Mathematics

Solution

Q. 1 **(2)**

$$f^{1}(x)=\frac{\alpha}{x}+2\beta x+1$$

when $x=-1,\ f^{1}(x)=0\Rightarrow -\alpha-2\beta+1=0$ ------ (1)

$x=2\ \ f^{1}(x)=0\Rightarrow\ \ \frac{\alpha}{2}+4\beta+1=0$ ------ (2)

$\alpha+8\beta+2=0$ ----- (3)

$(1)+(2)\ \Rightarrow\ 6\beta+3=0\ \Rightarrow\ \beta=-1/2,\ \ \alpha=2$

Solution

Q. 2 **(2)**

Sol : $\frac{x^2}{6}+\frac{y^2}{2}=1$

Let equation tanget : $\frac{x}{\sqrt{6}}\cos\theta+\frac{y}{\sqrt{2}}\sin\theta=1$ -------- (1)

equation of perpendicular line passing through origin

$\frac{x\sin\theta}{\sqrt{2}}-\frac{y}{\sqrt{6}}\cos\theta=0$ ----- (2)

from (1) and (2)

$\cos\theta=\frac{x\sqrt{6}}{x^2+y^2}\ ,\ \ \sin\theta=\frac{\sqrt{2}y}{x^2+y^2}$

$\cos^2\theta+\sin^2\theta=1$

$6x^2+2y^2=(x^2+y^2)^2$

Solution

Q. 3 **(3)**

$$f_4(x)-f_6(x)=\frac{1}{4}\left(\sin^4x+\cos^4x\right)-\frac{1}{6}\left(\sin^6x+\cos^6x\right)$$

$$=\frac{1}{4}\left(1-\frac{1}{2}\sin^2 2x\right)-\frac{1}{6}\left[1-\frac{3}{4}\sin^2 2x\right]$$

$$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

Solution

Q. 4 **(3)**

As $X\subset Y\ \Rightarrow\ X\cup Y=Y$

Solution

Q. 5 **(1)**

Given $A.A^1 = A^1.A$

$B.B^1 = (A^{-1}.A^1).(A^{-1}.A^1)^1$

$= (A^{-1}A^1).(A.A^{1^{-1}})$

$= A^{-1}(A^1A)A^{1^{-1}} = A^{-1}AA^{-1}A^{1^{-1}}$

$= (A^{-1}A)\ (A^1.A^{1^{-1}}) = I.I$

$= I^2 = I$

Solution

Q. 6 **(1)**

$$\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}\,dx$$

$$\int e^{x+\frac{1}{x}}\left[x\left(x-\frac{1}{x^2}\right)+1\right]dx$$

$$x\,e^{x+\frac{1}{x}}+c$$

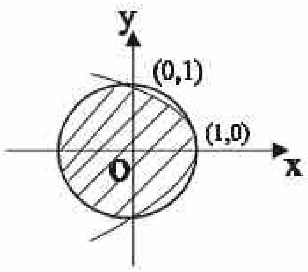
Solution

Q. 7 **(4)**

Area $= \frac{1}{2}\pi.1^2 + 2.\int_0^1(1-y^2)dy$

$= \frac{\pi}{2} + 2.\left[y-\frac{y^3}{3}\right]^1$

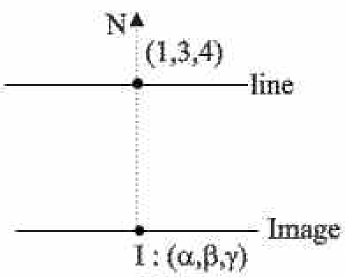
$= \frac{\pi}{2}+\frac{4}{3}$



Solution

Q. 8 **(4)**

line is || to the plane (not in the plane)



line $\perp$ to plane passing through (1, 3, 4)

$$\Rightarrow\ \frac{x-1}{2}=\frac{y-3}{-1}=\frac{2-4}{1}=r$$

P lies on the plane => $r = -1 \Rightarrow P:(-1,4,3)$

$$\frac{\alpha+1}{2} = -1, \ \frac{\beta+3}{2} = 4, \ \frac{r+4}{2} = 3 \Rightarrow (\alpha, \beta, \gamma):(-3,5,2)$$

$$\Rightarrow \quad \text{Image: } \frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$$

Solution

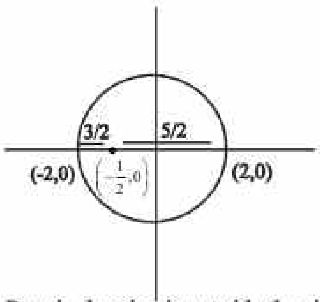
Q. 9 **(1)**

As we know variance of first n natural numbers is $\frac{n^2-1}{12}$

Hence variance of first 50 even natural numbers will be $\frac{50^2-1}{12} = 833$

