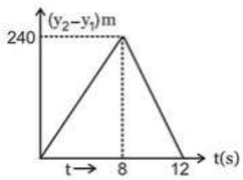
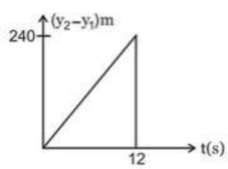
Physics

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10ms^{-2}$)
(the figure are schematic and not drawn to scale)

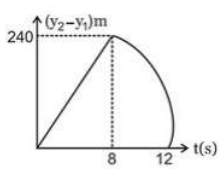
(A)



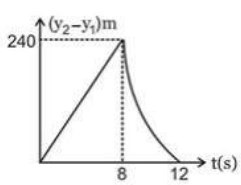
(B)



(C)



(D)



Answer: (C)

Solution: $S_1 = 10t - \frac{1}{2}gt^2$

When $S_1 = -240$

$$\Rightarrow -240 = 10t - 5t^2$$

$$\Rightarrow t = 8s$$

So at $t = 8$ seconds first stone will reach ground

$$S_2 = 20t - \frac{1}{2}gt^2$$

Till $t = 8 seconds$

$$S_2 - S_1 = 30t$$

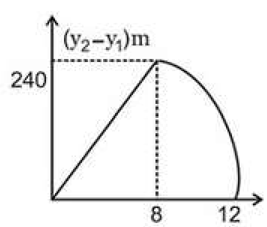
But after 8 second S_1 is constant -240

Relative to stone $t_1 > 8 seconds$ displacements of stone 2 $S_2 + 240$

$$\Rightarrow S_2 + 240 = 20t - \frac{1}{2}gt^2$$

And at t = 12s seconds stone will reach ground

The corresponding graph of relative position of second stone w.r.t. first is



Topic: Kinematics

Difficulty: Moderate (Embibe predicted high weightage)

Ideal time: 240

2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is:

(A) 2%
(B) 3%
(C) 1%
(D) 5%

Answer: (B)

Solution: $\therefore T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = 4\pi^2\frac{L}{T^2}$

∴ Error in g can be calculated as

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

∴ Total time for n oscillation is $t = n$ where T= time for oscillation.

$$\Rightarrow \frac{\Delta t}{t} = \frac{\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta t}{t}$$

Given that $\Delta L = 1mm = 10^{-3}m, L = 20 \times 10^{-2}m$

$$\Delta t = 1s, t = 90s$$

$$error \in g$$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta L}{L} + \frac{2\Delta t}{t}\right) \times 100$$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta L}{L} + \frac{2\Delta t}{t}\right) \times 100$$

$$\left(\frac{10^{-3}}{20 \times 10^{-2}} + \frac{2 \times 1}{90}\right) \times 100$$

$$\frac{1}{2} + \frac{20}{9}$$

$$0.5 + 2.22$$

2.72

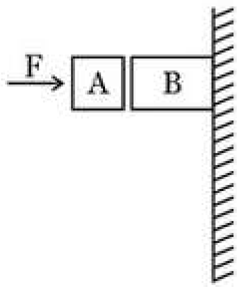
$$\cong 3$$

Topic: Unit & Dimensions

Difficulty: level: Easy (embibe predicted easy to score)

Ideal time: 90

3.

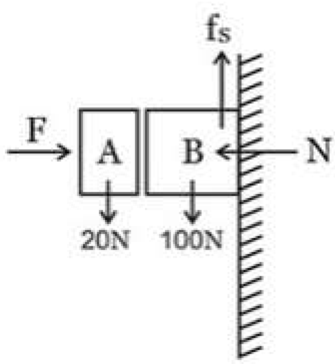


Given in the figure are two blocks A and B of weight 20N and 100N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is:

(A) 100 N
(B) 80 N
(C) 120 N
(D) 150 N

Answer: (C)

Solution:



For complete state equilibrium of the system. The state friction on the block B by wall will balance the total weight 120 N of the system.

Topic: Laws of Motion

Difficulty: level: Moderate (embibe predicted Low Weightage)

Ideal time: 60

4. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass 2m moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:

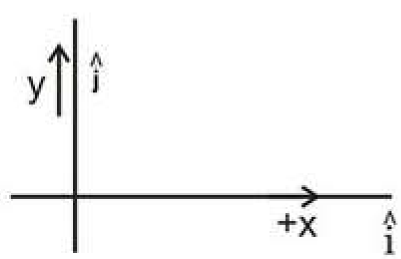
(A) 44%
(B) 50%
(C) 56%
(D) 65%

Answer: (C)

Solution:

The initial momentum of system is $\vec{P_i} = m(2V)\hat{i} + (2m)v\hat{j}$

According to question as



On perfectly inelastic collision the particles stick to each other so.

$$\vec{P_f} = 3m\vec{V_f}$$

By conservation of linear momentum principle

$$\vec{P_f} = \vec{P_i} \Rightarrow 3m\vec{V_f} = m2V\hat{i} + 2mV\hat{j}$$

$$\Rightarrow \vec{V_f} = \frac{2V}{3}(\hat{i} + \hat{j}) \Rightarrow V_f = \frac{2\sqrt{2}}{3}V$$

∴ loss in KE. of system $K_{initial} - K_{final}$

$$\frac{1}{2}m(2V)^2 + \frac{1}{2}(2m)V^2 - \frac{1}{2}(3m)\left(\frac{2\sqrt{2}V}{3}\right)^2$$

$$2mV^2 + mV^2 - \frac{4}{3}mV^2 = 3mV^2 - \frac{4mV^2}{3}$$

$$\frac{5}{3}mV^2$$

% change in KE $100 \times \frac{\Delta K}{K_i} = \frac{\frac{5}{3}mv^2}{3mV^2} = \frac{5}{9} \times 100$

$$\frac{500}{9} = 56$$

Topic: Conservation of Momentum

Difficulty: level: Moderate (embibe predicted easy to score (Must Do))

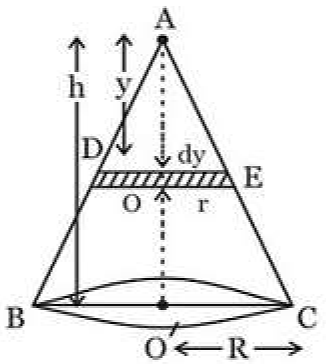
Ideal time: 90

5. Distance of the centre of mass of a solid uniform cone its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to:

(A) $\frac{h^2}{4R}$
(B) $\frac{3h}{4}$
(C) $\frac{5h}{8}$
(D) $\frac{3h^2}{8R}$

Answer: (B)

Solution:



Consider an elementary disc of radius r and thickness dy.

If total mass of cone $= \mathbf{M}$ and density $= \rho$

Then mass of elementary disc is $\mathbf{dm} = \rho\, \mathbf{dv} = \rho \times \pi \mathbf{r}^2\, \mathbf{dy}$ ..... (1)

In similar $\Delta's$ AOE and AO'C

$\frac{y}{h} = \frac{r}{R} \Rightarrow r = \frac{y}{h} R$ ..... (2)

Put (2) in (1)

$$dm = \rho(\pi)\left(\frac{y}{h} R\right)^2 dy$$

$$dm = \rho \times \frac{\pi R^2}{h^2} y^2\, dy$$

$\therefore$ The centre of mass of cone lying on the line AO' at a distance y_{cm} from A can be calculated as

$$y_{cm} = \frac{\int (dm) y}{\int dm} = \frac{\int \rho \pi R^2}{h^2} \frac{y^3 dy}{\int dm}$$

$$= \frac{\rho \pi R^2}{h^2 M} \int_0^h y^3\, dy$$

$$\because M = \rho \times \frac{1}{3} \pi R^2 h$$

$$\Rightarrow \quad y_{cm} = \frac{\rho \pi R^2}{h^2 \rho \times \pi R^2 h} \times \frac{h^4}{4} = \frac{3h}{4}$$

Topic: Centre of Mass

Difficulty: Easy (embibe predicted Low Weightage)

Ideal time: 30

6. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

(A) $\frac{MR^2}{32\sqrt{2}\pi}$
(B) $\frac{MR^2}{16\sqrt{2}\pi}$
(C) $\frac{4MR^2}{9\sqrt{3}\pi}$
(D) $\frac{4MR^2}{3\sqrt{3}\pi}$

Answer: (C)

Solution: Let a be length of cube for cube with maximum possible volume diagonal length = 2R

$$\Rightarrow \sqrt{3}a = 2R \Rightarrow a = \frac{2R}{\sqrt{3}}$$

As densities of sphere and cube are equal. Let M’ be mass of cube

$\frac{M}{\frac{4}{3}\pi R^3} = \frac{M'}{a^3}$

$$M' = \frac{3Ma^3}{4\pi R^3}$$

Moment of inertial of cube about an axis passing through centre

$\frac{M'(2a^2)}{12}$

$$\frac{3Ma^3}{4\pi R^3} \times \frac{2a^2}{12}$$

$$\frac{Ma^5}{8\pi R^3}$$

$$a = \frac{2}{\sqrt{3}}R$$

$$\frac{M \times 32R^5}{8\pi \times 9\sqrt{3}R^3}$$

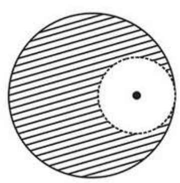
$$\frac{4MR^2}{9\sqrt{3}\pi}$$

Topic: Rotational Mechanics

Difficulty: Moderate (embibe predicted Low Weightage)

Ideal time: 300

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ and $r = \infty$, the potential at the center of the cavity thus formed is:
(G = gravitational constant)



(A) $\frac{-G}{2R}$

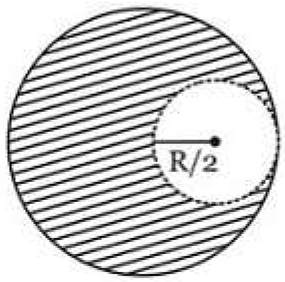
(B) $\frac{-G}{R}$

(C) $\frac{-2GM}{3R}$

(D) $\frac{-2GM}{R}$

Answer: (B)

Solution:



Potential due to whole sphere if cavity is not there at distance $\frac{R}{2}$ from centre

$$= \frac{-GM}{R^3}\left(\frac{3}{2}R^2 - 0.5\,r^2\right)_{r=\left(\frac{R}{2}\right)}$$

$$= \frac{-GM}{R^3}\left(\frac{3}{2}R^2 - \frac{R^2}{8}\right)$$

$$= \frac{-GM}{R^3}\left(\frac{12\,R^2 - R^2}{8}\right)$$

$$= \frac{-11GM}{8R} \qquad \_\_\_\_(1)$$

Potential due to sphere of radius $\frac{R}{2}$ at its centre let M' be mass of this sphere (equating densities)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{M^1}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$M' = \frac{M}{8}$$

Potential due to the sphere of $\frac{R}{2}$ radius at its centre is

$$= \frac{-3}{2}\frac{GM'}{\frac{R}{2}}$$

$$= \frac{-3}{2}\frac{GM\times 2}{8R}$$

$$= \frac{-3}{8}\frac{GM}{R} \qquad \_\_\_\_(2)$$

$\therefore$ Potential at $r = \frac{R}{2}$ is = (1) - (2)

$$= \frac{-11}{8}\frac{GM}{R} + \frac{3}{8}\frac{GM}{R} = \frac{-GM}{R}$$

Topic: Gravitation

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 360

8. A pendulum made of a uniform wire of cross sectional area A has time period T. When and additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:
(g = gravitational acceleration)

(A) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

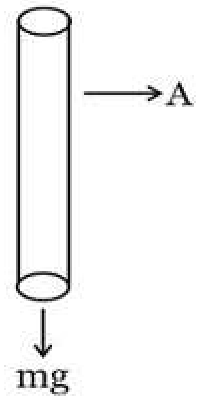
(B) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

(C) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

(D) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

Answer: (A)

Solution:



Initial length $= \ell$

Time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ ....(i)

After suspending mass M,

Youngs modulus $Y = \frac{\text{Stress}}{\text{Strain}}$

$$= \frac{\frac{F}{A}}{\Delta\frac{\ell}{\ell}} = \frac{F\ell}{\Delta\ell A}$$

Change in length of wire $\Delta\ell = \frac{F\ell}{Ay}$

Now Time period $T_M = 2\pi\sqrt{\frac{\ell+\Delta\ell}{g}}$ ....(ii)

$$\frac{T}{T_M} = \frac{2\pi\sqrt{\frac{\ell}{g}}}{2\pi\sqrt{\frac{(\ell+\Delta\ell)}{g}}} \quad \left[\because \frac{i}{ii}\right]$$

$$\frac{T^2}{T_M^2} = \frac{\ell}{\ell+\Delta\ell}$$

$$\frac{T^2}{T_M^2} = \frac{\ell}{\ell+\frac{F\ell}{Ay}} \quad \text{[putting } \Delta\ell \text{ value]}$$

$$\left(\frac{T}{T_M}\right)^2 = \frac{1}{1+\frac{F}{Ay}}$$

$$1+\frac{F}{Ay} = \left(\frac{T_M}{T}\right)^2$$

$$\frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{F}$$

F = mg

$$\Rightarrow \frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

Topic: Simple Harmonic Motion

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 120

9. Consider a spherical shell of radius R at temperature T. the black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

