2). Few can think option (3) also but remember side b and angle B is behaving differently in the question, so they should behave differently in the answer.

Q.4 In a ΔABC , if $2s = a+b+c$ and $(s-b)(s-c) = x\sin^2\frac{A}{2}$, (where a, b and c are sides of the triangle and A,B, and C are the respective angles), then x =

(1) bc (2) ca (3) ab (4) abc

Explanation -This question is symmetric about b and c{$(s-b)(s-c)$} on L.H.S. and A is {$x\sin^2\frac{A}{2}$} on R.H.S. behaving differently so the correct answer is (1).Some students can think option (4) but it is not the correct answer as A should behave differently.

Q.5 In $\Delta ABC, (a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2} =$

(1) a^2 (2) b^2 (3) c^2 (4) ac

Explanation -This question is symmetric about side a and b (as a and b are interchangeable in the question) and angle C is behaving differently, so the correct answer is option (3). Remember if option (1) is correct then option (2) is also correct from symmetry and if it is an single option correct question you will directly tick option (3).

Q 6. The value of the determinant $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} =$

(1) a + b (2) 4 bc (3) $4\,a^2b^2c^2$ (4) b^2c^2

Explanation -From the question it is clear that a, b and c are on the same status. That is question is symmetric about a, b and c. So option (1), (2) and (4) are not correct. The correct answer is option (3).

Q 7. The complex numbers z_1, z_2, z_3 are the vertices of a triangle. Then the complex number/ numbers z which make the triangle into a parallelogram is

(1) $z_1+z_2-z_3$ (2) $z_1-z_2+z_3$ (3) $z_2+z_3-z_1$ (4) All the above

Explanation -The complex numbers z_1, z_2, z_3 ***are symmetric in the questions, as if they are interchanged the essence of the question will not change, so the correct answer would be option (4), all of above.***

Q 8. If z_1, z_2, z_3 are the vertices A, B and C respectively of a triangle ABC having centroid at G such that $z = 0$ is the midpoint of AG, then

(1) $z_1 + z_2 + z_3 = 0$ (2) $z_1 + 4z_2 + z_3 = 0$ (3) $z_1 + z_2 + 4z_3 = 0$ (4) $4z_1 + z_2 + z_3 = 0$

Explanation -From the question it is clear that z_2, z_3 ***are on the same status and*** z_1 ***is behaving differently (As z = 0 is the midpoint of AG. . So*** z_1 ***(affix of A) will behave differently and*** z_2, z_3 ***(vertices of B and C) will be interchangeable in the correct option, hence correct answer is option (4) as in it*** z_2, z_3 ***are interchangeable and*** z_1 ***is behaving differently.***

Q 9. One of the values of x which satisfies the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$ is

(1) $-(a+b)$ (2) $-(b+c)$ (3) $-a$ (4) $-(a+b+c)$

Explanation -This question is symmetric about a, b and c. On interchanging a and b, it will not affect the essence of the question so the correct answer is (4), as option (1) ,(2) , and (3) are not symmetric about a, b and c.

Q 10. If a, b and c are vectors, then $(\mathbf{b}\times\mathbf{c})\times(\mathbf{c}\times\mathbf{a}) =$ (where [ ] is scalar triple product)

(1) [b c a] a (2) [c a b] b (3)[a b c] c (4) [a c b] b

Explanation -In this questions vector c is behaving differently (mentioned twice), so the correct answer would be option (3).

Practice Questions – Single Option Correct

Q.1 In a triangle ABC, $a^2 \cos 2B + b^2 \cos 2A + 2ab \cos(A - B) =$

(Where a, b and c are sides of the triangle and A, B and C are the respective angles)

(1) a^2 (2) c^2 (3) b^2 (4) $3b^2$

Q.2 In a triangle ABC, $bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + abc\cos^2 \frac{C}{2} =$

(Where a, b and c are sides of the triangle and A, B and C are the respective angles)

(1) $(s - a)^2$ (2) $(s - b)^2$ (3) $(s - c)^2$ (4) s^2

Q.3. In any Δ ABC, $\left(\frac{b-c}{a}\right)\cos^2\left(\frac{A}{2}\right) + \left(\frac{c-a}{b}\right)\cos^2\left(\frac{B}{2}\right) + \left(\frac{a-b}{2}\right)\cos^2\left(\frac{C}{2}\right) =$

(Where a, b and c are sides of the triangle and A, B and C are the respective angles)

(1) $\frac{b^2 - c^2}{a^2}$ (2) $\frac{c^2 - a^2}{b^2}$ (3) $\frac{a^2 - b^2}{c^2}$ (4) 0

Q.4 If in a ΔABC, $(\sin A + \sin B + \sin C).(\sin A + \sin B - \sin C) = 3 \sin A \sin B$, then

(1) $A = 60°$ (2) $B = 60°$ (3) $C = 60°$ (4) $A = 90°$

Q.5 In a ΔABC, let $\angle C = \frac{\pi}{2}$. If r and R are the inradius and the circumradius of the triangle respectively , then $2(r + R)$ is equal to -

(1) a + b (2) b + c (3) c + a (4) a + b + c

Q.6 In a ΔABC, $r\, r_1 + r_2\, r_3 =$

(Where r is inradius and r_1, r_2 and r_3 are the ex-radius in front of sides a, b and c)

(1) ba (2) ac (3) bc (4) abc

1 (2)	2 (4)	3 (4)	4 (3)	5 (1)	6 (3)

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