Solution

Q. 10 **(1)**



Required region is out side the circle

from figure it is clear that $\left|z+\frac{1}{2}\right| \geq \frac{3}{2}$

Hence, minimum value lies in $(1, 2)$

Solution

Q. 11 **(3)**

Let a, ar, ar^2 are in G.P with r > 1

Now a, 2ar, ar^2 are in A.P

$\Rightarrow r^2 + 1 = 4r$

$\Rightarrow (r-2)^2 = 3$

$\Rightarrow r - 2 = \pm\sqrt{3} \Rightarrow r = \sqrt{3} + 2$

Solution

Q. 12 **(3)**

Coefficient of $x^3 = {}^{18}C_3(-2)^3 + a\,{}^{18}C_2(-2)^2 + b\,{}^{18}C_1(-2)^1 = 0$

$51a - 3b = 544$ ----------- (1)

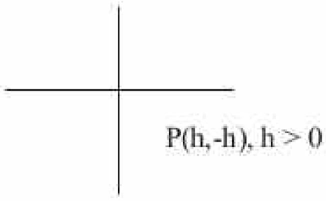
Coefficient of $x^4 = {}^{18}C_4(-2)^4 + a\,{}^{18}C_3(-2)^3 + b\,{}^{18}C_2(-2)^2 = 0$

544a - 51b = 4080 -------(2)

Solving (1) and (2)

$a = 16 \quad b = \frac{272}{3}$

Solution
Q.13 **(2)**



Intersection point between $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ is $\left(\frac{bc - ad}{ab}, \frac{4ad - 5bc}{2ab}\right)$

Now $\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab}$

$\Rightarrow 3bc - 2ad = 0$

Solution
Q. 14 **(3)**

$\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]$

$= (\vec{a}\times\vec{b}).\left[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\right]$

$= (\vec{a}\times\vec{b}).\left[(\vec{b}\times\vec{c}\cdot\vec{a})\vec{c} - (\vec{b}\times\vec{c}\cdot\vec{c})\vec{a}\right]$

$= \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$

$\lambda = 1$

Solution
Q. 15 **(2)**

$P(A') = \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(B \cap A') = 1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{6}$

$= \frac{1}{12}$

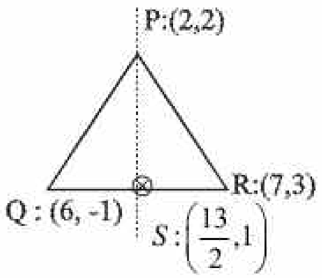
$P(B) = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$

$P(A \cap B) = \frac{1}{4} = P(A).P(B)$

A and B are independent but not equally likely

Solution

Q. 16 **(1)**



Equation of PS : $y-2=\frac{-1}{9/2}(x-2)$

$y-2=\frac{-2}{9}(x-2)$

$9y-18=-2x+4$

2x = 9y - 22 = 0

line || to PS $\Rightarrow$ $2x+9y+\lambda=0$

passing through $(1,-1) \Rightarrow \lambda=7$

$\Rightarrow 2x+9y+7=0$

Solution

Q. 17 **(3)**

$$\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}, \frac{0}{0}\ form$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin\left(\pi(1-\sin^2 x)\right)}{x^2}=\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{x^2}$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{(\pi\sin^2 x)}.\ \pi\frac{\sin^2 x}{x^2}$$

$\Rightarrow 1.\ \pi.\ 1=\pi$

Solution

Q. 18 **(3)**

Given condition $\alpha+\beta=4\alpha\beta$

$$\Rightarrow -\frac{q}{p}=4.\frac{r}{p}\Rightarrow q=-4r$$

p, q, r : in A.P $\Rightarrow$ p + r = 2q

$\Rightarrow$ p + r = -8r $\Rightarrow$ p = -9r

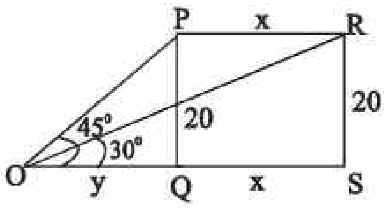
$$\Rightarrow \frac{p}{9}=\frac{q}{4}=\frac{r}{-1}$$

$\Rightarrow$ equation is $9x^2+4x-1=0$

$$|\alpha-\beta| = \sqrt{(\alpha+\beta)^2-4\alpha\beta} = \sqrt{\frac{16}{81}+\frac{4}{9}}$$

$$= \frac{2\sqrt{13}}{9}$$

Solution
Q. 19 : **(3)**



$$\frac{20}{y} = \tan 45 \Rightarrow 20 = y$$

$$\frac{20}{x+y} = \tan 30$$

$$20\sqrt{3} = 20 + x$$

$$x = 20(\sqrt{3}-1)$$

$$\text{speed} = \frac{x}{t} = \frac{20(\sqrt{3}-1)}{1} = 20(\sqrt{3}-1)$$

Solution
Q. 20 **(4)**

$$x-[x] = \{x\} = \frac{-2\pm\sqrt{4+12a^2}}{-6}$$

$$\Rightarrow \{x\} = \frac{2\mp 2\sqrt{1+3a^2}}{6}, \quad x \neq I$$

$$0 < \{x\} < 1$$

$$0 < \frac{1\mp\sqrt{1+3a^2}}{3} < 1$$

$$\Rightarrow a \in (-1,0) \cup (0,1)$$

Solution
Q.21 **(3)**

$$\int_0^{\pi}\left|1-2\sin\frac{x}{2}\right| dx$$

$$= 2\int_0^{\pi/2}|1-2\sin t|\, dt \text{ where } \frac{x}{2} = t \text{ ; } dx = 2dt$$