(A) $T \propto e^{-R}$
(B) $T \propto e^{-3R}$
(C) $T \propto \frac{1}{R}$
(D) $T \propto \frac{1}{R^3}$

Answer: (C)

Solution: ∵ in an adiabatic process.

$$dQ = 0$$

So by first law of thermodynamics

$$dQ = dU + dW$$

$$\Rightarrow 0 = dU + d$$

$$\Rightarrow dW = -dU$$

$$\therefore dW = PdV$$

$\Rightarrow PdV = -dU$ ....(i)

Given that $\frac{U}{V} \propto T^4 \Rightarrow U = kVT^4$

$$\Rightarrow dU = k \ \ (VT^4) = K(T^4dV + 4T^3VdT)$$

Also, $P = \frac{1}{3}\frac{U}{V} = \frac{1}{3}\frac{kV \quad ^4}{V} = \frac{KT^4}{3}$

Putting these values in equation

$$\Rightarrow \frac{KT^4}{3}dV = -k(T^4dV \quad 4T^3VdT)$$

$$\Rightarrow \frac{TdV}{3} = -TdV \quad 4VdT$$

$$\Rightarrow \frac{4T}{3}dV = -4VdT$$

$$\Rightarrow \frac{\frac{1}{3}dV}{V} = \frac{-dT}{T}$$

$\Rightarrow \frac{1}{3}lnV = -lnT \Rightarrow lnV = l \quad ^{-3}$

$$\Rightarrow VT^3 = constant$$

$$\frac{4}{3}\pi R^3T^3 = constant$$

$$RT = constan$$

$$\Rightarrow T \propto \frac{1}{R}$$

Topic: Heat &Thermodynamics

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 120

10. A solid body of constant heat capacity $1 J/°C$ is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat

In both the cases body is brought from initial temperature 100^oC to final temperature 200^oC. Entropy change of the body in the two cases respectively is:

(A) $ln2, 4ln2$
(B) $ln2, ln2$
(C) $ln2, 2ln2$
(D) $2ln2, 8ln2$

Answer: (B)

Solution:

Change in entropy $ds = \frac{dQ}{T}$

$\Delta Q =$ heat supplied $= C\,\Delta T$

$dQ = cdT$

$ds = \frac{CdT}{T}$

Integrating both sides

$$\int_{S_i}^{S_f} ds = C\int \frac{dT}{T}$$

$$S_f - S_i = \Delta S = C.\ln T\Big|_{100}^{200}$$

$$= C[\ln 200 - \ln 100]$$

$$\Delta S = C \ln 2$$

$C = 1J/^{\circ}C$

$\Rightarrow \quad \Delta S = \ln 2$

Entrpoy change in same for both cases as C in constant, and temperature change (i.e. from 100 to 200) in same.

Topic: Heat &Thermodynamics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increase as V^q, where V is the volume of the gas. The value of q is:

$$\left(\gamma = \frac{C_P}{C_v}\right)$$

(A) $\frac{3\gamma+5}{6}$

(B) $\frac{3\gamma-5}{6}$

(C) $\frac{\gamma+1}{2}$

(D) $\frac{\gamma-1}{2}$

Answer: (C)

Average time of collision

$t = \frac{mean\ free\ path(\lambda)}{average\ speed\ (v)}$

$t \propto \frac{\lambda}{v}$

$\because \lambda \propto \frac{1}{no.\ of\ molecules\ per\ unit\ volume}$

$\lambda \propto \frac{1}{\left(\frac{N}{v}\right)}$

$\Rightarrow \lambda \propto V$

And $\bar{v} \propto \sqrt{T}$

$\Rightarrow \quad \bar{v} \propto \sqrt{PV}$

$\because \quad P \propto V^{-\gamma}$

for adiabatic process where $\gamma =$ adiabatic coefficient

$\Rightarrow \quad \bar{v} \propto \sqrt{V^{-\gamma} V}$

$\Rightarrow \quad \bar{v} \propto V^{\frac{1-\gamma}{2}}$

So average time

$\therefore t_{avg} \propto \frac{V}{V^{\frac{1-\gamma}{2}}}$

$t_{avg} \propto V^{1-\left(1-\frac{\gamma}{2}\right)}$

$t_{avg} \propto V^{\frac{1+\gamma}{2}}$

$\therefore q = \frac{1+\gamma}{2}$

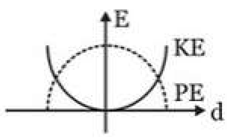
Topic: Heat &Thermodynamics

Difficulty: Difficult (embibe predicted high weightage)

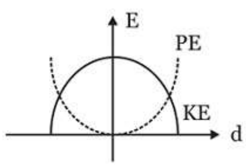
Ideal time: 120

12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. which one of the following represents these correctly? (Graphs are schematic and not drawn to scale)

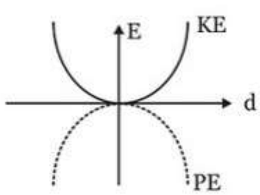
(A)



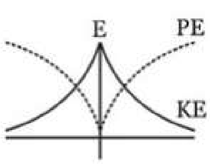
(B)



(C)



(D)



Answer: (B)

Solution:

For simple pendulum performing simple harmonic motion, displacement

$$y = A \sin \omega t$$

Velocity $\frac{dy}{dt} = V = \omega A \cos \omega t$

$$= A\omega\sqrt{1-\sin^2\omega t}$$

$$= A\omega\sqrt{1-\frac{y^2}{A^2}}$$

$$= \omega\sqrt{A^2 - y^2}$$

Kinetic energy $= \frac{1}{2} mv^2$

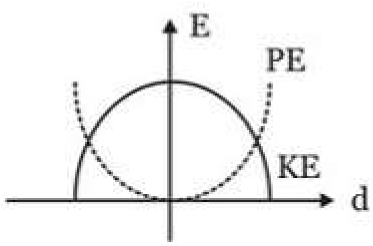
$$= \frac{1}{2} \times m \times \omega^2 (A^2 - y^2)$$

at $\quad y = A$ (extream positions)

Kinetic energy $= \frac{1}{2}\omega^2 m (A^2 - A^2) = 0$

Similarly potential energy $= \frac{1}{2} m\omega^2 y^2$

On plotting graphs of potential energy & Kinetic energy



Topic: Simple Harmonic Motion

Difficulty: Easy (embibe predicted high weightage)

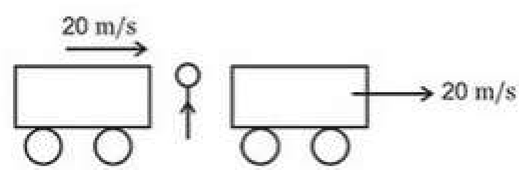
Ideal time: 60

13. A train is moving on a straight track with speed $20ms^{-1}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound $320ms^{-1}$) close to:

(A) 6%
(B) 12%
(C) 18%
(D) 24%

Answer: (B)

Solution:



Before $f_0 = 1000\,Hz$

$$f' = \left(\frac{v}{v - v_s}\right) \times f_0$$

$$= \left(\frac{320}{320-20}\right) \times f_0$$

$$= \left(\frac{320}{300}\right) \times f_0$$

$$= \frac{16f_0}{15}$$

$$f'' = \left(\frac{v}{v+v_s}\right) \times f_0$$

$$f'' = \left(\frac{320}{320+20}\right) f_0$$

$$= \left(\frac{320}{340}\right) f_0$$

$$= \left(\frac{16}{17}\right) f_0$$

Change in frequency

$$= \left(\frac{16}{15} - \frac{16}{17}\right) f_0$$

$\therefore$ Percentage change in frequency

$$= \frac{\left(\frac{16}{15} - \frac{16}{17}\right) f_0}{f_0} \times 100 \approx 12\% \text{ nearly}$$

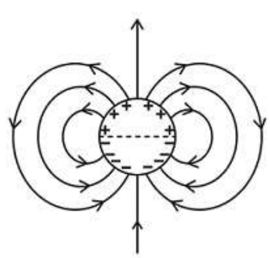
Topic: Wave & Sound

Difficulty: Moderate (embibe predicted high weightage)

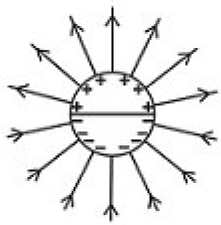
Ideal time: 200

14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in:
(Figures are schematic and not drawn to scale)

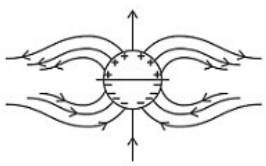
(A)



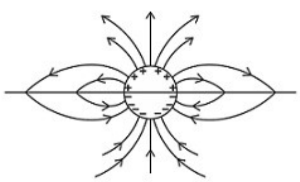
(B)



(C)

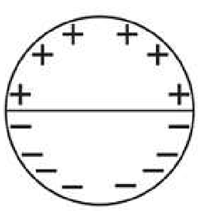


(D)

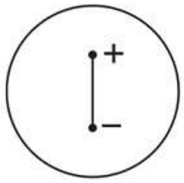


Answer: (A)

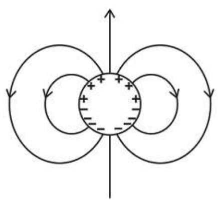
Solution:



Consider cross section of cylinders which is circle the half part of circle which has positive charge can be assume that total positive charge is at centre of mass of semicircle. In the same way we can assume that negative charge is at centre of mass of that semicircle.



Now it acts as a dipole now by the properties of dipole and lows of electric field line where two lines should not intersect the graph would be



Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

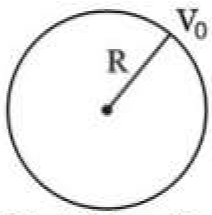
Ideal time: 90

15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potential $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(A) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(B) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(C) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(D) $2R < R_4$

Answer: (C)

Solution:



Potential for uniformly charged solid sphere

$v = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ outside i.e r > R

$v = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ on the surface

$v = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]$ inside i.e. r < R

Clearly potential in decreasing with r.

$\Rightarrow \frac{3v_0}{2}, \frac{5v_0}{4}$ are inside potentials $[\because > v_0]$

$\frac{3v_0}{4}, \frac{v_0}{4}$ are outside potentials $[\because < v_0]$

To get R_1: $\frac{3v_0}{2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{R_1^2}{R^2}\right]$

$v_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$

$\frac{3}{2\times 4\pi\epsilon_0}\frac{Q}{R} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{R_1^2}{R^2}\right]$

$\frac{3}{2} = \frac{3}{2} - \frac{1}{2}\frac{R_1^2}{R^2} \Rightarrow R_1 = 0$

To get R_2: $\frac{5}{4}v_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{R_2^2}{R^2}\right]$

$\frac{5}{4}\frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}\left[\frac{3}{2} - \frac{1}{2}\frac{R_2^2}{R^2}\right]$

$\frac{5}{4} = \frac{3}{2} - \frac{1}{2}\frac{R_2^2}{R^2}$

$\frac{1}{2}\frac{R_2^2}{R^2} = \frac{1}{4}$

$R_2^2 = \frac{R^2}{2}$

$R_2 = \frac{R}{\sqrt{2}}$

To get R_3 : $\frac{3v_0}{4} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_3}$

$\frac{3}{4}\frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_3}$

$\frac{3}{4R} = \frac{1}{R_3}$

$R_3 = \frac{4}{3}R$

$\frac{1}{4} \times \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_4}$

$R_4 = 4R$

$R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3} > R_2$

$R_1 = 0$ and $R_2 < (R_4 - R_3)$

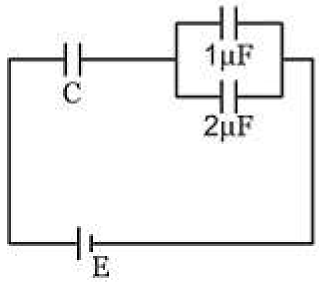
Both options are correct.

Topic: Electrostatics

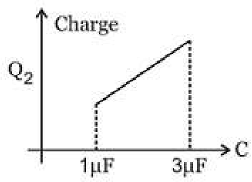
Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 210

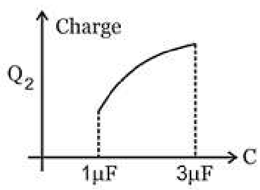
16. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by: (figure are drawn schematically and are not to scale)



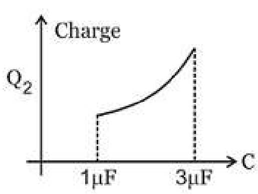
(A)



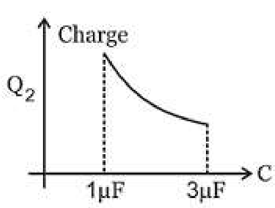
(B)



(C)

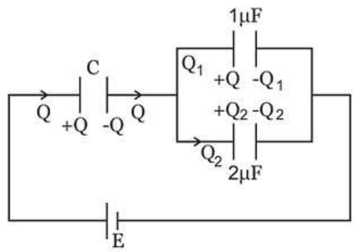


(D)

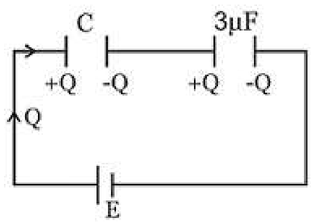


Answer: (B)

Solution:



$\because$ $1 \wedge 2\mu f$ are in parallel.



$\therefore$Equivalent capacitance of the series combination is

C_{eq} is$\frac{3c}{C+3}$

So total charge supplied by battery is $Q = C_{eq} = \frac{3CE}{C+3}$

$\therefore$ Potential difference across parallel combination of $1\mu f$ ad $2\mu f$ is

$\Delta V = \frac{Q}{3} = \frac{CE}{C+3}$

So charge on $2\mu f$ capacitor is

$Q_2 = C_2 \Delta V = \frac{2CE}{C+3}$

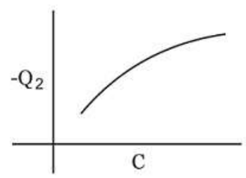
$\Rightarrow \frac{Q_2}{2E} = \frac{C}{C+3} \Rightarrow \frac{Q_2}{2E} = \frac{C+3-3}{C+3}$

$\Rightarrow \frac{Q_2}{2E} = 1 - \frac{3}{C+3} \Rightarrow \left(\frac{Q_2}{2E} - 1\right) = \frac{-3}{C+3}$

$\Rightarrow (Q_2 - 2E)(C+3) = -6E$

Which is of the form $(y - \alpha)(x + \beta) < 0$

So the graph in hyperbola. With down ward curve line. i.e



Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

17. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is $2.5 \times 10^{-4} ms^{-1}$. If the electron density in the wire is $8 \times 10^{28} m^{-3}$, the resistivity of the material is close to:

(A) $1.6 \times 10^{-8} \Omega m$
(B) $1.6 \times 10^{-7} \Omega m$
(C) $1.6 \times 10^{-6} \Omega m$
(D) $1.6 \times 10^{-5} \Omega m$

Answer: (D)

Solution:

Potential difference = 5V

length $= 0.1\text{m} = \ell$

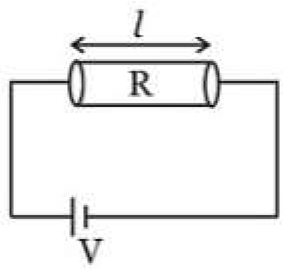
Electron speed = drift velocity $v_d = 2.5 \times 10^{-4} \text{m/s}$

electron density $(n) = 8 \times 10^{28} \text{m}^{-3}$

charge on each electron(e) $= 1.6 \times 10^{-19} \text{c}$

We know $i = n\,Ae\,v_d$ ...(i)

And v = iR ...(ii)



Resistance R is also equal $\frac{\rho\ell}{A}$

$R = \frac{\rho\ell}{A}$

$\rho = \frac{AR}{\ell}$ [ρ = Resistivity]

$= \frac{A}{\ell} \times \frac{v}{i}$ [from (ii)]

$= \frac{Av}{\ell \times nA\, ev_d}$ & [from (i)]

$= \frac{v}{\ell \times n \times e \times v_d}$

$= \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}$

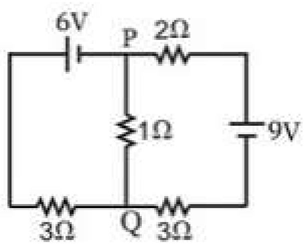
$= 0.16 \times 10^{-4}\Omega m = 1.6 \times 10^{-5}\Omega m$

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 120

18.



In the circuit shown, the current in the 1Ω resistor is:

(A) 1.3 A, from P to Q

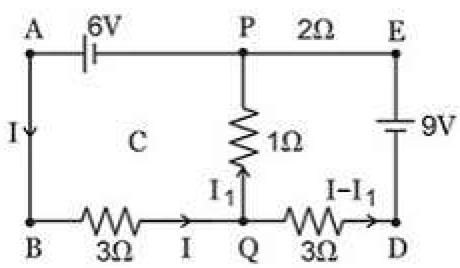
(B) 0 A

(C) 0.13 A, from Q to P

(D) 0.13 A, from P to Q

Answer: (C)

Solution:



The distribution of current according to Kirchhoff's first law is as shown in the circuit.
By Kirchoff's second law (voltage rule)

In loop APQBA using sign curve line

$6 - 3I - I_1 = 0$

$\Rightarrow \quad 3I + I_1 = 6 \qquad \text{...(i)}$

In loop QD & PQ

$\Rightarrow -3(I - I_1) + 9 - 2(I - I_1) + 1 \times I_1 = 0$

$\Rightarrow 9 - 5(I - I_1) + I_1 = 0$

$\Rightarrow 9 + 6I_1 - 5I = 0$

$\Rightarrow 5I - 6I_1 = 9 \quad ...(ii)$

(Multiplying (i) by 5) - (Multiplying (ii) by 3)

$$\begin{array}{lcccccc} \Rightarrow & 15\,I & + & 5\,I_1 & = & 30 \\ & 15\,I & - & 18\,I_1 & = & 27 \\ - & & + & & - & \end{array}$$

$23\,I_1 = 3 \Rightarrow I_1 = \frac{3}{23}\,A = 0.13\,A$

+ve sign of I_1 shows that current 0.13 A flows from Q to P.

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

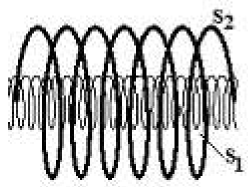
Ideal time: 90

19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then:

(A) $\vec{F_1} = \vec{F_2} = 0$
(B) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards
(C) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$
(D) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$

Answer: (A)

Solution:



S_2 is solenoid with more radius than S_1 field because of S_1 on S_2 is o

$\therefore$ force on S_2 by $S_1 = 0$

In the uniform field of S_2 S_1 behaves as a magnetic dipole

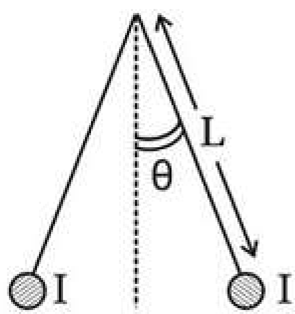
$\therefore$ force on S_1 by S_2 is zero because force on both poles are equal in magnitude and opposite indirection.

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

20.



Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is:
(g = gravitational acceleration)

(A) $sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0 cos\theta}}$

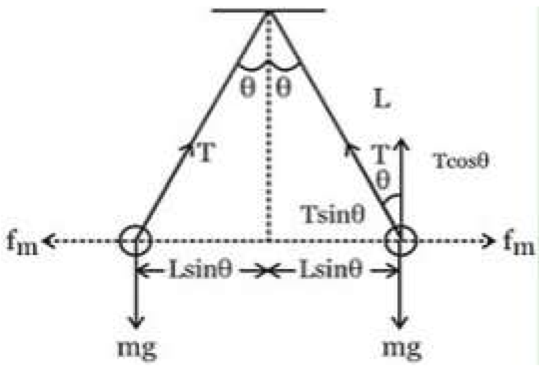
(B) $2sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0 cos\theta}}$

(C) $2\sqrt{\frac{\pi gL}{\mu_0}tan\theta}$

(D) $\sqrt{\frac{\pi\lambda gL}{\mu_0}tan\theta}$

Answer: (B)

Solution



Two wires will repel each other due to current in the same direction and due to magnetic force.

$\therefore$ magnetic force per unit length is

$$\frac{df}{dl} = \frac{\mu_0 I^2}{2\pi(2L\sin\theta)} = \frac{\mu_0 I^2}{4\pi L\sin\theta}$$

and mass per unit length of each mix $= \frac{dm}{dl} = \lambda$

So, magnetic force on total length ℓ' of the mix is $f_m = \frac{\mu_0 I^2 \ell'}{4\pi L\sin\theta}$

and weight $= \lambda\ell' g$

By equilibrium of mix,

$T\sin\theta = f_m$ and $T\cos\theta = mg$

$\Rightarrow \frac{T\sin\theta}{T\cos\theta} = \frac{f_m}{mg} \Rightarrow f_m = mg\tan\theta$

$\Rightarrow \frac{\mu_0 I^2}{4\pi L\sin\theta}\ell' = mg\frac{\sin\theta}{\cos\theta} = \lambda\ell' g\frac{\sin\theta}{\cos\theta}$

$\Rightarrow \frac{\mu_0 I^2}{4\pi L\sin\theta}\ell' = \lambda\ell' g\frac{\sin\theta}{\cos\theta}$

$\Rightarrow I^2 = \frac{\lambda g\pi L}{\mu_0\cos\theta}4\sin^2\theta$

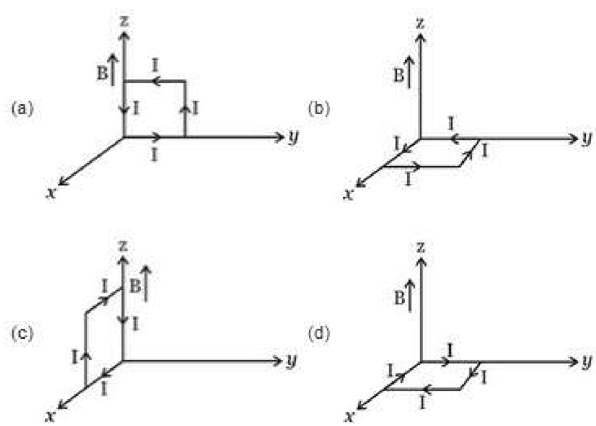
$I = 2\sin\theta\sqrt{\frac{\lambda\pi gL}{\mu_0\cos\theta}}$

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

21. A rectangular loop of sides 10 cm and 5cm carrying a current I of 12 A is placed in different orientations as shown in the figure below:



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(A) (a) and (b), respectively
(B) (a) and (c), respectively
(C) (b) and (d), respectively
(D) (b) and (c), respectively

Answer: (C)

Solution: For a magnetic dipole placed in a uniform magnetic field the torque is given by $\vec{\tau} = \vec{M} \times \vec{B}$ and potential energy U is given as

$$U = -\vec{M}.\vec{B} = -MBcos\theta$$

When $\vec{M}$ is in the same direction as $\vec{B}$ then $\vec{\tau} = 0$ and U is min = - MB as $\theta = 0^o$

⇒ Stable equilibrium is (b). and when $\vec{M}$ then $\vec{\tau} = 0$ and U is max = + MB

As $\theta = 180^o$

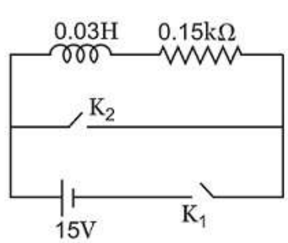
Unstable equilibrium in (d).

Topic: Magnetism

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

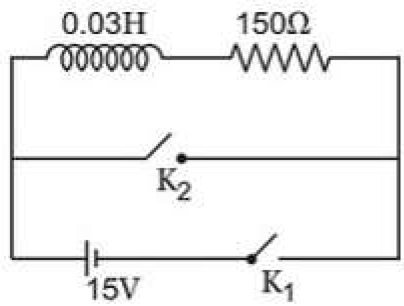
22. An inductor $(L = 0.03H)$ and a resistor $(R = 0.15k\Omega)$ are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$. K_1 is opened and key K_2 is closed simultaneously. At $t = 1ms$, the current in the circuit will be : $(e^5 \cong 150)$



(A) 100 mA
(B) 67 mA
(C) 6.7 mA
(D) 0.67 mA

Answer: (D)

Solution:



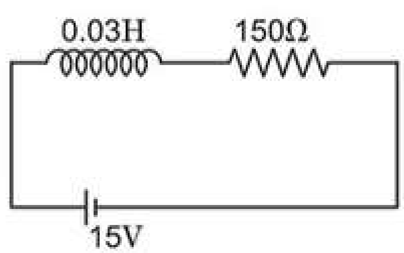
L = 0.03H

$R = 0.15k\Omega = 150\Omega$

$(e^5 \cong 150 \text{ given })$

$E = 15V$

Case I: K_1 is closed for long tiime



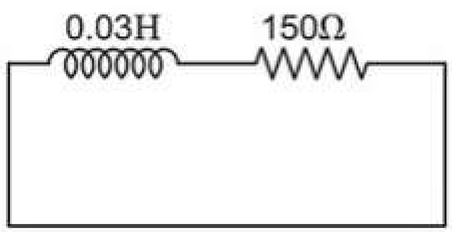
for long time, inductor acts as a conducting wire.