$$= 2\left[\int_0^{\pi/6}(1-2\sin t)dt + \int_{\pi/6}^{\pi/2}(2\sin t - 1)dt\right]$$

$$= 2\left[\int [t+2\cos t]_0^{\pi/6} + [-2\cos t - t]_{\pi/6}^{\pi/2}\right]$$

$$= 2\left[\left(\frac{\pi}{6}+\sqrt{3}-2\right)-\left(\frac{\pi}{2}-\sqrt{3}-\frac{\pi}{6}\right)\right]$$

$$= 2\left[2\sqrt{3}-2-\frac{\pi}{6}\right]$$

$$= 4\sqrt{3} - 4 - \frac{\pi}{3}$$

Solution
Q. 22 **(3)**

Using Mean Value Theorem

$$f^1(c) = \frac{f(1) - f(0)}{1-0} = \frac{6-2}{1} = 4$$

and $g^1(c) = \dfrac{g(1) - g(0)}{f - 0} = \dfrac{2-0}{1} = 2$

$$\Rightarrow f^1(c) = 2g^1(c)$$

Solution
Q. 23 **(3)**

$$g(x) = f^{-1}(x)$$

$$\Rightarrow f\left[g(x)\right] = x$$

$$\Rightarrow f'[g(x)] \times g'(x) = 1$$

$$\Rightarrow g^1(x) = \frac{1}{f^1\left[g(x)\right]} = \frac{1}{\frac{1}{1 + \left[g(x)\right]^5}} = 1 + \left[g(x)\right]^5$$

Solution
Q. 24 **(2)**

$$10^9 + 2(11)^1 \times 10^8 + 3(11)^2(10)^1 + ----+10\times(11)^9 = 10^9 k$$

$$\Rightarrow 10^9\left[1 + 2\times\frac{11}{10} + 3\times\left(\frac{11}{10}\right)^2 + ---+10\times\left(\frac{11}{10}\right)^9\right] = 10^9 k$$

$$\Rightarrow k = 1 + 2\times\frac{11}{10} + 3\times\left(\frac{11}{10}\right)^2 + ----10\times\left(\frac{11}{10}\right)^9 \quad \text{-----------------} \ (1)$$

Now $\dfrac{11}{10}k = \dfrac{11}{10} + 2\times\left(\dfrac{11}{10}\right)^2 + -----+9\left(\dfrac{11}{10}\right)^9 + 10\times\left(\dfrac{11}{10}\right)^{10}$ ------(2)

equation (1) - equation (2)

$$\Rightarrow \frac{-1}{10}k = 1 + \frac{11}{10} + \left(\frac{11}{10}\right)^2 + -----+\left(\frac{11}{10}\right)^9 - 10\times\left(\frac{11}{10}\right)^{10}$$

$$= \frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1} - 10\times\left(\frac{11}{10}\right)^{10}$$

$$\Rightarrow k = -10^2\times\left(\frac{11}{10}\right)^{10} + 10^2 + 10^2\times\left(\frac{11}{10}\right)^{10} = 100$$

Solution
Q. 25 **(2)**

$$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^2+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = (1-\alpha)^2\ (\alpha-\beta)^2\ (\beta-1)^2$$

Hence k = 1

Solution
Q. 26 **(4)**

Equation of tangent of $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$

If it is tangent of $x^2 = -32y$ then

$$\frac{1}{m} = 8m^2$$

$$m^3 = \frac{1}{8} \Rightarrow m = \frac{1}{2}$$

Solution
Q. 27 **(4)**

P	q	$p \Leftrightarrow q$	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	T	T	F	T

Hence $\sim (p \leftrightarrow \sim q) = p \Leftrightarrow q$

Solution
Q. 28 **(4)**

$$\frac{dp}{dt} = \frac{p}{2} - 200 \Rightarrow \quad \int \frac{dp}{\frac{p}{2} - 200} = \int dt + c$$

$$\Rightarrow \ell n \left| \frac{p}{2} - 200 \right| = t + C$$

$$\frac{p}{2} - 200 = \alpha . e^{t/2}$$

$p = 2\alpha e^{t/2} + 400$

at t = 0, p = 100

$\Rightarrow 2\alpha = -300$

$\Rightarrow p = 400 - 300.e^{t/2}$

Solution

Q. 29 **(3)**

$y + 1 = \sqrt{1^2 + (y-1)^2}$

$(y+1)^2 - (y-1)^2 = 1$

4y = 1

$y = \frac{1}{4}$

radius $= \frac{1}{4}$

Solution

Q.30 **(4)**

$(m+n)^2 = m^2 + n^2 \Rightarrow 2mn = 0 \Rightarrow m = 0 \; or \; n = 0$

1) If m = 0 $\Rightarrow$ l = -n $\Rightarrow$ D.R $\langle 1, 0, -1 \rangle$

2) If n = 0 $\Rightarrow$ l = -m $\Rightarrow$ D.R $\langle 1, -1, 0 \rangle$

Now $\ell_1 \ell_2 + m_1 m_2 + n_1 n_2 = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + 0 + 0 = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