$\Rightarrow$ current in the circuit $= \frac{V}{R}$

$= \frac{15}{150}$

$i_0 = 0.1 \text{ A}$

Case II: K_1 is open and K_2 is closed



Current in the circuit

$i = i_0 e^{\frac{-t}{\tau}}$; $\tau = \frac{L}{R}$

After $t = 1\text{ms} = 10^{-3}\text{s}$

$i = i_0 e^{-\left(\frac{10^{-3} \times 150}{3 \times 10^{-2}}\right)}$

$= 0.1 e^{\frac{-15}{3}}$

$= 0.1 \frac{1}{e^5} = \frac{0.1}{150}$

$= 6.67 \times 10^{-4}$

$= 0.67 \times 10^{-3}\text{A}$

$= 0.67\text{ mA}$

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

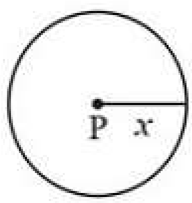
23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:

(A) 1.73 V/m
(B) 2.45 V/m

(C) 5.48 V/m
(D) 7.75 V/m

Answer: (B)

Solution:



For a point source of power = P, then intensity at a point at a separation x from the source is

$I = \frac{Power}{Area} = \frac{P}{4\pi x^2}$

$\because$ Average intensity of EM wave is given by

$I = \frac{1}{2} C \epsilon_0 E_0^2$

$\Rightarrow E_0 = \sqrt{\frac{2P}{4\pi \epsilon_0 C x^2}}$

$\because \frac{1}{4\pi \epsilon_0 \epsilon} = 9 \times 10^9,\ P = 0.1\,W,\ x = 1\,m$

$C = Speed\ of\ light = 3 \times 10^8 m/s$

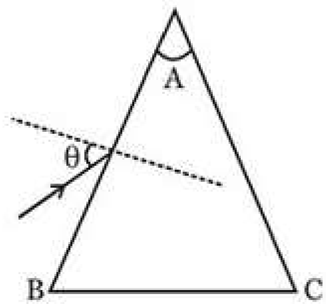
$\Rightarrow E_0 = \sqrt{\frac{2 \times 0.1 \times 9 \times 10^9}{3 \times 10^8 \times 1^2}} = \sqrt{6} = 2.45\ v/m$

Topic: Optics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 120

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided:



(A) $\theta > sin^{-1}\left[\mu sin\left(A - sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

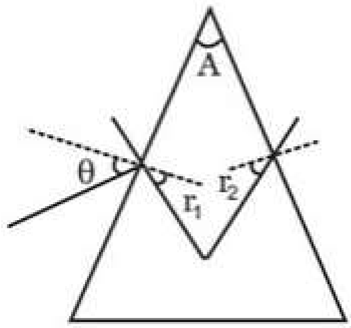
(B) $\theta < sin^{-1}\left[\mu sin\left(A - sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(C) $\theta > co\ \ ^{-1}\left[\mu sin\left(A + sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(D) $\theta < cos^{-1}\left[\mu sin\left(A + sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Answer: (A)

Solution:



For emergence r_2 < critical angle

$\Rightarrow \quad r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$

$A = r_1 + r_2$

$\Rightarrow A - r_1 = r_2$

$\Rightarrow A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow A - \sin^{-1}\left(\frac{1}{\mu}\right) < r_1$

$\because$ By shells law

$\sin\theta = \mu \sin r_1$

$\Rightarrow r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$

$\Rightarrow A - \sin^{-1}\left(\frac{1}{\mu}\right) < \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$

$\Rightarrow \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right) < \frac{\sin\theta}{\mu}$

$\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right) < \sin\theta$

$\Rightarrow \theta > \sin^{-1}\left(\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)$

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

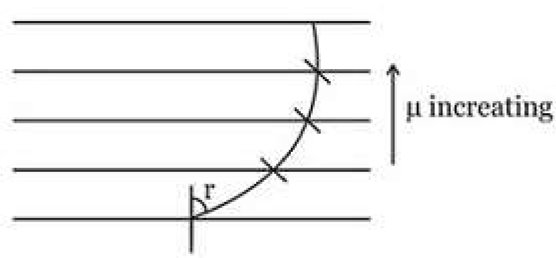
Ideal time: 240

25. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam:

(A) Becomes narrower
(B) Goes horizontally without any deflection
(C) Bends downwards
(D) Bends upwards

Answer: (D)

Solution: Consider air layers with increasing refractive index.



At critical angle it will bend upwards at interface. This process continues at each layer, and light ray bends upwards continuously.

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

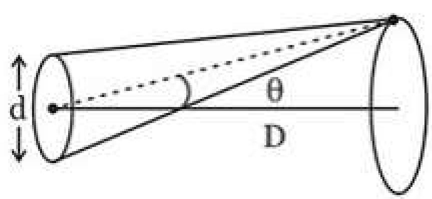
Ideal time: 60

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:

(A) $1\mu m$
(B) $30\mu m$
(C) $100\mu m$
(D) $300\mu m$

Answer: (B)

By fraunhofer diffraction through a circular aperture $\theta = \frac{1.22\lambda}{d}$



D = diameter of pupil $= 2 \times 0.25 = 0.5\ cm$

$\lambda = 500\ nm$

First dark ring is formed by the light diffracted from the hole at an angle θ with the axis

Viewing distance D = 25 cm

$\therefore$ minimum separation between

2 objects $= D\theta$

$= \frac{25 \times 10^{-2} \times 1.22 \times 500 \times 10^{-9}}{5 \times 10^{-1}}$

$= 30 \times 10^{-6} m$

$= 30\ \mu m$

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

27. As an electron makes transition from an excited state to the ground state of a hydrogen like atom/ion:

(A) Its kinetic energy increases but potential energy and total energy decrease
(B) Kinetic energy, potential energy and total energy decrease
(C) Kinetic energy decreases, potential energy increases but total energy remains same
(D) Kinetic energy and total energy decreases but potential energy increases

Answer: (A)

Solution: $U = \frac{-e^2}{4\pi\varepsilon_0 r}$

U = potential energy

$$k = \frac{e^2}{8\pi\varepsilon_0} r$$

K = kinetic energy

$E = U + k = \frac{-e^2}{8\pi\varepsilon_0 r}$

E = Total energy

∴ as electron de-excites from excited state to ground state k increases, U & E decreases

Topic: Modern Physics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

28. Match list-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list:

	List-I		List-II
A	Franck-Hertz Experiment	(i)	Particle nature of light
B	Photo-electric experiment	(ii)	Discrete energy levels of atom
C	Davison-Germer Experiment	(iii)	Wave nature of electron

D		(iv)	Structure of atom

(A) $A-(i); B-(iv); C-(iii)$
(B) $A-(ii); B-(iv); C-(iii)$
(C) $A-(ii); B-(i); C-(iii)$
(D) $A-(iv); B-(iii); C-(ii)$

Answer: (C)

Solution: Frank-Hertz experiment demonstrated the existence of excited states in mercury atoms helping to confirm the quantum theory which predicted that electrons occupied only discrete quantized energy states.

Phot-electric experiment = Demonstrate that photon is the field particle of light which can transfer momentum and energy due to collision.

Davisson-Germer experment = this experiment shows the wave nature of electron.

Topic: Modern Physics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequency of the resultant signal is/are:

(A) 2 MHz only
(B) 2005 kHz, and 1995 kHz
(C) 2005 kHz, 2000 kHz and 1995 kHz
(D) 2000 kHz and 1995 kHz

Answer: (C)

Solution:

Frequency of single wave $= 5\text{kHz} = \text{f}$

Carrier wave frequency = 2MHz

$= 2000\ \text{kHz} = \text{f}_c$

Resultant signal maximum frequency

$= \text{f} + \text{f}_c$

$= 5 + 2000\ \text{kHz}$

$= 2005\ \text{kHz}$

Resultant signal minimum frequency

$= \text{f}_c = \text{f}$

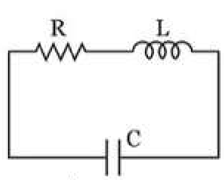
$= 2000 - 5\text{kHz}$

$= 1995\ \text{kHz}$

Topic: Communication Systems

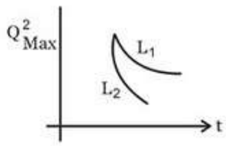
Difficulty: Easy (embibe predicted high weightage)
Ideal time: 60

30. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

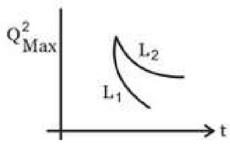


If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and $L_2 (L_1 > L_2)$ of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

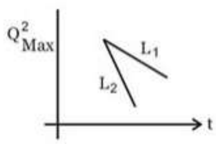
(A)



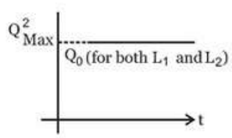
(B)



(C)



(D)



Answer: (A)

Solution:

Comparing to damped pendulum

We write

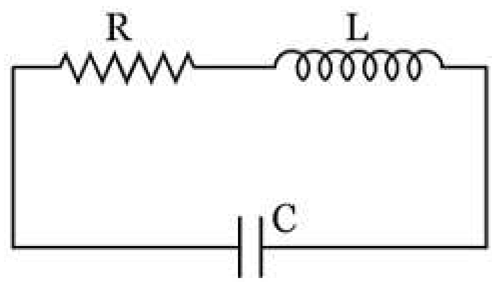
$m\frac{dv}{dt} = -kx - bv$; bv is resistive force

$\Rightarrow$ Amplitude $A = A_0 e^{-\frac{bt}{2m}}$

Comparing results, we can write

$\frac{q}{c} = +L\frac{di}{dt} + iR$

as charge decreasing



$$\frac{q}{c} = L\frac{d^2q}{dt^2} - \frac{dq}{dt}R$$

$$\Rightarrow \quad A = q; R = b, m = L$$

$$q = q_0 e^{-\frac{Rt}{2L}}$$

$$q^2 = q_0^2 e^{-\frac{Rt}{L}}$$

Exponentially decreasing function more the 'L' losses will $\left(\frac{R}{L}\right)$ and more will be $e^{-\left(\frac{Rt}{L}\right)t}$

L_1 graph has higher values than L_2

Topic: Magnetism

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 150

Chemistry

1. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin?

(A) $\frac{1}{103}$

(B) $\frac{1}{206}$

(C) $\frac{2}{309}$

(D) $\frac{1}{412}$

Solution: (D)

$$2C_8H_7SO_3-Na^{+} + Ca^{2+}_{(aq)} \rightarrow (C_8H_7SO_3)_2Ca + 2Na^{+}$$

$2moles(resin) = 412g \equiv 40gCa^{2+}ions$

$1molesofCa^{2+i}$

$Molesof\ Ca^{2+}/g\ ofresin = \frac{1}{412}$

2. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of $4.29Å$. The radius of sodium atom is approximately:

(A) $1.86Å$
(B) $3.22Å$
(C) $5.72Å$
(D) $0.93Å$

Solution: (A) For B.C.C,

$4r = \sqrt{3}a$

$r = \frac{\sqrt{3}}{4}a = \frac{1.732}{4} \times 4.29$

1.86Å

3. Which of the following is the energy of a possible excited state of hydrogen?
   a. +13.6 eV
   b. -6.8 eV
   c. -3.4 eV
   d. +6.8 eV

Solution: (C) $E_n = \frac{-13.6}{n^2} eV$

Where $n = 2 \Rightarrow E_2 = -3.40 eV$

4. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is:

(A) Ion-ion interaction
(B) Ion-dipole interaction
(C) London force
(D) Hydrogen bond

Solution: (D) $Ion - ionintera \propto \frac{1}{r^2}$

$$Ion - dipoleinteraction \propto \frac{1}{r^4}$$

$$Londonforces \propto \frac{1}{r^6}$$

And $Hydrogenbond \propto \frac{1}{r^3}$

5. The following reaction is performed at 298 K.

$$2NO_{(g)} + O_{2(g)} \rightleftharpoons 2NO_{2(g)}$$

The standard free energy of formation of $NO_{(g)}$ is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_{2(g)}$ at 298 K? $(K_P = 1.6 \times 10^{12})$

(A) $R(298)ln(1.6 \times 10^{12}) = 86,600$
(B) $86,600 + R(298)ln(1.6 \times 10^{12})$
(C) $86,600 - \frac{ln(1.6 \times 10^{12})}{R(298)}$