PHYSICS

Solution:

Q. 31 (2)

$$W = \int_0^L F dx$$

$$= \int_0^L \left(ax + bx^2\right) dx$$

$$= \left[\frac{ax^2}{2} + \frac{bx^3 3}{}\right]_0^L$$

$$= \frac{aL^2}{2} + \frac{bL^3}{3}$$

Solutoin :

Q.32 (2)

$$B = \mu_0 ni$$

$$\frac{B}{\mu_0} = H = ni$$

$$3 \times 10^3 = \frac{100}{10^{-1}} i$$

$$i = 3amp$$

Solutions

Q.33 (2)

Total power $= 15 \times 40 + 5 \times 100 + 5 \times 80 + 1 \times 1000$

$= 2500 W$

Applied voltage $= 220 V$

Current $= \frac{2500}{220} \simeq 11.36 A$

Hence minimum capacity of fuse wire is 12 A

Solution :

Q.34 (4)

$76 \times 8 = P \times x$

Where P is new pressure exerted by air and x is length of air column

or $P = \frac{76 \times 8}{x}$

(Pressure exerted by air column) + (Pressure exerted by mercury column) = Atmospheric pressure

$$\frac{76\times 8}{x}+(54-x)=76$$

Solving the obtain $x=16cm$

Solution :

Q.35 (4)

Net torque acting on bodies zero, so angular momentum is conserved

Solutoin :

Q.36 (4)

$$I=e^{\frac{1000V}{T}-1}\simeq e^{\frac{1000V}{T}}$$

$$\Delta I\simeq\frac{dI}{dV}\times\Delta V=\frac{1000}{T}\times e^{\frac{1000V}{T}}\times\Delta V$$

$$=\frac{1000}{300}\times(5mA)\times(0.01)$$

$$\frac{1}{6}mA\simeq 0.2mA$$

Soluiton :

Q. 37 (2)

Time taken by the particle to reach maximum height $=\frac{u}{g}$

Time taken by the particle to reach ground $=\frac{u}{g}+\sqrt{\frac{2}{g}\left(H+\frac{u^2}{2g}\right)}$

According to the given problem $\frac{nu}{g}=\frac{u}{g}+\sqrt{\frac{2}{g}\left(H+\frac{u^2}{2g}\right)}$

or $\frac{(n-1)^2u^2}{g^2}=\frac{2}{g}\left(H+\frac{u^2}{2g}\right)$

or $u^2n(n-2)=2gH$

Solution :

Q.38 (1)

$$\frac{1}{f_1}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{9}{8}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{8}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_2}=\left(\left(\frac{3/2}{5/3}\right)-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{9}{10}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)_2=-\frac{1}{10}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

So $f_1 > f$ and f_2 is Negative

Solution :
Q.39 (4)

$$E=\frac{\sigma}{K\in_0}$$

$$3\times10^4=\frac{\sigma}{2.2\times8.85\times10^{-12}}$$

or $\sigma = 3\times10^4\times2.2\times8.85\times10^{-12}$

$\simeq 5.8\times10^{-7}\, c/m^2$

Solution :
Q.40 (2)

Decay current $i=i_0e^{-\frac{t}{\tau}}$

Voltage across the resistance is

$$V_R=iR=V_0e^{-\frac{t}{\tau}}$$

Voltage across the inductor is

$$V_L=L\frac{di}{dt}=L\left(-\frac{i_0}{\tau}\cdot e^{-\frac{t}{\tau}}\right)=-V_0e^{-\frac{t}{\tau}}$$

The Ratio $V_R/V_L=-1$

Solutoin :
Q.41 (3)

$$I_A\cos^2\theta_1=I_B\cos^2\theta_2$$

$$I_A\cos^2 30^\circ=I_B\cos^2 60^\circ$$

$$I_A\left(\frac{3}{4}\right)=I_B\left(\frac{1}{4}\right)$$

$$\frac{I_A}{I_B}=\frac{1}{3}$$

Solution :
Q.42 (2)

$OB = R\cos\alpha \quad CD = R\sin\alpha$

$OA = R\sin\alpha \quad DE = R\cos\alpha$

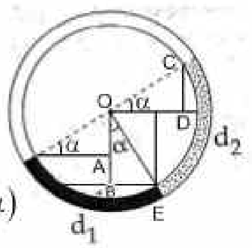
$P_1 = P_0 + d_1 g(AB)$

$P_1 = P_0 + d_1 g(OB - OA)$

$P_1 = P_0 + d_1 gR(\cos\alpha - \sin\alpha)$

$P_2 = P_0 + d_2 g(CE) = P_0 + d_2 g(CD + De)$

$P_2 = P_2 + d_2 g(R\sin\alpha + R\cos\alpha) = P_0 + d_2 gR(\sin\alpha + \cos\alpha)$



As system is in equlibrium $P_1 = P_2$

$P_0 + d_1 gR(\cos\alpha - \sin\alpha) = P_0 + d_2 gR(\sin\alpha + \cos\alpha)$

$d_1(\cos\alpha - \sin\alpha) = d_2(\sin\alpha + \cos\alpha)$

$\frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$

Solution :
43. (1)

Pressure $= \frac{f}{A} =$ thermal sterss

$= Y\alpha\Delta T$

$= 2\times 10^{11} \times 1.1\times 10^{-5} \times 100 = 2.2\times 10^{9}\, Pa$

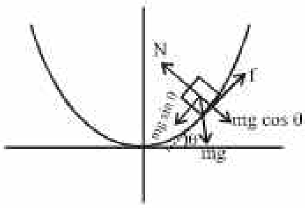
Solution :
44. (4)

$y = \frac{x^3}{6}$

$\tan\theta = \frac{dy}{dx} = \frac{3x^2}{6} = \frac{x^2}{2}$

$f = mg\sin\theta$

$\tan\theta = \mu$

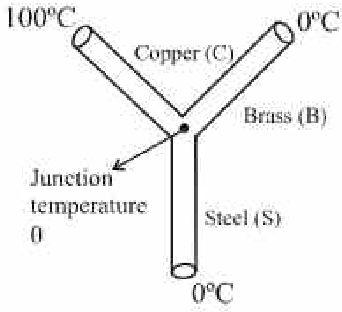


$\frac{x^2}{2} = 0.5 \Rightarrow x^2 = 1$

$x = 1$

height of the block would be $= \frac{x^3}{6} = \frac{1}{6} m$

Solution
Q. 45 (4)



Using law of junction, we obtain

$$\frac{K_C A(100-\theta)}{46} = \frac{K_B A(\theta-0)}{13} + \frac{K_S A(\theta-0)}{12}$$

Solving above equation we obtain.

$\theta \simeq 41.03°C$

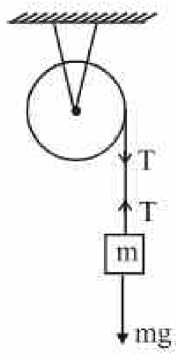
Rate of flow of heat through the copper rod

$$= \frac{K_C A(100-\theta)}{l_C}$$

$$= \frac{0.96 \times 4(100-41.03)}{46}$$

$\simeq 1.2 \text{ cal/s}$

Solution
Q. 46 (1)



For block,

$mg - T = ma$ ....(1)

For cylinder, $TR = mR^2 \times \frac{a}{R}$ ....(2)

Solving above two equations, we obtain

$a = \frac{g}{2}$

Q.47. (3)

Solution :

Q.48 (1)

The energy of the photon released in the transition corresponding to $3 \rightarrow 2$ is

$$h\nu = E_3 - E_2 = 13.6\left(\frac{1}{4}-\frac{1}{9}\right) \approx 2eV$$

if K.E of electron ejected from the metal plate is K

$$R = \frac{\sqrt{2mK}}{qB}$$

$$K = \frac{R^2q^2B^2}{2m} = \frac{\left(10\times10^{-3}\right)^2\left(1.6\times10^{-19}\right)^2\left(3\times10^{-4}\right)^2}{2\times9.1\times10^{-31}}J \;= 0.79eV.$$

$h\nu = w + K.E.$

$2eV = W + 0.79eV$

$W \approx 1.1eV$

Solution

Q. 49 (3)

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

and $u_B = \frac{1}{2}\frac{B^2}{\mu}$

But $E = cB$

$\therefore u_E = u_B$

Solution :

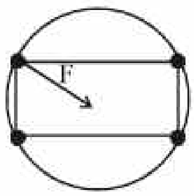
Q.50. (2)

$\left\{f \propto \frac{1}{\lambda}\right\}$ and $\mu = A + \frac{B}{\lambda^2}$ $\sin\theta_c = \frac{1}{\mu}$

$f\uparrow, \lambda\downarrow, \mu\uparrow, \theta_c\downarrow$

Solution :

Q. 51. (3)



Resultant gravitational force on one particle due to other three particles $= F = \frac{GM^2}{4R^2}\left(2\sqrt{2}+1\right)$

F provides necessary centripetal force

$$\therefore \frac{Mv^2}{R} = \frac{GM^2}{4R^2}\left(2\sqrt{2}+1\right)$$

Or $V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(2\sqrt{2}+1\right)}$

Solution :

Q.52. (3)