(D) $2 \times 86{,}600 - R(298)ln\dfrac{(1.6 \times 10^{12})}{0.5}$

Solution: (D) $\Delta G^o_{reac} = -2.303RTlogK_P$

$-RTlnK_P$

$-R(298)ln(1.6 \times 10^{12})$

$\Delta G^o_{reac} = 2\Delta G^o_{f(NO_2)} - 2\Delta G^o_{f(NO)g}$

$2\Delta G^o_{f(NO_2)} - 2 \times 86.6 \times 10^3$

$2\Delta G^o_{f(NO_2)} = -R(298)ln(1.6 \times 10^{12}) + 2 \times 86{,}600$

$\Delta G^o_{f(NO_2)} = 86{,}600 - \frac{R(298)}{2} ln(1.6 \times 10^{12})$

$0.5[2 \times 86{,}600 - R(298)ln(1.6 \times 10^{12})]$

6. The vapour pressure of acetone at 20^oC is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20^oC, its vapour pressure was 183 torr. The molar mass $(gmol^{-1})$ of the substance is :

(A) 32

(B) 64

(C) 128

(D) 488

Solution: (B) $\Delta P \quad 185 - 183 = 2torr$

$$\frac{\Delta P}{P^o} = \frac{2}{185} = X_B = \frac{\frac{1.2}{M}}{\left(\frac{1.2}{M}\right) + \frac{100}{58}}$$

$$M_{(CH_3)_2CO} = 15 \times 2 + 16 + 12 = 58\, g/mol$$

$$\frac{1.2}{M} \ll \frac{100}{58}$$

$$\Rightarrow \frac{2}{185} = \frac{1.2}{M} \times \frac{58}{100}$$

$$M = \frac{58 \times 1.2}{100} \times \frac{185}{2}$$

$$64.38 \approx 64\, g/mole$$

7. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}, [B] = 2$ and $[C] = \frac{1}{2}$. The reaction proceeds in the: $K - mol, e = 2.718$
$R = 8.314 J/$

(A) Forward direction because $Q > K_C$
(B) Reverse direction because $Q > K_C$
(C) Forward direction because $Q < K_C$
(D) Reverse direction because $Q < K_C$

Solution: (B) $\Delta G^o = -RT \in K_C$

$$2494.2 = -8.314 \times 300 \in K_C$$

$$2494.2 = -8.314 \times 300 \times 2.303 log K_C$$

$$\frac{-249\ \ .2}{2.303 \times 300 \times 8.314} = -0.44 = log\ \ _C$$

$$log K_C = -0.44 = \acute{1}.56$$

$$K_C = 0.36$$

8. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is: (Atomic mass of Cu = 63.5 amu)
(A) 0 g
(B) 63.5 g
(C) 2 g
(D) 127 g

Solution: (B) $2F \equiv 2 Eqs of Cu$

$$2 \times \frac{63.5}{2} = 63.5g$$

9. Higher order (3) reactions are rare due to:
(A) Low probability of simultaneous collision of all the reacting species
(B) Increase in entropy and activation energy as more molecules are involved
(C) Shifting of equilibrium towards reactants due to elastic collisions.
(D) Loss of active species on collision

Solution: (A) Probability of an event involving more than three molecules in a collision are remote.

10. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is:

(A) 18 mg
(B) 36 mg
(C) 42 mg
(D) 54 mg

Solution: (D) Meqs of CH_3COOH (initial) $50 \times 0.06 = 3Meqs$

Meqs CH_3COOH (final) $50 \times 0.042 = 2.1Meqs$

$$CH_3COOHadsorbed = 3 - 2.1 = 0.9Meqs$$

$9 \times 10^{-1} \times 60\, g/Eq \times 10^{-3}$ g

$$540 \times 10^{-4} = 0.054g$$

$$54mg$$

$Pergram = \frac{54}{3} = 18\, mg/g$ of Charcoal

11. The ionic radii (Å) of $\underset{N}{3-}, \overset{2-}{O}$ and $\overset{-}{F}$ are respectively:

(A) 1.36, 1.40 and 1.71
(B) 1.36, 1.71 and 1.40
(C) 1.71, 1.40 and 1.36
(D) 1.71, 1.36 and 1.40

Solution: (C) As $\frac{Z}{e} \uparrow$ ionic radius decreases for isoelectronic species.

$\underset{N}{3 - \left(\frac{Z}{e}\right)} = \frac{7}{10}$

$\underset{O}{2 - \left(\frac{Z}{e}\right)} = \frac{8}{10}$

$\underset{F}{- \left(\frac{Z}{e}\right)} = \frac{9}{10}$

$-$
$2 -> F$
$3 -> O$
N

12. In the context of the Hall-Heroult process for the extraction of Al, which of the following statements is false?

(A) CO and CO_2 are produced in this process.
(B) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity.
(C) $\frac{3+}{Al}$ is reduced at the cathode to form Al.
(D) Na_2AlF_6 serves as the electrolyte.

Solution: (D) Hall-Heroult process for extraction of Al. Al_2O_3 is electrolyte Na_3AlF_6 reduces the fusion temperature and provides good conductivity.

13. From the following statements regarding H_2O_2 , choose the incorrect statement:

(A) It can act only as an oxidizing agent
(B) It decomposes on exposure to light
(C) It has to be stored in plastic or wax lined glass bottles in dark.
(D) It has to be kept away from dust

Solution: (A) It can act both as oxidizing agent and reducing agent.

14. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?

(A) $CaSO_4$
(B) $BeSO_4$
(C) $BaSO_4$
(D) $SrSO_4$

Solution: (B) $\Delta H_{Hydration} > \Delta H_{Lattice}$

Salt is soluble. $BeSO_4$ is soluble due to high hydration energy of small $\frac{2+}{Be}$ ion. K_{sp} for $BeSO_4$ is very high.

15. Which among the following is the most reactive?

(A) Cl_2
(B) Br_2

(C) I_2
(D) ICI

Solution: (D) $I - Cl$ bond strength is weaker than I_2, Br_2 and Cl_2 (Homonuclear covalent).

16. Match the catalysts to the correct process:

	Catalyst		Process
A.	$TiCl_3$	i.	Wacker process
B.	$PdCl_2$	ii.	Ziegler – Natta polymerization
C.	$CuCl_2$	iii.	Contact process
D.	V_2O_5	iv.	Deacon's process

(A) $A \rightarrow iii, B \rightarrow ii, C \rightarrow iv, D \rightarrow i$
(B) $A \rightarrow ii, B \rightarrow i, C \rightarrow iv, D \rightarrow iii$
(C) $A \rightarrow ii, B \rightarrow iii, C \rightarrow iv, D \rightarrow i$
(D) $A \rightarrow iii, B \rightarrow i, C \rightarrow ii, D \rightarrow iv$

Solution: (B) The Wacker process originally referred to the oxidation of ethylene to acetaldehyde by oxygen in water in the presence of tetrachloropalladate (II) as the catalyst.
In contact process, Platinum used to be the catalyst for this reaction, however as it is susceptible to reacting with arsenic impurities in the sulphur feedstock, vanadium (V) oxide (V_2O_5) is now preferred.
In Deacon's process, the reaction takes place at about 400 to 450^oC in the presence of a variety of catalysts, including copper chloride$(CuCl_2)$.
In Ziegler-Natta catalyst catalyst, Homogenous catalysts usually based on complexes of Ti, Zr or Hf used. They are usually used in combination with different organ aluminium co-catalyst.

17. Which one has the highest boiling point?
(A) He
(B) Ne
(C) Kr
(D) Xe

Solution: (D) Due to higher Vander Waal's forces. Xe has the highest boiling point.

18. The number of geometric isomers that can exist for square planar $\overset{+}{[Pt(Cl)(py)(NH_3)(NH_2OH)]}$ is (py = pyridine):
(A) 2

(B) 3
(C) 4
(D) 6

Solution: (B) Complexes with general formula $\overset{+}{[Mabcd]}$ square planar complex can have three isomers.

19. The color of $KMnO_4$ is due to:
(A) $M \rightarrow L$ charge transfer transition
(B) $d - d$ transition
(C) $L \rightarrow M$ charge transfer transition
(D) $\sigma - \sigma$ transition

Solution: (C) Charge transfer from $\overset{2-}{O}$ to empty d-orbitals of metal ion (Mn^{+7})

20. Assertion: Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.
Reason: The reaction between nitrogen and oxygen requires high temperature.

(A) Both Assertion and Reason are correct and the Reason is the correct explanation for the Assertion.
(B) Both Assertion and Reason are correct, but the Reason is not the correct explanation for the Assertion.
(C) The Assertion is incorrect but the Reason is correct.
(D) Both the Assertion and Reason are incorrect.

Solution: (B) $N_{2(g)}$ & $O_{2(g)}$ react under electric arc at 2000^oC to form$NO_{(g)}$. Both assertion and reason are correct and reason is correct explanation.

21. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is : (Atomic mass : Ag = 108, Br = 80)
(A) 24
(B) 36
(C) 48
(D) 60

Solution: (A) $R - Br \underset{\rightarrow}{Cariusme} \quad AgBr$

250 mg organic compound is RBr

$141 mgAgBr \Rightarrow 141 \times \frac{80}{188} mgBr$

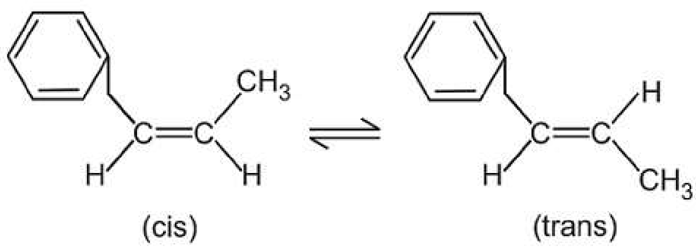
Br in organic compound

$141 \times \frac{80}{188} \times \frac{1}{250} \times 100 = 24$

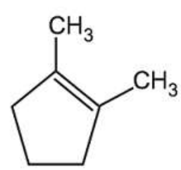
22. Which of the following compounds will exhibit geometrical isomerism?

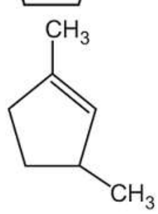
(A) 1 - Phenyl - 2 - butane
(B) 3 - Phenyl - 1 – butane
(C) 2 - Phenyl - 1 – butane
(D) 1, 1 - Diphenyl - 1 – propane

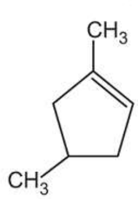
Solution: (A) 1 - Phenyl - 2 - butene:

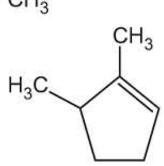


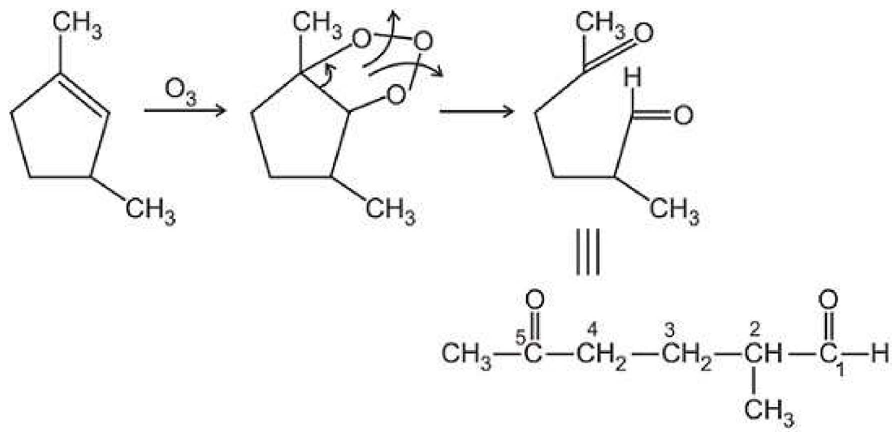
23. Which compound would give 5-keto-2-methyl hexanal upon ozonolysis:

(A)


(B)


(C)


(D)




Solution: (B)

24. The synthesis of alkyl fluorides is best accomplished by:

(A) Free radical fluorination
(B) Sandmeyer's reaction
(C) Finkelstein reaction
(D) Swarts reaction

Solution: (D) Alkyl fluroides is best accomplished by swarts reaction i.e. heating an alkyl chloride/bromide in the presence of metallic fluoride such as $AgF, Hg_2F_2, CoF_2, SbF_3$.

$$CH_3Br + Ag \quad \rightarrow CH_3 - F + Ag$$

The reaction of chlorinated hydrocarbons with metallic fluorides to form chlorofluoro hydrocarbons, such as CCl_2F_2 is known as swarts reaction.