Consider equation of given SHM as

$$X = A\cos\omega t$$

According to given problem

$$A - a = A\cos\omega t \quad ...(1)$$

and $$A - 3a = A\cos 2\omega t \quad ...(2)$$

$$\therefore \quad \cos\omega t = \frac{A-a}{A} \text{ and } \cos 2\omega t = \frac{A-3a}{A}$$

$$\cos 2wt = 2\cos^2 wt - 1$$

or $\left(\frac{A-3a}{A}\right)^2 = 2\left(\frac{A-a}{A}\right)^2 - 1$

or $A = 2a$

substituting $A = 2a$ in equation (1)

we obtain $a = 2a\cos wt$

or $\cos wt = \frac{1}{2}$

or $\frac{2\pi}{T}\tau = \frac{\pi}{3}$

or $T = 6\tau$

Solution:

Q.53. (1)

Force acting on the rod at any instant is

$$f = i(\ell B) = 10\times 3\times 3\times 10^{-4} e^{-0.2x}$$

$$\vec{f} = -9\times 10^{-3} e^{-0.2x} N\,\hat{a}_x$$

Total work done in moving from $x = 0$ to $x = 2m$ is

$$w\int_0^2 f dx = \int_0^2 9\times 10^{-3} e^{-0.2x} dx$$

$$= 9\times 10^{-3}\left(\frac{e^{-0.2x}}{-0.2}\right)_0^2$$

$$= 9\times 10^{-3}\left(\frac{e^{-0.4}}{-0.2} + \frac{1}{0.2}\right)$$

$$w = \frac{9\times10^{-3}}{0.2}\left(1-e^{-0.4}\right)$$

Power required is $P = \frac{w}{f} = \frac{9\times10^{-3}}{0.2\times5\times10^{-3}}\left(1-e^{-0.4}\right)$

$$P = 9\left(1-e^{-0.4}\right) = 9(1-0.67)$$

$$= 9(0.33) = 2.97w$$

Solution:

Q.54. (4)

In forward bias connetion, P - n junction diode should connect such that P should be at higher potention than N. so desired connection is option : 4

Solution

Q. 55 (2)

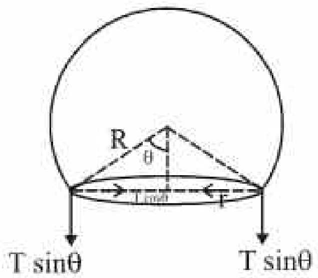
$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

or $\lambda Z^2 =$ constant

or $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

Solution

Q.56 (4)



Bubble starts to move up if

unbalance force due to excess pressure and force exerted by base due to surface tension = Buoyancy force

$$T\sin\theta \times 2\pi r + \frac{2T}{R}\times \pi r^2 = \frac{4}{3}\pi R^3 \rho g$$

or $\frac{4T\times \pi r^3}{R} = \frac{4\pi R^3 \rho g}{3}$

or $r = R\sqrt{\frac{\rho g}{3T}}$

Solutions

Q.57. (2)

$$(2n+1)\frac{V}{4\ell} \le 1250$$

$$(2n+1) \le 1250 \times \frac{4 \times 0.8}{340}$$

or $(2n+1) \le 12.5$

$\therefore$ Possible no of harmonics $= 6$

Solution:

Q.58. (2)

$$dV = -\vec{E} \cdot \vec{dr}$$

$$\int dV = -\int_{x=0}^{x=2m} E\, dx \cos 0$$

$$\int_{V_0}^{V_A} dV = -\int_0^2 30x^2 dx = -30.\left(\frac{x^3}{3}\right)_0^2$$

$$V_A - V_0 = -30\left(\frac{8}{3} - 0\right) = -80V$$

Q.59. (1)

Solution:

Q.60. (3)

$$\Delta U_{C\to A} < 0$$

$$\Delta U_{A\to B} > 0$$

$$\Delta U_{B\to C} = \frac{fnR\,\Delta T}{2}$$

$$= \frac{5}{2} \times 1 \times R(-200)$$

$$= -500R$$

CHEMISTRY

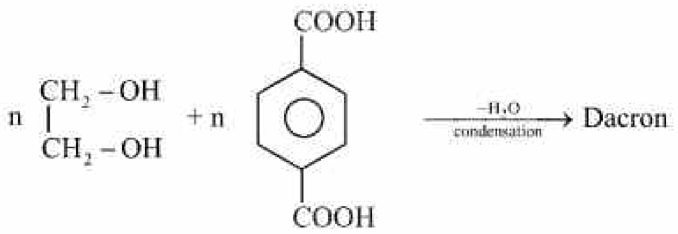
Solution

Q.61 (2)

1) $CH_2=CH-C\equiv N \xrightarrow{Add^n} \left(CH_2-\underset{CN}{\underset{|}{CH}}\right)_n$

Acrylonitrile

2) Dacron is condensation polymer



Neopreone

3) $nCH_2=CH-\underset{Cl}{\underset{|}{C}}=CH_2 \xrightarrow{Add^n} \left(CH_2-CH=\underset{Cl}{\underset{|}{C}}-CH_2\right)$