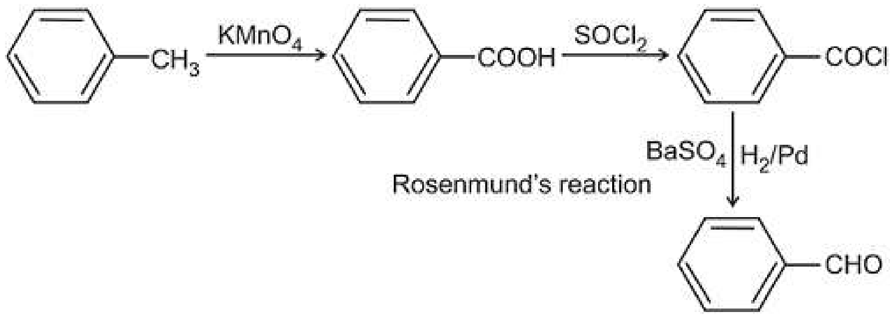
25. In the following sequence of reactions:

$$Toluene \underset{\rightarrow}{KMnO_4} A \underset{\rightarrow}{SOC} {}_2 B\ H_2/Pd\, , \underset{\rightarrow}{BaSO_4} C$$

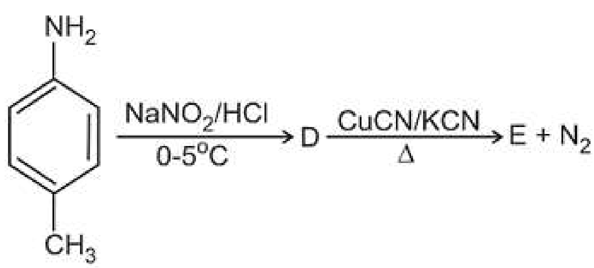
The Product C is:

(A) C_6H_5COOH
(B) $C_6H_5CH_3$
(C) $C_6H_5CH_2OH$
(D) C_6H_5CHO

Solution: (D)

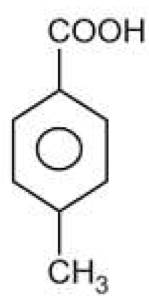


26. In the reaction,

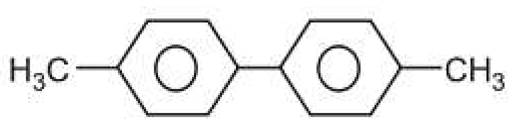


The product E is:

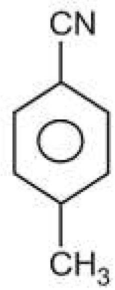
(A)



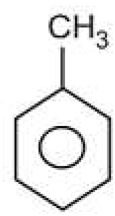
(B)



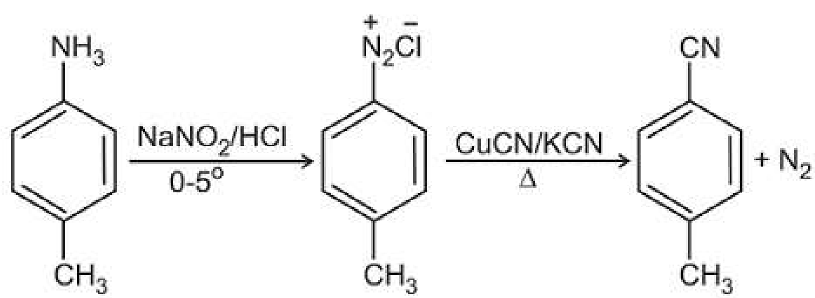
(C)



(D)



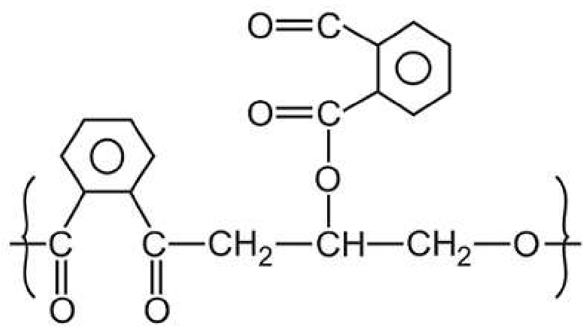
Solution: (C)



27. Which polymer is used in the manufacture of paints and lacquers?

(A) Bakelite
(B) Glyptal
(C) Polypropene
(D) Poly vinyl chloride

Solution: (B) Glyptal is polymer of glycerol and phthalic anhydride.



28. Which of the vitamins given below is water soluble?

(A) Vitamin C
(B) Vitamin D
(C) Vitamin E
(D) Vitamin K

Solution: (A) B complex vitamins and vitamin C are water soluble vitamins that are not stored in the body and must be replaced each day.

29. Which of the following compounds is not an antacid?

(A) Aluminium hydroxide
(B) Cimetidine
(C) Phenelzine
(D) Rantidine

Solution: (C) Ranitidine, Cimetidine and metal hydroxides i.e. Aluminum hydroxide can be used as antacid but not phenelzine. Phenelzine is not an antacid. It is an antidepressant. Antacids are a type of medication that can control the acid levels in stomach. Working of antacids: Antacids counteract (neutralize) the acid in stomach that's used to aid digestion. This helps reduce the symptoms of heartburn and relieves pain.

30. Which of the following compounds is not colored yellow?

(A) $Zn_2[Fe(CN)_6]$
(B) $K_3[Co(NO_2)_6]$
(C) $(NH_4)_3[As(Mo_3O_{10})_4]$
(D) $BaCrO_4$

Solution: (A) Cyanides not yellow.

$$BaCrO_4 -$$
$$K_3[Co(NO_2)_6] -$$
$$(NH_4)_3[As(Mo_3O_{10})_4] -$$

Mathematics

1. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is:

(A) 219

(B) 256

(C) 275

(D) 510

Answer: (A)
Solution:

$$n(A) = 4$$
$$n(B) = 2$$

Number of elements in $A \times B = 2 \cdot 4 = 8$
Number of subsets having at least 3 elements

$$- \;\; {}^{8}C_0 - {}^{8}C_1 - {}^{8}C_2$$
$$2^8$$
$$256 - 1 - 8 - 28$$
$$219$$

2. A complex number z is said to be unimodular if $|z| = 1$. suppose $z_1 \wedge z_2$ are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 \acute{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a:

(A) Straight line parallel to x-axis.

(B) Straight line parallel to y-axis.

(C) Circle of radius 2

(D) Circle of radius $\sqrt{2}$

Answer: (C)
Solution:

Given, $\frac{z_{1-2Z_2}}{2-z_1\acute{z}_2}$ is unimodular

$$\Rightarrow \left|\frac{z_{1-2Z_2}}{2-z_1\acute{z}_2}\right| = 1$$

$$|z_{1-2Z_2}| = |2 - z_1\acute{z}_2|$$

Squaring on both sides.

$$|z_{1-2Z_2}|^2 = |2 - z_1\acute{z}_2|^2$$

$$(z_{1-2Z_2})(\acute{z}_1 - 2\acute{z}_2) = (2 - z_1\acute{z}_2)(2 - \acute{z}_1 z_2)$$

$$\left(\because |z|^2 = z\,\bar{z}\right)$$

$$z_1\bar{z}_1 - 2z_1\,\bar{z}_2 - 2\bar{z}_1\,z_2 + 4z_2\,\bar{z}_2$$

$$= 4 - 2\bar{z}_1\,z_2 - 2z_1\,\bar{z}_2 + z_1\,\bar{z}_1\,z_2\,\bar{z}_2$$

$$|z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2\,|z_2|^2$$

$$|z_1|^2 - 4 + 4|z_2|^2 - |z_1|^2\,|z_2|^2 = 0$$

$$\left(|z_1|^2 - 4\right)\left(1 - |z_2|^2\right) = 0$$

$\Rightarrow \quad |z_1| = 2$ or $|z_2|^2 = 1$ Given, z_2 is not unimodular

$\therefore \quad |z_1| = 2$

$\therefore$ Point z_1 lies on a circle of radius 2.

3. Let $\alpha \wedge \beta$ be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to :

(A) 6

(B) -6

(C) 3

(D) -3

Answer: (C)
Solution:
Given, α, β are roots of $x^2 - 6x - 2 = 0$
$\Rightarrow \alpha^2 - 6\alpha - 2 = 0$ and $\beta^2 - 6\beta - 2 = 0$
$\Rightarrow \alpha^2 - 6 = 6\alpha$ and $\beta^2 - 2 = 6\beta$ .....(1)

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$\frac{(\alpha^{10} - 2\alpha^8) - (\beta^{10} - 2\beta^8)}{2(\alpha^9 - \beta^9)}$$

$$\frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$\frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{2(\alpha^9 - \beta^9)} \qquad [\because (1)]$$

$$\frac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)} = 3$$

4. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where $I is 3 \times 3$ identity matrix, then the ordered pairs (a, b) is equal to:

(A) $(2, -1)$

(B) $(-2, 1)$

(C) $(2, 1)$

(D) $(-2, -1)$

Answer: (D)
Solution:
$AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{array}{llllllll} 1(1) & +2(2)+2(2)1(2) & +2(1)+2(-2) & & & & 2(2) & +2(b) \\ 2(1) & +1(2)-2(2)2(2) & +1(1)-2(-2)1(a) & +2(a) & +a(a) & +1(2) & -2(b) \\ a(1) & +2(2)+b(2)a(2) & +2(1)+b(-2) & & & & 2(2) & +b(b) \end{array}$$

$$\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 0 & a+2b+4 \\ 0 & 9 & 2a-2b+2 \\ a+2b+4 & 2a-2b+2 & a^2+b^2+4 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$Comparingthecorrespondingelements$$

$a+2b \quad +4=0$
$2a-2b \quad +2=0$ and $a^2+b^2+4=9$

$3a+6=0$
$a=-2 \Rightarrow -2+2b+4=0$
$b=-1$

The third equation is useful to verify whether this multiplication is possible.

5. The set of all values of λ for which the system of linear equations:

$2x_1-2x_2+x_3=\lambda x_1$
$2x_1-3x_2+2x_3=\lambda x_2$
$-x_1+2x_2=\lambda x_3$ has a non-trivial solution,

(A) Is an empty set

(B) Is a singleton

(C) Contains two elements.

(D) Contains more than two elements

Answer: (C)
Solution:

$$(2-\lambda)x_1 = 2x_2 + x_3 = 0$$
$$2x_1 - (\lambda+3)x_2 + 2x_3 = 0$$
$$-x_1 + 2x_2 - \lambda x_3 = 0$$

The systems of linear equations will have a non-trivial solution

$$\Rightarrow \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -\lambda-3 & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$
$$\Rightarrow (2-\lambda)[\lambda^2+3\lambda-4] + 2[-2\lambda+2] + 14 - \lambda - 3 = 0$$
$$2\lambda^2 + 6\lambda - 8 - \lambda^3 - 3\lambda^2 + 4\lambda - 4\lambda + 4 + 1 - \lambda = 0$$
$$-\lambda^3 - \lambda^2 + 5\lambda - 3 = 0$$
$$\lambda^3 + \lambda^2 - 5\lambda + 3 = 0$$
$$(\lambda-1)(\lambda^2+2\lambda-3) = 0$$
$$(\lambda-1)(\lambda+3)(\lambda-1) = 0$$
$$\Rightarrow \lambda = 3, -1, -1$$

6. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is:

(A) 216

(B) 192

(C) 120

(D) 72

Answer: (C)
Solution:
4 digit and 5 digit numbers are possible

4 digit numbers:

$\underset{\uparrow}{6/7/8} \quad \underset{\uparrow}{\ } \uparrow 343 \overset{\uparrow}{\underset{2}{}}$ $\qquad Total numbers possible = 3 \cdot 4 \cdot 3 \cdot 2 = 72$

5 digit numbers:

$$\begin{matrix} \uparrow & \uparrow & \uparrow\uparrow & \uparrow \\ 5 & 4 & 32 & 1 \end{matrix} \qquad Totalnumberspossible \quad 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$\therefore Totalnumber \quad 72 + 120 = 192.$$

7. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1 - 2\sqrt{x}\right)^{50}$ is:

(A) $(\quad \frac{1}{2}\big|_{}^{3} 50 + 1)$

(B) $\frac{1}{2}(3^{50})$

(C) $\frac{1}{2}(3^{50} - 1)$

(D) $\frac{1}{2}(2^{50} + 1)$

Answer: (A)
Solution:
Integral powers $\Rightarrow$ odd terms

$$oddterms = \frac{\left(1 - 2\sqrt{x}\right)^{50} + \left(1 + 2\sqrt{x}\right)^{50}}{2}$$

$$\sum of\ coefficie \quad \frac{(1-2)^{50} + (1+2)^{50}}{2}$$

$$\frac{1 + 3^{50}}{2}$$

8. If m is the A.M. of two distinct real numbers $l \wedge n (l, n > 1) \wedge G_1, G_2 \wedge G_3$ are three geometric means between $l \wedge n, then G_1^4 + 2G_2^4 + G_3^4$ equals.