Neoprene

4) $nCF_2=CF_2 \xrightarrow{Add^n} \left(CF_2-CF_2\right)_n$

Teflon

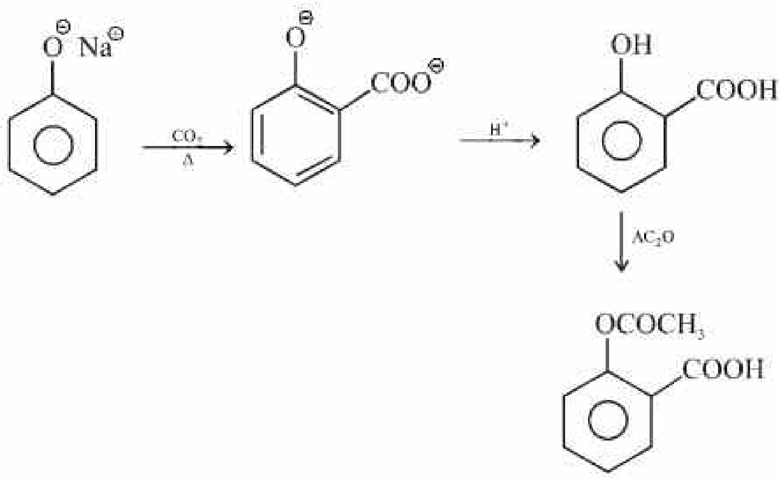
Solution

Q.62 (2)

NO is paramagnetic with 1 unpaired electron.

Solution

Q.63 (2)



Solution

Q.64 (2)

$$E^{o}_{cell} = \underset{\text{Cathod}}{E^{o}_{R.P}} + \underset{\text{Anode}}{\cdot}$$

$$= -1.18 - 1.51$$

$$= -2.69$$

Since E^{o}_{cell} is negative the reaction will not occur.

Solution

Q.65 (2)

$$C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$$

In bomb calorimeter heat liberated = 1364.47 kJ

So, $\Delta U = -1364.47\,kJ$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -1364.47 + (-1) \times \frac{8.314}{1000} \times 298$$

$$= -1366.95\ kJ/mol.$$

Solution

Q.66 (3)

$$\begin{tikzcd}
\underset{(1.4\,g)}{\text{Organic compound}} \arrow[r, "\text{kjeldahl method}"] & NH_3 \arrow[d, "H_2SO_4\ (6\ \text{milli moles})"] \\
\begin{array}{l}\text{So milli moles of } H_2SO_4 \\ \text{used with } NH_3 = 5\end{array} & \begin{array}{l}\text{Remaining} \\ H_2SO_4\end{array} \arrow[l, "\text{NaOH}", "\text{2 milli moles}"']
\end{tikzcd}$$

So milli moles of $NH_3 = 5 \times 2$ (1 H_2SO_4 requires 2 NH_3)

$= 10$

So moles of $NH_3 = \frac{10}{1000}$

(same will be moles of N)

so mass of $N = \frac{14 \times 10}{1000} = 0.14\,g$

so % $N = \frac{0.14}{1.4} \times 100 = 10$

Solution

Q.67 (4)

$$2CH_3 - CCl_3 \xrightarrow[-6AgCl]{Ag} \underset{\text{2-Butyne}}{CH_3 - C \equiv C - CH_3}$$

Solution

Q.68 (3)

Let mass of $O_2 = W$ g

mass of $N_2 = 4W$ g

so moles ratio = molecules ratio = $\dfrac{\text{moles of } O_2}{\text{moles of } N_2}$

$$= \frac{W \times 28}{32 \times 4W}$$

$$= \frac{7}{32}$$

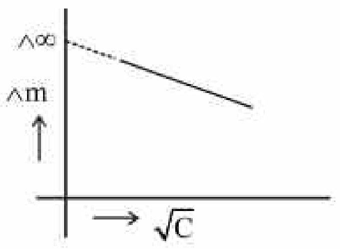
Solution

Q.69 (3)

Alkali and Alkaline earth metals can be extracted only in molten state.

Solution

Q.70 (4)



$\wedge m = \wedge\infty - B\sqrt{C}$

Solution

Q.71 (2)

Electronic configuration of Cs = $[Xe]5s^1$

$\therefore$ valence electron comes in 5s orbital.

Solution

Q.72 (2)

All solutions have same osmotic pressure because they have same number of particles in the solution.

$0.5\,M\ C_2H_5OH$ (Non electrolyte)

$0.1M\ Mg_3(PO_4)_2 \equiv 0.1 \times 5 = 0.5M$

$0.25M\ KBr \equiv 0.25 \times 2 = 0.5M$

$0.125\,M\ Na_3PO_4 \equiv 0.125 \times 4 = 0.5M$

Solution
Q.73 (1)

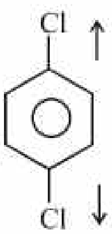
$$R-CH_2-OH \xrightarrow{K_2Cr_2O_7} R-COOH$$
$$R-CH_2-OH \xrightarrow{CrO_3} R-COOH + R-CHO$$
$$R-CH_2-OH \xrightarrow{PCC} R-CHO \quad \text{only}$$
$$R-CH_2-OH \xrightarrow{KMnO_4} R-COOH$$

Solution
Q.74 (4)
For CsCl the crystalline structure is bodycentred.