(A) $4l^2mn$

(B) $4lm^2n$

(C) $4lmn^2$

(D) $4l^2m^2n^2$

Answer: (B)
Solution:
m is the A.M. of l, n

$\Rightarrow m = \frac{l+n}{2}$ ....(1)

$G_{1,}G_{2,}G_3$ are G.M. of between l and n

$\Rightarrow l, G_{1,}G_2, G_{3,}n$ are in G.P.

$n = l(r)^4$ $\Rightarrow r^4 = \frac{n}{l}$ ....(2)

$G_1 = lr,$ $G_2 = lr^2,$ $G_3 = lr^3$

$G_1^4 + 2G_2^4 + G_3^4 = l^4r^4 + 2 \times l^4r^8 + l^4 \times r^{12}$

$l^4r^4[1 + 2r^4 + r^8]$

$l^4 \times \frac{n}{l}[1 + r^4]^2 = l^3n \times \left[1 + \frac{n}{l}\right]^2$

$l^3 \times n \times \frac{(l+n)^2}{l^2}$

$ln \times (2m)^2$ $[\because |(1)]$

$4lm^2n$

9. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots is:$

(A) 71

(B) 96

(C) 142

(D) 192

Answer: (B)
Solution:

$$T_n = \frac{1^3+2^3+\cdots+n^3}{1+3+\cdots+(2n-1)} = \frac{\frac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4}$$

Sum of 9 terms = $\sum_{n=1}^{9} \frac{(n+1)^2}{4}$

$$\frac{1}{4} \times [2^2 + 3^2 + \cdots + 10^2]$$

$$\frac{1}{4}[(1^2 + 2^2 + \cdots + 10^2) - 12]$$

$$\frac{1}{4}\left[\frac{10.11.21}{6} - 1\right]$$

$$\frac{1}{4}[384] = 96$$

10. $lim \quad x \to 0 \frac{(1-cos2x)(3+cosx)}{xtan4}$ is equal to:

(A) 2

(B) 3

(C) 2

(D) $\frac{1}{2}$

Answer: (C)
Solutions :

$$2x$$

$$x$$

$$3 + cos$$

$$1 - cos$$

$$\lim_{x \to o}$$

$$\lim_{x \to o} \frac{2sin^2 x \cdot (3 + cosx)}{x \cdot tan4x}$$

$$x$$
$$3 + cos$$

$$2 \cdot \left(\frac{sinx}{x}\right)^2 \cdot$$

$$\lim_{x \to o}$$
$$\frac{2 \cdot (1)^2 \cdot (3+1)}{1 \cdot 4}$$

2.

11. If the function. $g(x) = \begin{cases} k\sqrt{x+1}, & 0 \le x \le 3 \\ mx+2, & 3 < x \le 5 \end{cases}$ is differentiable, then the value of k + m is :

(A) 2

(B) $\frac{16}{5}$

(C) $\frac{10}{3}$

(D) 4

Answer: (A)
Solution:
$g(x)$ is differentiable at x = 3
⇒ $g(x)$ is continuous at x = 3
∴ LHL = RHL $g(3)$

$$x \to 3^+ g(x) = g(3)$$
$$x \to 3^- g(x) = lim$$

$$lim$$

$$k\sqrt{3+1} = m(3) + 2$$
$$2k = 3m + 2$$

$k = \frac{3}{2}m + 1$ ....(1)
$g(x)$ is differentiable

$$\Rightarrow LHD = RHD$$
$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}}, & 0 < x < 3 \\ m, & 3 < x < 5 \end{cases}$$

$$g'(x) = \lim_{x\to 3^+ g'(x)}$$
$$x \to 3^-$$
$$lim$$

$$\frac{k}{2\sqrt{3+1}} = m$$

$$k = 4m$$

From (1), $4m = \frac{3}{2}m + 1$

$$\frac{5m}{2} = 1$$
$$\Rightarrow m = \frac{2}{5}, k = \frac{8}{5}$$

$k + m = \frac{2}{5} + \frac{8}{5} = 2.$

12. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1):

(A) Does not meet the curve again

(B) Meets the curve again in the second quadrant

(C) Meets the curve again in the third quadrant

(D) Meets the curve again in the fourth quadrant

Answer: (D)
Solution:
$x^2 + 2xy - 3y^2 = 0$
$(x + 3y)(x - y) = 0$
Pair of straight lines passing through origin.
$x + 3y = 0$ or $x - y = 0$
Normal exists at (1, 1), which is on $x - y = 0$
⇒Slope of normal at $(1,1) = -1$
∴Equation of normal will be
$y - 1 = -1(x - 1)$
$$y - 1 = -x + 1$$
$x + y = 2$
Now, find the point of intersection with $x + 3y = 0$
$x + y = 2$
$x + 3y = 0$

$$-2y = 2 \Rightarrow y = -1, x = 3$$

(3, 1) lies in fourth quadrant.

13. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1 \wedge x = 2$. If $lim \quad \rightarrow 0\left[1 + \frac{f(x)}{x^2}\right] = 3, then f(2)$ is equal to:

(A) -8

(B)-4

(C) 0

(D) 4

Answer: (C)

Solution:

Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$$lim \quad x \longrightarrow 0\left[1 + \frac{f(x)}{x^2}\right] = 3$$

$$lim \quad x \longrightarrow 0\left[1 + ax^2 + bx + c + \frac{d}{x} + \frac{e}{x^2}\right] = 3$$

This limit exists when d = e = 0

So, $lim \quad x \longrightarrow 0[1 + ax^2 + bx + c] = 3$

$$\Rightarrow c + 1 = 3$$

$$c = 2$$

It is given, $x = 1 \wedge x = 2$ are solutions of $f'(x) = 0$

$$f'(x) = 4ax^3 + 3bx^2 + 2cx$$

$$x(4ax^2 + 3bx + 2c) = 0$$

1,2 are roots of quadratic equation

$$\Rightarrow \sum of\ rooots = \frac{-3b}{4a} = 1 + 2 = 3$$

$$\Rightarrow b = -4a$$

Product of roots = $\frac{2c}{4a} = 1.2 = 2$

$$\Rightarrow a = \frac{c}{4}$$

$$a = \frac{1}{2}, b = -2$$

$$\therefore f(x) = \frac{1}{2}x^4 - 2x^3 + 2x^2$$

$$f(2) = 8 - 16 + 8$$

$$0$$

14. The integral $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$ equals:

(A) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

(B) $(x^4 + 1)^{\frac{1}{4}} + c$

(C) $-(x^4 + 1)^{\frac{1}{4}} + c$

(D) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

Answer: (D)
Solution:

$$I = \int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$$

$$\int \frac{dx}{x^2 \times x^3\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}$$

$Put 1 + \frac{1}{x^4} = t$

$\Rightarrow \frac{-4}{x^5} dx = d$

$\therefore I = \int \frac{\frac{-dt}{4}}{t^{\frac{3}{4}}}$

$-\frac{1}{4} \times \int t^{\frac{-3}{4}} dt$

$\frac{-1}{4} \times \left(\frac{t^{\frac{1}{4}}}{\frac{1}{4}}\right) + c$

$-\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}}$ + c

15. The integral $\int_2^4 \frac{log\ \ ^2}{logx^2 + log(36-12x+x^2)} dx$ is equal to :

(A) 2

(B) 4

(C) 1

(D) 6

Answer: (C)

Solution:

$$I = \int_2^4 \frac{logx^2}{logx^2 + log(6-x)^2} dx$$
$$I = \int_2^4 \frac{log(6-x)^2}{log(6-x)^2 + logx^2} dx$$
$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$
$$2I = \int_2^4 \frac{logx^2 + log(6-x)^2}{log(6-x)^2 + log\ \ ^2} dx$$
$$2I = \int_2^4 \frac{logx^2 + log(6-x)^2}{log(6-x)^2 + logx^2} dx$$

$$2I = \int_{2}^{4} 1dx$$
$$2I = [x]_2^4$$
$$2I = 4 - 2$$

$I = 1.$

16. The area (in sq. units) of the region described by $[(x, y): y^2 \leq 2x \wedge y \geq 4x - 1] is$

(A) $\frac{7}{32}$

(B) $\frac{5}{64}$

(C) $\frac{15}{64}$

(D) $\frac{9}{32}$

Answer: (D)

Solution:

Let us find the points intersections of $y^2 = 2x$ and $y = 4x - 1$

$$(4x - 1)^2 = 2x$$

$$16x^2 - 10x + 1 = 0$$

$$(8x - 1)(2x - 1) = 0$$

$$x = \frac{1}{2}, \frac{1}{8}$$

$x = \frac{1}{2} \Rightarrow y = 4\left(\frac{1}{2}\right) - 1 = 1$

$x = \frac{1}{8} \Rightarrow y = 4\left(\frac{1}{8}\right) - 1 = \frac{-1}{2}$

Required area $\int_{\frac{-1}{2}}^{1} (x_2 - x_1)dy$

$$\int_{\frac{-1}{2}}^{1}\left[\frac{y^2}{2}-\left(\frac{y+1}{4}\right)\right]dy$$

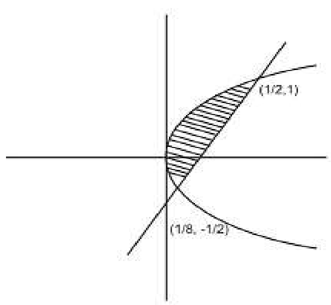
$\frac{1}{6}\times\lfloor y^3\rfloor_{\frac{-1}{2}}^{1}-\frac{1}{8}\lfloor y^2\rfloor_{\frac{-1}{2}}^{1}-\frac{1}{4}\lfloor y\rfloor_{\frac{-1}{2}}^{1}$

$$\frac{1}{6}\left[1+\frac{1}{8}\right]-\frac{1}{8}\left[1-\frac{1}{4}\right]-\frac{1}{4}\left[1+\frac{1}{2}\right]$$

$$\frac{1}{6}\times\frac{9}{8}-\frac{1}{8}\times\frac{3}{4}-\frac{1}{4}\times\frac{3}{2}$$

$$\frac{3}{16}-\frac{3}{32}-\frac{3}{8}$$

$$\frac{6-3-12}{32}=\frac{-9}{32}sq.units$$



17. Let $y(x)$ be the solution of the differential equation $(xlogx)\frac{dy}{dx}+y=2xlogx,(x\geq 1)$. Then $y(e)$ is equal to:

(A) e

(B) 0

(C) 2

(D) 2e

Answer: (B)

Solution:

$\frac{dy}{dx} + \left(\frac{1}{xlogx}\right) y = 2$

Integrating factor = $e^{\int Pdx}$

$e^{\int \frac{1}{xlogx} dx}$

x
log

log
e

$logx$

The solution will be

$$y \cdot e^{\int Pdx} = \int Q \cdot e^{\int Pdx} + c$$

$$y \cdot logx = \int 2 \cdot logx + c$$

$$x - x$$
$$xlog +$$

$$x = 2$$
$$y \cdot log$$

Let $P(1, y_1)$ be any point on the curve

$$y_1(0) = 2(0 - 1) + c$$

$$c = 2$$

e
$e - e$
$elog + 2$
When

$log = 2$
$x = e, y$

$$y = 2$$

18. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is:

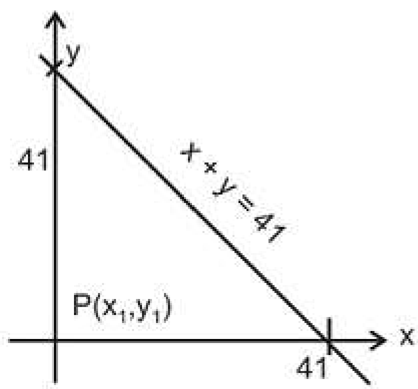
(A) 901

(B) 861

(C) 820

(D) 780

Answer: (D)
Solution:



$P(x_1, y_1)$ lies inside the triangle $\Rightarrow x_1, y_1 \epsilon N$

$$x_1 + y_1 < 41$$
$$\therefore 2 \leq x_1 + y_1 \leq 40$$

Number of points inside = Number of solutions of the equation

$$x_1 + y_1 = n$$
$$2 \leq n \leq 40$$
$$x_1 \geq 1, y_1 \geq 1$$
$$(x_1 - 1) + (y_1 - 1) = (n - 2)$$
$$r$$
$$Numberofnon - negativeintegralsolutionsof \ _1 + x_2 + \cdots . +x_n = ris^{n+r-1}C$$

Number of solutions $\quad {}^{2+(n-2)-1}C_{n-2}$

$\quad {}^{n-1}C_{n-2}$

$n - 1$

We have $\quad 2 \leq n \leq 40$

$\therefore$ Number of solutions = 1 + 2 +....+39

$\frac{39\times40}{2}$

780

19. Locus of the image of the point (2, 3) in the line $(2x-3y+4)+k(x-2y+3)=0, k \in R$,is a :

(A) Straight line parallel to x-axis

(B) Straight line parallel to y-axis

(C) Circle of radius $\sqrt{2}$

(D) Circle of radius $\sqrt{3}$

Answer: (C)
Solution:
Given, family of lines $L_1 + L_2 = 0$
Let us take the lines to be

$$L_2 + \lambda(L_1 - L_2) = 0$$
$$(x-2y+3)+\lambda(x-y+1)=0$$
$$(1+\lambda)x-(2+\lambda)y+(3+\lambda)=0$$

Let, Image of (2, 3) be (h, k)

$$\frac{h-2}{1+\lambda}=\frac{k-3}{-(2+\lambda)}=\frac{-2(2+2\lambda-6-3\lambda+3+\lambda)}{(1+\lambda)^2+(2+\lambda)^2}$$

$\frac{h-2}{\lambda+1}=\frac{k-3}{-(\lambda+2)}=\frac{-2(-1)}{(1+\lambda)^2+(2+\lambda)^2}$ (1)

$$\frac{h-2}{\lambda+1}=\frac{k-3}{-(\lambda+2)} \Rightarrow \frac{\lambda+2}{\lambda+1}=\frac{3-k}{h-2}$$

$1+\frac{1}{\lambda+1}=\frac{3-k}{h-2}$

$\frac{1}{\lambda+1}=\frac{5-h-k}{h-2}$ (2)

From (1): $(h-2)^2+(k-3)^2=\frac{4}{(\lambda+1)^2+(\lambda+2)^2}$ (3)

From (1) and (3):

$$\frac{h-2}{\lambda+1}=\frac{2}{(\lambda+1)^2+(\lambda+2)^2}$$

$$5 - h - k = \frac{1}{2}[(h-2)^2 + (k-3)^2]$$
$$10 - 2h - 2k = h^2 + k^2 - 4h - 6k + 13$$
$$x^2 + y^2 - 2x - 4y + 3 = 0$$

Radius $\sqrt{1+4-3}$
$\sqrt{2}$.

20. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(A) 1

(B) 2

(C) 3

(D) 4

Answer: (C)
Solution:

$$x^2 + y^2 - 4x - 6y - 12 = 0$$
$$C_1(2,3), r_1 = \sqrt{2^2 + 3^2 + 12} = \sqrt{25} = 5$$
$$x^2 + y^2 + 6x + 18y + 26 = 0$$
$$C_2(-3,-9), r_2 = \sqrt{3^2 + 9^2 - 26}$$
$$\sqrt{90 - 26} = 8$$
$$C_1C_2 = \sqrt{5^2 + 12^2} = 13$$
$$C_1C_2 = r_1 + r_2$$

⇒ Externally touching circles
⇒ 3 common tangents.

21. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is :

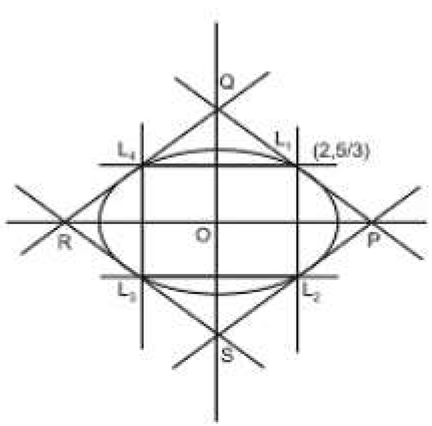
(A) $\frac{27}{4}$

(B) 18

(C) $\frac{27}{2}$

(D) 27

Answer: (D)
Solution:



$$\frac{x^2}{9}+\frac{y^2}{5}=1$$
$$e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$$
$$a^2=9, b^2=5$$

Focii = $(\pm ae, 0)=(\pm 2,0)$
Ends of latus recta $\left(\pm ae, \pm\frac{b^2}{a}\right)$

$$\left(\pm 2, \pm\frac{5}{3}\right)$$

Tangent at 'L' is T = 0

$$\frac{2\cdot x}{9}+\frac{5}{3}\cdot\frac{y}{5}=1$$

It cut coordinates axes at $P\left(\frac{9}{2},0\right)\wedge Q(0,3)$
Area of quadrilateral PQRS = 4(Area of triangle OPQ)

$$4\left(\frac{1}{2}\cdot\frac{9}{2}\cdot 3\right)=27 squareunits.$$

22. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is:

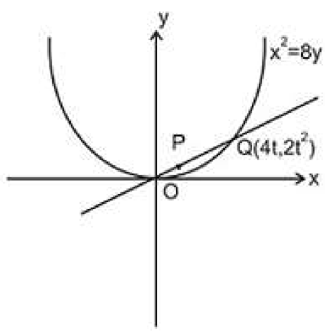
(A) $x^2 = y$

(B) $y^2 = x$

(C) $y^2 = 2x$

(D) $x^2 = 2y$

Answer: (D)
Solution:



General point on $x^2 = 8y$ is $Q(4t, 2t^2)$

Let P(h, k) divide OQ in ratio 1:3

(h, k) = $\left(\frac{1(4t)+3(0)}{1+3}, \frac{1(2t^2)+3(0)}{1+3}\right)$

(h, k) $\left(t, \frac{2t^2}{4}\right)$

h = t and $k = \frac{t^2}{2}$

$$k = \frac{h^2}{2}$$

$\Rightarrow x^2 = 2y$ is required locus.

23. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is

(A) $2\sqrt{14}$

(B) $3\sqrt{21}$

(C) 8

(D) 13

Solution: (D)
$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-1}{12} = t(say)$

$\Rightarrow$ General point is $(2 + 3t, -1 + 4t, 2 + 12t)$

It lies on the plane $x - y + z = 16$

$\Rightarrow 2 + 3t + 1 - 4t + 2 + 12t = 16$

$\Rightarrow 11t = 11$

$\Rightarrow t = 1$

$\therefore$ The point of intersection will be

$$\left(2 + 3(1), -1 + 4(1), 2 + 12(1)\right)$$
$$(5,3,14)$$

Distance from $(1,0,2) = \sqrt{(5-1)^2 + (3-0)^2 + (14-2)^2}$

$\sqrt{4^2 + 3^2 + 12^2}$

13

24. The equation of the plane containing the line $2x - 5y + z = 3; x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is:

(A) $2x + 6y + 12z = 13$

(B) $x + 3y + 6z = -7$

(C) $x + 3y + 6z = 7$

(D) $2x + 6y + 12z = -13$

Solution (C)

Any plane containing the line of intersection of $2x - 5y + z = 3, x + y + 4z = 5$ will be of the form

$$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$$

$$(2 + \lambda)x - (5 - \lambda)y + (1 + 4\lambda)z - (3 + 5\lambda) = 0$$

$$(2 + \lambda)x - (5 - \lambda)y + (1 + 4\lambda)z - (3 + 5\lambda) = 0$$

It is parallel to plane $x + 3y + 6z = 1$

$$\Rightarrow \frac{2 + \lambda}{1} = \frac{\lambda - 5}{3} = \frac{1 + 4\lambda}{6} \neq \frac{-(3 + 5\lambda)}{11}$$

$$\frac{2 + \lambda}{1} = \frac{\lambda - 5}{3} \Rightarrow 6 + 3\lambda = \lambda - 5$$

$$2\lambda = -11$$

$$\lambda = \frac{-11}{2}$$

∴ Required plane is

$$(2x - 5y + z - 3)\frac{-11}{2}(x + y + 4z - 5) = 0$$

$$2(2x - 5y + z - 3) - 11(x + y + 4z - 5) = 0$$

$$4x - 10y + 2z - 6 - 11x - 11y - 44z + 55 = 0$$

$$-7x - 21y - 42z + 49 = 0$$

$$\Rightarrow x + 3y + 6z - 7 = 0$$

25. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}||\vec{a}|$. If θ is the angle between vectors $\vec{b} \wedge \vec{c}$, then a value of $sin\theta$ is:

(A) $\frac{2\sqrt{2}}{3}$

(B) $\frac{-\sqrt{2}}{3}$

(C) $\frac{2}{3}$

(D) $\frac{-2\sqrt{3}}{3}$

Answer: (A)
Solution:

$$(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$\Rightarrow \vec{a} \cdot \vec{c} = 0 \wedge \vec{b} \cdot \vec{c} = \frac{-1}{3}|\vec{b}||\vec{c}|$$

$$|\vec{b}||\vec{c}|cos\theta \quad \frac{-1}{3}|\vec{b}||\vec{c}|$$

$$cos\theta = \frac{-1}{3}$$

$$sin\theta = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

26. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is:

(A) $22\left(\frac{1}{3}\right)^{11}$

(B) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$

(C) $55\left(\frac{2}{3}\right)^{10}$

(D) $220\left(\frac{1}{3}\right)^{12}$

Solution: (A)

B_1	B_2	B_3
12	0	0
11	1	0
10	1	1
10	2	0
9	**3**	**0**
9	2	1
8	4	0
8	**3**	**1**
8	2	2
7	5	0
7	4	1
7	**3**	**2**
6	6	0
6	5	1

6 4 2

<u>6 3 3</u>

5 5 2

<u>5 4 3</u>

4 4 4

Total number of possibilities = 19.

Number of favorable case = 5

Required probability $\frac{5}{19}$

INCORRECT SOLUTION

Required Probability $\quad {}^{12}C_9 \cdot \left(\frac{1}{3}\right)^3 \cdot \left(\frac{2}{3}\right)^9$

$$55 \cdot \left(\frac{2}{3}\right)^{11}$$

The mistake is “we can’t use ${}^{n}C_r$ for identical objects”.

27. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is :

(A) 16.8

(B) 16.0

(C) 15.8

(D) 14.0

Answer: (D)

Solution:

Given, $\frac{\sum_{i=1}^{15} xi+16}{16} = 16$

$$\Rightarrow \sum_{i=1}^{15} xi + 16 = 256$$

$$\sum_{i=1}^{15} xi \quad 240$$

Required mean = $\frac{\sum_{i=1}^{15} xi+3+4+5}{18} = \frac{240+3+4+5}{18}$

$$\frac{252}{18} = 14$$

28. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are $30°, 45° \wedge 60°$ respectively, then the ratio, $AB: BC$,is:

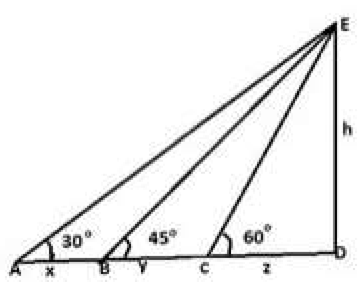
(A) $\sqrt{3}: 1$

(B) $\sqrt{3}: \sqrt{2}$

(C) $1: \sqrt{3}$

(D) $2: 3$

Answer: (A)
Solution: (B)



$$tan30^o = \frac{h}{x+y+z}, tan45^0 = \frac{h}{y+z}, tan60^0 = \frac{h}{z}$$
$$\Rightarrow x+y+z = \sqrt{3}h$$
$$y+z = h$$

$$z = \frac{h}{\sqrt{3}}$$
$$y = h\left(1 - \frac{1}{\sqrt{3}}\right)$$
$$x = (\sqrt{3} - 1)h$$
$$\frac{x}{y} = \frac{(\sqrt{3} - 1)h}{h\left(\frac{\sqrt{3} - 1}{\sqrt{3}}\right)}$$
$$\frac{\sqrt{3}}{1}$$

29. Let $tan^{-1}y = t \quad {}^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$,Then a value of y is:

(A) $\frac{3x-x^3}{1-3x^2}$

(B) $\frac{3x+x^3}{1-3x^2}$

(C) $\frac{3x-x^2}{1+3x^2}$

(D) $\frac{3x+x^3}{1+3x^2}$

Answer: (A)
Solution:

$$tan^{-1}y \; tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < \frac{1}{\sqrt{3}}$$

$$tan^{-1}x + 2tan^{-1}x$$

$$3tan^{-1}x$$

$$tan^{-1}y \quad tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

$$\Rightarrow \quad y = \frac{3x-x^3}{1-3x^2}$$

30. The negation of $s \vee (r \wedge s)$ is equivalent to:

(A) $s \wedge \sim r$

(B) $s \wedge (r \wedge \sim s)$

(C) $s \vee (r \vee \sim s)$

(D) $s \wedge r$

Answer: (D)
Solution:

$$\big((s) \vee (\sim r \wedge s)\big) = s \wedge \big[\sim \big((\sim r) \wedge s\big)\big] \qquad [\because \ (p \wedge q) = (\sim p) \vee (\sim q)]$$

$$s \wedge \big(r \vee (\sim s)\big) \qquad [\sim (p \vee q) = (\sim p) \wedge (\sim q)]$$

$$(s \wedge r) \vee \big(s \wedge (\sim s)\big)$$

$$(s \wedge r) \vee F$$

$s \wedge r \quad (F \rightarrow Fallacy)$