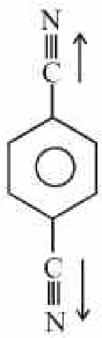
$$2r^+ + 2r^- = a\sqrt{3}$$
$$r^+ + r^- = \frac{a\sqrt{3}}{2}$$

Solution
Q.75 (1)
In (b) and (d) oxidation number of oxygen is increasing from – 1 to 0 i.e. oxidation of H_2O_2 is taking place. Therefore H_2O_2 is acting as reducing agent.

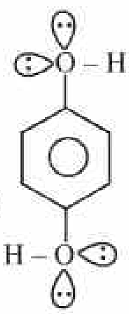
Solution
Q.76 (1)
For symmetrycally substituted molecules $\mu = 0$ when it is symmetrical

(a) 

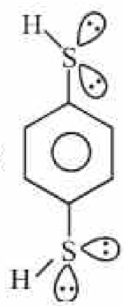
$\mu = 0$

(b) 

$\mu = 0$

(c) 

unsymmetrical $\therefore \mu \neq 0$

(d) 

unsymmetrical $\therefore \mu \neq 0$

Q.77 (4)

$$\underset{\text{Aliphatic } 1^{\circ}\text{ amine}}{R-CH_2-NH_2} \xrightarrow[C_2H_5O^-\ Na^+]{CHCl_3} \underset{\text{Isocyanide}}{R-CH_2-NC}$$

Solution
Q.78 (3)

Rate of $SN^2 \ \alpha \dfrac{1}{\text{crowding}}$

$\therefore CH_3Cl > CH_3-CH_2-Cl > (CH_3)_2CH-Cl > (CH_3)_3C-Cl$

Solution
Q.79 (3)

Strength of a ligand is directly proportional to energy and inversly proportional to wavelength of the radiation absorbed. $\left(E \propto \dfrac{1}{\lambda}\right)$

Solution
Q.80 (3)

In acidic medium complex is unstable and decomposes.

Solution
Q.81 (4)

$$CH_3COOH \xrightarrow{LiAlH_4} \underset{(A)}{CH_3CH_2OH} \xrightarrow{PCl_5} \underset{(B)}{CH_3CH_2Cl} \xrightarrow{\text{Alc KOH}} \underset{\text{(C) Ethylene}}{CH_2=CH_2}$$

Solution
Q.82. (3)

Cesium can form compounds only in $+1$ oxidation state and Iodine forms I_3^- with Iodide iron.

Solution
Q.83 **(3)**

$K_p = K_c(RT)^{\Delta n_g}$

$\Delta n_g = x = -\dfrac{1}{2}$

Solution

Q.84 (1)

From the given data it is clear that,

rate of reaction does not depend upon the concentration of B therefore order of reaction with respect to B will be 'zero'

While on doubling the concentration of A alone rate of reaction is also doubled therefore orders of reaction with respect to A will be 'one'

$\therefore r = k[A]$

Solution

Q.85 (2)

$$G = \frac{k}{C} \qquad k = \text{specific conductance}$$

$$C = \text{conductance}\left(\frac{1}{R}\right)$$

$$\frac{k_1}{C_1} = \frac{k_2}{C_2}$$

$$k_2 = \frac{1.4 \times 50}{280}$$

$$\wedge_m = \frac{10^{-3} \times k_2}{M} \text{ S m}^2 \text{ mol}^{-1}$$

$$= \frac{10^{-3} \times 1.4 \times 50}{280 \times 0.5} = 5 \times 10^{-4} \text{ Sm}^2 \text{ mol}^{-1}$$

Solution

Q.86 (4)

The strength of acidic character of an oxyacid is directly proportional to the oxidation state of central atom.

Solution

Q.87 (2)

DNA contains

1) Adenine

2) Guanine

3) Thymine

4) Cytosine

Quinoline doesn't present in DNA

Solution

Q.88 (2)

Basic strength $\alpha K_b \alpha \frac{1}{p^{k_b}}$

Basic strength of amines in H_2O

$(CH_3)_2NH > CH_3-NH_2 > (CH_3)_3N > Ph-NH_2$

Solution

Q.89 (3)

van der Waal equation

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

for 1 mole at low pressure b can be neglected

$$\left(P+\frac{a}{V^2}\right)V=RT$$

$$PV=RT-\frac{a}{V}$$

$$\frac{PV}{RT}=1-\frac{a}{VRT}$$

$$Z=1-\frac{a}{VRT}$$

Solution

Q.90 (1)

$$Fe\xrightarrow{O_2,\text{Heat}}Fe_3O_4\xrightarrow{CO,600^\circ C}FeO\xrightarrow{CO,700^\circ C}Fe$$

Carbon monoxide reduces Iron oxide.